Faculty of Engineering
Civil Engineering Department
2nd Class

Strength of Materials II
(Mechanics of Materials)
(SI Units)

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2017-2018

Mechanics of Material II

Text Books

- Mechanics of Materials, 10th edition (SI version), by: R. C. Hibbeler, 2017
- Mechanics of Materials, 2nd edition (SI version), by: E. Popov, 1990

References

- Strength of Materials, fifth edition,(SI units), Schaum's outlines, by: W. Nash and M. Potter, 1977
- Mechanics of Materials, eighth edition,(SI units), by: James M. Gere, and Barry J. Goodno, 2013.
- Strength of Materials Lectures, Civil Engineering Department, Faculty of Eng., UOM, by: Dr Ali Al-Ghalib, 2014.

Syllabus of Mechanics of Material II – Course No. (50601202):

Ch1. Shearing Stresses in Beam

Preliminary remarks, Shear flow, The shear stress formula for beams, Shear stresses in beam flanges.

Ch2. Compound Stresses

Normal stresses, Superposition of shearing stresses, Combined direct and torsional shear stresses

Ch3. Analysis of Plane Stress and Strain

Principal Stresses, Maximum shearing stresses, Mohr's Circle of Stress, Construction of Mohr's circle of stress, Transformation Equations of Plane Strain, Principal strains, Maximum shear strain, Mohr's Circle of Plane Strain.

Ch4. Deflection of Beams

Double Integration Method, Macaulay's Method (Singularity Functions).

COURSE OBJECTIVES (Learning Outcomes):

- Analyze and design structural members subjected to tension, compression, torsion, bending and combined stresses using the fundamental concepts of stress, strain and elastic behavior of materials.
- Utilize appropriate materials in design considering engineering properties, sustainability, cost and weight.
- Perform engineering work in accordance with ethical and economic constraints related to the design of structures and machine parts.

INTRODUCTION

Strength of materials is a branch of applied mechanics that deals with the behavior of solid bodies subjected to various types of loading. Other names for this field of study are mechanics of materials and solid mechanics. The solid bodies considered in this book include bars with axial loads, shafts in torsion, beams in bending, and columns in compression. The principal objective of mechanics of materials is to determine the stresses, strains, and displacements in structures and their components due to the loads acting on them. If we can find these quantities for all values of the loads up to the loads that cause failure, we will have a complete picture of the mechanical behavior of these structures

SYMBOLS AND SI UNITS

GREEK ALPHABET

Α	α	Alpha	Ν	ν	Nu
Β	β	Beta	Ξ	ξ	Xi
Γ	γ	Gamma	Ο	o	Omicron
Δ	δ	Delta	Π	π	Pi
Ε	ε	Epsilon	Ρ	ρ	Rho
Ζ	ζ	Zeta	Σ	σ	Sigma
Η	η	Eta	Τ	τ	Tau
Θ	θ	Theta	Υ	υ	Upsilon
Ι	ι	Iota	Φ	ϕ	Phi
Κ	κ	Kappa	Χ	χ	Chi
Λ	λ	Lambda	Ψ	ψ	Psi
Μ	μ	Mu	Ω	ω	Omega

Prefixes	Value	Standard form	Symbol
Tera	1 000 000 000 000	10^{12}	T
Giga	1 000 000 000	10^{9}	G
Mega	1 000 000	10^{6}	M
Kilo	1 000	10^{3}	k
deci	0.1	10^{-1}	d
centi	0.01	10^{-2}	c
milli	0.001	10^{-3}	m
micro	0.000 001	10^{-6}	µ
nano	0.000 000 001	10^{-9}	n
pico	0.000 000 000 001	10^{-12}	p

Quantity	U.S. Customary	SI Equivalent
Force	lb. kip	4.448N 4.448kN
Length	in ft	25.4mm 0.3048m
Area	in^2 ft^2	$645.2mm^2$ $0.0929m^2$
Stress	Ib/in^2 (psi)	$6.895kN/m^2$ (kPa)

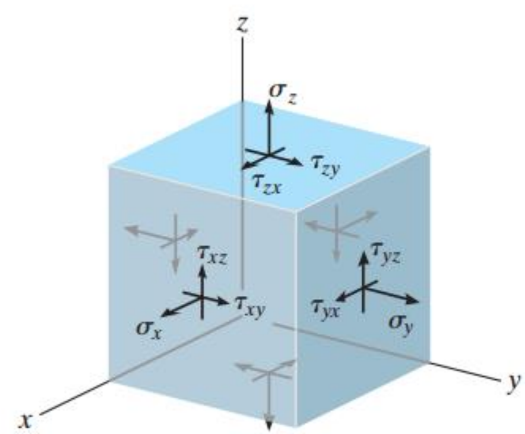
CHAPER ONE – SHEARING STRESSES IN BEAM

Preliminary Remarks:

The **stress describes the intensity of the internal force acting on a specific plane (area) passing through a point.**

General State of Stress:

The state of stress is characterized by three components acting on each face of the element.



