

**5/28** The design characteristics of a gear-reduction unit are under review. Gear B is rotating clockwise with a speed of 300 rev/min when a torque is applied to gear A at time  $t = 2$  s to give gear A a counterclockwise acceleration  $\alpha$  which varies with time for a duration of 4 seconds as shown. Determine the speed  $N_B$  of gear B when  $t = 6$  s.

Solution

$N_B = 300 \text{ r.p.m.}$  at  $t = 2 \text{ Sec}$   
 $N_B ?$  at  $t = 6 \text{ sec}$

$$\omega_B = \omega_A = \frac{N_B}{N_A} = \frac{R_A}{R_B} \Rightarrow N_B = 0.5 N_A \Rightarrow N_A = 2 N_B = 2(300) = 600 \text{ r.p.m.}$$

$$\alpha_A = \frac{d\omega}{dt} \Rightarrow \int d\omega = \alpha dt$$

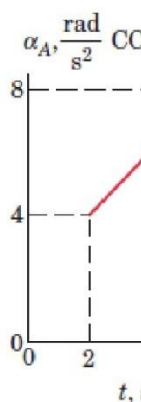
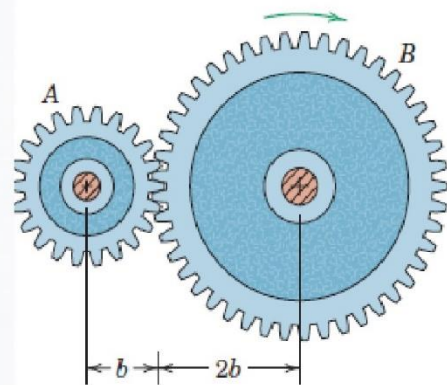
$$\therefore \int_{\omega_0}^{\omega_A} d\omega = \int_2^6 \alpha_A dt = \text{Area under the curve}$$

$$\therefore \int_{600 \times \frac{2\pi}{60}}^{\omega_A} d\omega = \left[ 4(4) + \frac{1}{2} * 4(4) \right]$$

$$\therefore \omega_A = 86.83 \text{ rad/sec}$$

$$\therefore \omega_B = 0.5 \omega_A = 0.5(86.83) = 43.41 \text{ rad/sec}$$

$$\therefore N_B = 43.41 * \frac{60}{2\pi} \approx 415 \text{ rev/min}$$



**5/27** A clockwise variable torque is applied to a flywheel at time  $t = 0$  causing its clockwise angular acceleration to decrease linearly with angular displacement  $\theta$  during 20 revolutions of the wheel as shown. If the clockwise speed of the flywheel was 300 rev/min at  $t = 0$ , determine its speed  $N$  after turning the 20 revolutions. (Suggestion: Use units of revolutions instead of radians.)

5/27 )  $t=0 \Rightarrow N_0 = 300 \text{ r.p.m.} \text{ } \omega \text{ at } \theta = 20 \text{ rev}$

solve

$$\omega = \frac{d\theta}{dt} \text{ } \alpha = \frac{d\omega}{dt}$$

$$\therefore \omega d\omega = \alpha d\theta$$

$$\therefore \int_{\omega_0}^{\omega} \omega d\omega = \int_{\theta_0=0}^{\theta=20 \text{ rev}} \alpha d\theta = \text{Area under the curve}$$

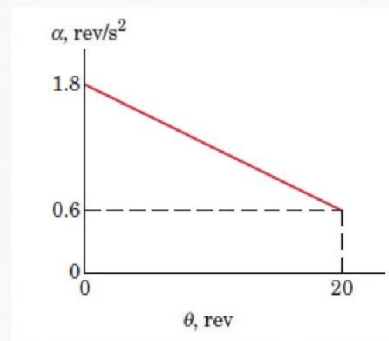
$$\therefore \frac{1}{2} \omega^2 \Big|_{\omega_0}^{\omega} = \left[ (0.6)(2\pi) \times (20)(2\pi) + \frac{1}{2} (1.2)(2\pi) \times (20)(2\pi) \right]$$

