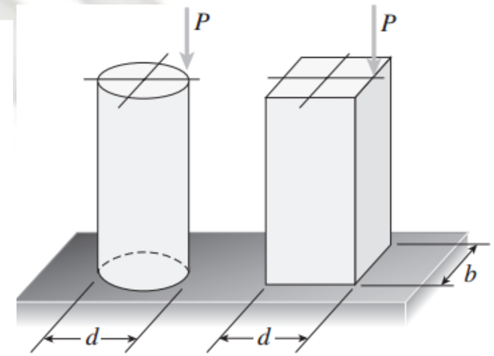
EXAMPLE 2-7

A circular post and a rectangular post are each compressed by loads that produce a resultant force P acting at the edge of the cross section (see figure). The diameter of the circular post and the depth of the rectangular post are the same.

(a) For what width b of the rectangular post will the maximum tensile stresses be the same in both posts?

(b) Under the conditions described in part (a), which post has the larger compressive stress?

Solution:



CIRCULAR POST

$$A = \frac{\pi d^2}{4} \quad S = \frac{\pi d^3}{32} \quad M = \frac{Pd}{2}$$

Tension: $$\sigma_t = -\frac{P}{A} + \frac{M}{S} = -\frac{4P}{\pi d^2} + \frac{16P}{\pi d^2} = \frac{12P}{\pi d^2}$$

Compression: $$\sigma_c = -\frac{P}{A} - \frac{M}{S} = -\frac{4P}{\pi d^2} - \frac{16P}{\pi d^2} = -\frac{20P}{\pi d^2}$$

RECTANGULAR POST

$$A = bd \quad S = \frac{bd^2}{6} \quad M = \frac{Pd}{2}$$

Tension: $$\sigma_t = -\frac{P}{A} + \frac{M}{S} = -\frac{P}{bd} + \frac{3P}{bd} = \frac{2P}{bd}$$

Compression: $$\sigma_c = -\frac{P}{A} - \frac{M}{S} = -\frac{P}{bd} - \frac{3P}{bd} = -\frac{4P}{bd}$$

EQUAL MAXIMUM TENSILE STRESSES

$$\frac{12P}{\pi d^2} = \frac{2P}{bd} \quad \text{or} \quad \frac{6}{\pi d} = \frac{1}{b}$$

Determine the width b of the rectangular post

$$b = \frac{\pi d}{6}$$

Compressive stresses

Circular post: $$\sigma_c = -\frac{20P}{\pi d^2}$$

Rectangular post: $$\sigma_c = -\frac{4P}{bd} = -\frac{4P}{(\pi d/6)d} = -\frac{24P}{\pi d^2}$$

Rectangular post has the larger compressive stress.

EXAMPLE 2-8

A short column of wide-flange shape is subjected to a compressive load that produces a resultant force P 60 kN acting at the midpoint of one flange (see figure).

(a) Determine the maximum tensile and compressive stresses σ t and σc, respectively, in the column.

(b) Locate the neutral axis under this loading condition.

Solution:

$b = 160$ mm $\quad t_w = 8$ mm

$h = 200$ mm $\quad t_f = 12$ mm

$P = 60$ kN $\quad e = \frac{h}{2} - \frac{t_f}{2} = 94$ mm

$A = 2bt_f + (h - 2t_f)\,t_w = 5248\ \text{mm}^2$

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b - t_w)(h - 2t_f)^3$$
$$= 37.611 \times 10^6\ \text{mm}^4$$