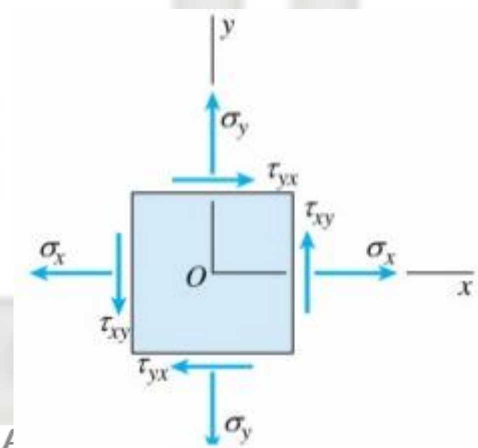
SI Stress Units:

Pascal = $\dfrac{N}{m^2}$

Kilo Pascal = $1000 \times \dfrac{N}{m^2} = \dfrac{kN}{m^2}$

Mega Pascal = $1000000 \times \dfrac{N}{m^2} = \dfrac{N}{mm^2}$

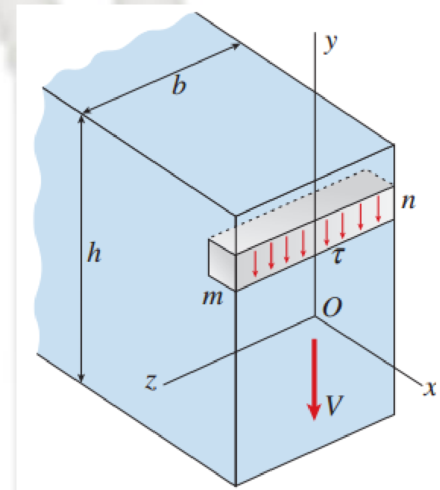
In plane, there are 4 components of stresses; 2 normal stresses and 2 shear stresses, as in the plane element below:

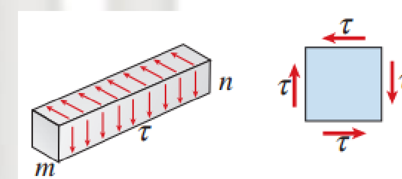


Shear Stress In Beams of Rectangular Cross-Section

When a beam is in pure bending, the only stress resultants are the bending moments and the only stresses are the bending stresses acting on the cross sections. However, most beams are subjected to loads that produce both bending moments and shear forces. In these cases, both bending and shear stresses are developed in the beam. The bending stresses are calculated from the flexure formula, provided the beam is constructed of a linearly elastic material.