$$\therefore \omega^2 - \left( 300 \times \frac{2\pi}{60} \right)^2 = 2 (1894.964)$$

$$\therefore \omega = 53.6833 \text{ rad}$$

$$\therefore N = 53.6833 \times \frac{60}{2\pi}$$

$$= 512.14 \text{ rev/min}$$



**5/11** The angular acceleration of a body which is rotating about a fixed axis is given by  $\alpha = -k\omega^2$ , where the constant  $k = 0.1$  (no units). Determine the angular displacement and time elapsed when the angular velocity has been reduced to one-third its initial value  $\omega_0 = 12$  rad/s.

$$\alpha = -k\omega^2 \quad k = 0.1 = \text{const.} \quad \omega_0 = 12 \text{ rad/s}$$

Solution  $\omega = \frac{1}{3}\omega_0$  at  $\theta$  after  $t$

$$\alpha = \frac{d\omega}{dt} = -k\omega^2 \Rightarrow d\omega = -k\omega^2 dt$$

$$\therefore \int_{\omega_0}^{\omega} \frac{d\omega}{\omega^2} = -k \int_0^t dt \Rightarrow \frac{\omega^{-2+1}}{-2+1} \Big|_{\omega_0}^{\omega} = -kt$$

$$\therefore -\frac{1}{\omega} \Big|_{\omega_0}^{\omega} = -kt \Rightarrow -\frac{1}{\omega} + \frac{1}{\omega_0} = -kt \quad \text{--- (1)}$$

$$\therefore -\frac{1}{\frac{12}{3}} + \frac{1}{12} = -0.1t \Rightarrow t = 1.667 \text{ Sec}$$

$$\text{From eq (1)} \Rightarrow -\frac{1}{\omega} = -\left(kt + \frac{1}{\omega_0}\right) \Rightarrow$$

$$\omega = \frac{1}{kt + \frac{1}{\omega_0}} \Rightarrow \frac{d\theta}{dt} = \frac{1}{kt + \frac{1}{\omega_0}}$$

$$\therefore \int_{\theta_0}^{\theta} d\theta = \int_0^t \frac{dt}{kt + \frac{1}{\omega_0}}$$

$$\therefore \theta - \theta_0 = \frac{1}{k} \ln \left( kt + \frac{1}{\omega_0} \right) \Big|_0^t$$

$$\theta - \theta_0 = \frac{1}{k} \left[ \ln \left( kt + \frac{1}{\omega_0} \right) - \ln \left( \frac{1}{\omega_0} \right) \right]$$

**5/12** The angular position of a radial line in a rotating disk is given by the clockwise angle  $\theta = 2t^3 - 3t^2 + 4$ , where  $\theta$  is in radians and  $t$  is in seconds. Calculate the angular displacement  $\Delta\theta$  of the disk during the interval in which its angular acceleration increases from  $42 \text{ rad/s}^2$  to  $66 \text{ rad/s}^2$ .

$$\theta = 2t^3 - 3t^2 + 4 \quad \Delta\theta ? \quad \alpha_1 = 42 \text{ rad/s}^2 \text{ to}$$

$$\alpha_2 = 66 \text{ rad/s}^2$$

Solution

$$\theta = 2t^3 - 3t^2 + 4$$

$$\omega = \frac{d\theta}{dt} = 6t^2 - 6t$$

$$\alpha = \frac{d\omega}{dt} = 12t - 6$$

$$\therefore \text{at } \alpha = 42 \Rightarrow 42 = 12t - 6 \Rightarrow t_1 = 4 \text{ Sec}$$

$$\text{at } \alpha = 66 \Rightarrow 66 = 12t - 6 \Rightarrow t_2 = 6 \text{ Sec}$$

$$\therefore \theta_1 = 2(4)^3 - 3(4)^2 + 4 = 84 \text{ rad}$$

$$\therefore \theta_2 = 2(6)^3 - 3(6)^2 + 4 = 328 \text{ rad}$$

$$\therefore \Delta\theta = \theta_2 - \theta_1 = 328 - 84 = 244 \text{ rad}$$

}  
Ans.

