

## EXAMPLE 1-11

A steel cantilever beam is made of two structural tee-section beams welded together as shown in the figure below. Determine the allowable safe load ( $P$ ) that the beam can carry. The allowable stresses are:  $\sigma=150\text{MPa}$  in tension and compression,  $\tau=100\text{MPa}$  in shear, and  $f=2000\text{N/mm}$  on the welded joint.

$$y_1 = \frac{150(25)(12.5) + 25(200)(125) + 200(25)(237.5)}{150(25) + 25(200) + 200(25)} = 135.2\text{mm}$$

$$I_{NA} = \frac{150(25)^3}{12} + 150(25)(135.2 - 12.5)^2 + \frac{25(200)^3}{12} + 25(200)(135.2 - 125)^2 + \frac{200(25)^3}{12} + 200(25)(237.5 - 135.2)^2 = 1.26 * 10^8 \text{mm}^4$$

- The allowable force ( $P$ ) based on the bending stresses:

$$\sigma = \frac{M C}{I_{NA}}$$

$\sigma_{allow} = 150\text{MPa}$ , for both tension and compression

$$150\text{MPa} = \frac{1000P * 135.2}{1.26 * 10^8} \Rightarrow P = 140\text{kN}$$

- The allowable force ( $P$ ) based on the shearing stresses:

$$V=P;$$

$$\tau = \frac{VQ}{I_{NA}b}$$

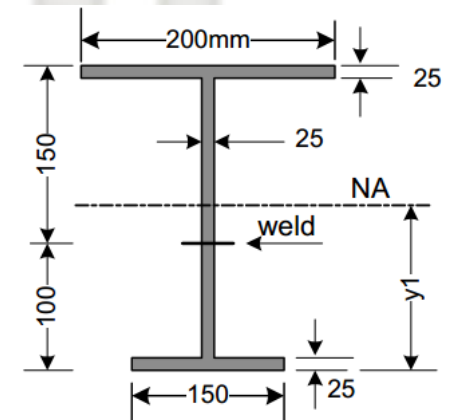
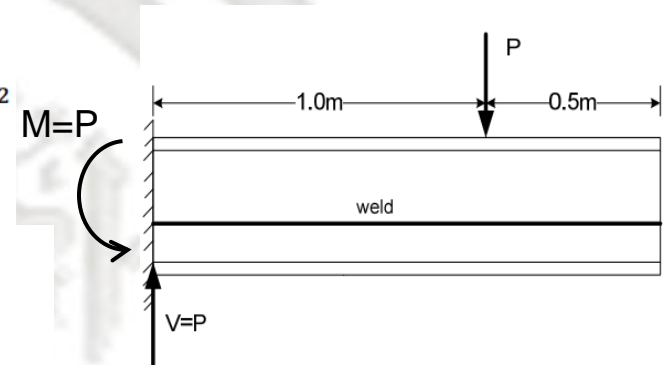
$$\tau_{NA} = \frac{P * [200 * 25 * 102.3 + 25 * 89.8 * 44.9]}{1.26 * 10^8 * 25} = 100\text{MPa} \Rightarrow P = 514.5\text{kN}$$

- The allowable force ( $P$ ) based on the shear flow in the welding joint:

$$f = \frac{VQ}{I_{NA}}$$

$$Q = 200(25)(102.3) + 25(125)(27.3) = 5.97 * 10^5 \text{mm}^3$$

$$f_{weld} = \frac{P * 5.97 * 10^5}{1.26 * 10^8} = 2000 \Rightarrow P = 422.2\text{kN}$$



**The safe allowable load  $P_{safe} = 140\text{kN}$**

### EXAMPLE 1-12

A wood box beam shown in the figure is constructed of two boards, each 180x40mm in cross section, that serve as flanges and two plywood webs, each 15mm thick. The total height of the beam is 280mm. The plywood is fastened to the flanges by wood screws having an allowable load in shear of  $F=800\text{N}$  each. If the shear force  $V$  acting on the cross section is 10.5kN, determine the maximum permissible longitudinal spacing ( $S$ ) of the screws.

$$I_{NA} = \frac{210(280)^3}{12} - \frac{180(200)^3}{12} = 2.64 * 10^8 \text{ mm}^4$$