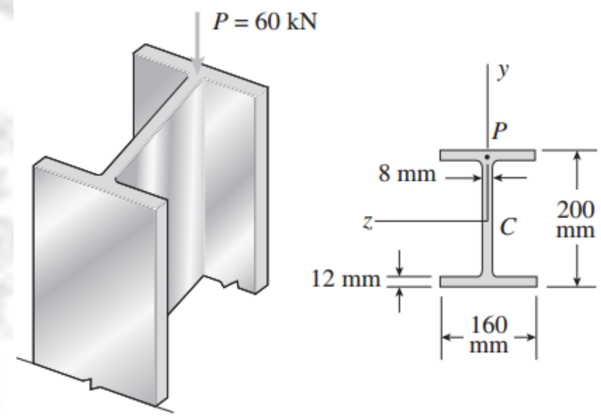
(a) MAXIMUM STRESSES

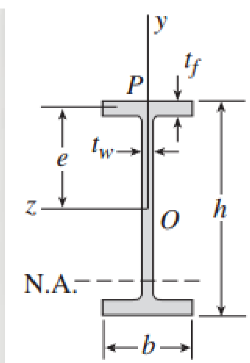
$$\sigma_t = -\frac{P}{A} + \frac{Pe(h/2)}{I}$$
$$= -\frac{60000}{5248} + \frac{(60000)(94)(100)}{37.611 \times 10^6}$$
$$= -11.43\ \text{MPa} + 15.00\ \text{MPa}$$
$$= 3.57\ \text{MPa}$$

$$\sigma_c = -11.43\ \text{MPa} - 15.00\ \text{MPa}$$
$$= -26.4\ \text{MPa}$$

(b) NEUTRAL AXIS (SEE FIGURE)

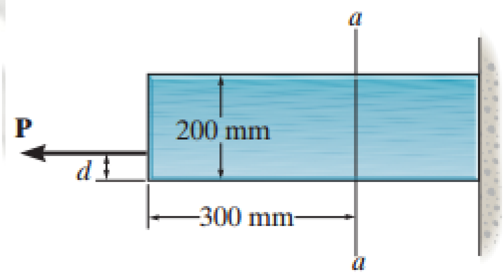
$$y_0 = -\frac{I}{Ae} = -\frac{37.611 \times 10^6\ \text{mm}^4}{(5248\ \text{mm}^2)(94\ \text{mm})}$$
$$= -76.2\ \text{mm}$$





EXAMPLE 2-9

Determine the smallest distance d to the edge of the plate at which the force **P** can be applied so that it produces no compressive stresses in the plate at section a–a. The plate has a thickness of 20 mm and **P** acts along the centerline of this thickness.



Solution:

Consider the equilibrium of the FBD of the left cut segment in Fig. *a*,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N - P = 0 \qquad N = P$

$\circlearrowleft + \Sigma M_C = 0; \qquad M - P(0.1 - d) = 0 \qquad M = P(0.1 - d)$

$A = 0.2\,(0.02) = 0.004\ \text{m}^4 \qquad I = \frac{1}{12}(0.02)(0.2^3) = 13.3333(10^{-6})\ \text{m}^4$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

Since no compressive stress is desired, the normal stress at the top edge fiber must be equal to zero. Thus,

$$0 = \frac{P}{0.004} \pm \frac{P(0.1 - d)(0.1)}{13.3333\,(10^{-6})}$$

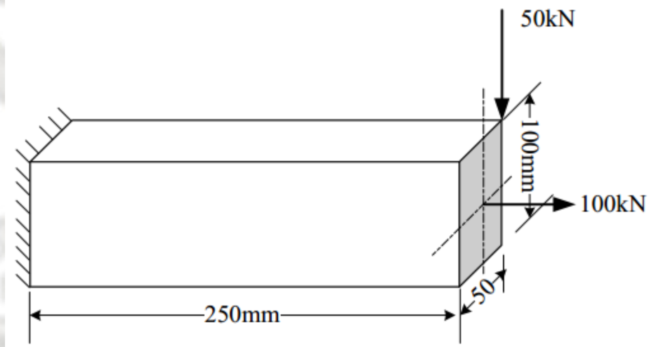
$$0 = 250\,P - 7500\,P\,(0.1 - d)$$

$$d = 0.06667\ \text{m} = 66.7\ \text{mm}$$

EXAMPLE 2-10

Find the maximum normal and shearing stresses due to the applied forces in the fixed end of the beam shown.