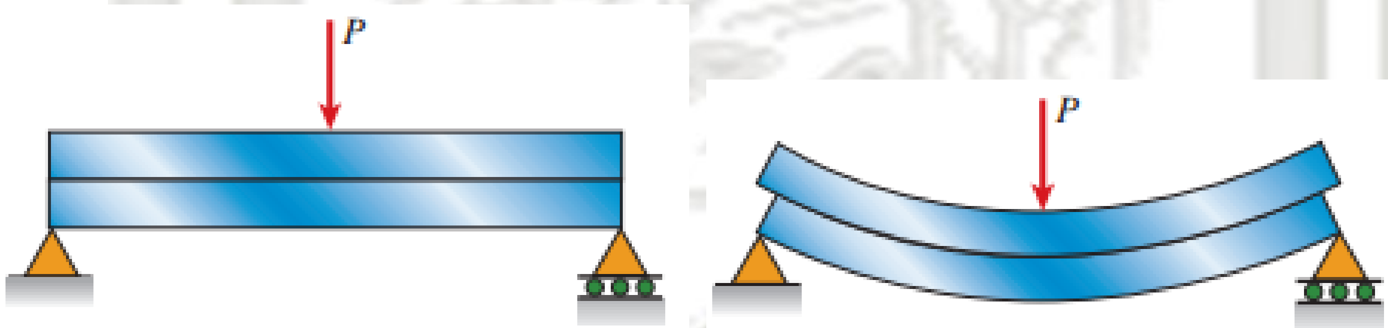
Vertical and Horizontal Shear Stresses





Consider a beam of rectangular cross section (width b and height h) subjected to a positive shear force V as shown in the figure. It is reasonable to assume that the shear stresses τ acting on the cross section are parallel to the shear force, that is, parallel to the vertical sides of the cross section. It is also reasonable to assume that the shear stresses are uniformly distributed across the width of the beam, although they may vary over the height. Using these two assumptions, the intensity of the shear stress at any point on the cross section can be determined. The shear stresses τ acting on the front face of this element are vertical and uniformly distributed from one side of the beam to the other. Also, we know that shear stresses acting on one side of an element are accompanied by shear stresses of equal magnitude acting on perpendicular faces of the element. Thus, there are horizontal shear stresses acting between horizontal layers of the beam as well as vertical shear stresses acting on the cross sections. At any point in the beam, these complementary shear stresses are equal in magnitude.
The horizontal shear stresses must vanish at either the top or the bottom, because there are no stresses on the outer surfaces of the beam. It follows that the vertical shear stresses must also vanish at those locations; in other words, where y=+h/2 and y=-h/2.

The existence of horizontal shear stresses in a beam can be demonstrated by a simple experiment. Place two identical rectangular beams on simple supports and load them by a force P, as shown in Figure. If friction between the beams is small, the beams will bend independently. Each beam will be in compression above its own neutral axis and in tension below its neutral axis, and therefore the bottom surface of the upper beam will slide with respect to the top surface of the lower beam. Now suppose that the two beams are glued along the contact surface, so that they become a single solid beam. When this beam is loaded, horizontal shear stresses must develop along the glued surface in order to prevent the sliding. Because of the presence of these shear stresses, the single solid beam is much stiffer and stronger than the two separate beams.



Derivation of Shear Formula

$$\sigma_1 = -\frac{My}{I} \quad \text{and} \quad \sigma_2 = -\frac{(M + dM)y}{I}$$

$$\sigma_1 dA = \frac{My}{I} dA$$

$$F_1 = \int \sigma_1 dA = \int \frac{My}{I} dA$$

$$F_2 = \int \sigma_2 dA = \int \frac{(M + dM)y}{I} dA$$

$$F_3 = F_2 - F_1$$

$$F_3 = \int \frac{(M + dM)y}{I} dA - \int \frac{My}{I} dA$$

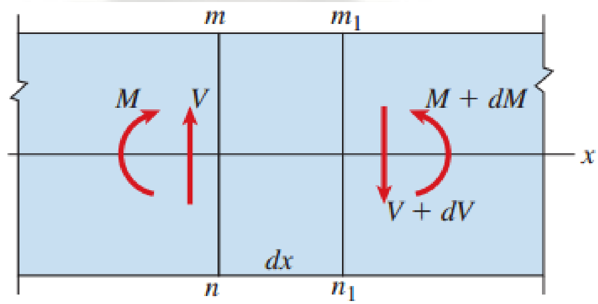
$$F_3 = \int \frac{(dM)y}{I} dA$$

$$F_3 = \tau b\, dx$$

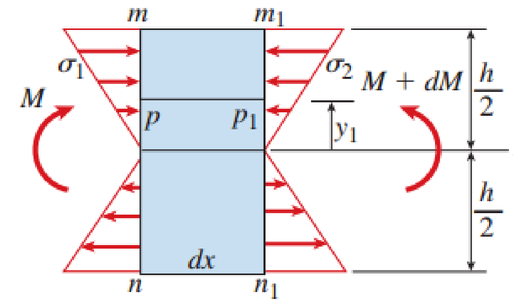
$$\tau = \frac{dM}{dx}\left(\frac{1}{Ib}\right)\int y\, dA$$

$$\tau = \frac{V}{Ib}\int y\, dA$$

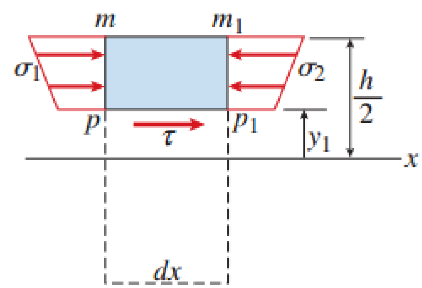
$\tau = \dfrac{VQ}{Ib}$, Q is the first moment of area separated by the fiber about NA