**5/20** Point A of the circular disk is at the angular position  $\theta = 0$  at time  $t = 0$ . The disk has angular velocity  $\omega_0 = 0.1 \text{ rad/s}$  at  $t = 0$  and subsequently experiences a constant angular acceleration  $\alpha = 2 \text{ rad/s}^2$ . Determine the velocity and acceleration of point A in terms of fixed  $i$  and  $j$  unit vectors at time  $t = 1 \text{ s}$ .

$$\theta = 0 \text{ at } t = 0 \quad \& \quad \omega_0 = 0.1 \text{ rad/s at } t = 0$$

Solution  $\alpha = \text{const} = 2 \text{ rad/s}^2 \quad \& \quad v_A \& a_A \text{ at } t = 1 \text{ sec}$

$$\text{At } t = 0 \Rightarrow \theta = 0$$

$$\text{At } t = 1 \text{ sec} \Rightarrow \theta = ?$$

$$\therefore \omega = \omega_0 + \alpha t \Rightarrow \omega = 0.1 + 2(1) \Rightarrow \omega = 2.1 \text{ rad/s}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow 2.1^2 = 0.1^2 + 2(2)\theta$$

$$\therefore \theta = 1.1 \text{ rad} \Rightarrow \theta = 1.1 \times \frac{180}{\pi} = 63^\circ$$

$$\therefore \text{at } t = 1 \text{ sec} \Rightarrow \theta_A = 63^\circ$$

$$v_A = \omega r_A = 2.1 \times 0.2 = 0.42 \text{ m/s} \quad \begin{matrix} \nearrow \theta \\ \searrow \end{matrix}$$

$$a_A = \alpha r_A = 2 \times 0.2 = 0.4 \text{ m/s}^2 \quad \begin{matrix} \nearrow \theta \\ \searrow \end{matrix}$$

$$a_A^n = \frac{v_A^2}{r_A} = \omega^2 r_A = \frac{0.42^2}{0.2} = 0.88 \text{ m/s}^2 \quad \begin{matrix} \nearrow \theta \\ \searrow \end{matrix}$$

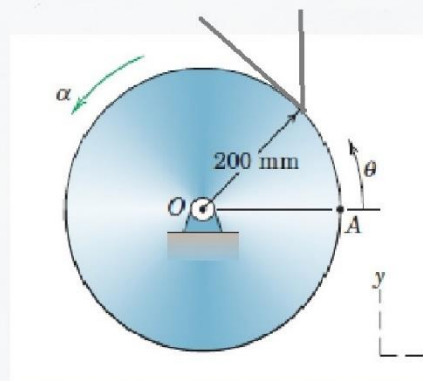
$$\therefore v_A = (v_A)_x i + (v_A)_y j$$

$$= (-0.42 \sin(63)) i + (0.42 \cos(63)) j$$

$$= -0.3772 i + 0.19 j$$

$$a_A = (-a_A^n \cos \theta - a_A^t \sin \theta) i + (-a_A^n \sin \theta + a_A^t \cos \theta) j$$

$$a_A = (-0.88 \cos 63 - 0.4 \sin 63) i + (-0.88 \sin 63 + 0.4 \cos 63) j$$



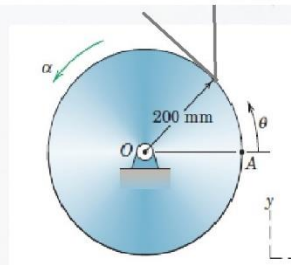
**5/22** Repeat Prob. 5/20, except now the angular acceleration of the disk is given by  $\alpha = 2\omega$ , where  $\omega$  is in radians per second and  $\alpha$  is in radians per second squared. Determine the velocity and acceleration of point A in terms of fixed  $i$  and  $j$  unit vectors at time  $t = 1$  s.