Solution:



Torque (T) = 50* 0.05/2 = 1.25kN.m

Shear force (V) = 50kN

Bending moment (M) = 50*0.25 = 12.5kN.m

Axial force (P) = 100kN

- Normal stresses

Axial stress:

$$\sigma_{axial} = \frac{P}{A} = \frac{100000}{50*100} = 20MPa$$

Bending stresses:

$$\sigma_{bending} = \frac{MC}{I} = \frac{12.5*10^6*50}{\frac{50*100^3}{12}} = 150MPa$$

Using the superposition principal:

$$\sigma_{top} = +\frac{P}{A} + \frac{MC}{I} = 20 + 150 = 170MPa$$

$$\sigma_{bot} = +\frac{P}{A} - \frac{MC}{I} = 20 - 150 = -130MPa$$

- Shear stresses

a) Due to the shear force

τ @ A, B, E and F =0;

τ @ C, O and D is τ_{max} .

$$\tau = \frac{VQ}{Ib}$$

$$\tau_{max} = \frac{50*10^3*[50*50*25]}{\frac{50(100)^3}{12}*50}$$

$$\tau_{max} = 15MPa$$

b) Due to the torque

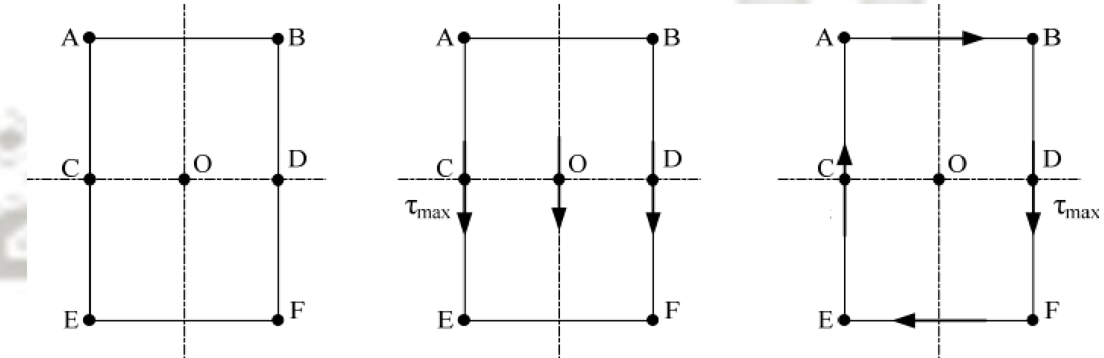
τ @ A, B, E, F and O =0;

τ @ C, and D is τ_{max}

$$\tau_{max} = \frac{T}{\alpha\, a\, b^2}$$

a /b = 100/50 = 2 (α = 0.246)

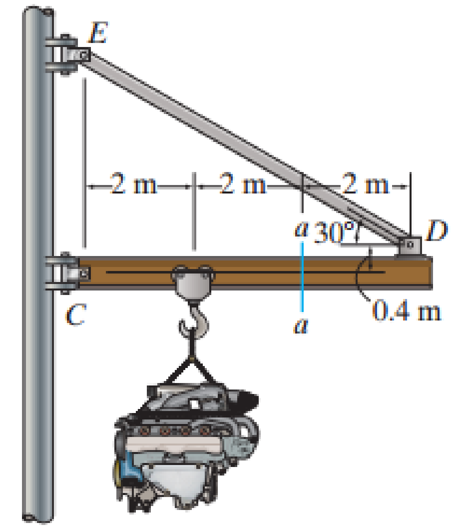
$$\tau_{max} = \frac{1.25*10^6}{0.246*100*(50)^2} = 20.33MPa$$



$$\tau_{max})_{compound} @ D = \tau\ due\ to\ V + \tau\ due\ to\ T = 15 + 20.33 = 35.33MPa$$

EXAMPLE 2-12

The 500-kg engine is suspended from the crane at the position shown. Determine the state of stress at points A and B on the cross section of the boom at section a–a.



Solution:

$\circlearrowleft + \Sigma M_C = 0; \quad F_{DE} \sin 30°(6) + F_{DE} \cos 30°(0.4) - 500(9.81)(2) = 0\ ; \quad F_{DE} = 2931.50 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N - 2931.50 \cos 30° = 0 \qquad N = 2538.75 \text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad 2931.50 \sin 30° - V = 0 \qquad V = 1465.75 \text{ N}$

$\circlearrowleft + \Sigma M_O = 0; \qquad 2931.50 \sin 30°(2) + 2931.50 \cos 30°(0.4) - M = 0\ ; \quad M = 3947.00 \text{ N}\cdot\text{m}$

$A = 0.15(0.3) - 0.13(0.26) = 0.0112 \text{ m}^2$

$$I = \frac{1}{12}(0.15)(0.3^3) - \frac{1}{12}(0.13)(0.26^3) = 0.14709(10^{-3}) \text{ m}^4$$

