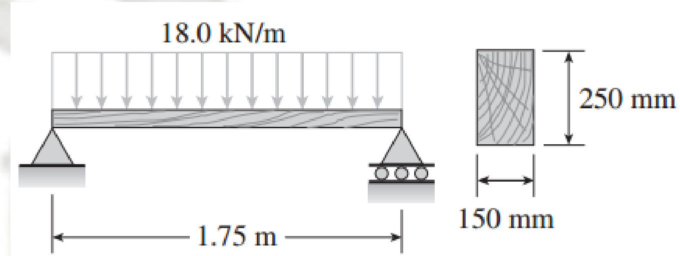
EXAMPLE 1-4

Calculate the maximum shear stress τ_{max} in a simply supported wood beam carrying a uniform load of (w=18 kN/m) if the length is 1.75 m and the cross section is rectangular with width 150 mm and height 250 mm.

Solution:

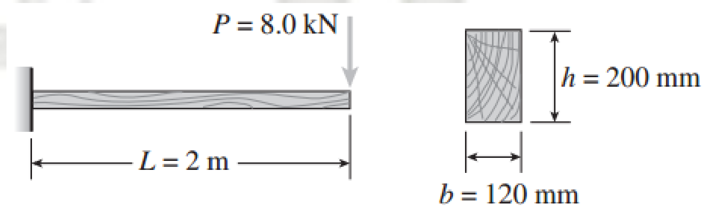


$$V=\frac{wL}{2}=\frac{18\times1.75}{2}=15.75kN$$

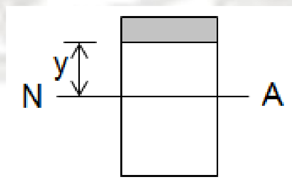
$$\tau_{\max}=1.5\frac{V}{A}=1.5\times\frac{15.75\times1000}{150\times250}=0.63MPa$$

EXAMPLE 1-5

A cantilever beam is made of wood with cross-sectional dimensions as shown in the Figure. Calculate the shear stresses due to the load *P* at points located 25 mm, 50 mm, 75 mm, and 100 mm from the top surface of the beam. From these results, plot a graph showing the distribution of shear stresses from top to bottom of the beam.



Solution:



$$V=P$$

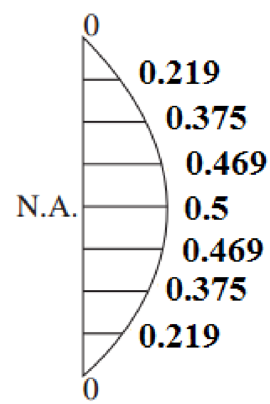
$$I=\frac{bh^3}{12}=\frac{120\times200^3}{12}=80\times10^6\,mm^4$$

$$\tau=\frac{VQ}{Ib}=\frac{V}{2I}\left(\frac{h^2}{4}-y^2\right)$$

$$\tau=\frac{V}{2I}\left(\frac{h^2}{4}-y^2\right)=\frac{8\times1000}{2\times80\times10^6}\left(\frac{200^2}{4}-y^2\right)$$

$$\tau=50\times10^{-6}(10000-y^2)$$

Distance from the top surface (mm)	y_1 (mm)	τ (MPa)
0	100	0
25	75	0.219
50	50	0.375
75	25	0.469
100 (N.A.)	0	0.500



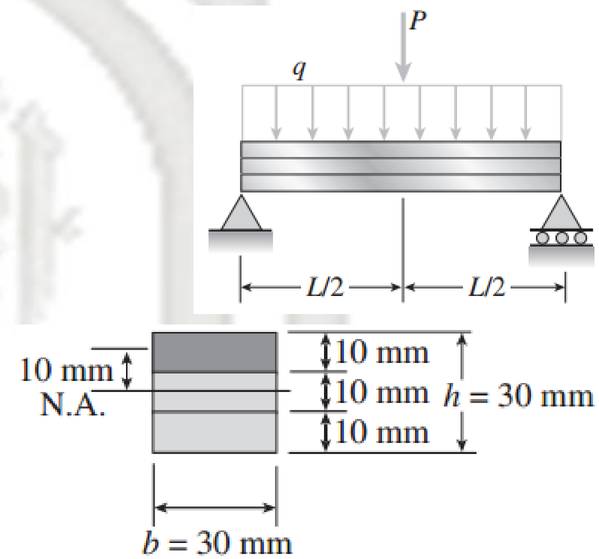
EXAMPLE 1-6

A laminated plastic beam of square cross section is built up by gluing together three strips, as shown in the figure. The beam has a total weight of 3.2 N and is simply supported with span length $L = 320$ mm. Considering the weight of the beam, calculate the maximum permissible load P that may be placed at the midpoint if:

(a) the allowable shear stress in the glued joints is 0.3 MPa.

(b) the allowable bending stress in the plastic is 8 MPa.

Solution:



(a) $w = \dfrac{3.2}{320} = 10 N/m$

$$I = \frac{bh^3}{12} = \frac{30 \times 30^3}{12} = 67500 mm^4$$

$$\tau_{allow} = 0.3 MPa$$

$$V = \frac{P}{2} + \frac{wL}{2} = \frac{P}{2} + 1.6$$

$$Q = 30 \times 10 \times 10 = 3000 mm^3$$

$$\tau = \frac{VQ}{Ib} = \frac{(P/2 + 1.6) \times 3000}{67500 \times 30}$$

$$0.3 = \frac{(P/2 + 1.6) \times 3000}{67500 \times 30}$$

$$P = 2\left[\frac{0.3 \times 67500 \times 30}{3000} - 1.6\right] = 401.8N$$

(b) $\sigma_{allow} = 8 MPa$

$$M_{\max} = \frac{PL}{4} + \frac{wL^2}{8} = 0.08P + 0.128$$

$$\sigma = \frac{My}{I} = \frac{(0.08P + 0.128) \times 15}{67500}$$

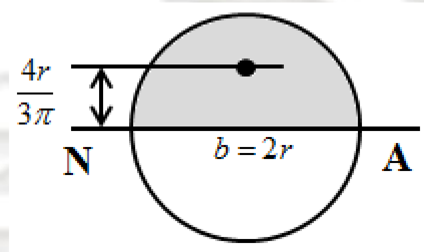
$$8 = \frac{(0.08P + 0.128) \times 10^3 \times 15}{67500}$$

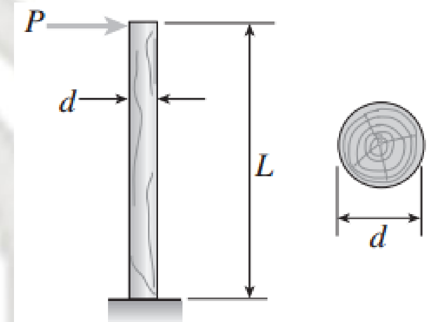
$$P = \frac{1}{80} \times \left[\frac{8 \times 67500}{15} - 128\right] = 448.4N$$

EXAMPLE 1-7

A wood pole of solid circular cross section is subjected to a horizontal force P= 450 lb. The length of the pole is L=6 ft, and the allowable stresses in the wood are 1900 psi in bending and 120 psi in shear. Determine the minimum required diameter of the pole based upon the allowable shear stress.

Solution:





$$I=\pi\frac{r^4}{4}$$

$$b=2r$$

$$Q=\frac{4r}{3\pi}\times\frac{1}{2}\pi r^2=\frac{2}{3}r^3$$

$$\tau=\frac{VQ}{Ib}=\frac{V\times\frac{2}{3}r^3}{\pi\frac{r^4}{4}\times 2r}=\frac{4V}{3A}$$

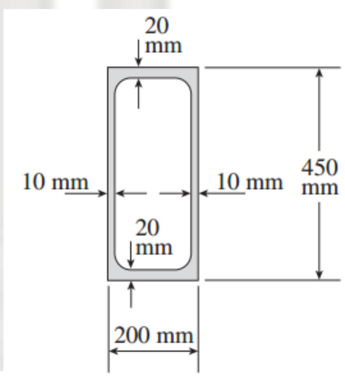
$$\tau=\frac{16V}{3\pi d^2}\rightarrow d^2=\frac{16V_{\max}}{3\pi\tau_{allow}}=6.366$$

$$d_{\min}=2.52in$$

EXAMPLE 1-8

A hollow steel box beam has the rectangular cross section shown in the figure. Determine the maximum allowable shear force *V* that may act on the beam if the allowable shear stress is 36 MPa.