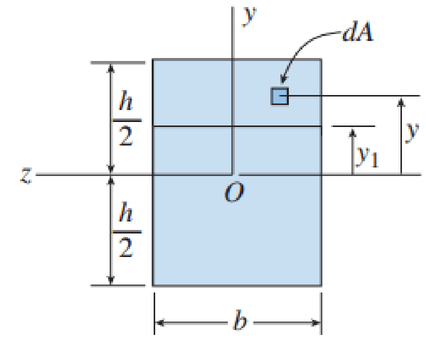
Side view of beam



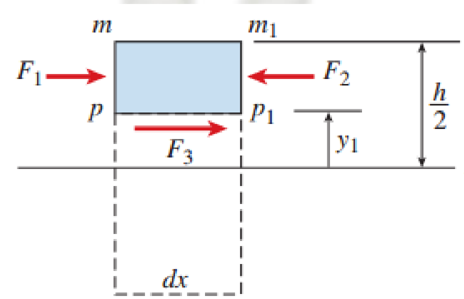
Side view of element



Side view of subelement



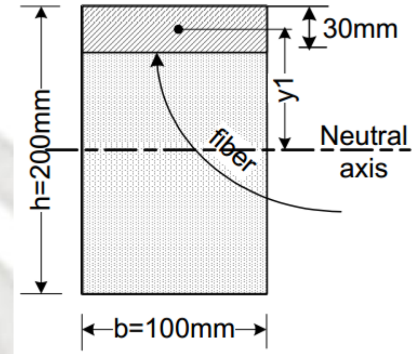
Cross section of beam at subelement



$\tau = \dfrac{\frac{dF}{dX}}{b} = \dfrac{q}{b} = \dfrac{VQ}{I_{NA}b}$, $\dfrac{dF}{dX}$: shear flow is the shear force per unit length

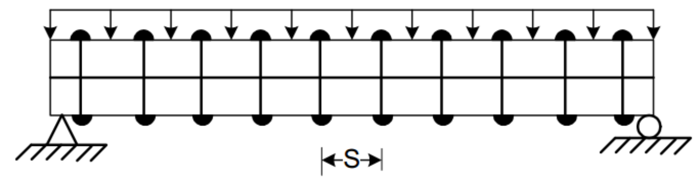
Q= 100*30*85= 255000mm^3 or:

Q=100*170*15= 255000mm^3



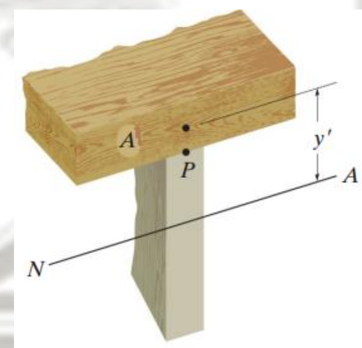
The Shear Flow (q) would help to find the spacing between bolts, as follow:

S=F/q

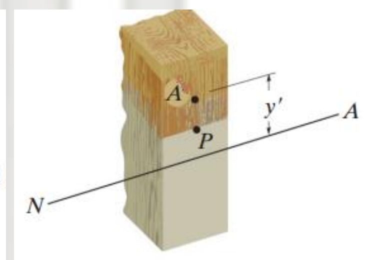


Where:
S= spacing between the bolts, and
F= allowable shearing force per bolt.

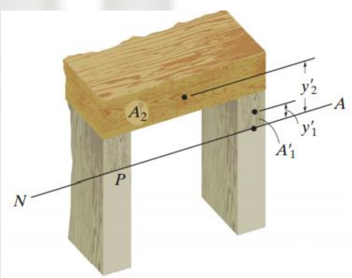
Q represents the moment of the cross-sectional area that is above or below the point where the shear stress is to be determined. It is this area A that is “held onto” the rest of the beam by the longitudinal shear stress as the beam undergoes bending. The examples shown in the Figure will help to illustrate this point. Here the stress at point P is to be determined, and so A represents the dark shaded region. The value of Q for each case is reported under each figure. These same results can also be obtained for Q by considering A to be the light shaded area below P, although here y′ is a negative quantity when a portion of A is below the neutral axis.