$Q_A = 0.065(0.13)(0.2) + 0.14(0.02)(0.15) = 0.589(10^{-3}) \text{ m}^3$

Normal Stress:

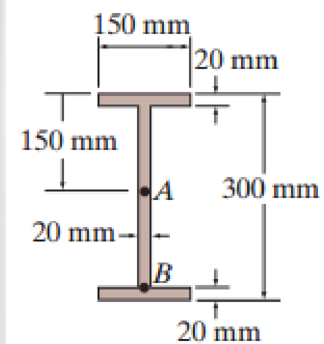
$$\sigma = \frac{N}{A} + \frac{My}{I}$$

For point A. $y = 0$. Then

$$\sigma_A = \frac{-2538.75}{0.0112} + 0 = -0.2267 \text{ MPa} = 0.227 \text{ MPa (C)}$$

Shear Stress:

$$\tau_A = \frac{VQ_A}{It} = \frac{1465.75[0.589(10^{-3})]}{0.14709(10^{-3})(0.02)} = 0.293 \text{ MPa}$$



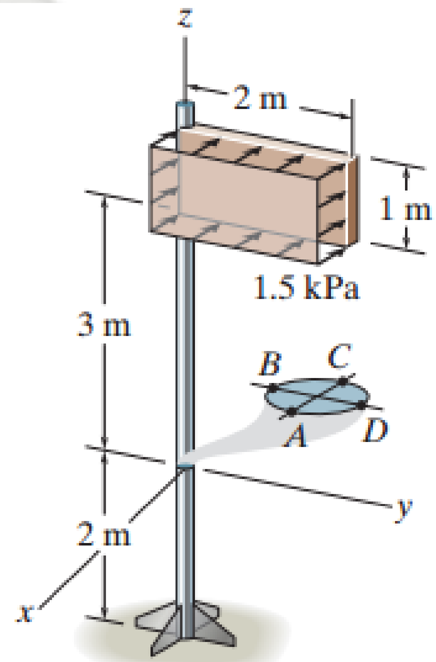
Section $a-a$

Complete the solution for point B.

EXAMPLE 2-11

The sign is subjected to the uniform wind loading. Determine the stress components at points A, B, C and D on the100-mm-diameter supporting post.

Solution:



Point A:

$$\sigma_A = \frac{Mc}{I} = \frac{10.5(10^6)\ (50)}{\frac{\pi}{4}(50)^4} = 107\ \text{MPa (T)}$$

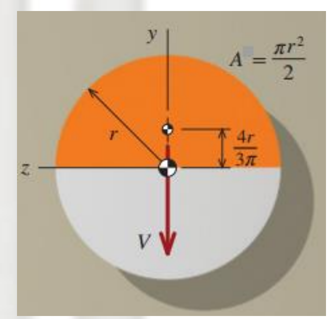
$$\tau_A = \frac{Tc}{J} = \frac{3(10^6)(50)}{\frac{\pi}{2}(50)^4} = 15.3\ \text{MPa}$$

Point B:

$$\sigma_B = 0$$

$$\tau_B = \frac{Tc}{J} - \frac{VQ}{It} = 15.3 - \frac{3000(4(50)/3\pi))(\frac{1}{2})(\pi)(50)^2}{\frac{\pi}{4}(50)^4(100)}$$

$$\tau_B = 14.8\ \text{MPa}$$

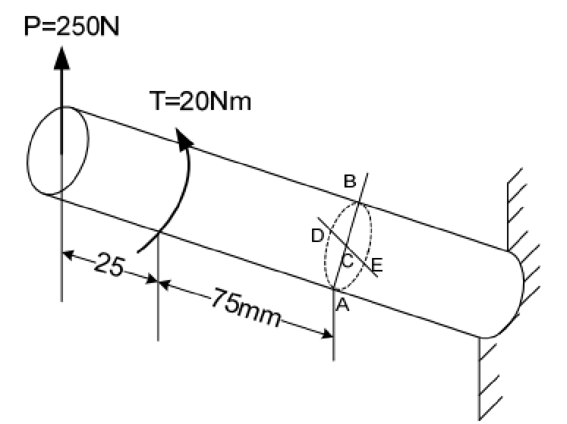


Complete the solution for points C and D

EXAMPLE 2-13

Find the maximum shearing stress due to the applied forces in the plane AB of the 10mm diameter circular shaft.