$$\alpha = 2\omega \quad \text{at } t=0 \Rightarrow \theta_0 = 0 \Rightarrow \omega_0 = 0.1 \text{ rad/s}$$

Solution  $v_A$  &  $a_A$  at  $t = 1 \text{ sec}$

$$\alpha = 2\omega \Rightarrow \frac{d\omega}{dt} = 2\omega \Rightarrow \frac{d\omega}{\omega} = 2dt$$

$$\int_{\omega_0}^{\omega} \frac{d\omega}{\omega} = \int_0^t 2dt \Rightarrow \ln \omega - \ln \omega_0 = 2t$$



$$\therefore \ln \omega = 2t + \ln(\omega_0) \Rightarrow \omega = e^{2t + \ln \omega_0}$$

$$\therefore \omega \text{ at } t = 1 \text{ sec} \Rightarrow \omega = e^{2(1) + \ln 0.1} = 0.739 \text{ rad/s}$$

$$\therefore \alpha = 2\omega = 2(0.739) = 1.478 \text{ rad/s}^2$$

$$\omega = \frac{d\theta}{dt} = e^{2t + \ln 0.1} \Rightarrow \int_{\theta_0}^{\theta} d\theta = \int_0^t e^{2t + \ln 0.1} dt$$

$$\therefore \theta - 0 = \frac{1}{2} \left[ e^{2(1) + \ln 0.1} - e^{\ln 0.1} \right]$$

$$\therefore \theta = 0.31945 \text{ rad} \Rightarrow \theta_A = 18.3^\circ$$

$$\therefore v_A = \omega \cdot r_A = 0.739(0.2) = 0.1478 \text{ m/s}$$

$$v_A = (-0.1478 \sin \theta) i + (0.1478 \cos \theta) j$$

$$= -0.0464 i + 0.14 j$$

$$a_A^t = \alpha \cdot r_A = 1.478(0.2) = 0.2956 \text{ m/s}^2$$

$$a_A^n = \frac{v_A^2}{r_A} = \frac{0.1478^2}{0.2} = 0.10922 \text{ m/s}^2$$

**5/10** The bent flat bar rotates about a fixed axis through point  $O$ . At the instant depicted, its angular properties are  $\omega = 5 \text{ rad/s}$  and  $\alpha = 8 \text{ rad/s}^2$  with directions as indicated in the figure. Determine the instantaneous velocity and acceleration of point  $A$ .

solution  $\omega = 5 \text{ rad/sec}$  &  $\alpha = 8 \text{ rad/s}^2$  &  $v_A \perp r_{OA}$  ?

$$OA = r_A = \sqrt{0.3^2 + 0.5^2 - 2(0.3)(0.5)\cos 105} = 0.646 \text{ m}$$

$$\frac{0.646}{\sin 105} = \frac{0.3}{\sin \alpha} \Rightarrow \alpha = 26.65^\circ$$

$$\therefore \theta = \alpha + 30 = 56.65^\circ$$

$$v_A = \omega \cdot r_A = 5 \times 0.646 = 3.23 \text{ m/s}$$

$$v_A = (v_A)_x i + (v_A)_y j$$

$$v_A = (-3.23 \cos 56.65) i + 3.23 \sin 56.65 j$$

$$\therefore v_A = -1.776 i + 2.7 j$$

$$a_A = \alpha \cdot r_A = 8 \times 0.646 = 5.168 \text{ m/s}^2$$

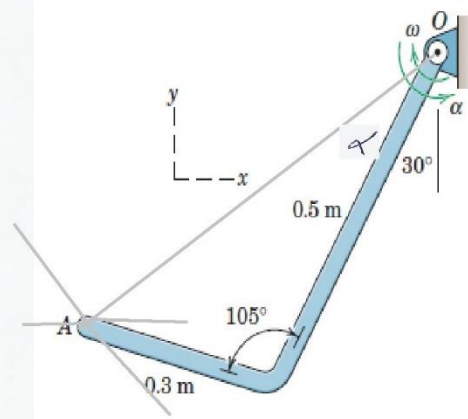
$$a_A = \frac{v_A^2}{r_A} = \omega \cdot r_A = \frac{3.23^2}{0.646} = 16.15 \text{ m/s}^2$$

