EX1: Test the convergence of the following series:

1- $\sum_{n=1}^{\infty} \frac{n^2+2n+1}{3^n+2}$

$$a_n = \frac{n^2+2n+1}{3^n+2}$$

$$a_{n+1} = \frac{(n+1)^2+2(n+1)+1}{3^{n+1}+2} = \frac{(n+1)^2+2n+2}{3^n \cdot 3+2}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^2+2n+3}{3^n \cdot 3+2} \cdot \frac{3^n+2}{n^2+2n+1}$$

$$= \lim_{n\to\infty} \frac{(n^2+2n+1)+2n+3}{n^2+2n+1} \cdot \frac{3^n+2}{3^n \cdot 3+2}$$

$$= \lim_{n\to\infty} \frac{n^2+4n+4}{n^2+2n+1} \cdot \frac{3^n+2}{3^n \cdot 3+2}$$

$$= \lim_{n\to\infty} \frac{1+\frac{4}{n}+\frac{4}{n^2}}{1+\frac{2}{n}+\frac{1}{n^2}} \cdot \frac{1+\frac{2}{3^n}}{3+\frac{2}{3^n}} = 1 \times \frac{1}{3} = \frac{1}{3}$$

$\frac{1}{3} < 1$ ∴ The series is converges.

2- $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{n\,4^{2n+1}}$

$$a_n = \frac{10^n}{n \cdot 4^{2n+1}}$$

$$a_{n+1} = \frac{10^{n+1}}{(n+1)\cdot 4^{2n+3}} = \frac{10^n \cdot 10}{(n+1)\,4^{2n+3}}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\cancel{10^n}\cdot 10}{(n+1)\,4^{2n+3}} \cdot \frac{n \cdot 4^{2n+1}}{\cancel{10^n}}$$

$$\frac{4^{2n+1}}{4^{2n+3}} = \frac{1}{4^2}$$ عند القسمة تطرح الاسس

$$\lim_{n\to\infty} \frac{10}{16} \frac{n}{(n+1)} = \frac{10}{16} < 1$$

$\therefore$ converges.

EX2 : Find the interval of the convergence and the center of the convergence of the following Power series :

1. $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n\,2^n}$

$$a_n(x-c)^n = \frac{(x-3)^n}{n\;2^n}$$

$$a_{n+1}(x-c)^{n+1} = \frac{(x-3)^{n+1}}{(n+1)\;2^{n+1}} = \frac{(x-3)^n\cdot(x-3)}{(n+1)\;2^n\cdot 2}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}(x-c)^{n+1}}{a_n(x-c)^n}\right| = \lim_{n\to\infty}\left|\frac{\cancel{(x-3)^n}\cdot(x-3)}{(n+1)\;\cancel{2^n}\cdot 2}\cdot\frac{n\,\cancel{2^n}}{\cancel{(x-3)^n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x-3}{2}\right|\frac{n}{n+1}$$

1

$$-1 < \frac{x-3}{2} < 1$$

$$-2 < x-3 < 2$$

$$1 < x < 5$$

if $x = 1$

$$\sum_{n=1}^{\infty} \frac{(1-3)^n}{n \cdot 2^n} = \sum_{n=1}^{\infty} \frac{(-2)^n}{n \cdot 2^n} = \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n\, 2^n}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$ alternative series

① all a_n's positive
② $|a_n| \geq |a_{n+1}|$
③ $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{1}{n} = 0$