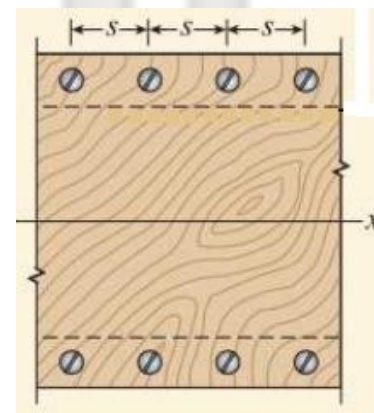
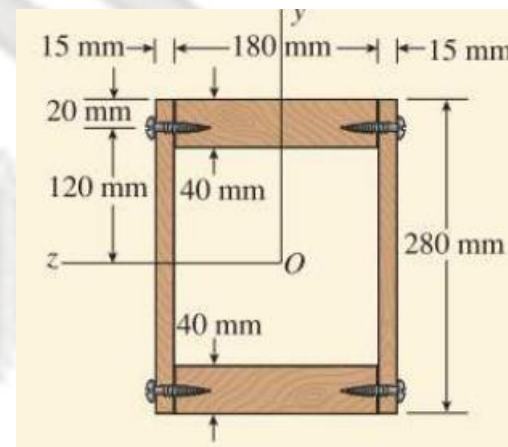
$$Q = \frac{1}{2}(180 * 40 * 120) = 432000 \text{ mm}^3$$

$$f = \frac{VQ}{I_{NA}}$$

$$f = \frac{10.5 * 10^3 * 432000}{2.64 * 10^8} = \frac{17.18 \text{ N}}{\text{mm}}$$

$$S = \frac{F}{f} = \frac{800}{17.18} = 46.6 \text{ mm}$$

For practical fabrication of the beam, use spacing between screws  $S=45\text{mm}$



### EXAMPLE 1-13

A beam is loaded so that the moment diagram of it varies as shown in the figure.

- Find the maximum longitudinal shearing force in the 12mm diameter bolts spaced 300mm apart.
- Find the maximum shearing stress in the glued joint.

$$y_1 = \frac{200(50)(25) + 100(150)(125)}{200(50) + 100(150)} = 85\text{mm}$$

$$I_{NA} = \frac{200(50)^3}{12} + 200(50)(85 - 25)^2 + \frac{100(150)^3}{12} + 100(150)(125 - 85)^2$$

$$I_{NA} = 9 * 10^7 \text{mm}^4$$

- Find Q @ the red fiber, which represents Q for the bolt

$$Q = 100(50)(-60) + 150(50)(40) = -300000 + 300000 = 0$$

$$f = 0$$

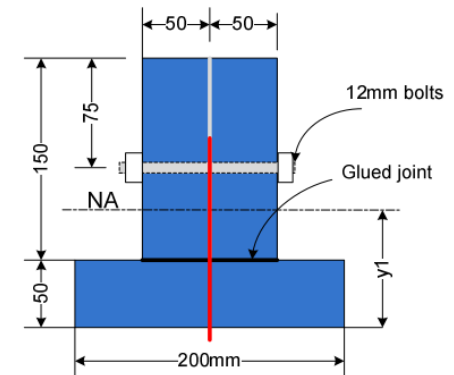
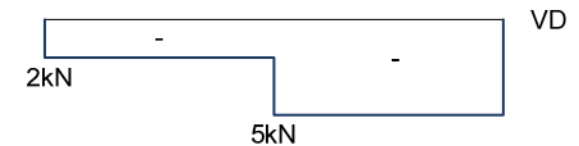
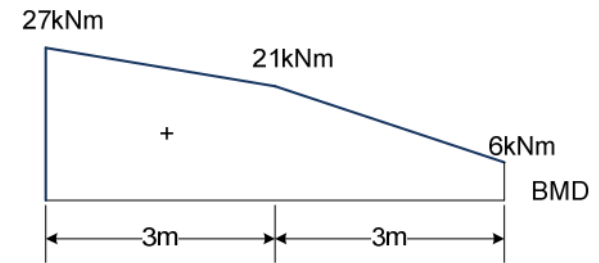
- Find Q @ the glued joint