$$\therefore (a_A)_x = (a_t \cos \theta + a_n \cos(90-\theta)) i$$

$$= (5.168 \cos 56.65 + 16.15 \cos 33.35) i$$

$$(a_A)_y = (-a_t \sin \theta + a_n \sin(90-\theta)) j$$

$$= (-5.168 \sin 56.65 + 16.15 \sin 33.35) j$$



**5/9** A shaft is accelerated from rest at a constant rate to a speed of 3600 rev/min and then is immediately decelerated to rest at a constant rate within a total time of 10 seconds. How many revolutions  $N$  has the shaft turned during this interval?

Solution  $\alpha = \text{Const.}$  &  $\omega_0 = 0$  &  $\omega_1 = 3600 \text{ r.p.m.}$  &  $\omega_2 = 0$   
 $t_{\text{total}} = t_1 + t_2 = 10 \text{ Sec}$

Case (1)

$$\omega_1 = \cancel{\omega_0} + \alpha_1 t_1 \Rightarrow t_1 = \frac{\omega_1}{\alpha_1}$$

$$\omega_1^2 = \cancel{\omega_0^2} + 2\alpha_1 \theta_1 \Rightarrow \alpha_1 = \frac{\omega_1^2}{2\theta_1}$$

$$\therefore t_1 = \frac{\omega_1}{\alpha_1} = \frac{\omega_1}{\frac{\omega_1^2}{2\theta_1}} = \frac{2\theta_1}{\omega_1}$$

Case (2)

$$\omega_2 = \cancel{\omega_1} + \alpha_2 t_2 \Rightarrow t_2 = -\frac{\omega_1}{\alpha_2}$$

$$\omega_2^2 = \omega_1^2 + 2\alpha_2 \theta_2 \Rightarrow \alpha_2 = -\frac{\omega_1^2}{2\theta_2}$$

$$\therefore t_2 = -\frac{\omega_1}{\alpha_2} = \frac{\omega_1}{\frac{\omega_1^2}{2\theta_2}} = \frac{2\theta_2}{\omega_1}$$

$$\therefore t_1 + t_2 = 10 \text{ Sec}$$

$$\therefore \frac{2\theta_1}{\omega_1} + \frac{2\theta_2}{\omega_1} = 10 \Rightarrow 2(\theta_1 + \theta_2) = 10\omega_1$$

$$\therefore \theta_1 + \theta_2 = 5 \left( 3600 \times \frac{2\pi}{60} \right) = 600\pi \text{ rad}$$



**5/8** If the rectangular plate of Prob. 5/7 starts from rest and point  $B$  has an initial acceleration of  $5.5 \text{ m/s}^2$ , determine the distance  $b$  if the plate reaches an angular speed of  $300 \text{ rev/min}$  in 2 seconds with a constant angular acceleration.

$$v_0 = 0 \text{ at } B \quad \& \quad a_0^B = 5.5 \text{ m/s}^2 \quad \& \quad b? \quad \& \quad N = 300 \text{ r.p.m.}$$

