العنوان الالكتروني للمكتبة المجانية

Электронная библиотека. Скачать книги бесплатно. Поиск книг

Fundamentals of Linear Vibrations

1. Single Degree-of-Freedom Systems
2. Two Degree-of-Freedom Systems
3. Multi-DOF Systems
4. Continuous

Single Degree-of-Freedom Systems

A spring-mass system

1- Newton law

- General solution for any simple oscillator

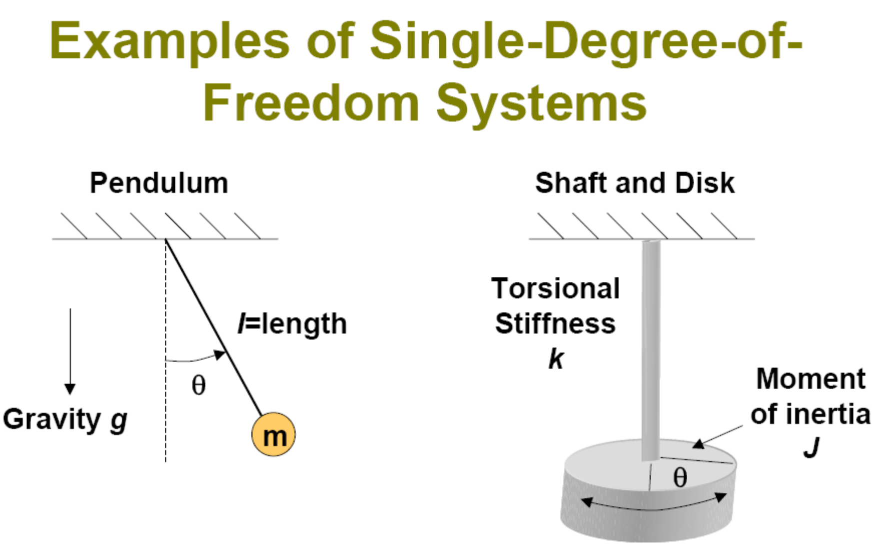
2. Equivalent springs

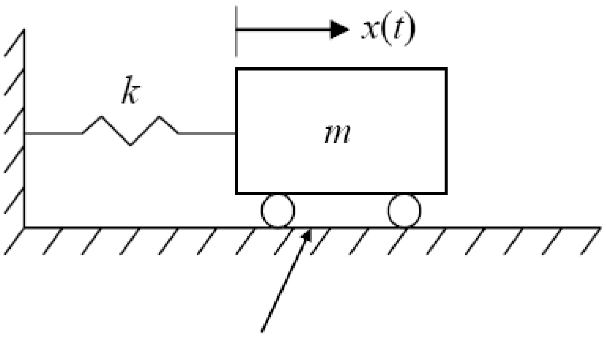
- Spring in series and in parallel

3. Energy Methods

- Strain energy & kinetic energy (Work-energy statement)

Undamped Free Vibrations of Single Degree of Freedom Systems :





Equation of Motion: $m\ddot{x} + kx = 0$

Or, in another form: $\ddot{x} + \omega_n^2 x = 0$

Any simple oscillator

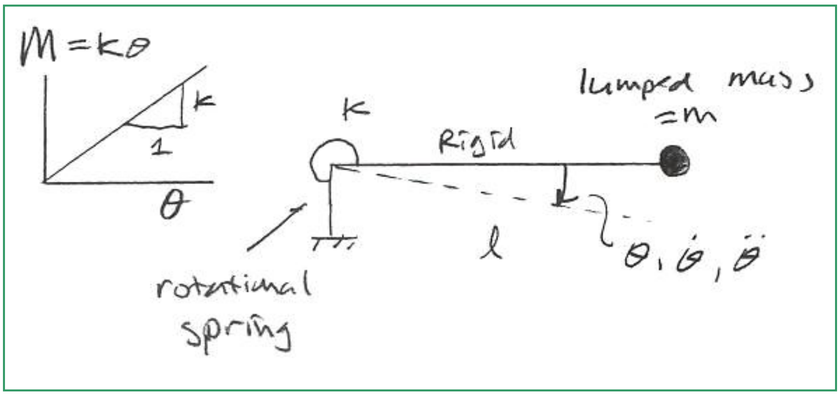
General approach:

1. Select coordinate system
2. Apply small displacement
3. Draw FBD
4. Apply Newton's Laws:

$$\Sigma F = \frac{d}{dt}(m\dot{x})$$

$$\Sigma M = \frac{d}{dt}(I\dot{\theta})$$

Simple oscillator – Example 1



$$I = mass\ moment\ of\ inertia = ml^2$$

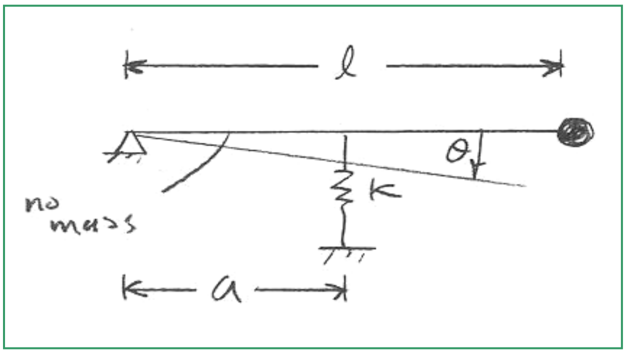
$$\Sigma M = I\ddot{\theta}$$

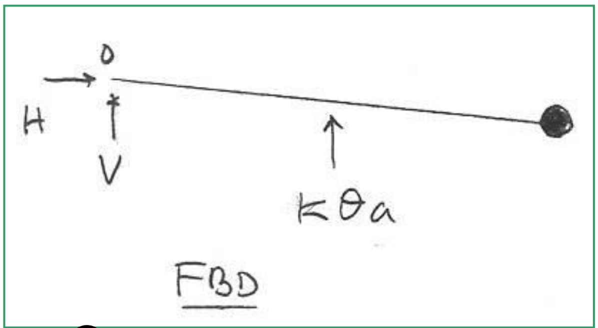
$$-K\theta = I\ddot{\theta}$$

$$\boxed{ml^2\ddot{\theta} + K\theta = 0}$$

$$\omega_n = \sqrt{\frac{K}{ml^2}}$$

Simple oscillator – Example 2





$$\Sigma M_o = I_o\ddot{\theta}$$

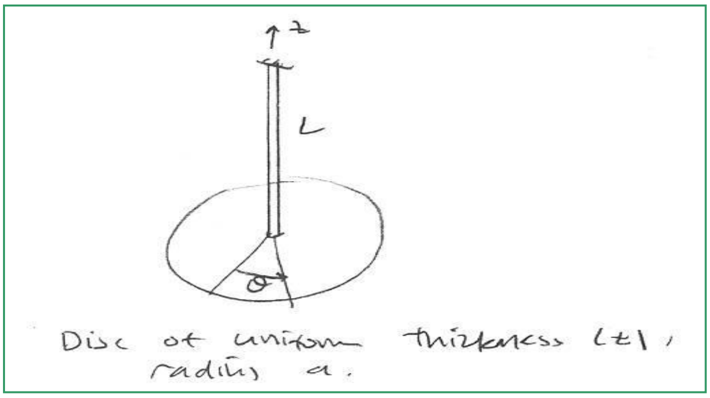
$$-(k\theta a)a = ml^2\ddot{\theta}$$

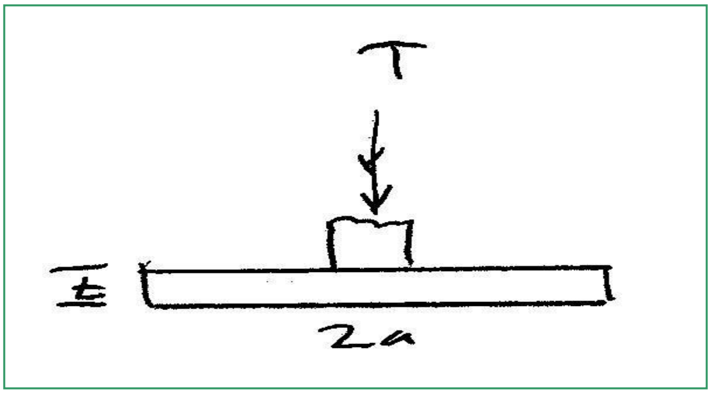
$$\boxed{ml^2\ddot{\theta} + ka^2\theta = 0}$$

$$I = I_{cg} + md^2 = ml^2$$

$$\omega_n = \sqrt{\frac{k}{m}\left(\frac{a}{l}\right)}$$

Simple oscillator – Example 3





$$I = \frac{ma^2}{2}$$

$$\theta = \frac{TL}{JG} \quad \Rightarrow \quad \left(\frac{JG}{L}\right)\theta = T$$

