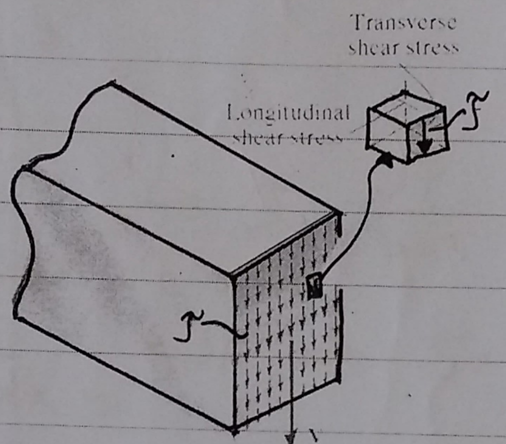
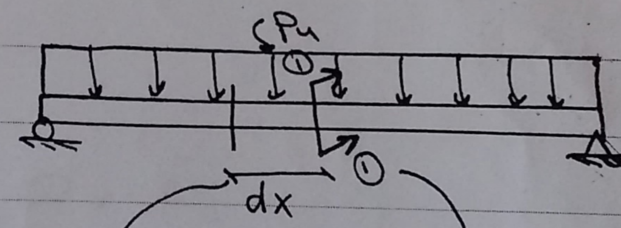
Shearing Stresses in Beams

In general, a beam will support both shear and moment. The shear (V) is a result of a shear stress distribution that acts over the beam's cross section.



The Shear Formula :-

In order to derive the shear formula, consider the simply supported beam shown below:

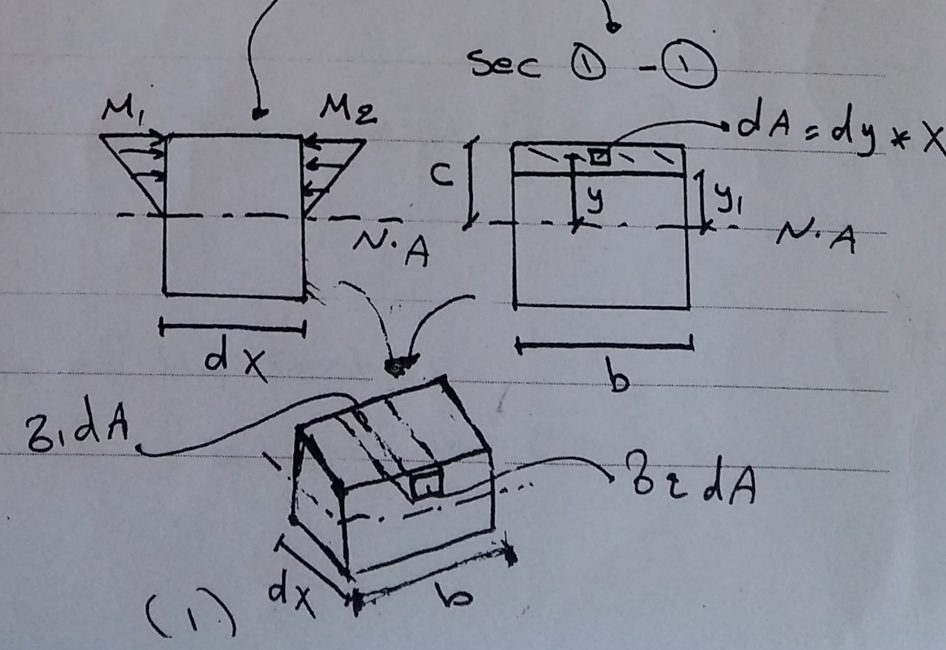


Assume df = resistance shear

$\sum f_x = 0$

$df = H_2 - H_1$

$= \int_{y_1}^{c} \sigma_2 dA - \int_{y_1}^{c} \sigma_1 dA$



Now

$$\sigma = \frac{M \cdot y}{I}$$

$$df = \frac{M_2}{I} \int_{y_1}^{c} y\,dA - \frac{M_1}{I} \int_{y_1}^{c} dA \cdot y$$

$$df = \frac{M_2 - M_1}{I} \int_{y_1}^{c} y\,dA$$

$$df = \tau\,dx\,b \Rightarrow \tau = \frac{df}{b\,dx}$$

$$\tau = \frac{M_2 - M_1}{I \cdot b \cdot dx} \int_{y_1}^{c} y\,dA \Rightarrow \tau = \frac{V}{I \cdot b} \int_{y_1}^{c} y\,dA$$

$\int_{y_1}^{c} y\,dA$ = Sum of moments of different area about N.A

$$\int_{y_1}^{c} y\,dA = \bar{A}\,\bar{y}$$

$$\tau = \frac{V \acute{A} \bar{y}}{I \cdot b} \quad , \text{let } Q = \acute{A}\bar{y}$$

$$\tau = \frac{V \cdot Q}{I \cdot b}$$ ⇒ The shear formula.

where:

V: Shearing force of the section.