at  $t = 2 \text{ sec}$   $\&$   $\alpha = \text{const.}$

Solution

$$\text{at } \omega_0 = 0 \Rightarrow a_0 = 0$$

$$\therefore a_B = a_t = 5.5 \text{ m/s}^2$$

(عُباره كذا)  $\Leftarrow a_n = 0 \Leftarrow \text{سرعة} = 0$

$$\omega = \omega_0 + \alpha t$$

$$\therefore \left(300 \times \frac{2\pi}{60}\right) = 0 + \alpha(2)$$

$$\therefore \alpha = 15.708 \text{ rad/s}^2$$

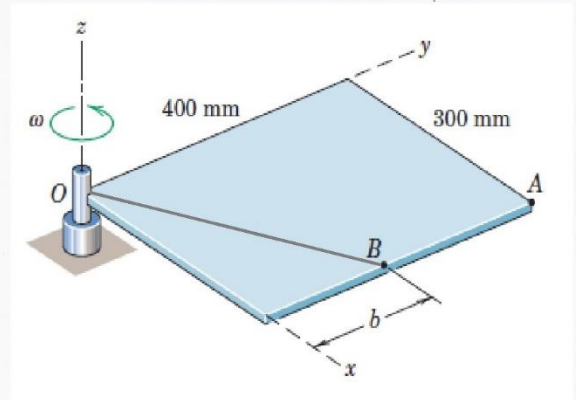
$$a_t^B = \alpha \cdot R_B$$

$$R_B = \sqrt{0.3^2 + b^2}$$

$$\therefore 5.5 = 15.708 \times \sqrt{0.3^2 + b^2}$$

$$\therefore b = 180.55 \text{ mm}$$

Ans.



**5/4** A torque applied to a flywheel causes it to accelerate uniformly from a speed of 200 rev/min to a speed of 800 rev/min in 4 seconds. Determine the number of revolutions  $N$  through which the wheel turns during this interval. (Suggestion: Use revolutions and minutes for units in your calculations.)

$$\alpha = \text{const.} \quad \& \quad N_1 = 200 \text{ r.p.m} \quad \& \quad N_2 = 800 \text{ r.p.m}$$
$$\Delta t = 4 \text{ Sec}$$

Solution

$$\omega = \omega_0 + \alpha t$$

$$\left( \frac{800 \times 2\pi}{60} \right) = \left( \frac{200 \times 2\pi}{60} \right) + \alpha(4)$$

$$\therefore \alpha = 15.7 \text{ rad/s}^2$$

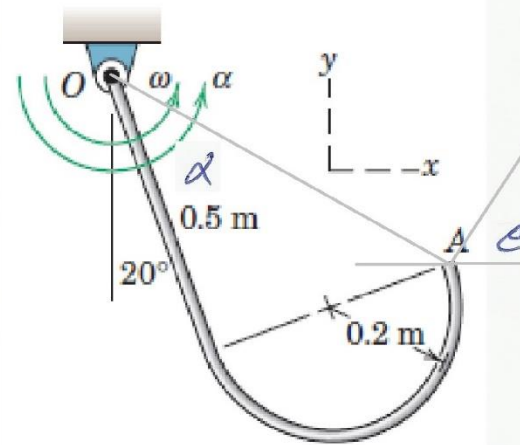
$$\omega^2 = \omega_0^2 + 2\alpha \theta$$

$$\left( \frac{800 \times 2\pi}{60} \right)^2 = \left( \frac{200 \times 2\pi}{60} \right)^2 + 2(15.7)\theta$$

$$\therefore \theta = 209.54 \text{ rad}$$

$$\therefore \text{No. of Revolutions} = \frac{209.54}{2\pi} = 33.35 \text{ rev}$$

**5/3** The body is formed of slender rod and rotates about a fixed axis through point  $O$  with the indicated angular properties. If  $\omega = 4 \text{ rad/s}$  and  $\alpha = 7 \text{ rad/s}^2$ , determine the instantaneous velocity and acceleration of point  $A$ .



$$r_A = \sqrt{0.5^2 + 0.4^2} = 0.64 \text{ m}$$

$$\alpha = \tan^{-1} \frac{0.4}{0.5} = 38.65^\circ$$

$$\theta = 38.65 + 20 = 58.66^\circ$$

$$v_A = \omega \cdot r_A = 4 \times 0.64 = 2.56 \text{ m/s}$$

$$(v_A)_x = 2.56 \cos \theta \quad ; \quad (v_A)_y = 2.56 \sin \theta$$

$$\therefore v_A = 2.56 \cos 58.66^\circ \mathbf{i} + 2.56 \sin 58.66^\circ \mathbf{j}$$

$$= (1.33 \mathbf{i} + 2.18 \mathbf{j}) \text{ m/sec}$$

Ans.

