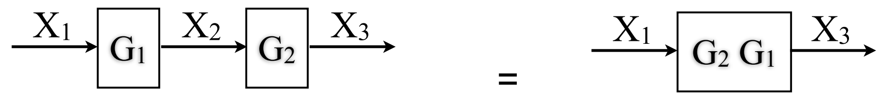
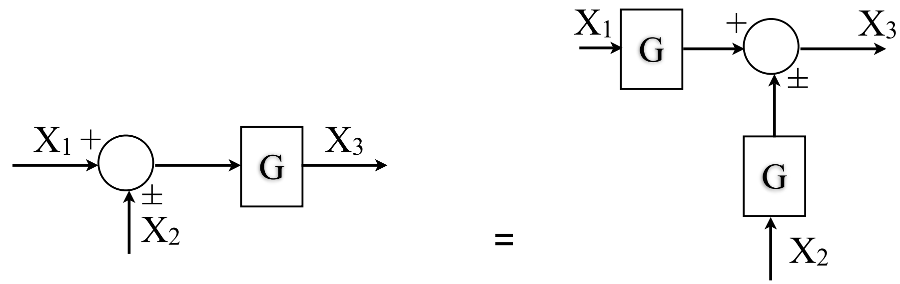
Block Diagram Models, Signal Flow Graphs and Simplification Methods

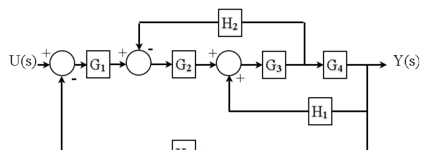
Dynamic Systems and Control
Lavi Shpigelman

Block Diagram Manipulation Rules





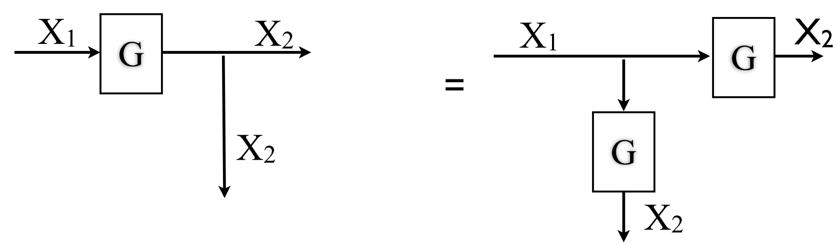
Block Diagram Models

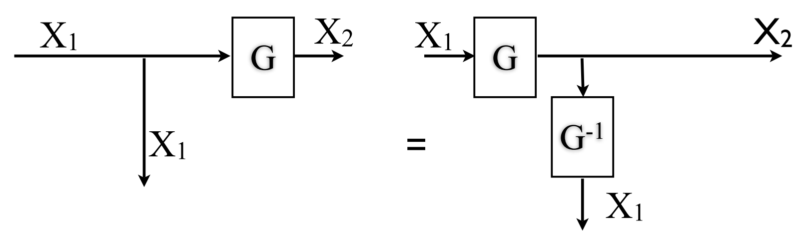


- Visualize input output relations
- Useful in design and realization of (linear) components
- Helps understand flow of information between internal variables.
- Are equivalent to a set of linear algebraic equations

  (of rational functions).
- Mainly relevant where there is a cascade of information flow