Solution:



$$I=\frac{bh^3}{12}=\frac{200\times450^3}{12}-\frac{180\times410^3}{12}=484.9\times10^6\,mm^4$$

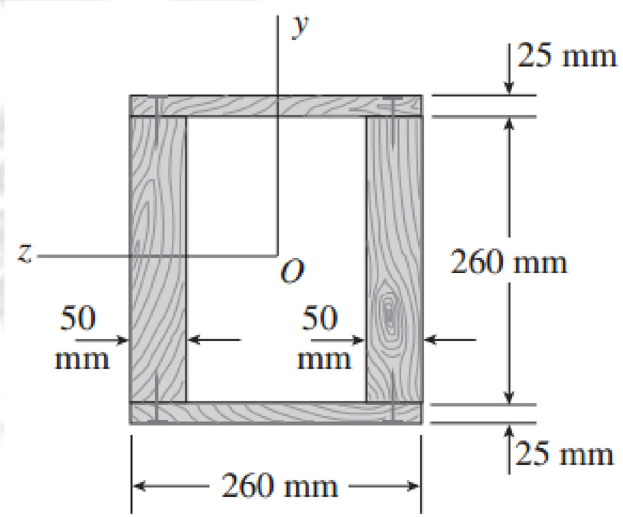
$$Q=200\times\frac{450}{2}\times\frac{450}{4}-180\times\frac{410}{2}\times\frac{410}{4}=1.28\times10^6\,mm^3$$

$$\tau_{allow}=\frac{V_{allow}Q}{Ib}$$

$$V_{allow}=\frac{\tau_{allow}Ib}{Q}=\frac{36\times484.8\times10^6\times20}{1.28\times10^6}=273kN$$

EXAMPLE 1-9

A box beam of wood is constructed of two (260 mm x 50 mm) boards and two 260 mm x 25 mm) boards as shown in the figure. The boards are nailed at a longitudinal spacing (S=100 mm). If each nail has an allowable shear force F=1200 N, what is the maximum allowable shear force V max?



Solution:

$b = 260 \qquad b_1 = 260 - 2(50) = 160$

$h = 310 \qquad h_1 = 260$

s = nail spacing = 100 mm

F = allowable shear force for one nail = 1200 N

f = shear flow between one flange and both webs

$$f_{\text{allow}} = \frac{2F}{s} = \frac{2(1200\text{ N})}{100\text{ mm}} = 24\text{ kN/m}$$

$$f = \frac{VQ}{I} \qquad V_{\max} = \frac{f_{\text{allow}} I}{Q}$$

$$I = \frac{1}{12}(bh^3 - b_1 h_1^3) = 411.125 \times 10^6\text{ mm}^4$$

$$Q = Q_{\text{flange}} = A_f d_f = (260)(25)(142.5) = 926.25 \times 10^3\text{ mm}^3$$

$$V_{\max} = \frac{f_{\text{allow}} I}{Q} = \frac{(24\text{ kN/m})(411.125 \times 10^6\text{ mm}^4)}{926.25 \times 10^3\text{ mm}^3}$$

$$= 10.7\text{ kN}$$

EXAMPLE 1-10

Two wood box beams (beams A and B) have the same outside dimensions (200 mm x 360 mm) and the same thickness (t=20 mm) throughout, as shown in the figure. Both beams are formed by nailing, with each nail having an allowable shear load of 250 N. The beams are designed for a shear force V =3.2 kN.

(a) What is the maximum longitudinal spacing (SA) for the nails in beam A?

(b) What is the maximum longitudinal spacing (SB) for the nails in beam B?

(c) Which beam is more efficient in resisting the shear force?