$$a_t = \alpha \cdot r_A = 7 \times 0.64 = 4.48 \text{ m/s}^2$$

$$a_n = \frac{v_A^2}{r_A} = \omega^2 r_A = \frac{2.56^2}{0.64} = 10.24 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = 11.18 \text{ m/s}^2$$

$$(a_A)_x = (a_t \cos \theta - a_n \cos(90 - \theta)) \mathbf{i}$$

$$(a_A)_y = (a_t \sin \theta + a_n \sin(90 - \theta)) \mathbf{j}$$

$$\therefore (a_A)_x = (4.48 \cos 58.66 - 10.24 \cos 31.34) \mathbf{i}$$

$$(a_A)_y = (4.48 \sin 58.66 + 10.24 \sin 31.34) \mathbf{j}$$

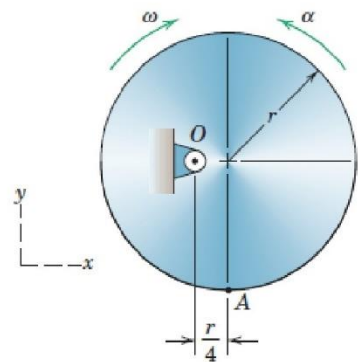
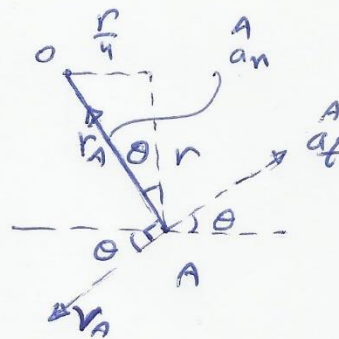
**5/1** The circular disk of radius  $r = 0.16$  m rotates about a fixed axis through point  $O$  with the angular properties  $\omega = 2$  rad/s and  $\alpha = 3$  rad/s<sup>2</sup> with directions as shown in the figure. Determine the instantaneous values of the velocity and acceleration of point  $A$ .

Solution

$$r_A = \sqrt{\left(\frac{r}{4}\right)^2 + r^2}$$

$$= \sqrt{\left(\frac{0.16}{4}\right)^2 + 0.16^2}$$

$$= 0.165 \text{ m}$$



$$v_A = \omega r_A = 2 \times 0.165$$

$$= 0.33 \text{ m/sec}$$



$$\theta = \tan^{-1} \left( \frac{0.16}{4} \right) = 14^\circ$$

$$\therefore (v_A)_x = -v_A \cos \theta \quad \& \quad (v_A)_y = -v_A \sin \theta \mathbf{j}$$

$$\therefore v_A = -0.33 \cos 14^\circ \mathbf{i} - 0.33 \sin 14^\circ \mathbf{j}$$

$$v_A = -0.32 \mathbf{i} - 0.08 \mathbf{j} \quad \leftarrow \text{Ans.}$$

$$a_A^t = \alpha \cdot r_A = 3 \times 0.165 = 0.495 \text{ m/s}^2$$

$$a_A^n = \frac{v_A^2}{r_A} = \omega^2 \cdot r_A = \frac{0.33^2}{0.16} = 0.66 \text{ m/s}^2$$

$$a_A = \sqrt{(a_A^t)^2 + (a_A^n)^2} = \sqrt{0.66^2 + 0.495^2} = 0.825 \text{ m/s}^2$$

$$(a_A)_x = (a_t \cos \theta - a_n \sin \theta) \mathbf{i} \quad \& \quad (a_A)_y = (a_t \sin \theta + a_n \cos \theta) \mathbf{j}$$