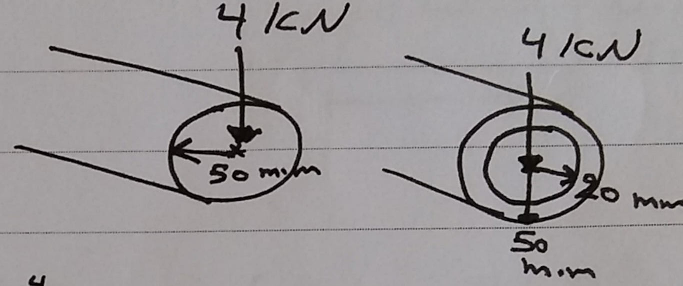
Ex: The solid shaft and tube shown in the figure below are subjected to the shear force of 4 kN. Determine the shear stress acting over the diameter of each cross section.



Sol:

$$I_{solid} = \frac{\pi r^4}{4} = \frac{\pi * 0.05^4}{4} = 4.909 * 10^{-6}\ m^4$$

$$I_{tube} = \frac{\pi}{4}\left[r_o^4 - r_i^4\right] = \frac{\pi}{4}\left[0.05^4 - 0.02^4\right] = 4.783 * 10^{-6}\ m^4$$

$$\tau_{solid} = \frac{VQ}{Ib} = \frac{VA\bar{y}}{Ib} = \frac{V * (\frac{1}{2}\pi r^2)(\frac{4}{3\pi}r)}{I\ b}$$

$$\tau_{solid} = \frac{4 * 10^3 * (\frac{1}{2}\pi * 0.05^2) * (\frac{4}{3\pi} * 0.05)}{4.909 * 10^{-6} * 0.1}$$

$$= 679\ kpa$$

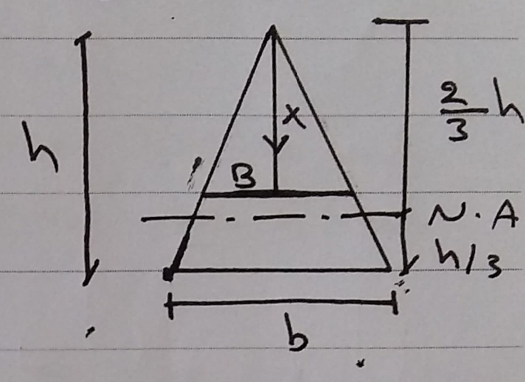
$$\tau_{tube} = \frac{VQ}{Ib} = \frac{VA\bar{y}}{Ib} = \frac{V * \left[\frac{1}{2}\pi r_o^2 * \frac{4}{3\pi} r_o - \frac{4}{3\pi} r_i * \frac{\pi}{2} r_i^2\right]}{I\ b}$$

$$= \frac{4 * 10^3 * \left[\frac{\pi}{2} * 0.05^2 * \frac{4}{3\pi} * 0.05 - \frac{4}{3\pi} * 0.02 * \frac{\pi}{2} * 0.02^2\right]}{4.783 * 10^{-6} * 0.06}$$

$$= 1.09\ Mpa$$

Ex: For the section shown below, Prove that:

$$\tau_{max} = \frac{3V}{bh} \quad , \text{ if } I = \frac{bh^3}{36}$$



Sol:

$$B = \frac{x \cdot b}{h}$$

$$\tau = \frac{V}{I \cdot b} \bar{A}\bar{y}$$

$$= \frac{V}{\frac{bh^3}{36} * \frac{x \cdot b}{h}} * \left(\frac{1}{2} \frac{x \cdot b}{h} * x\right)\left(\frac{2}{3}h - \frac{2}{3}x\right)$$

$$= \frac{36V}{b^2 h^2 x}\left[\frac{bx^2}{3} - \frac{bx^3}{3h}\right]$$

$$= \frac{36V}{3h^2 b}\left[x - \frac{x^2}{h}\right] = \frac{12V}{h^2 b}\left(x - \frac{x^2}{h}\right) \text{ — (1)}$$

$$\frac{d\tau}{dx} = \frac{12V}{h^2 b}\left[1 - \frac{2x}{h}\right] = 0 \Rightarrow 1 - \frac{2x}{h} = 0 \Rightarrow \boxed{x = \frac{h}{2}}$$

Sub in (1)

$$V_{max} = \frac{12V}{h^2 b}\left[\frac{h}{2} - \frac{h^2}{4h}\right] = \frac{3V}{h \cdot b}$$

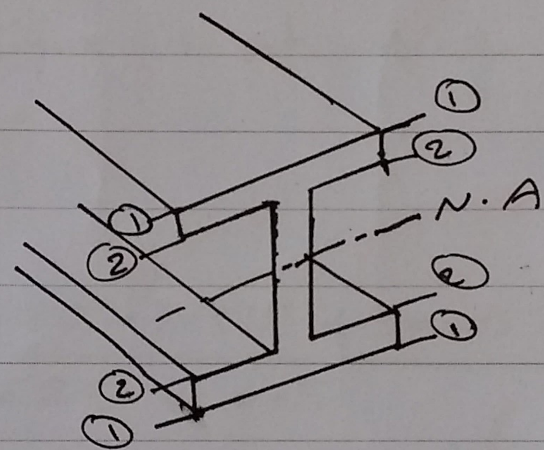
Shear Stress in I section beams.

For the I section beams, the following remarks can be drawn:-

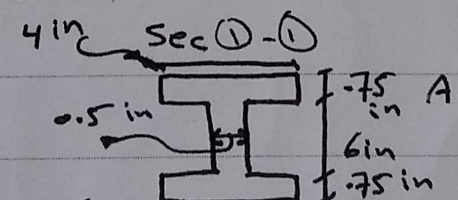
① The shear stress is zero at level (1-1).