Solution:

$$b = 200 \qquad b_1 = 200 - 2(20) = 160$$

$$h = 360 \qquad h_1 = 360 - 2(20) = 320$$

$$t = 20$$

F = allowable load per nail = 250 N

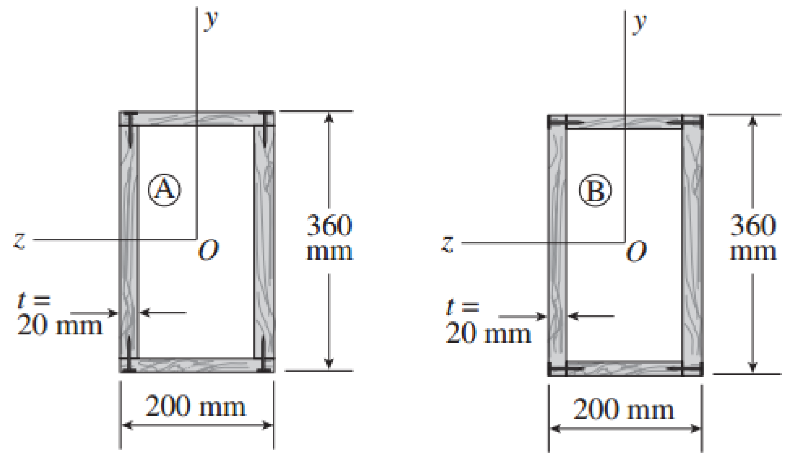
V = shear force = 3.2 kN

$$I = \frac{1}{12}(bh^3 - b_1h_1^3) = 340.69 \times 10^6 \text{ mm}^4$$

s = longitudinal spacing of the nails

f = shear flow between one flange and both webs

$$f = \frac{2F}{s} = \frac{VQ}{I} \qquad \therefore\ s_{\max} = \frac{2FI}{VQ}$$



(a) Beam A

$$Q = A_p d_p = (bt)\left(\frac{h-t}{2}\right) = (200)(20)\left(\frac{1}{2}\right)(340)$$
$$= 680 \times 10^3 \text{ mm}^3$$

$$s_A = \frac{2FI}{VQ} = \frac{(2)(250 \text{ N})(340.7 \times 10^6 \text{ mm}^4)}{(3.2 \text{ kN})(680 \times 10^3 \text{ mm}^3)}$$
$$= 78.3 \text{ mm}$$

(b) Beam B

$$Q = A_f d_f = (b - 2t)(t)\left(\frac{h-t}{2}\right) = (160)(20)\frac{1}{2}(340)$$
$$= 544 \times 10^3 \text{ mm}^3$$

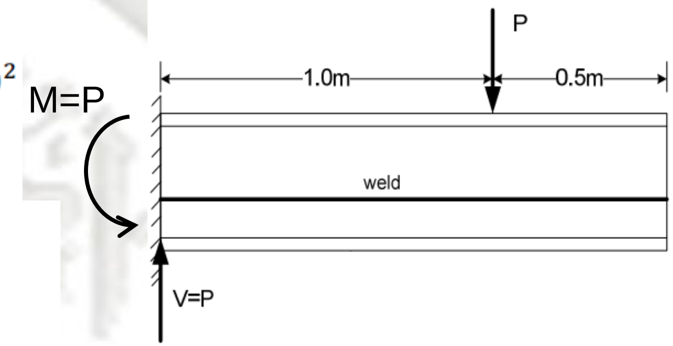
$$s_B = \frac{2FI}{VQ} = \frac{(2)(250 \text{ N})(340.7 \times 10^6 \text{ mm}^4)}{(3.2 \text{ kN})(544 \times 10^3 \text{ mm}^3)}$$
$$= 97.9 \text{ mm}$$

(c) BEAM B IS MORE EFFICIENT because the shear flow on the contact surfaces is smaller and therefore fewer nails are needed.