$$Q = 200(50)(60) = 600000 \text{mm}^3$$

$$\text{Or: } Q = 100(150)(40) = 600000 \text{mm}^3$$

$$\tau = \frac{VQ}{I_{NA}b}$$

$$\tau_{glue} = \frac{5000 * 600000}{9 * 10^7 * 100} = 0.333 \text{MPa}$$



### EXAMPLE 1-14

A beam of T cross section is formed by nailing together two boards having the dimensions shown in the figure. If the total shear force  $V$  acting on the cross section is 1600 N and each nail may carry 750 N in shear, what is the maximum allowable nail spacing  $s$ ?

$$y = [(200)(50)(100) + (200)(50)(225)] / 20 \times 10^3$$

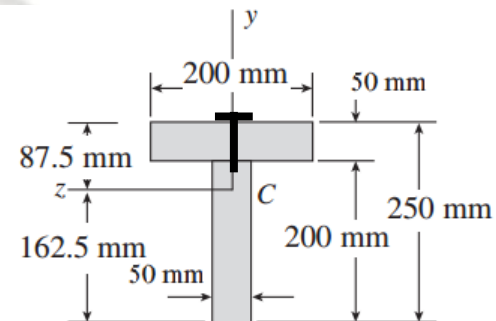
$$= 162.5 \text{ mm}$$

$$I = \frac{1}{3}(50)(162.5)^3 + \frac{1}{3}(50)(37.5)^3 + \frac{1}{12}(200)(50)^3 + (200)(50)(62.5)^2 = 113.541 \times 10^6 \text{ mm}^4$$

$$Q = (200)(50)(62.5) = 625 \times 10^3 \text{ mm}^3$$

$$f = \frac{VQ}{I} = \frac{F}{s}$$

$$s_{\max} = \frac{F_{\text{allow}} I}{VQ} = \frac{(750 \text{ N})(113.541 \times 10^6 \text{ mm}^4)}{(1600 \text{ N})(625 \times 10^3 \text{ mm}^3)} = 85.2 \text{ mm}$$



### EXAMPLE 1-15

Find the shear stress in the glue joint and the nail spacing if the applied shear force is 1000 N and the shear capacity of the nail is 96 kN.

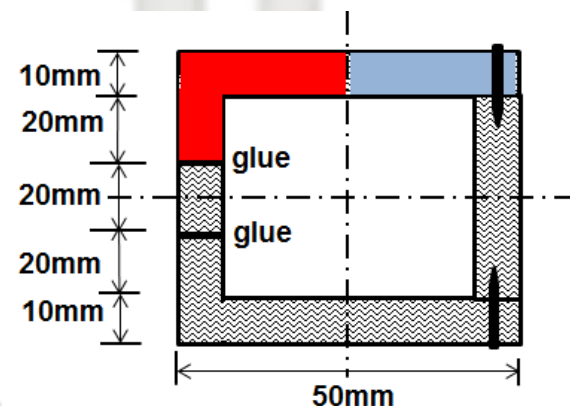
$$I = 1.593 \times 10^6 \text{ mm}^4$$

$$\tau_{\text{glue}} = \frac{VQ}{Ib} = \frac{1000 \times (25 \times 10 \times 35 + 20 \times 10 \times 20)}{1.593 \times 10^6 \times 10} = 0.8 \text{ MPa}$$

$$f = \frac{VQ}{I} = \frac{1000 \times 25 \times 10 \times 35}{1.593 \times 10^6} = 5.5 \text{ N/mm}$$

$$S = \frac{F}{f} = \frac{96}{5.5} = 17.45 \text{ mm}$$

$$S = 17 \text{ mm}$$



### EXAMPLE 1-16

Find the allowable uniform distributed load  $w$  if the allowable force for nail = 0.5 kN, allowable compressive bending stress = 5 MPa, allowable tensile bending stress = 4 MPa and spacing for nails = 30 mm.