Block Diagram Manipulation Rules





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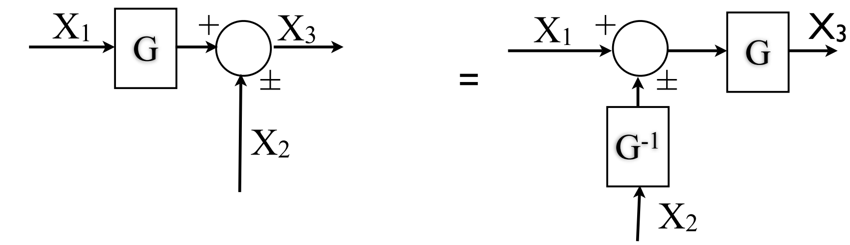
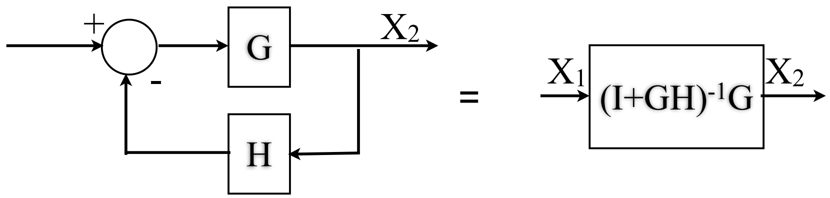
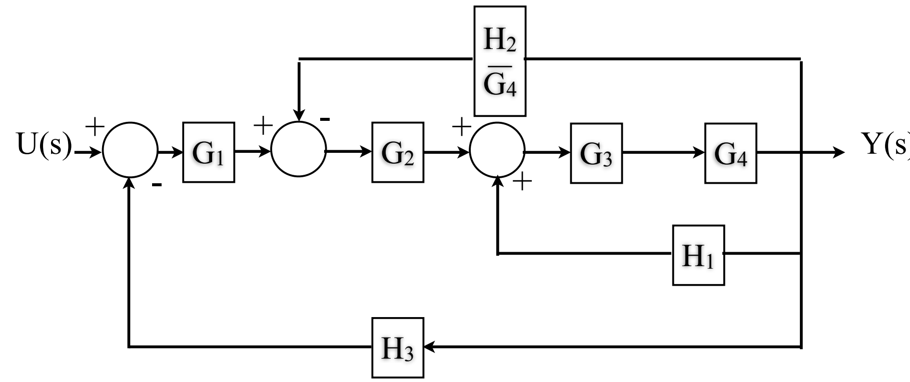




Block Diagram Reduction

(with SISO components)



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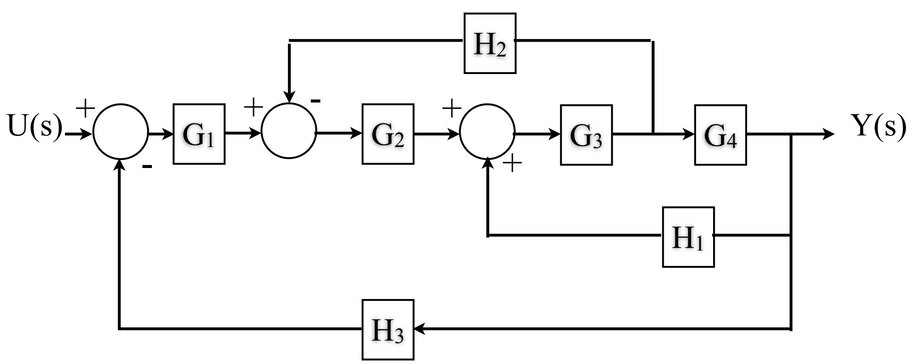

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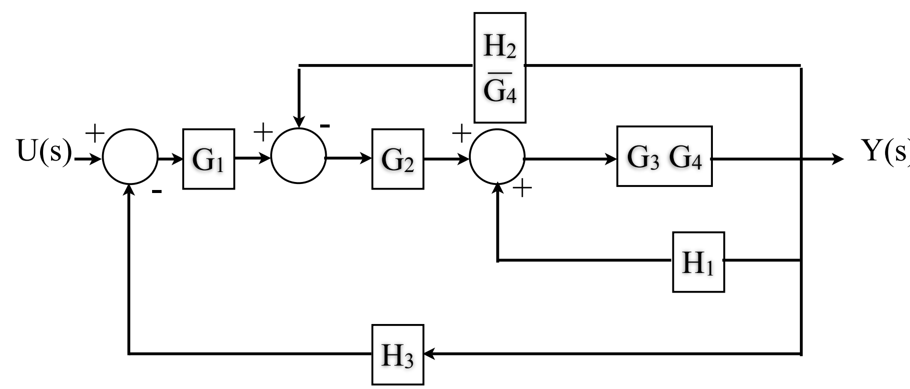