EXAMPLE 1-11

A steel cantilever beam is made of two structural tee- section beams welded together as shown in the figure below. Determine the allowable safe load (P) that the beam can carry. The allowable stresses are: σ=150MPa in tension and compression, τ=100MPa in shear, and q=2000N/mm on the welded joint.

$$y1 = \frac{150(25)(12.5)+25(200)(125)+200(25)(237.5)}{150(25)+25(200)+200(25)} = 135.2mm$$



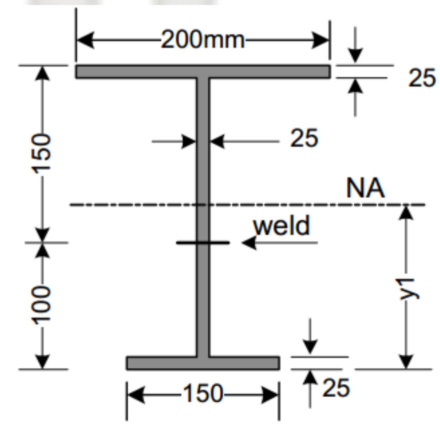
$$I_{NA} = \frac{150(25)^3}{12} + 150(25)(135.2-12.5)^2 + \frac{25(200)^3}{12} + 25(200)(135.2-125)^2 + \frac{200(25)^3}{12} + 200(25)(237.5-135.2)^2 = 1.26*10^8 mm^4$$

- The allowable force (P) based on the bending stresses:

$$\sigma = \frac{M\ C}{I_{NA}}$$

$$\sigma_{allow} = 150MPa, for\ both\ tension\ and\ compression$$

$$150MPa = \frac{1000P*135.2}{1.26*10^8} => P = 140kN$$



- The allowable force (P) based on the shearing stresses:

V=P;

$$\tau = \frac{VQ}{I_{NA}b}$$

$$\tau_{NA} = \frac{P*[200*25*102.3+25*89.8*44.9]}{1.26*10^8*25} = 100MPa => P = 514.5kN$$

- The allowable force (P) based on the shear flow in the welding joint:

$$q = \frac{VQ}{I_{NA}}$$

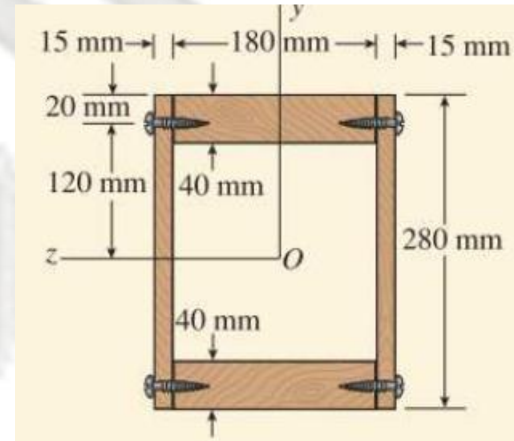
$Q=200(25)(102.3)+25(125)(27.3)= 5.97*10^5 mm^3$

$$q_{weld} = \frac{P*5.97*10^5}{1.26*10^8} = 2000 => P = 422.2kN$$

The safe allowable load P_{safe}= 140kN

EXAMPLE 1-12

A wood box beam shown in the figure is constructed of two boards, each 180x40mm in cross section, that serve as flanges and two plywood webs, each 15mm thick. The total height of the beam is 280mm. The plywood is fastened to the flanges by wood screws having an allowable load in shear of F=800N each. If the shear force V acting on the cross section is 10.5kN, determine the maximum permissible longitudinal spacing (S) of the screws.



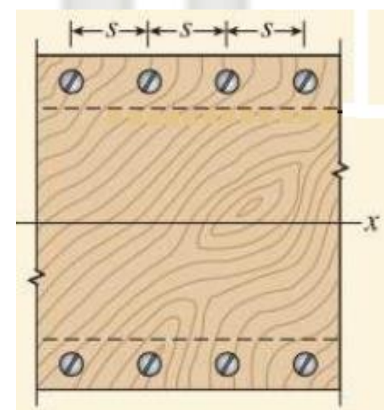
$$I_{NA} = \frac{210(280)^3}{12} - \frac{180(200)^3}{12} = 2.64 * 10^8 mm^4$$

$$Q = \frac{1}{2}(180 * 40 * 120) = 432000 mm^3$$

$$q = \frac{VQ}{I_{NA}}$$

$$q = \frac{10.5 * 10^3 * 432000}{2.64 * 10^8} = \frac{17.18N}{mm}$$

$$S = \frac{F}{q} = \frac{800}{17.18} = 46.6mm$$



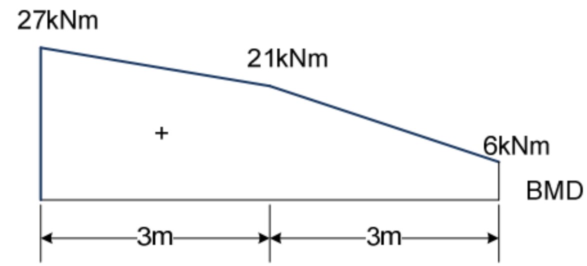
For practical fabrication of the beam, use spacing between screws S=45mm