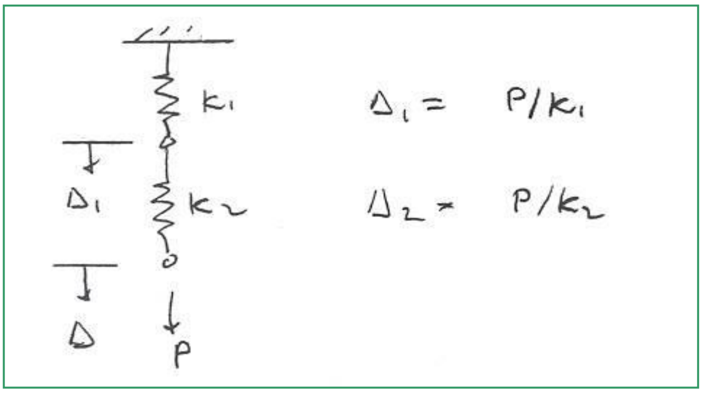
$$\text{Equivalent stiffness} : K = \frac{JG}{L}$$

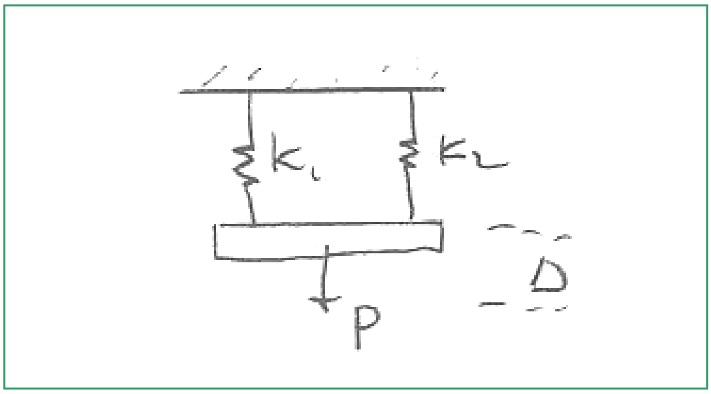
$$\omega_n^2 = \frac{2GJ}{ma^2 L}$$

$$\Sigma M_z = I\ddot{\theta}$$

$$-T = I\ddot{\theta}$$

$$\frac{ma^2}{2}\ddot{\theta} + \frac{GJ}{L}\theta = 0$$





Springs in series:

same force - flexibilities add

$$\Delta = \Delta_1 + \Delta_2 = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)P$$

$$= (f_1 + f_2)P = f_{eq}P$$

$$f_{eq} = f_1 + f_2$$

Springs in parallel:

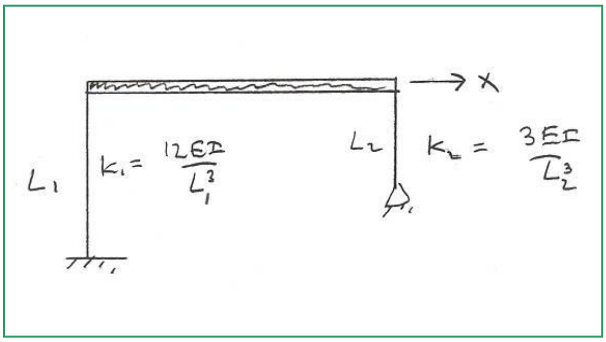
same displacement - stiffnesses

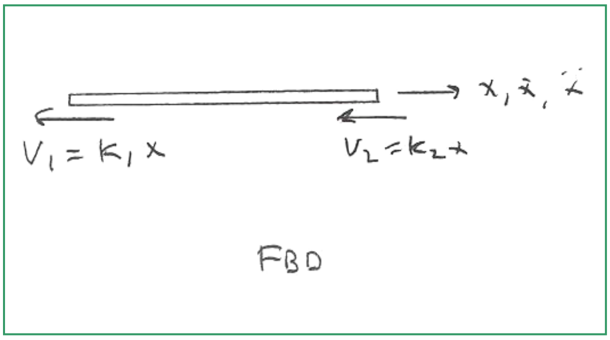
$$P = k_1\Delta + k_2\Delta$$

$$= (k_1 + k_2)\Delta = k_{eq}\Delta$$

$$k_{eq} = k_1 + k_2$$

Equivalent springs – Example 4

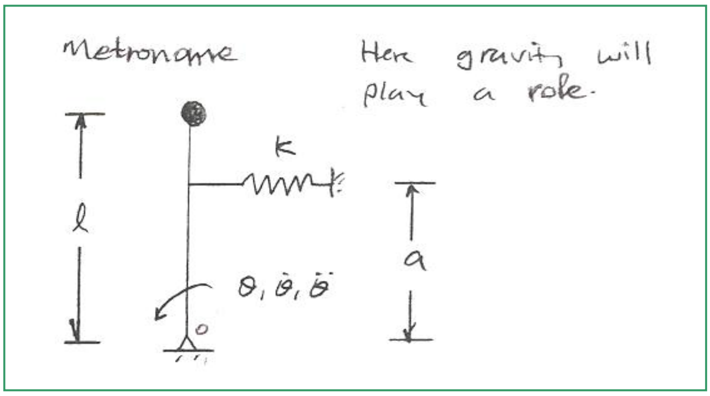


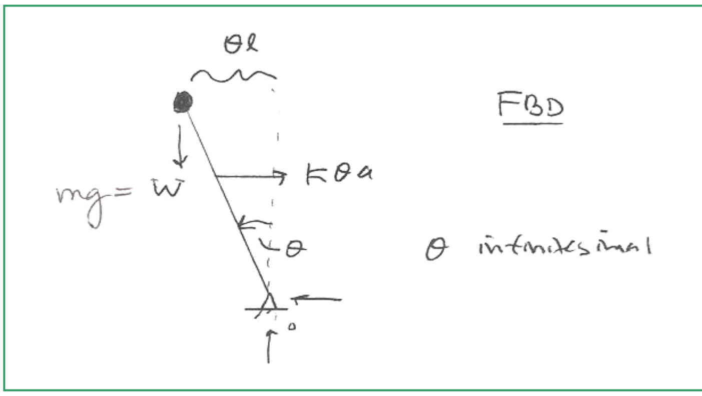


$$m\ddot{x} + K_{eq}x = 0$$

$$m\ddot{x} + \left(\frac{12EI}{L_1^3} + \frac{3EI}{L_2^3}\right)x = 0$$

Example 5 :





$$\Sigma M_o = I_o \ddot{\theta}$$

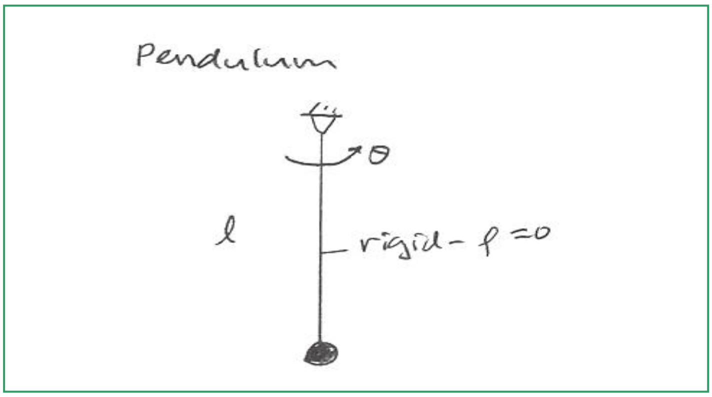
$$-(k\theta a)a + W\theta l = ml^2\ddot{\theta}$$

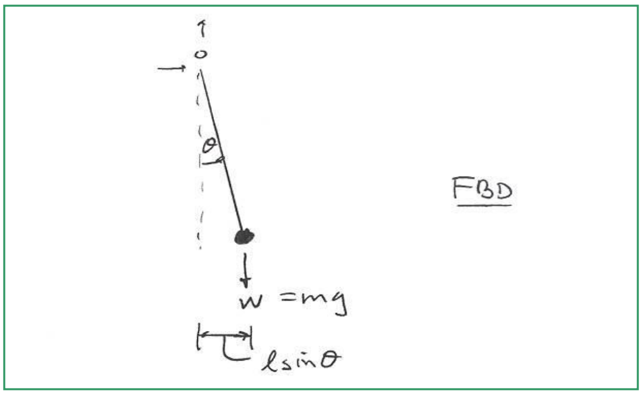
$$ml^2\ddot{\theta} + \theta(ka^2 - Wl) = 0$$

$$\omega_n^2 = \frac{ka^2 - Wl}{ml^2}$$

$$\omega_n = \omega_n(a)$$

Example 6 :