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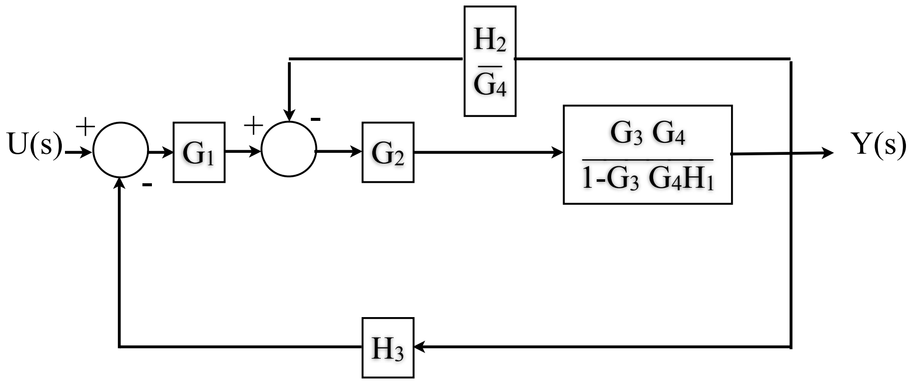

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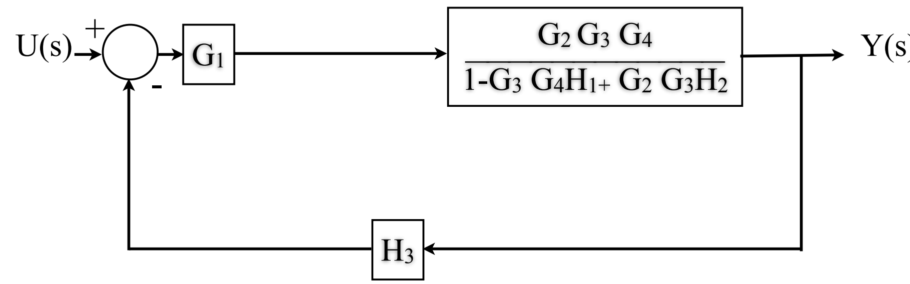

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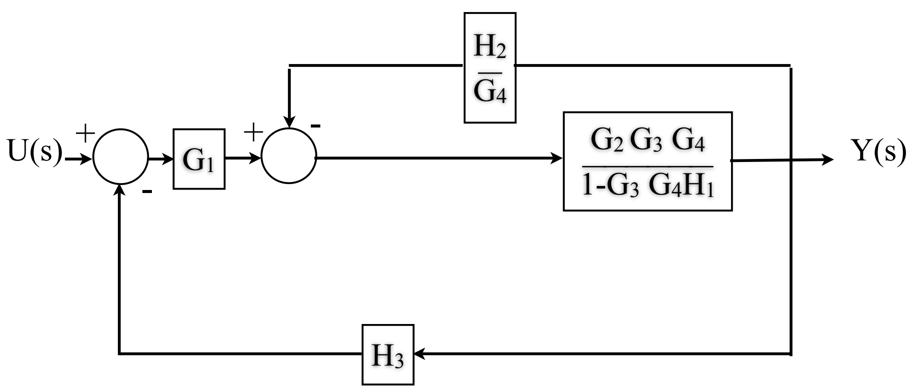

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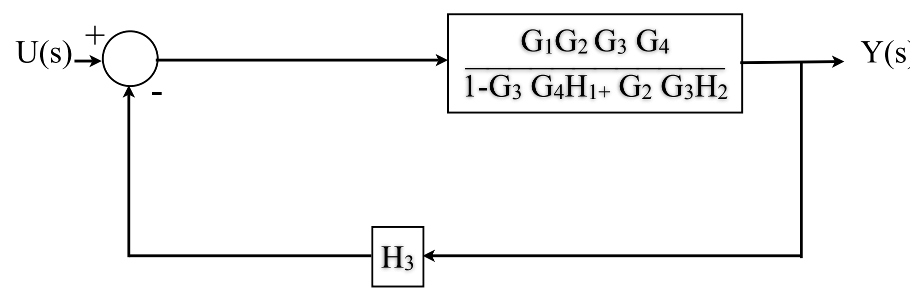