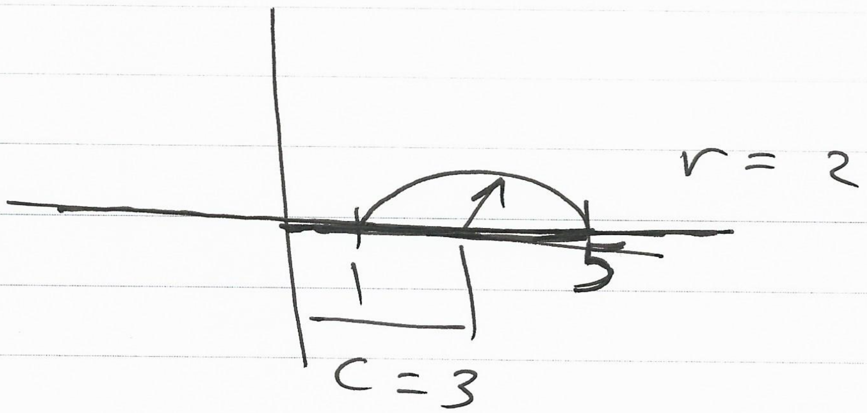
Converges.

if $x = 5$

$$\sum_{n=1}^{\infty} \frac{(5-3)^n}{n \cdot 2^n} = \sum_{n=1}^{\infty} \frac{\cancel{2^n}}{n \cdot \cancel{2^n}} = \sum_{n=1}^{\infty} \frac{1}{n}$$

P-series with $P = 1$ Diverges.

$[1, 5)$



2- $\sum_{n=1}^{\infty} \frac{(3x+2)^n}{n^2}$

$$a_n (x-c)^n = \frac{(3x+2)^n}{n^2}$$

$$a_{n+1} (x-c)^{n+1} = \frac{(3x+2)^{n+1}}{(n+1)^2} = \frac{(3x+2)^n (3x+2)}{(n+1)^2}$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}(x-c)^{n+1}}{a_n(x-c)^n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{(3x+2)^n}(3x+2)}{(n+1)^2} \cdot \frac{n^2}{\cancel{(3x+2)^n}} \right|$$

$$= \lim_{n\to\infty} |(3x+2)| \left(\frac{n}{n+1}\right)^2 \text{——} 1$$

$$-1 < 3x+2 < 1$$

$$-3 < 3x < -1$$

$$-1 < x < -\frac{1}{3}$$

if $x = -1$

$$\sum_{n=1}^{\infty} \frac{(-3+2)^n}{n^2} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \quad \text{altenative series}$$

① all a_n's positive
② $|a_n| \geq |a_{n+1}|$
③ $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n^2} = 0$

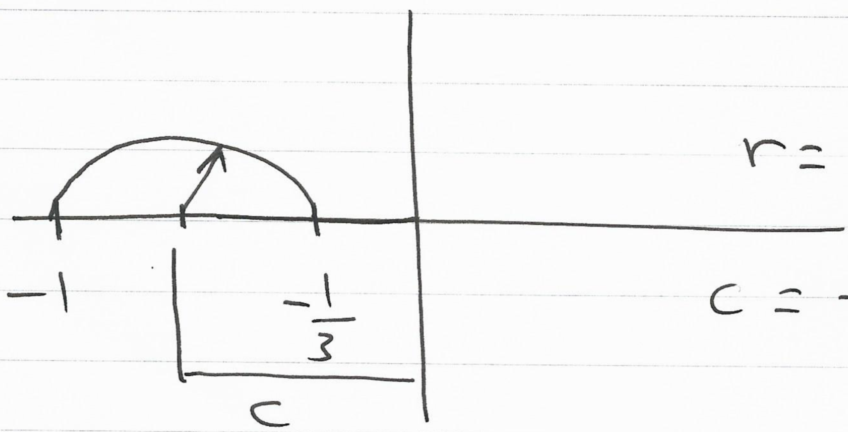
Converges

if $x = -\frac{1}{3}$

$$\sum_{n=1}^{\infty} \frac{(3(-\frac{1}{3})+2)^n}{n^2} = \sum_{n=1}^{\infty} \frac{(-1+2)^n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

p-series with $p = 2 > 1$ $\therefore$ Converges

$$\left[-\frac{1}{3}, -\frac{1}{3}\right]$$



$r = \frac{1}{3}$

$c = -\frac{2}{3}$