I: Second moment of area of the cross section.

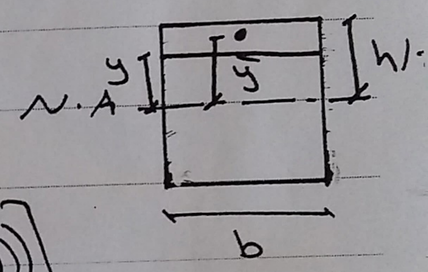
b: width of the beam at the level where the shear stress is being computed.

$\acute{A}$: The area of the section above (or below) the level at which the shearing stress is to be calculated.

$\acute{y}$: moment arm of the area $\acute{A}$ with respect to the N.A.

$Q = \acute{A}\acute{y}$ = moment of area.

- The shear stress formula for rectangular section.



$$\tau = \frac{V}{I \cdot b} \acute{A} \cdot \bar{y}$$

$$\tau = \frac{V}{I \cdot b}\left[\left(b*\left(\frac{h}{2}-y\right)\right)*\left(y+\frac{1}{2}\left(\frac{h}{2}-y\right)\right)\right]$$

$$= \frac{V}{I}\left(\frac{h}{2}-y\right)\left(\frac{y}{2}+\frac{h}{4}\right)$$

$$= \frac{V}{I}\left(\frac{h-2y}{2}\right)\left(\frac{2y+h}{4}\right) = \frac{V}{8I}(h-2y)(2y+h)$$

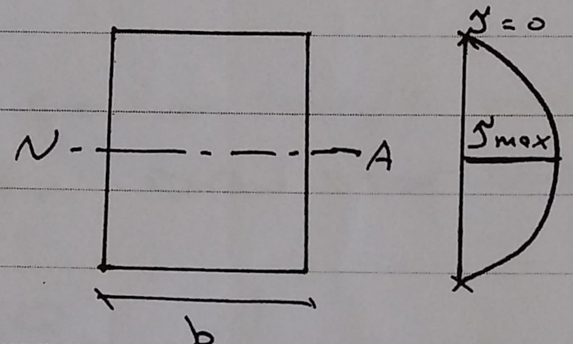
$$= \frac{V}{8I}(\cancel{2yh}+h^2-4y^2-\cancel{2yh}) = \frac{V}{8I}(h^2-4y^2)$$

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4}-y^2\right)$$

at $y=0$, $\tau_{max} = \frac{V h^2}{8 I}$

at $y = h/2 \Rightarrow \tau = \frac{V}{8I}\left(\frac{h^2}{4} - \frac{h^2}{4}\right) = 0$

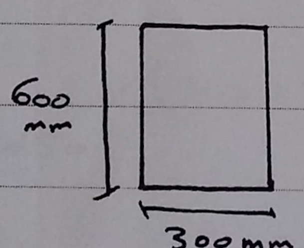
So, the distribution of shear stress $((\tau))$ across the section is illustrated in the figure below

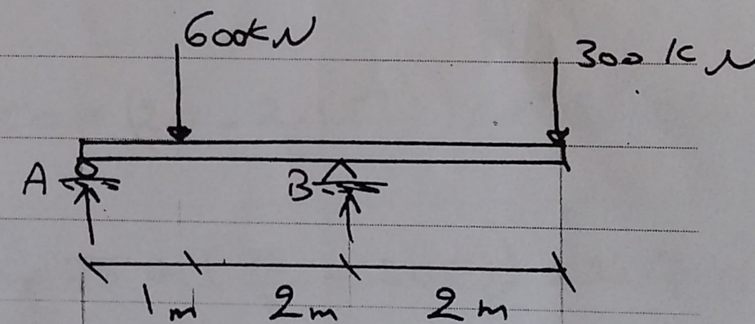


Examples:

① Determine the absolute max. shear stress developed in the beam shown below.