$Q = y'A$



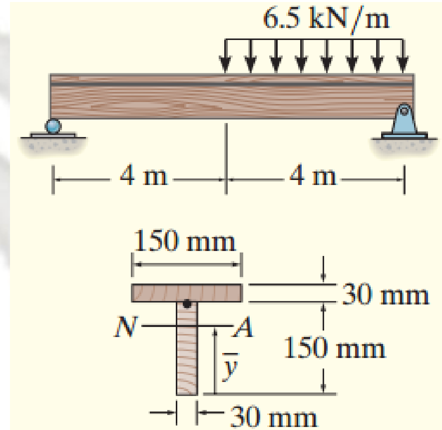
$Q = y'A$



$Q = 2y'_1A_1 + y'_2A$

EXAMPLE 1-1

The beam shown in Figure is made from two boards. Determine the maximum shear stress in the glue necessary to hold the boards together along the surface where they are joined.



Solution:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

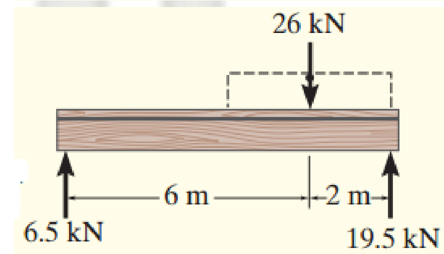
$$= \frac{[0.075 \text{ m}](0.150 \text{ m})(0.030 \text{ m}) + [0.165 \text{ m}](0.030 \text{ m})(0.150 \text{ m})}{(0.150 \text{ m})(0.030 \text{ m}) + (0.030 \text{ m})(0.150 \text{ m})} = 0.120 \text{ m}$$

The moment of inertia about the neutral axis is

$$I = \left[\frac{1}{12}(0.030 \text{ m})(0.150 \text{ m})^3 + (0.150 \text{ m})(0.030 \text{ m})(0.120 \text{ m} - 0.075 \text{ m})^2\right]$$

$$+ \left[\frac{1}{12}(0.150 \text{ m})(0.030 \text{ m})^3 + (0.030 \text{ m})(0.150 \text{ m})(0.165 \text{ m} - 0.120 \text{ m})^2\right]$$

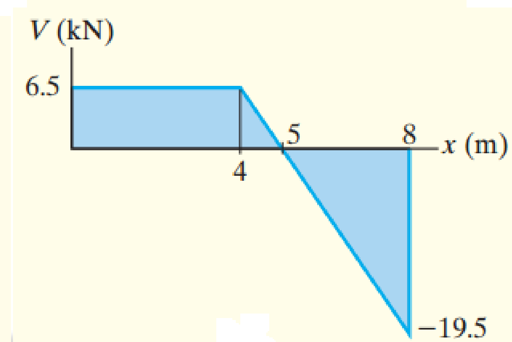
$$= 27.0(10^{-6}) \text{ m}^4$$

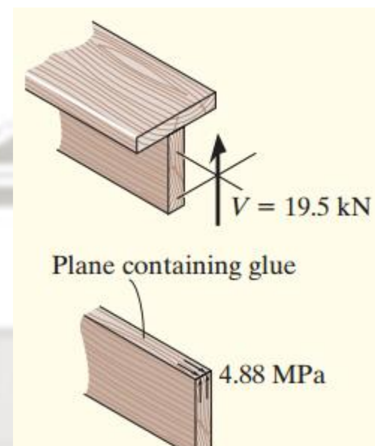


$$Q = y'A = [0.180 \text{ m} - 0.015 \text{ m} - 0.120 \text{ m}](0.03 \text{ m})(0.150 \text{ m})$$

$$= 0.2025(10^{-3}) \text{ m}^3$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{19.5(10^3) \text{ N}(0.2025(10^{-3}) \text{ m}^3)}{27.0(10^{-6}) \text{ m}^4(0.030 \text{ m})10^6} = 4.88 \text{ MPa}$$

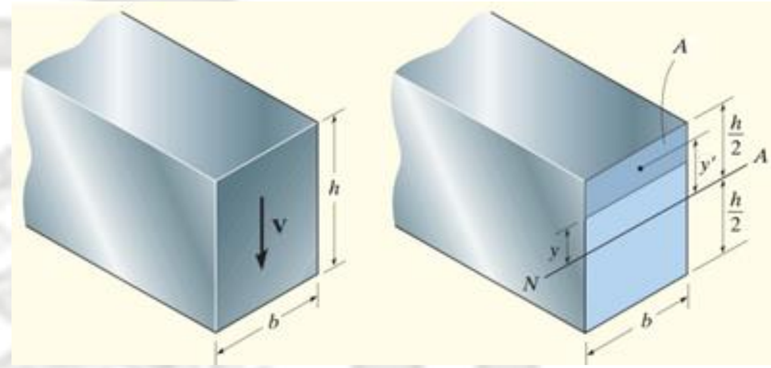




NOTE: It is the glue's resistance to this longitudinal shear stress that holds the boards from slipping at the right support.

