Chapter 2

Compound Stresses

In this chapter, we will discuss the solution of problems where axial load, torsion, bending, and shear loads occur simultaneously on a member's cross section. When this occurs, the method of superposition can be used to determine the resultant stress distribution. when the member is subjected to several different types of loadings simultaneously. It is assumed that the material is homogeneous and behaves in a linear elastic manner

Types of Forces:

1) Normal Force: the internal normal force is developed by a uniform normal stress distribution determined from:

$$\sigma = \frac{P}{A}$$

2) Shear Force: the internal shear force in a member is developed by :

$$\tau = \frac{V\,Q}{I\,b}$$

3) Bending Moment: this stress distribution is determined from the flexural formula:

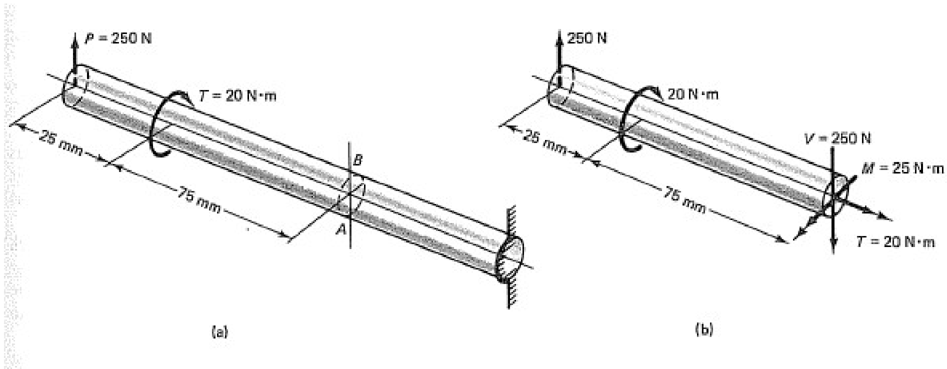
$$\sigma = \frac{M\,y}{I}$$

4) Torsional Moment: this stress distribution is determined from the torsional formula:

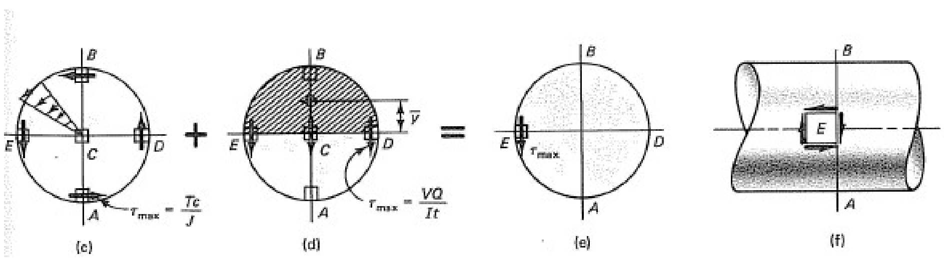
$$\tau = \frac{T\,r}{J}$$

Examples:

Ex1: Find the maximum shear stress due to the applied forces in plane A-B of the 10 mm diameter high strength steel shaft shown in Fig.:



Sol:



$$Q = \frac{\pi c^2}{2}\frac{4c}{3\pi} = \frac{2c^3}{3}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{V}{2c}\frac{2c^3}{3}\frac{4}{\pi c^4} = \frac{4V}{3\pi c^2} = \frac{4V}{3A}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi \times 10^4}{32} = 982 \text{ mm}^4$$

$$I = \frac{J}{2} = 491 \text{ mm}^4$$

$$A = \frac{1}{4}\pi d^2 = 78.5 \text{ mm}^2$$

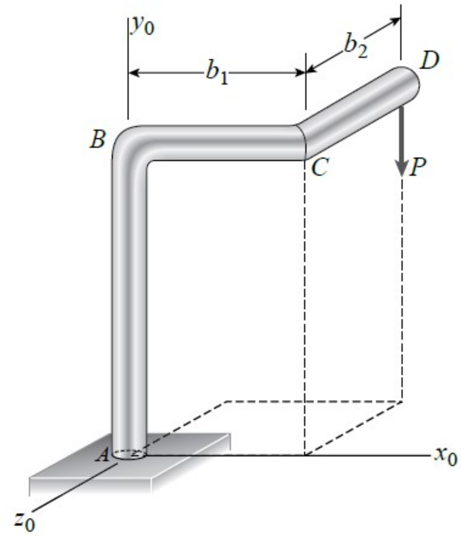
$$(\tau_{\max})_{\text{torsion}} = \frac{Tc}{J} = \frac{20 \times 10^3 \times 5}{982} = 102 \text{ MPa}$$

$$(\tau_{\max})_{\text{direct}} = \frac{VQ}{It} = \frac{4V}{3A} = \frac{4 \times 250}{3 \times 78.5} = 4 \text{ MPa}$$

$$\tau_E = 102 + 4 = 106 \text{ MPa}$$

EX2: A bracket ABCD shown in Fig. having a hollow circular cross Section. The arms BC and CD have lengths b1 =3.6 ft and b2 =2.2 ft, respectively. The outer and inner diameters of the bracket are d2 =7.5 in. and d1 =6.8 in. A vertical load P =1400 lb acts at point D. Determine the maximum tensile and compressive stresses in the vertical arm.