Sol:





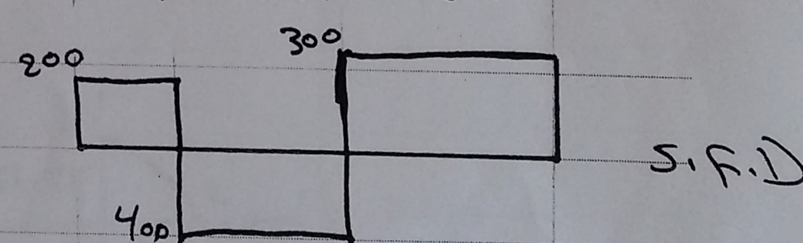
$\Sigma M @ A = 0$

$-By*3 + 600*1 + 300*5 = 0$

$By = 700\ kN$

$\Sigma Fy = 0 \Rightarrow Ay = 200\ kN$

$V = 400\ kN \Leftarrow$ From S.F.D



$$\tau_{max} = \frac{V \cdot Q}{I \cdot b}$$

$V = 400\ kN$, $b = 300\ mm$, $I = \frac{b \cdot h^3}{12} = \frac{300 * 600^3}{12} = 54 * 10^8\ mm^4$

$Q = \acute{A}\acute{y} = 300 * 300 * 150 = 13.5 * 10^6\ mm^3$

$$\tau_{max} = \frac{400 * 1000 * 13.5 * 10^6}{54 * 10^8 * 300} = 3.33\ MPa$$

(2) The section shown is subjected to a shear force of 20 kN. Draw shear stress distribution. ($I = 2.74 * 10^6\ mm^4$)

Sol:

$$\tau = \frac{VQ}{Ib}$$

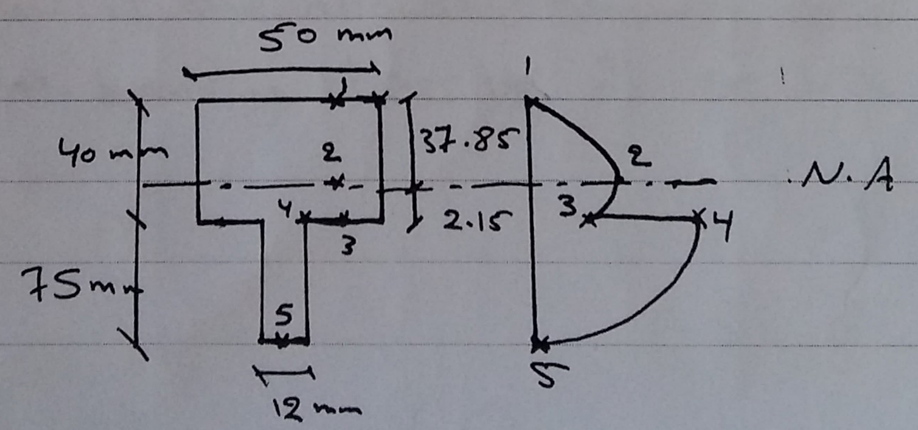
$\tau_1 = 0$, $\acute{A} = 0$

$$\tau_2 = \frac{20 * 10^3}{2.74 * 10^6 * 50} * \left(37.85 * 50 * \frac{37.85}{2}\right) = 5.23\ MPa$$

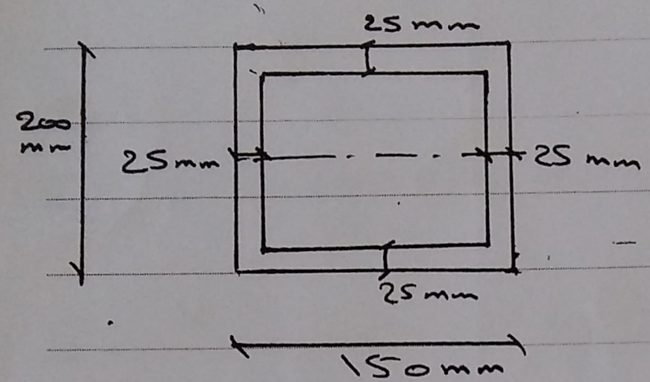
$$\tau_3 = \frac{20 * 10^3}{2.74 * 10^6 * 50} * \left(40 * 50 * (20 - 2.15)\right) = 5.2\ MPa.$$

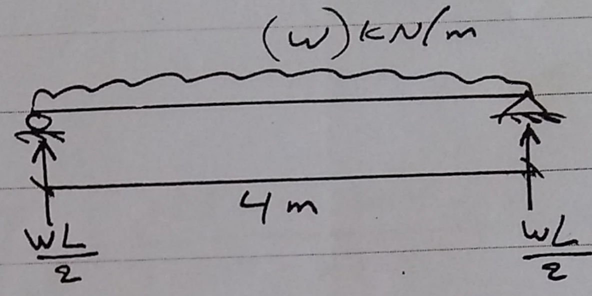
$$\tau_4 = \frac{20 * 10^3}{2.74 * 10^6 * 12} * \left(12 * 75 * \left(\frac{75}{2} + 2.15\right)\right) = 21.7\ MPa$$

