The Bisection Method (or Interval Halving Method):

The *bisection method,* which is alternatively called binary chopping, interval halving, is one type of incremental search method in which the interval is always divided in half. If a function changes sign over an interval, the function value at the midpoint is evaluated. The location of the root is then determined as lying at the midpoint of the subinterval within which the sign change occurs. The process is repeated to obtain refined estimates.

When applying the graphical technique, you have observed that $f(x)$ changed sign on opposite sides of the root. In general, if $f(x)$ is real and continuous in the interval from x_L to x_R and $f(x_L)$ and $f(x_R)$ have opposite signs, that is, $f(x_L)f(x_R) < 0$, then there is at least one real root between x_L and x_R. *Incremental search methods* capitalize on this observation by locating an interval where the function changes sign. Then the location of the sign change (and consequently, the root) is identified more precisely by dividing the interval into a number of subintervals.

Algorithm of Bisection Method:

Step1. Choose left x_L and right x_R guesses for the root such that the function changes sign over the interval. This can be checked by ensuring that $f(x_L)f(x_R) < 0$.

Step2. An estimate of the root x_{i+1} is determined by x_R

Step3. Make the following evaluations to determine in which subinterval the root lies:

(a) If $f(x_L)f(x_R) < 0$, the root lies in the left subinterval. Set $x_{i+1} = x_L$ and return to step 2.

(b) If $f(x_L)f(x_R) > 0$, the root lies in the right subinterval. Therefore, set $x_{i+1} = x_R$ and return to step 2.

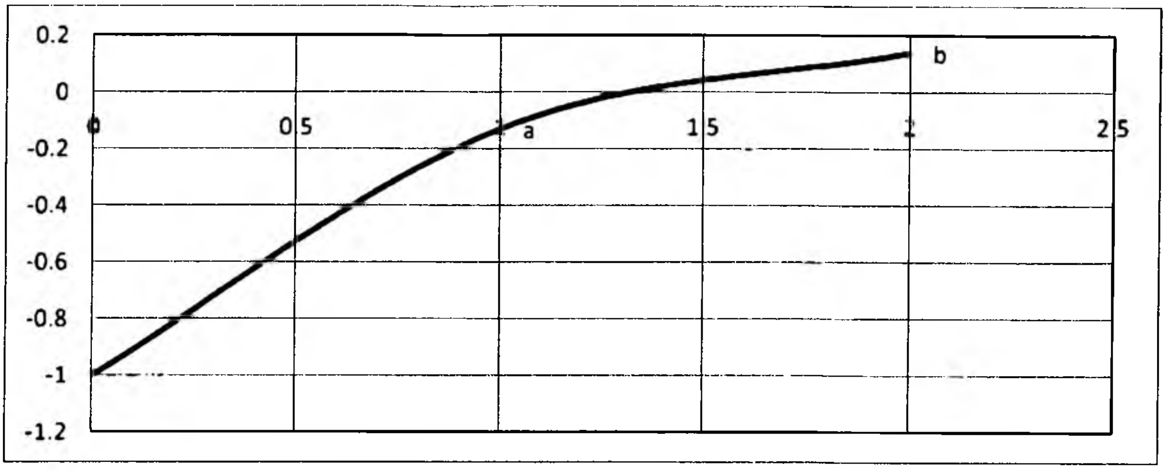
(c) If $f(x_L)f(x_R) = 0$, the root equals x_{i+1}; terminate the computation.

Example 1: Find the positive root of the following equation by (Bisection) method, $f(x) = \sin(\frac{x}{2\pi}) - cos^2(x)$ *to* 3 *dec.* ?