Sol:



$b_1 = 3.6$ ft $\quad b_2 = 2.2$ ft $\quad P = 1400$ lb

$d_2 = 7.5$ in. $\quad d_1 = 6.8$ in.

$$A = \frac{\pi}{4}\left(d_2^2 - d_1^2\right) \qquad A = 7.862 \text{ in.}^2$$

$$I = \frac{\pi}{64}\left(d_2^4 - d_1^4\right) \qquad I = 50.36 \text{ in.}^4$$

Vertical arm *AB*

$$M = P\sqrt{b_1^2 + b_2^2} \qquad M = 7.088 \times 10^4 \text{ lb} \cdot \text{in.}$$

Maximum tensile stress

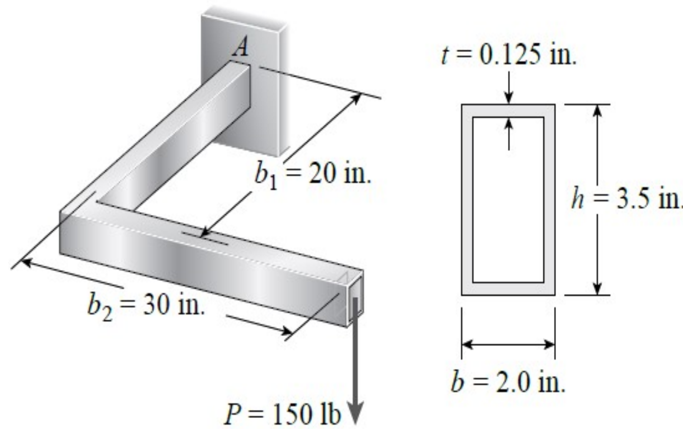
$$\sigma_t = -\frac{P}{A} + \frac{M\left(\frac{d_2}{2}\right)}{I} \qquad \sigma_t = 5100 \text{ psi} \quad \leftarrow$$

Maximum compressive stress

$$\sigma_c = -\frac{P}{A} - \frac{M\left(\frac{d_2}{2}\right)}{I}$$

$$\sigma_c = -5456 \text{ psi} \quad \leftarrow$$

Ex3: An L-shaped bracket lying in a horizontal plane supports a load P =150 lb (see figure). The bracket has a hollow rectangular cross section with thickness t=0.125 in. and outer dimensions b =2.0 in. and h =3.5 in. The centerline lengths of the arms are $b1$ =20 in. and $b2$ =30 in. Considering only the load P, calculate the maximum bending stress and maximum shear stress at point A, which is located on the top of the bracket at the support.

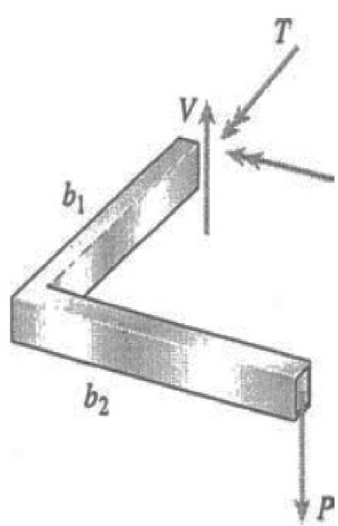


Sol:

$P = 150$ lb $\quad b_1 = 20$ in. $\quad b_2 = 30$ in.

$t = 0.125$ in. $\quad h = 3.5$ in. $\quad b = 2.0$ in.