Solution:



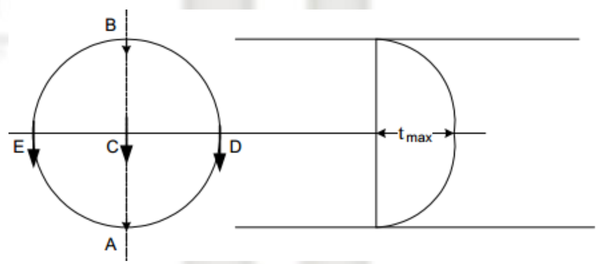
a. Shear stress due to the shear force

τ @ A and B =0;

τ @ C, D and E = τ_{max} .

$$\tau = \frac{V\,Q}{I\,b}$$

$$\tau_{max} = \frac{250*\left[\frac{\pi(5^2)}{2}*\left(\frac{4*5}{3\pi}\right)\right]}{\frac{\pi(5)^4}{4}*10} = 4.24MPa$$



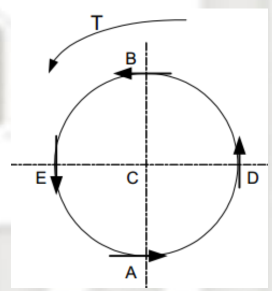
b. Shear stress due to the torque

τ @ C =0;

τ @ A, B, D and E = τ_{max} .

$$\tau_{max} = \frac{T\,R}{J}$$

$$\tau_{max} = \frac{20000*5}{\frac{\pi}{2}(5^4)} = 101.86MPa$$

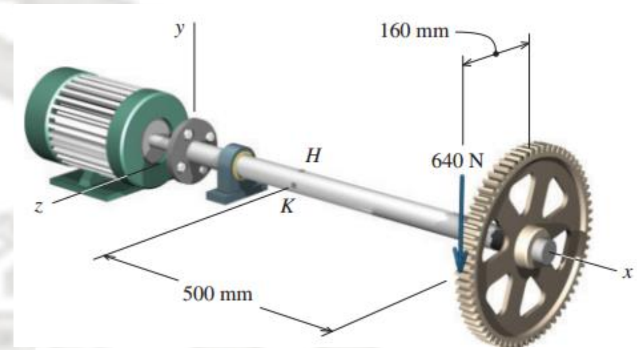


$$\tau_{max}\ occurs\ @\ point\ E = 4.24 + 101.86 = 106.1MPa$$

EXAMPLE 2-14

A 36 mm solid shaft supports a 640 N load as shown. Determine the principal stresses and the maximum shear stress at points *H* and *K*.

Solution:

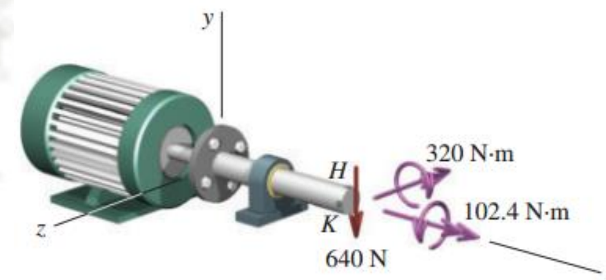


$$M_x = (640\ \text{N})(160\ \text{mm}) = 102{,}400\ \text{N}\cdot\text{mm} = 102.4\ \text{N}\cdot\text{m}.$$

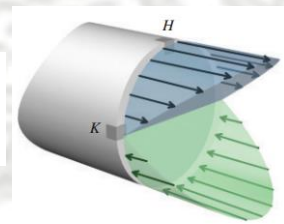
$$M_z = (640\ \text{N})\ (500\ \text{mm}) = 320{,}000\ \text{N}\cdot\text{mm} = 320\ \text{N}\cdot\text{m}.$$

$$J = \frac{\pi}{32}d^4 = \frac{\pi}{32}(36\ \text{mm})^4 = 164{,}896\ \text{mm}^4$$

$$I_z = \frac{\pi}{64}d^4 = \frac{\pi}{64}(36\ \text{mm})^4 = 82{,}448\ \text{mm}^4$$