Solution

$$f(x) = \sin(\frac{x}{2\pi}) - cos^2(x)$$

X	0	1	2
F(x)	-1	-0.13	0.14



The root lies between 1&2

$f(0) = \sin(\frac{x}{2\pi}) - cos^2(x) = \sin(\frac{0}{2\pi}) - cos^2(0) = -1$

$f(1) = \sin(\frac{x}{2\pi}) - cos^2(x) = \sin(\frac{1}{2\pi}) - cos^2(1) = -0.13$

$f(2) = \sin(\frac{x}{2\pi}) - cos^2(x) = \sin(\frac{2}{2\pi}) - cos^2(2) = 0.14$

$$Bisection = \frac{a+b}{2}$$

$$\frac{1+2}{2} = 1.5$$

$f(1.5) = \sin(\frac{x}{2\pi}) - cos^2(x) = \sin(\frac{1.5}{2\pi}) - cos^2(1.5) = 0.231$

The root lies between 1 & 1.5

$$\frac{1+1.5}{2} = 1.25$$

$f(1.25) = \sin(\frac{x}{2\pi}) - cos^2(x) = \sin(\frac{1.25}{2\pi}) - cos^2(1.25) = 0.098$

The root lies between 1 & 1.25

$$\frac{1+1.25}{2} = 1.125$$

$f(1.125) = \sin(\frac{x}{2\pi}) - cos^2(x) = \sin(\frac{1.125}{2\pi}) - cos^2(1.125) = -0.007$

The root lies between 1.125 & 1.25

$$\frac{1.125+1.25}{2} = 1.188$$

f(1.188) = $\sin(\frac{x}{2\pi}) - cos^2(x)$= $\sin(\frac{1.188}{2\pi}) - cos^2(1.188)$= 0.048

The root lies between 1.125 & 1.188

$$\frac{1.125 + 1.188}{2} = 1.157$$

f(1.157) = $\sin(\frac{x}{2\pi}) - cos^2(x)$= $\sin(\frac{1.157}{2\pi}) - cos^2(1.157)$= 0.021

The root lies between 1.125 & 1.157

$$\frac{1.125 + 1.157}{2} = 1.141$$

f(1.141) = $\sin(\frac{x}{2\pi}) - cos^2(x)$= $\sin(\frac{1.141}{2\pi}) - cos^2(1.141)$= 0.007

The root lies between 1.125 & 1.141

$$\frac{1.125 + 1.141}{2} = 1.133$$

f(1.133) = $\sin(\frac{x}{2\pi}) - cos^2(x)$= $\sin(\frac{1.133}{2\pi}) - cos^2(1.133)$= 0

The Approximation root is = 1.133

Or another solution

The root lies between 1&2

f(0) = $\sin(\frac{x}{2\pi}) - cos^2(x) = \sin(\frac{0}{2\pi}) - cos^2(0) = -1$

f(1) = $\sin(\frac{x}{2\pi}) - cos^2(x)$= $\sin(\frac{1}{2\pi}) - cos^2(1) = -0.13$

f(2) = $\sin(\frac{x}{2\pi}) - cos^2(x)$= $\sin(\frac{2}{2\pi}) - cos^2(2)$= 0.14

$$Bisection = \frac{a+b}{2}$$

$$\frac{1+2}{2} = 1.5$$

f(1.5) = $\sin(\frac{x}{2\pi}) - cos^2(x)$= $\sin(\frac{1.5}{2\pi}) - cos^2(1.5)$= 0.231

The root lies between 1 & 1.5

I	x_a	$f(x_a)$	x_b	$f(x_b)$	*Root lies between a,b*	x_{i+1}
0	1	-0.13	2	0.14	1,2	1.5
1	1	-0.13	1.5	0.231	1.1.5	1.25
2	1	-0.13	1.25	0.098	1,1.25	1.125
3	1.125	-0.0007	1.25	0.098	1.125,1.25	1.188
4	1.125	-0.0007	1.188	0.048	1.125,1.188	1.157
5	1.125	-0.0007	1.157	0.021	1.125,1.157	1.141
6	1.125	-0.0007	1.141	0.007	1.125,1.141	1.133

f(1.133) = $\sin(\frac{x}{2\pi}) - cos^2(x)$= $\sin(\frac{1.133}{2\pi}) - cos^2(1.133)$= 0