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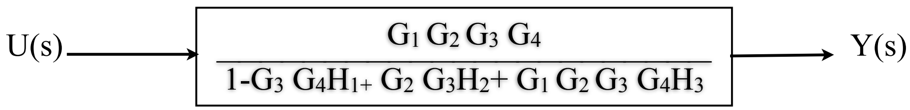
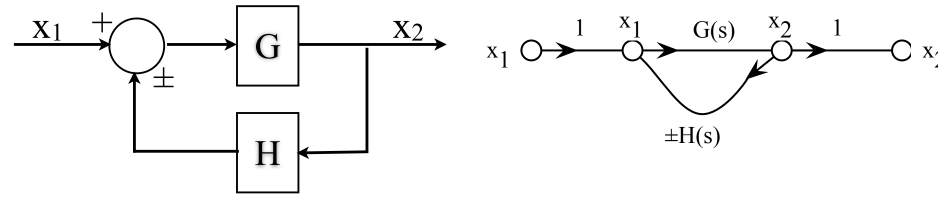


Block Diagram Vs. SFG



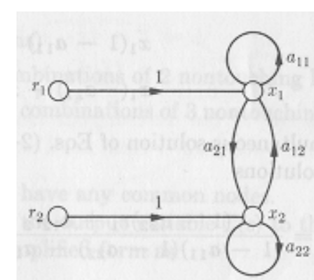
- Blocks ⇒ Edges (aka branches) (representing transfer functions)
- Edges + junctions ⇒ Vertices (aka nodes) (representing variables)

Signal Flow Graphs

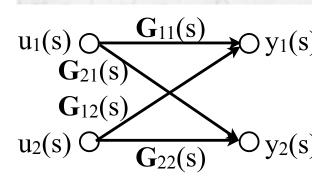
- Alternative to block diagrams
- Do not require iterative reduction to find transfer functions (using Mason's gain rule)
- Can be used to find the transfer function between any two variables (not just the input and output).
- Look familiar to computer scientists (?)

Algebraic Eq representation

- $\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{r}$
  $x_1 = a_{11}x_1 + a_{12}x_2 + r_1$
  $x_2 = a_{21}x_1 + a_{22}x_2 + r_2$



- $\mathbf{y}(s) = \mathbf{G}(s)\mathbf{u}(s)$



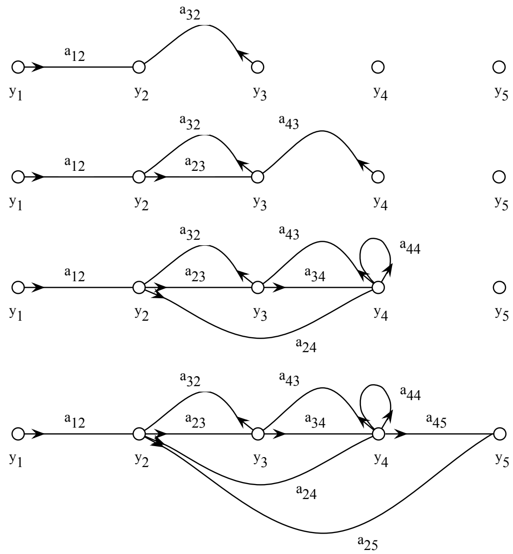
Another SFG Example

$y_2 = a_{12}y_1 + a_{32}y_3$

$y_3 = a_{23}y_2 + a_{43}y_4$

$y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4$

$y_5 = a_{25}y_2 + a_{45}y_4$

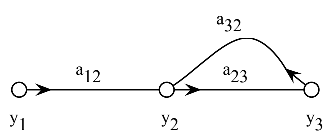


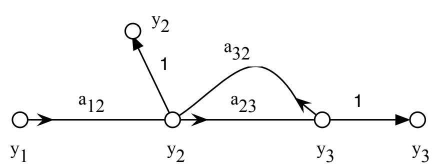
Definitions

- **Input:** (*source*) has only outgoing branches
- **Output:** (*sink*) has only incoming branches
- **Path:** (from node i to node j) has no loops.
- **Forward-path:** path connecting a source to a sink
- **Loop:** A simple graph cycle.
- **Path Gain:** Product of gains on path edges
- **Loop Gain:** Product of gains on loop
- **Non-touching Loops:** Loops that have no vertex in common (and, therefore, no edge.)