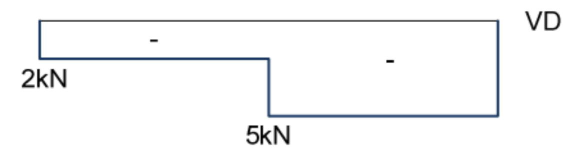
EXAMPLE 1-13

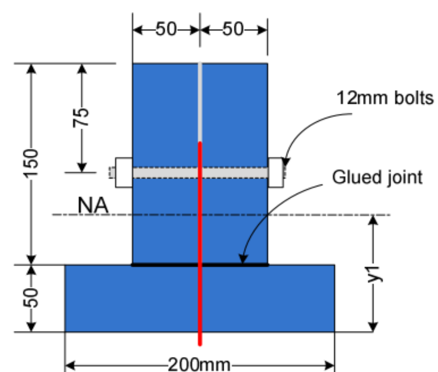
A beam is loaded so that the moment diagram of it varies as shown in the figure.

a) Find the maximum longitudinal shearing force in the 12mm diameter bolts spaced 300mm apart.

b) Find the maximum shearing stress in the glued joint.







$$y1 = \frac{200(50)(25) + 100(150)(125)}{200(50) + 100(150)} = 85mm$$

$$I_{NA} = \frac{200(50)^3}{12} + 200(50)(85-25)^2 + \frac{100(150)^3}{12} + 100(150)(125-85)^2$$

$$I_{NA} = 9 * 10^7 mm^4$$

a) Find Q @ the red fiber, which represents Q for the bolt

Q= 100(50) (-60) +150(50) (40) =-300000+300000=0

q=0

b) Find Q @ the glued joint

Q=200(50) (60) =600000mm^3

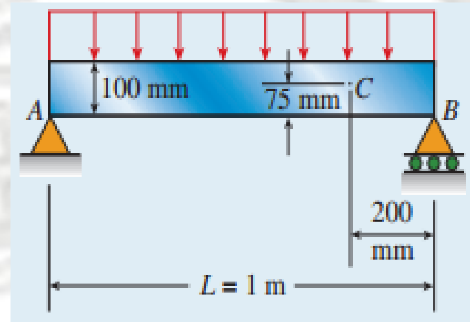
Or: Q=100(150) (40) =600000mm ^{3}

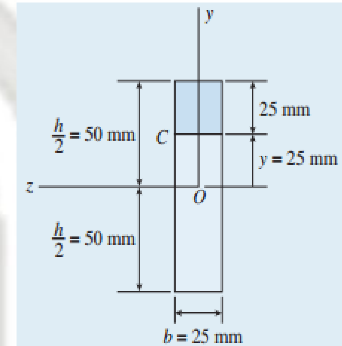
$$\tau = \frac{VQ}{I_{NA}b}$$

$$\tau_{glue} = \frac{5000 * 600000}{9 * 10^7 * 100} = 0.333MPa$$

H.W.1

A metal beam with span is simply supported at points A and B. The uniform load on the beam (including its own weight) is 28 kN/m. Determine the normal stress σc and shear stress τc at point C, which is located 25 mm below the top of the beam and 200 mm from the right support. Show these stresses on a sketch of a stress element at point C.





H.W.2

A wood beam AB supporting two concentrated loads P has a rectangular cross section of width b= 100 mm and height h= 150 mm. The distance from each end of the beam to the nearest load is a= 0.5 m. Determine the maximum permissible value Pmax of the loads if the allowable stress in bending is σ=11 MPa (for both tension and compression) and the allowable stress in horizontal shear is τ=1.2 MPa.