The Approximation root is = 1.133

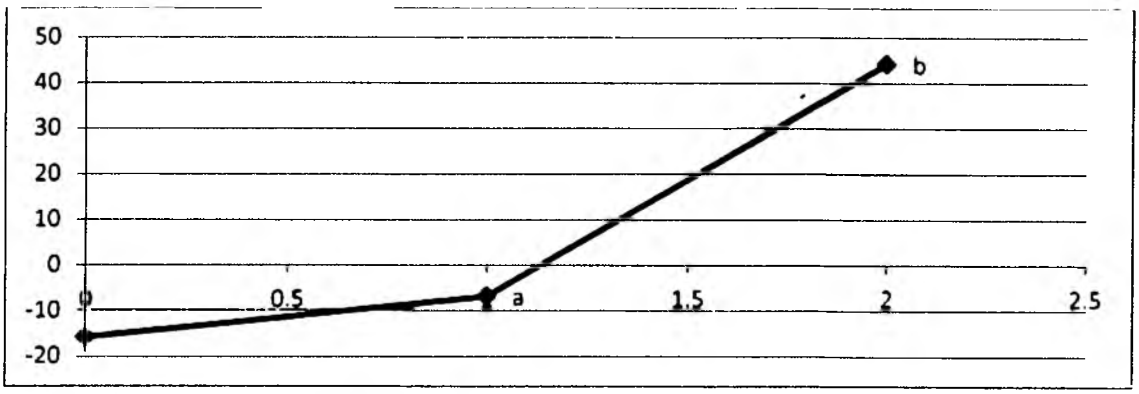
Example 2: Find the positive root of the following equation by (Bisection) method,

$$f(x) = 6x^3 + 3x^2 - 16 \; to \; 2 \; dec.?$$

Solution

$$f(x) = 6x^3 + 3x^2 - 16$$

X	0	1	2
F(x)	-16	-7	44



The root lies between 1 &2

$$Bisection = \frac{a+b}{2}$$

$$\frac{1+2}{2} = 1.5$$

$$f(1.5) = 6 * 1.5^3 + 3 * 1.5^2 - 16 = 11$$

The root lies between 1&1.5

$$\frac{1+1.5}{2} = 1.25$$

$$f(1.25) = 6 * 1.25^3 + 3 * 1.25^2 - 16 = 0.40$$

The root lies between 1 & 1.25

$$\frac{1+1.25}{2} = 1.12$$

$$f(1.12) = 6 * 1.12^3 + 3 * 1.12^2 - 16 = -3.80$$

The root lies between 1.12 & 1.25

$$\frac{1.12+1.25}{2} = 1.18$$

$$f(1.12) = 6 * 1.12^3 + 3 * 1.12^2 - 16 = -1.96$$

The root lies between 1.18 & 1.25

$$\frac{1.18+1.25}{2} = 1.21$$

$$f(1.21) = 6 * 1.21^3 + 3 * 1.21^2 - 16 = -0.97$$

The root lies between 1.21 & 1.25

$$\frac{1.21+1.25}{2} = 1.23$$

$$f(1.23) = 6 * 1.23^3 + 3 * 1.23^2 - 16 = -0.29$$

The root lies between 1.23 & 1.25

$$\frac{1.23+1.25}{2} = 1.24$$

$$f(1.24) = 6 * 1.24^3 + 3 * 1.24^2 - 16 = 0.05$$

The root lies between 1.23 & 1.24

$$\frac{1.23 + 1.24}{2} = 1.23$$

$$f(1.23) = 6 * 1.23^3 + 3 * 1.23^2 - 16 = -0.29$$

The root lies between 1.23 & 1.24

$$\frac{1.23 + 1.24}{2} = 1.23$$

$$f(1.23) = 6 * 1.23^3 + 3 * 1.23^2 - 16 = -0.29$$

The better approximation root is = 1.23