Input / Output

- Input (*source*) has only outgoing edges
- Output (*sink*) has only incoming edges
- *any* variable can be made into an output by adding a sink with "1" edge





Mason's Gain Rule (1956)

Given an SFG, a source and a sink, N forward paths between them and K loops, the gain (transfer function) between the source-sink pair is

$$T_{ij} = \frac{\sum P_k \Delta_k}{\Delta}$$

P_k is the gain of path k, Δ is the "graph determinant":

$\Delta = 1 - \sum$(all loop gains)

$+ \sum$(products of non-touching-loop gain pairs)

$- \sum$(products of non-touching-loop gain triplets)

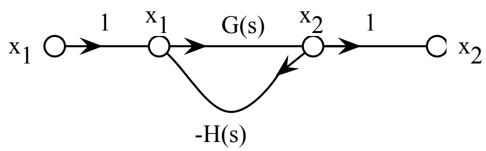
$+ \ldots$

$\Delta_k = \Delta$ of the SFG after removal of the k_{th} forward path

Mason's Rule for Simple Feedback loop

$P_1 = G(s)$

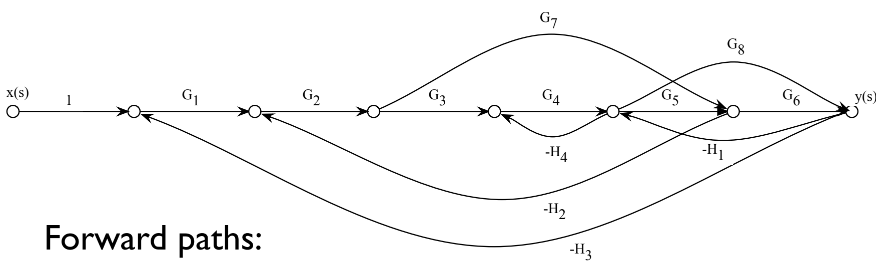
$L_1 = -G(s)H(s)$



$\Delta = 1 - (-G(s)H(s))$

$\Delta_1 = 1$

$$T(s) = \frac{P_1 \Delta_1}{\Delta} = \frac{G(s)}{\Delta} = \frac{G(s)}{1+G(s)H(s)}$$



Forward paths:

$P_1 = G_1G_2G_3G_4G_5G_6$ $P_2 = G_1G_2G_7G_6$ $P_3 = G_1G_2G_3G_4G_8$

Feedback loops:

$L_1 = -G_2G_3G_4G_5H_2$ $L_2 = -G_5G_6H_1$ $L_3 = -G_8H_1$

$L_4 = -G_7H_2G_2$ $L_5 = -G_4H_4$ $L_6 = -G_1G_2G_3G_4G_5G_6H_3$

$L_7 = -G_1G_2G_7G_6H_3$ $L_8 = -G_1G_2G_3G_4G_8H_3$

Loops $\{3,4\},\{4,5\}$ and $\{5,7\}$ don't touch

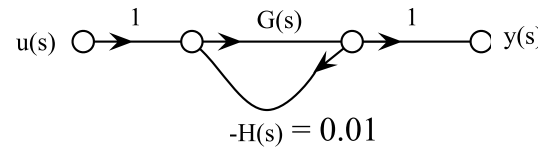
$\Delta = 1-(L_1+L_2+L_3+L_4+L_5+L_6+L_7+L_8)+(L_3L_4+L_4L_5+L_5L_7)$

$\Delta_1 = \Delta_3 = 1$, $\Delta_2 = 1-L_5 = 1 - G_4H_4$

$$T(s) = \frac{y(s)}{x(s)} = \frac{P_1+P_2\Delta_2+P_3}{\Delta}$$

A Feedback Loop Reduces Sensitivity To Plant Variations

$y(s)/u(s)=G/(1+GH)$



G=10000

y(s)/u(s)=10000/(1+10000*0.01)=99.01

G=20000

y(s)/u(s)=20000/(1+20000*0.01)=99.50