Modified Secant Method:

Rather than using two arbitrary values to estimate the derivative, an alternative approach involves a fractional perturbation of the independent variable to estimate $f(x_i)$,

$$f'(x_i) \cong \frac{f(x_i + \delta x_i) - f(x_i)}{\delta x_i}$$

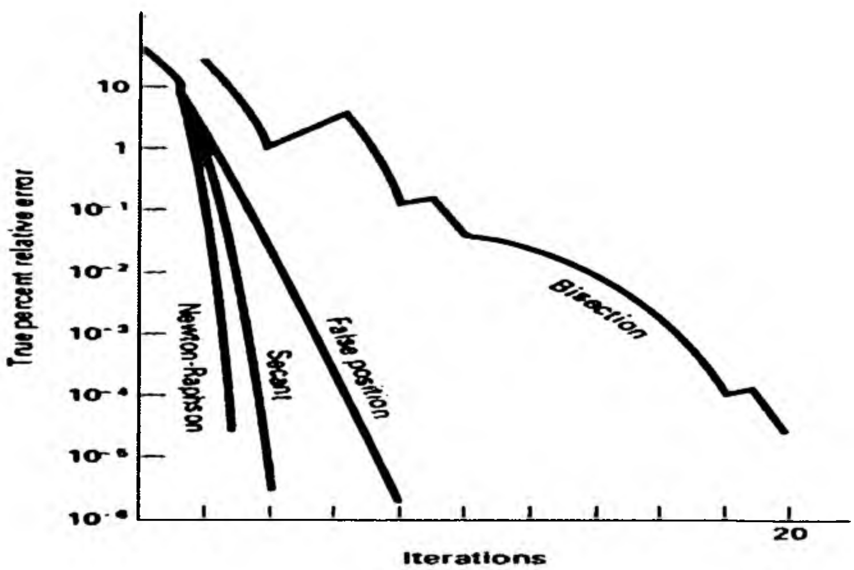

Figure (3)

Where δ = a small perturbation fraction. This approximation can be substituted into eqation above to yield the following iterative equation:

$$x_{i+1} = x_i - \frac{\delta_{x_i} * f(x_i)}{f(\delta_{x_i} + x_i) - f(x_i)}$$

Example 3: Use the modified secant method to estimate the root of $f(x) = e^{-x} - x$ Use a value of 0.01 for δ and start with $x_0 = 1$?

Solution

For i=0:

$x_0 = 1, f(x_0) = e^{-x_0} - x_0 = e^{-1} - 1 = -0.63212$

$\delta_{x_0} = 0.01$

$\delta_{x_0} = \delta_{x_0} * x_0 = 0.01 * 1 = 0.01$

$\delta_{x_0} + x_0 = 0.01 + 1 = 1.01, f(\delta_{x_0} + x_0) = f(1.01) = e^{-1.01} - 1.01 = -0.64578$

$x_1 = x_0 - \frac{\delta_{x_0} * f(x_0)}{f(\delta_{x_0}+x_0) - f(x_0)} = 1 - \frac{0.01 * -0.63212}{-0.64578-(-0.63212)} = 0.53725$

For i=1:

$x_1 = 0.53725, f(x_1) = e^{-x_1} - x_1 = e^{-0.53725} - 0.53725 = 0.04710$

$\delta_{x_0} = 0.01$

$\delta_{x_1} = \delta_{x_0} * x_1 = 0.01 * 0.53725 = 0.00537$

$\delta_{x_1} + x_1 = 0.00537 + 0.53725 = 0.54262$

$f(\delta_{x_1} + x_1) = f(0.54262) = e^{-0.54262} - 0.54262 = 0.03860$

$x_2 = x_1 - \frac{\delta_{x_1} * f(x_1)}{f(\delta_{x_1}+x_1) - f(x_1)} = 0.53725 - \frac{0.00537 * 0.04710}{0.03860 - 0.04710} = 0.56701$

I	x_i	$f(x_i)$	$\delta_{x_i} = \delta_{x_0} * x_i$	$\delta_{x_i} + x_i$	$f(\delta_{x_i} + x_i)$	x_{i+1}
0	1	-0.63212	0.01	1.01	-0.64578	0.53725
1	0.53725	0.04710	0.00537	0.54262	0.03860	0.56701
2	0.56701	0.00021	0.00567	0.57268	-0.00867	0.56714
3	0.56714	0.00001	0.00567	0.57281	-0.00887	0.56715
4	0.56715	-0.00001	0.00567	0.57282	-0.00889	0.56714
5	0.56714	0.00001	0.00567	0.57281	-0.00887	0.56715

The choice of a proper value for δ is not automatic. If δ is too small, the method can be swamped by round-off error caused by subtractive cancellation in the denominator of Eq. (6.8). If it is too big, the technique can become inefficient and even divergent. However, if chosen correctly, it provides a nice alternative for cases where evaluating the derivative is difficult and developing two initial guesses is inconvenient.