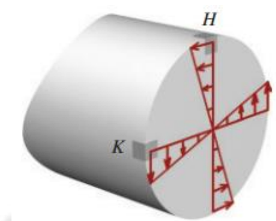


Normal Stresses at *H*

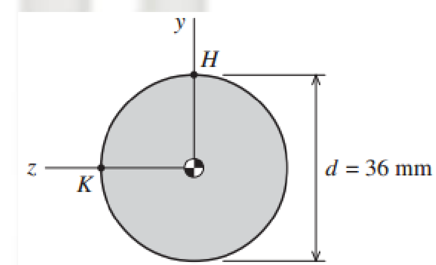


$$\sigma_x = \frac{Mc}{I_z} = \frac{(320{,}000\ \text{N}\cdot\text{mm})(18\ \text{mm})}{82{,}448\ \text{mm}^4} = 69.9\ \text{MPa (T)}$$

Shear Stress at *H*



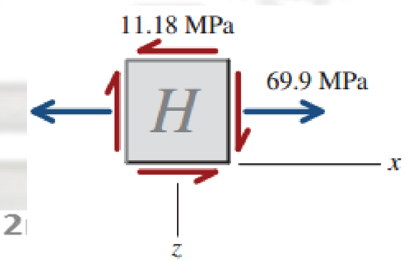
$$\tau = \frac{Tc}{J} = \frac{(102{,}400\ \text{N}\cdot\text{mm})(36\ \text{mm}/2)}{164{,}896\ \text{mm}^4} = 11.18\ \text{MPa}$$



Shaft cross section.

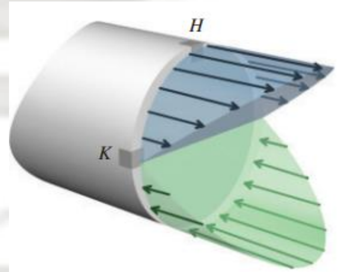
The transverse shear stress associated with the 640 N shear force is zero at *H*.

Combined Stresses at *H*

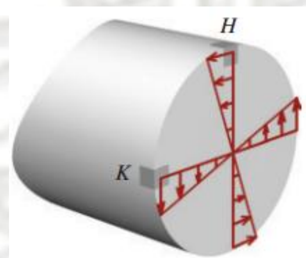


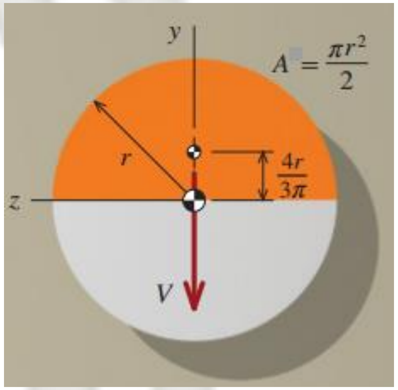
Normal Stresses at K

The bending stress at K is zero.



Shear Stresses at K

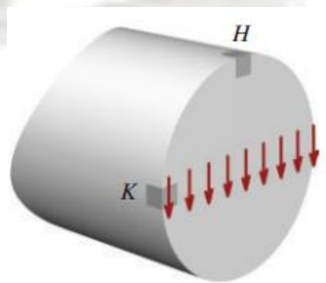




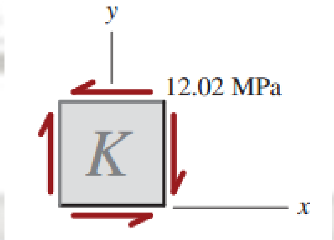
$$Q = \frac{4r}{3\pi}\frac{\pi r^2}{2} = \frac{2}{3}r^3 = \frac{1}{12}d^3$$

$$Q = \frac{d^3}{12} = \frac{(36\text{ mm})^3}{12} = 3{,}888\text{ mm}^3$$

$$\tau = \frac{VQ}{I_z t} = \frac{(640\text{ N})(3{,}888\text{ mm}^3)}{(82{,}448\text{ mm}^4)(36\text{ mm})} = 0.838\text{ MPa}$$



Combined Stresses at K



H.W. Determine the maximum allowable force **P**, if the column is made from material having an allowable normal stress of sallow = 100 MPa.