EXAMPLE 1-2

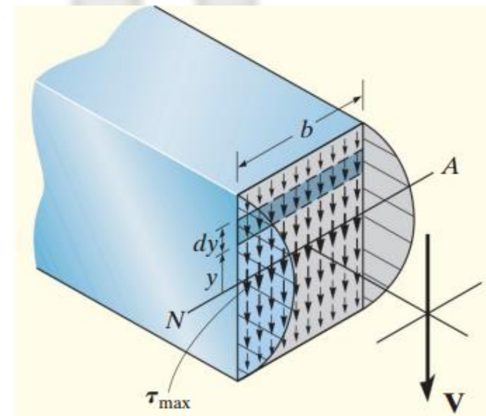
Determine the distribution of the shear stress over the cross section of the beam shown in Figure shown.



Solution: The distribution can be determined by finding the shear stress at an arbitrary height y from the neutral axis, and then plotting this function. Here, the dark colored area A will be used for Q, Hence

$$Q = y'A = \left[y + \frac{1}{2}\left(\frac{h}{2} - y\right)\right]\left(\frac{h}{2} - y\right)b = \frac{1}{2}\left(\frac{h^2}{4} - y^2\right)b$$

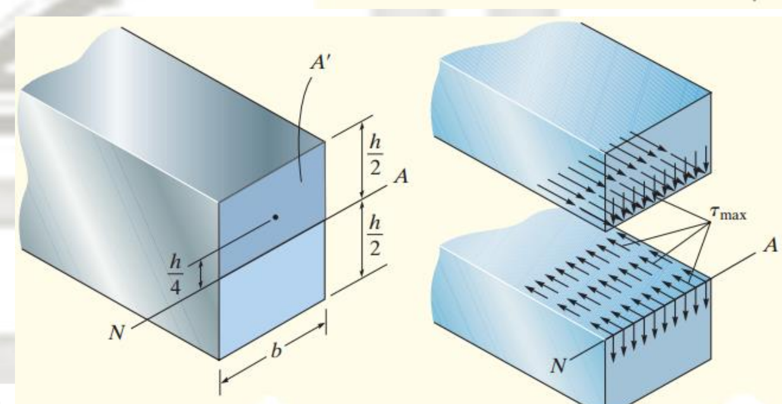
$$\tau = \frac{VQ}{It} = \frac{V\left(\frac{1}{2}\right)\left[(h^2/4) - y^2\right]b}{\left(\frac{1}{12}bh^3\right)b} = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$



This result indicates that the shear-stress distribution over the cross section is parabolic. The intensity varies from zero at the top and bottom, y =+h/2 and y=-h/2 , to a maximum value at the neutral axis, y = 0. Specifically, since the area of the cross section is (A = bh), then at y = 0 the above equation becomes:

$$\tau_{max} = 1.5\frac{V}{A}$$

This same value for τmax can be obtained directly from the shear formula, τ = VQ >It, by realizing that τmax occurs where Q is largest, since V, I, and b are constant. By inspection, Q will be a maximum when the entire area above (or below) the neutral axis is considered; that is, A = bh/2 and y = h/4 Thus,