$$\sum M_A = 0$$

$$w \times 3 \times 6 = 4B_y$$

$$B_y = 4.5w$$

$$A_y = 1.5w$$

$$y' = \frac{10 \times 4 \times 2 + 10 \times 4 \times 9}{40 + 40} = 55 \text{ mm}$$

$$I = 13.67 \times 10^6 \text{ mm}^4$$

$$\sigma_{comp.} = 5 = \frac{1.125w \times 10^6 \times 55}{13.67 \times 10^6} \Rightarrow w = 1.1 \text{ kN/m}$$

$$\sigma_{ten.} = 4 = \frac{1.125w \times 10^6 \times 85}{13.67 \times 10^6} \Rightarrow w = 0.57 \text{ kN/m}$$

$$\sigma_{comp.} = 5 = \frac{2w \times 10^6 \times 85}{13.67 \times 10^6} \Rightarrow w = 0.4 \text{ kN/m}$$

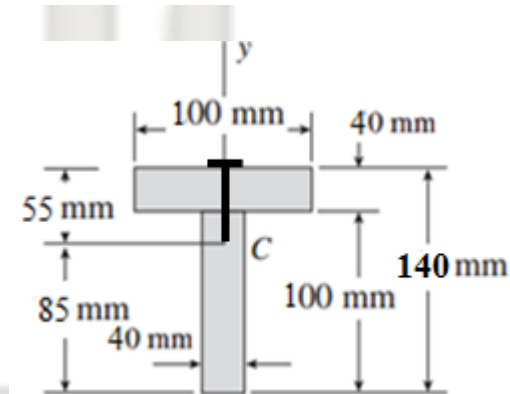
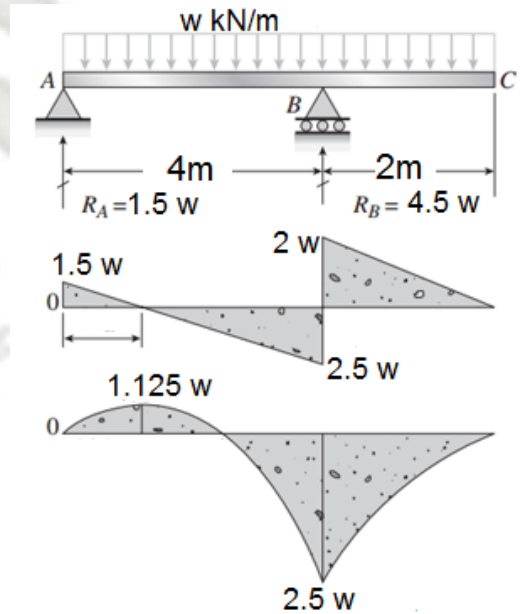
$$\sigma_{ten.} = 4 = \frac{2w \times 10^6 \times 55}{13.67 \times 10^6} \Rightarrow w = 0.5 \text{ kN/m}$$

$$S = \frac{F}{f} \Rightarrow f = \frac{F}{S} = \frac{1000}{30} = 33.33 \text{ N/mm}$$

$$f = \frac{VQ}{I} \Rightarrow 33.33 = \frac{2.5w \times 10^3 \times 40 \times 10 \times 35}{13.76 \times 10^6}$$

$$w = 13.02 \text{ kN/m}$$

$$\therefore w = 0.4 \text{ kN/m}$$



### EXAMPLE 1-17

For the beam shown in the figure, find the spacing for the screws if the flexural stress = 50 MPa, shearing stress for screw A = 50, diameter of the screw A = 5mm, , shearing stress for screw B = 75, and diameter of the screw B = 8 mm.

$$I = 7 \times 10^8 \text{ mm}^4$$

For screw A:

$$\sigma = 50 = \frac{5P \times 10^6 \times 150}{7 \times 10^8} \Rightarrow P = 46.67 \text{ kN}$$

$$f = \frac{VQ}{I} = \frac{46.67 \times 10^3 \times 200 \times 50 \times 125}{7 \times 10^8} = 83.33 \text{ N/mm}$$

$$\tau = \frac{F}{A} \Rightarrow 50 = \frac{F}{\pi \frac{5^2}{4}} \Rightarrow F = 981.74 \text{ N}$$

$$S = \frac{F}{f} = \frac{981.74}{83.33} = 11.78 \text{ mm}$$

For screw B:

Complete the solution