FREE-BODY DIAGRAM OF BRACKET



STRESS RESULTANTS AT THE SUPPORT

Torque: $\quad T = Pb_2 = (150 \text{ lb})(30 \text{ in.}) = 4500 \text{ lb-in.}$

Moment: $\quad M = Pb_1 = (150 \text{ lb})(20 \text{ in.}) = 3000 \text{ lb-in.}$

Shear force: $\quad V = P = 150$ lb

PROPERTIES OF THE CROSS SECTION

For torsion:

$$A_m = (b - t)(h - t) = (1.875 \text{ in.})(3.375 \text{ in.}) = 6.3281 \text{ in.}^2$$

For bending: $c = \dfrac{h}{2} = 1.75$ in.

$$I = \frac{1}{12}(bh^3) - \frac{1}{12}(b - 2t)(h - 2t)^3$$

$$= \frac{1}{12}(2.0 \text{ in.})(3.5 \text{ in.})^3 - \frac{1}{12}(1.75 \text{ in.})(3.25 \text{ in.})^3$$

$$= 2.1396 \text{ in.}^4$$

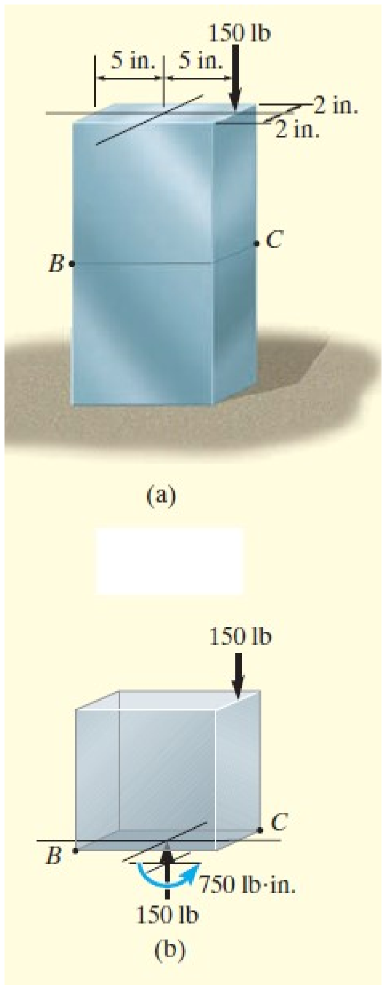
STRESSES AT POINT A ON THE TOP OF THE BRACKET

$$\tau = \frac{T}{2tA_m} = \frac{4500 \text{ lb-in.}}{2(0.125 \text{ in.})(6.3281 \text{ in.}^2)} = 2844 \text{ psi}$$

$$\sigma = \frac{Mc}{I} = \frac{(3000 \text{ lb-in.})(1.75 \text{ in.})}{2.1396 \text{ in.}^4} = 2454 \text{ psi}$$

(The shear force V produces no stresses at point A.)

Ex4: A force of 150 lb is applied to the edge of the member shown in Fig. Neglect the weight of the member and determine the state of stress at points *B* and *C*.



Sol:

Internal Loadings. The member is sectioned through B and C. For equilibrium at the section there must be an axial force of 150 lb acting through the centroid and a bending moment of about the centroid or principal axis, Fig b.

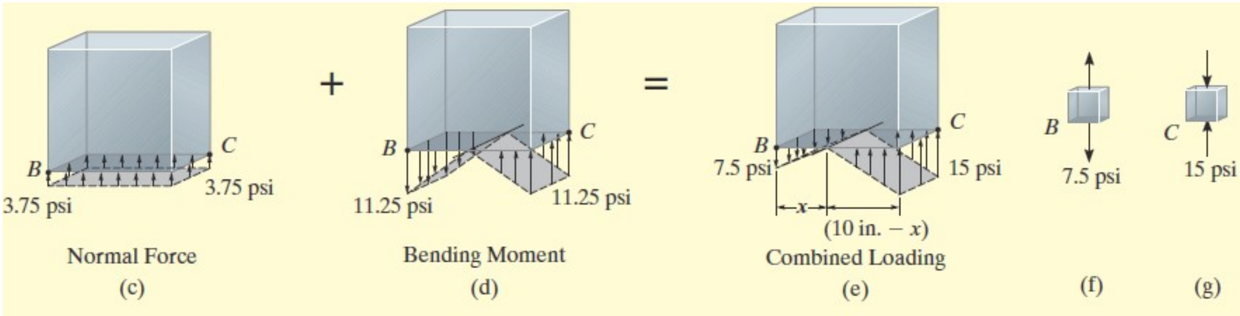
Stress Components:

Normal Force: The uniform normal-stress distribution due to the normal force is shown in Fig c. here:

$$\sigma = \frac{P}{A} = \frac{150 \text{ lb}}{(10 \text{ in.})(4 \text{ in.})} = 3.75 \text{ psi}$$

Bending Moment: The normal-stress distribution due to the bending moment is shown in Fig d. The maximum stress is:

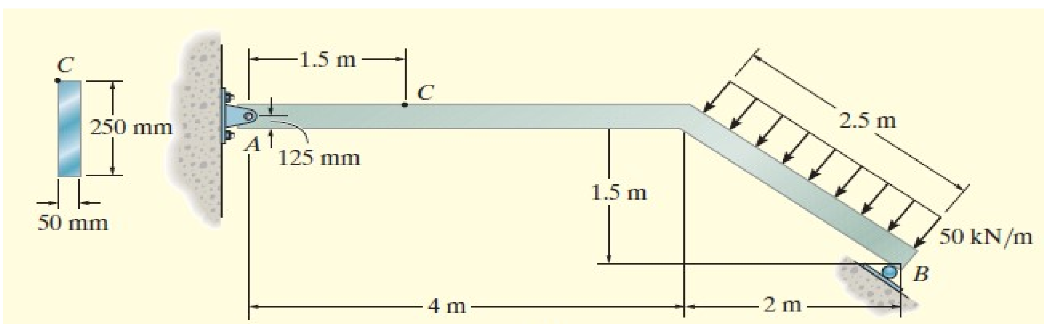
$$\sigma_{\max} = \frac{Mc}{I} = \frac{750 \text{ lb} \cdot \text{in.}(5 \text{ in.})}{\frac{1}{12}(4 \text{ in.})(10 \text{ in.})^3} = 11.25 \text{ psi}$$



$\sigma_B = 7.5$ psi (tension)

$\sigma_C = 15$ psi (compression)

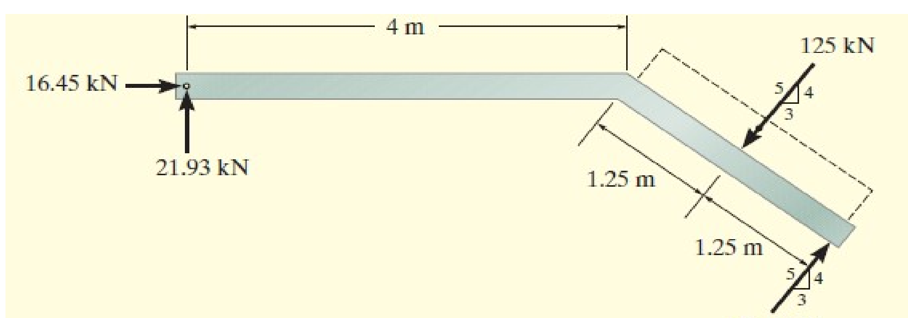
Ex5: The member shown in Fig. has a rectangular cross section. Determine the state of stress that the loading produces at point *C*.



(a)

Sol:

Find the support reactions as shown in fig. b



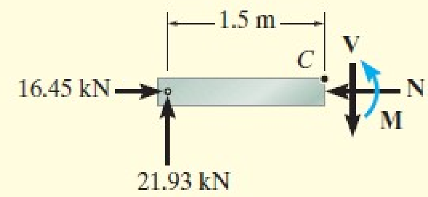
(b)

Cut the beam at point C and find the internal forces as shown in Fig. c.

N = 16.45 kN

V = 21.93 kN

M = 32.89 kN.m

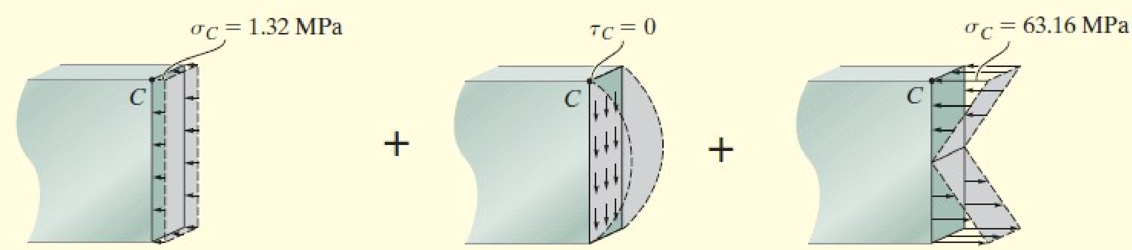


(c)

$$\sigma_C = \frac{P}{A} = \frac{16.45(10^3)\text{ N}}{(0.050\text{ m})(0.250\text{ m})} = 1.32\text{ MPa}$$

$\tau_C = 0$ since point C is located at the top of the member.

$$\sigma_C = \frac{Mc}{I} = \frac{(32.89(10^3)\text{ N}\cdot\text{m})(0.125\text{ m})}{\left[\frac{1}{12}(0.050\text{ m})(0.250\text{ m})^3\right]} = 63.16\text{ MPa}$$



Normal Force (d) Shear Force (e) Bending Moment (f)

Superposition: The shear stress is zero. Adding the normal stresses determined above gives a compressive stress at C having a value of

$$\sigma_C = 1.32\text{ MPa} + 63.16\text{ MPa} = 64.5\text{ MPa}$$