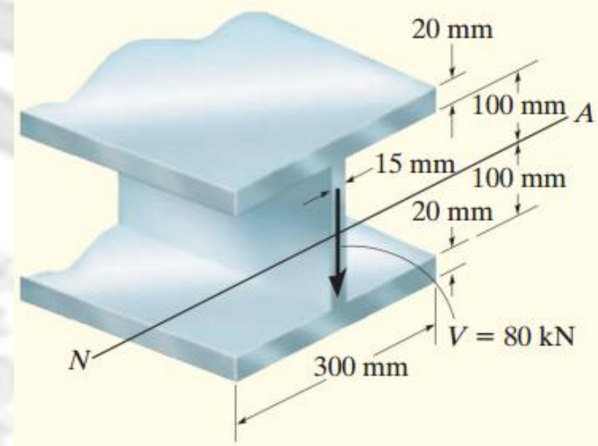


$$\tau_{max} = \frac{VQ}{It} = \frac{V(h/4)(bh/2)}{\left[\frac{1}{12}bh^3\right]b} = 1.5\frac{V}{A}$$,if $\tau_{avg} = V/A$

$$\tau_{max} = 1.5\tau_{ave}$$

EXAMPLE 1-3

A steel wide-flange beam has the dimensions shown in Figure. If it is subjected to a shear of V = 80 kN, plot the shear-stress distribution acting over the beam's cross section.



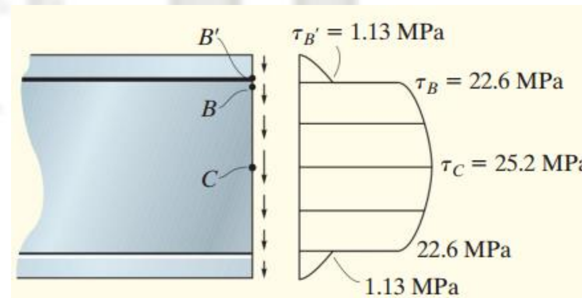
Solution:

Since the flange and web are rectangular elements, then like the previous example, the shear-stress distribution will be parabolic and in this case it will vary in the manner shown in Figure. Due to symmetry, only the shear stresses at points B, B, and C have to be determined. To show how these values are obtained, we must first determine the moment of inertia of the cross-sectional area about the neutral axis.

$$I = \left[\frac{1}{12}(0.015\text{ m})(0.200\text{ m})^3\right]$$

$$+ 2\left[\frac{1}{12}(0.300\text{ m})(0.02\text{ m})^3 + (0.300\text{ m})(0.02\text{ m})(0.110\text{ m})^2\right]$$

$$= 155.6(10^{-6})\text{ m}^4$$



For point B', $t_{B'} = 0.300$ m, and A is the dark shaded area shown in Fig.

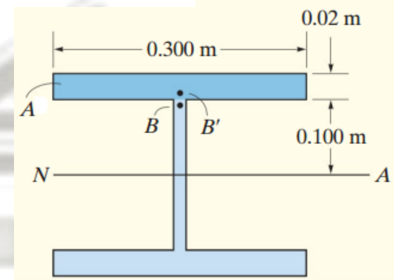
$$Q_{B'} = \bar{y}'A' = [0.110\text{ m}](0.300\text{ m})(0.02\text{ m}) = 0.660(10^{-3})\text{ m}^3$$

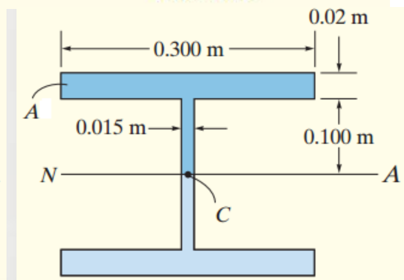
so that

$$\tau_{B'} = \frac{VQ_{B'}}{It_{B'}} = \frac{80(10^3)\text{ N}(0.660(10^{-3})\text{ m}^3)}{155.6(10^{-6})\text{ m}^4(0.300\text{ m})10^6} = 1.13\text{ MPa}$$

For point B, $t_B = 0.015$ m and $Q_B = Q_{B'}$,

$$\tau_B = \frac{VQ_B}{It_B} = \frac{80(10^3)\text{ N}(0.660(10^{-3})\text{ m}^3)}{155.6(10^{-6})\text{ m}^4(0.015\text{ m})10^6} = 22.6\text{ MPa}$$





For point C, $t_C = 0.015$ m and A is the dark shaded area

$$Q_C = \Sigma\bar{y}'A' = [0.110\text{ m}](0.300\text{ m})(0.02\text{ m} + [0.05\text{ m}](0.015\text{ m})(0.100\text{ m}) = 0.735(10^{-3})\text{ m}^3$$

$$\tau_C = \tau_{\max} = \frac{VQ_C}{It_C} = \frac{80(10^3)\text{ N}[0.735(10^{-3})\text{ m}^3]}{155.6(10^{-6})\text{ m}^4(0.015\text{ m})10^6} = 25.2\text{ MPa}$$