We cannot define ω_n

since we have $sin\theta$ term

If $\theta << 1, sin\theta \approx \theta$:

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

$$\omega_n = \sqrt{\frac{g}{l}}$$

$$\Sigma M_o = I_o \ddot{\theta}$$

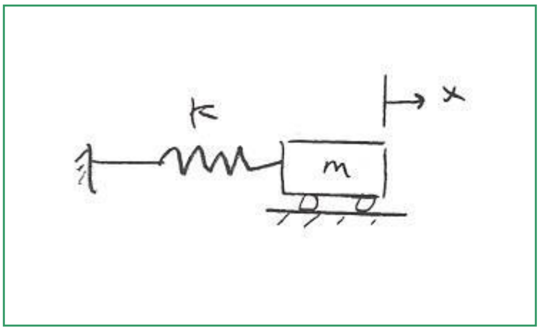
$$-Wl\, sin\theta = ml^2\ddot{\theta}$$

$$ml^2\ddot{\theta} + mgl\, sin\theta = 0$$

$$\ddot{\theta} + \frac{g}{l} sin\theta = 0$$

Energy methods

Work-Energy principles



$$U = \frac{1}{2}kx^2 \text{ ------ } (strain - energy)$$

$$T = \frac{1}{2}m\dot{x}^2 \text{ ------ } (Kinetic - energy)$$

$$E = U + T = \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2$$

Work-energy principles have many uses, but one of the most useful is to derive the equations of motion.

Conservation of energy: E = const.

$$\frac{d}{dt}(E) = 0$$

$$kx\dot{x} + m\dot{x}\ddot{x} = 0$$

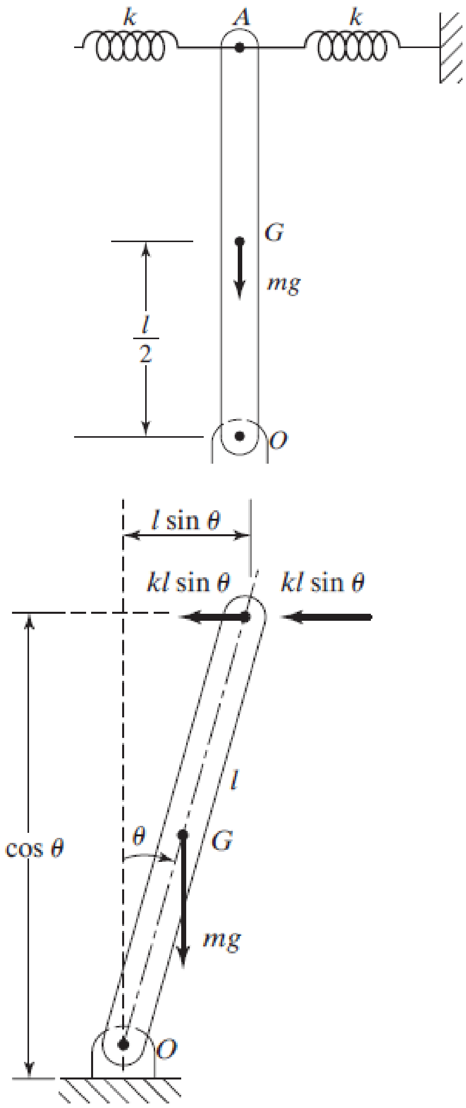
$$\boxed{m\ddot{x} + kx = 0}$$

Consider a uniform rigid bar, of mass m and length l, pivoted at one end and connected symmetrically by two springs at the other end, as shown in Fig. **below**. Assuming that the springs are unstretched when the bar is vertical, derive the equation of motion of the system for small angular displacements (θ) of the bar about the pivot point, and investigate the stability behavior of the system.

BSc

Sol:

$$\frac{ml^2}{3}\ddot{\theta} + (2kl \sin\theta)l \cos\theta - W\frac{l}{2}\sin\theta = 0$$



For small oscillations, Eq. (E.1) reduces to

$$\frac{ml^2}{3}\ddot{\theta} + 2kl^2\theta - \frac{Wl}{2}\theta = 0$$

or

$$\ddot{\theta} + \alpha^2\theta = 0$$

where

$$\alpha^2 = \left(\frac{12kl^2 - 3Wl}{2ml^2}\right)$$

$$\omega_n = \left(\frac{(12kl^2 - 3Wl}{2ml^2}\right)^{1/2}$$