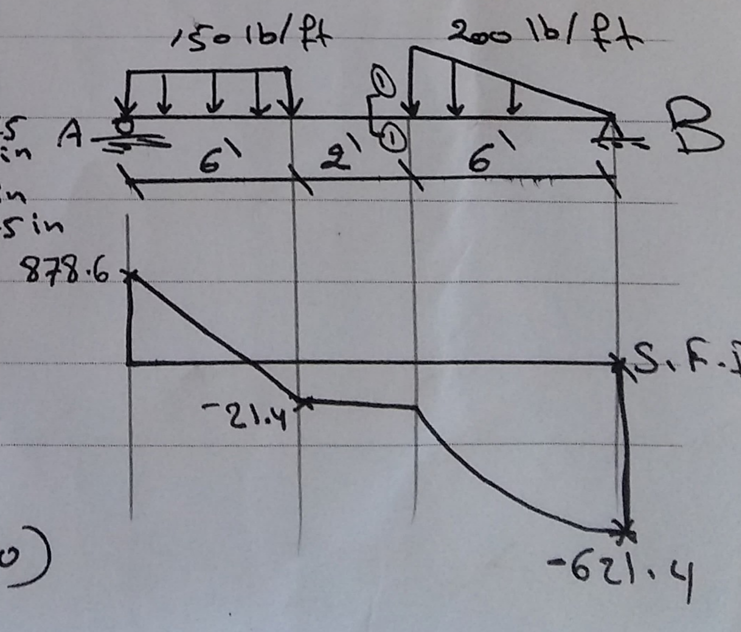
② At level (2-2), the value of shear stress (shear stress in the flange) is lower than that in the web.

③ The maximum shear stress occurs at the N.A.



Ex. Determine the max. shear stress acting in the fiberglass beam at the section where the internal shear force is maximum :-





Sol

$\Sigma M @ A = 0$

$-By * 14 + 150 * 6 * 3 +$

$\frac{1}{2} * 6 * 200 * 10 = 0$

$By = 621.4$ lb

$\Sigma F_y = 0$

$621.4 - 150 * 6 - \frac{1}{2} * 6 * 200 + A_y = 0$

$A_y = 878.6$ lb

From S.F.D., the max. shear force (V) is 878.6 lb

$I = \frac{4 * 7.5^3}{12} - \frac{3.5 * 6^3}{12} = 77.625 \ in^3$

The max shear τ_{max} is at N.A.

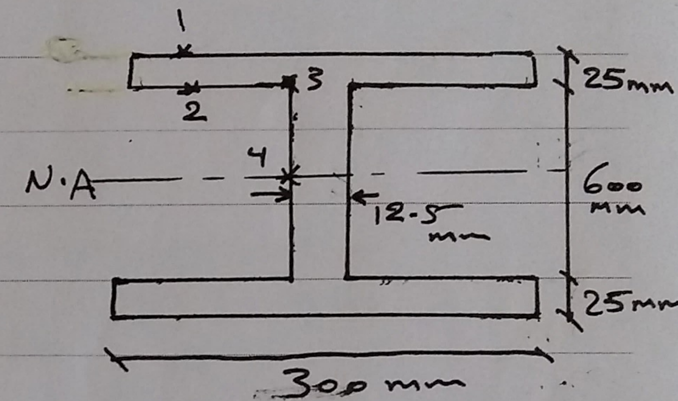
$\tau_{max} = \frac{V \bar{A} \bar{y}}{I b}$

$= \frac{878.6 * [4 * 0.75 * 3.375 + 3 * 0.5 * 1.5]}{77.625 * 0.5}$

$= 280.13 \ \frac{lb}{in^2}$

Ex: The section shown below is subjected to shear force of 1000 kN, Draw shear stress distribution and determine the percentage of shear carried by the web alone.

Sol:



$$I = \frac{300 * 650^3}{12} - \frac{287.5 * 600^3}{12}$$

$$= 1690 * 10^6 \ mm^4$$

$$\tau = \frac{V A \bar{y}}{I b}$$

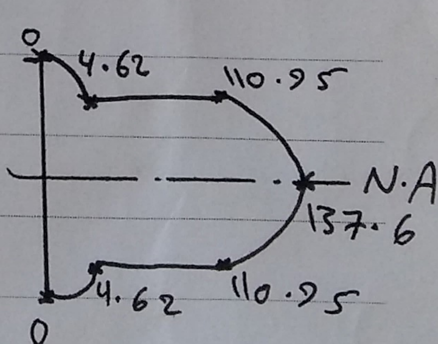
$$\tau_1 = 0$$

$$\tau_2 = \frac{1000 * 10^3 * (300 * 25 * 312.5)}{1690 * 10^6 * 300} = 4.62 \ MPa$$

$$\tau_3 = \frac{1000 * 10^3 * [300 * 25 * 312.5)}{1690 * 10^6 * 12.5} = 110.95 \ MPa$$

$$\tau_4 = \frac{1000 * 10^3 * [300 * 25 * 312.5 + 300 * 12.5 * 150]}{1690 * 10^6 * 12.5}$$

$$= 137.6 \ MPa.$$



$$\tau_{av} = \frac{2}{3}[137.61 - 110.95) + 110.95 = 128.72$$

Shear force carried by web only $= \tau_{av})_{web} * A_{web}$

$$= 128.72 * 600 * 12.5 = 965.4 * 10^3 \ N$$

$$V_{web} = \frac{965.4}{1000} * 100 = 96.5 \ \%$$