$\tau_5 = 0$



③ For the simply supported beam shown in fig., find the distributed load (w) if $\tau_{all} = 1.2$ MPa.





$$I = \frac{150 * 200^3}{12} - \frac{100 * (150)^3}{12} = 71.875 * 10^6 \text{ mm}^4$$

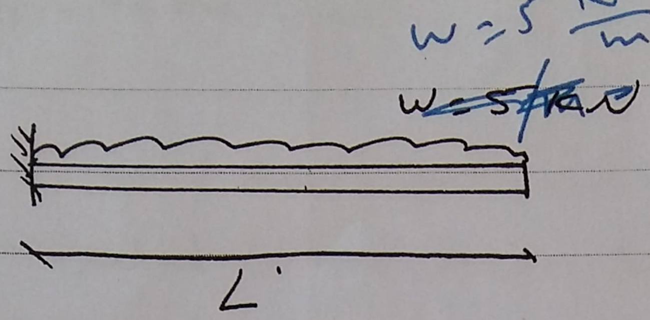
$$\tau_{max} = \frac{VQ}{Ib} = \frac{V * (150 * 25 * 78.5 + 2 * 25 * 75 * \frac{75}{2})}{71.875 * 10^6 * 2 * 25}$$

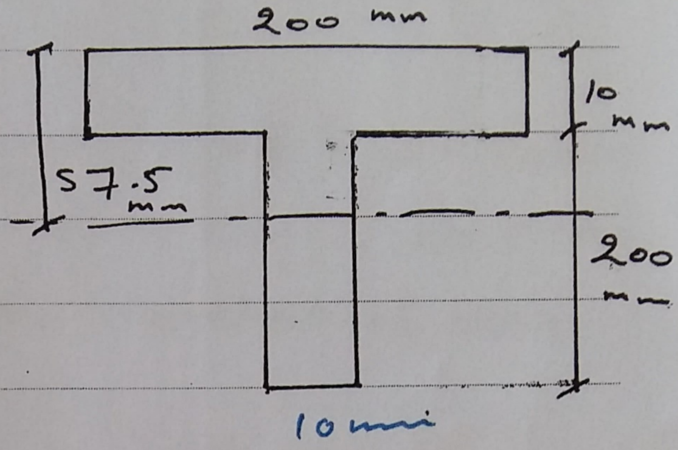
$$1.2 = \frac{V * 468.75 * 10^3}{71.875 * 10^6 * 2 * 25}$$

$V = 9.2$ KN

$$V = \frac{W * L}{2} \Rightarrow 9.2 = \frac{W * 4}{2} \Rightarrow W = 4.6 \text{ KN/m}$$

④ For the beam shown below, determine the length of the beam if the allowable shearing stress is 70 MPa and the max. bending stress is $\sigma = 125$ MPa.





Sol:

$I_{N.A} = 17.704 * 10^6 \ mm^4$

$\sigma = \frac{M \cdot y}{I} \Rightarrow M = \frac{\sigma \cdot I}{y} = \frac{125 * 17.704 * 10^6}{(210 - 57.5)} = 14.5 \ kN.m$

Max at support $= \frac{w \ell^2}{2}$

$14.5 = \frac{5 * \ell^2}{2} \Rightarrow \ell = 2.409 \ m$

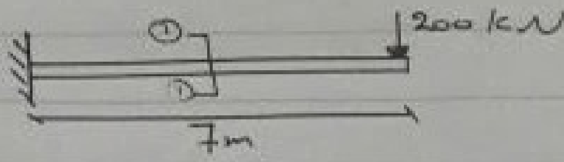
$\tau_{max} = \frac{VQ}{Ib} \Rightarrow V = \frac{\tau I b}{Q}$

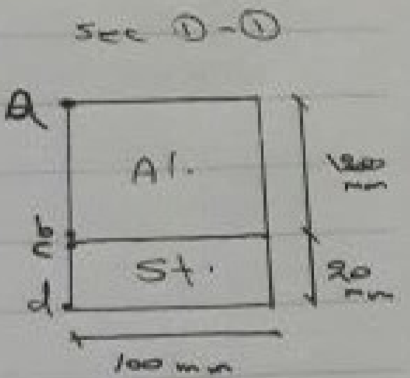
$V = \frac{70 * 17.704 * 10^6 * 10}{(210 - 57.5) * 10 * \frac{(210 - 57.5)}{2}} = 106.57 \ kN$

$V_{max} = w \ell \Rightarrow 106.57 = 5 \ell \Rightarrow \ell = 21.32 \ m$

$\therefore$ max $\ell = 2.409 \ m$

(2) For the cantilever beam shown in fig., determine the max stresses at points a, b, c and d. If $E_s = 200\ GPa$, $E_{al} = 70\ GPa$

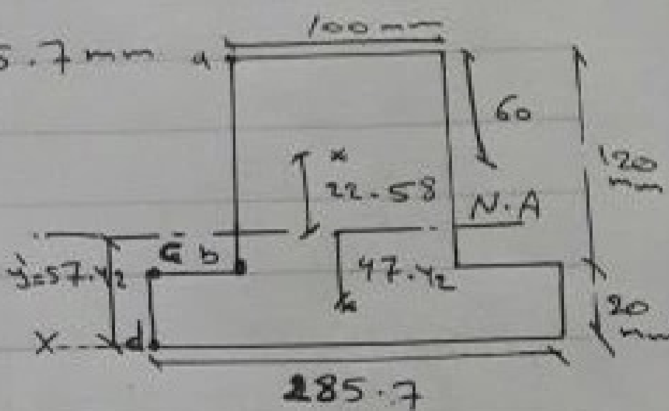




Sol:

$n = \frac{E_s}{E_{al}} = \frac{200}{70} = 2.857$

$b_1 = b * n = 100 * 2.857 = 285.7\ mm$



$\bar{y} = \frac{\Sigma A \cdot y}{\Sigma A}$

$= \frac{285.7 * 20 * 10 + 120 * 100 * 80}{285.7 * 20 + 100 * 120}$

$= 57.42\ mm$

$I_T = \frac{285.7 * 20^3}{12} + 285.7 * 20 * (47.42)^2 + \frac{100 * (120)^3}{12} +$
$100 * 120 * (22.58)^2$

$= 33557566\ mm^4$

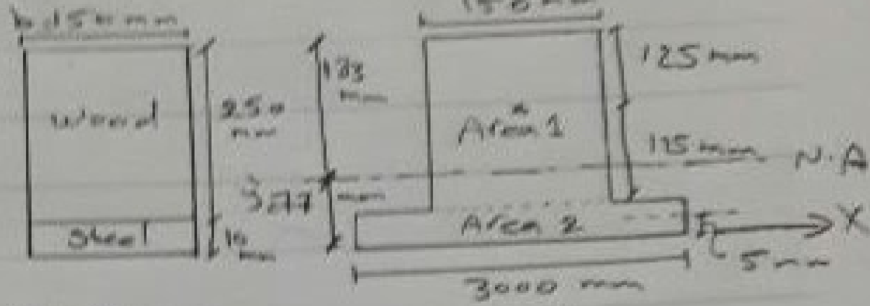
$M_{max} = 200 * 10^3 * 7 * 10^3 = 1400 * 10^6\ N.m$

$\sigma_a = \frac{1400 * 10^6 * (140 - 57.42)}{33557566} = 3445.18\ MPa.$

Examples :

① Consider a Composite beam of the cross-sectional dimensions shown in fig. is subjected to a bending moment of 30 KN.m , what are the maximum stresses in the steel and wood. $E_s = 200$ GPa , $E_W = 10$ GPa.

Sol :



$$n = \frac{E_s}{E_W} = \frac{200\,000}{10\,000} = 20$$

$$b_1 = 20 * b = 20 * 150 = 3000 \text{ mm}$$

$$\bar{y} = \frac{\Sigma A \cdot y}{\Sigma A} = \frac{150 * 250 * 135 + 3000 * 10 * 5}{150 * 250 + 3000 * 10} = 77 \text{ mm}$$

$$I_T = \frac{150 * 250^3}{12} + 150 * 250 * (183 - 125)^2$$

$$+ \frac{3000 * 10^3}{12} + 3000 * 10 * (72)^2 = 478 * 10^6 \text{ mm}^4$$

$$\sigma_{wood})_{max} = \frac{M \cdot C}{I} = \frac{30 * 10^3 * 10^3 * 183}{478 * 10^6} = 11.5 \text{ MPa}$$

$$\sigma_{st})_{max} = n\,\sigma_w = 20\,\frac{30 * 10^6 * 77}{478 * 10^6} = 96.7 \text{ MPa}$$