MOHR'S Circle For Plane Stress

The transformation equations for plane stress can be represented in graphical form by a plot known as **Mohr's circle**. It is so named in honor of the German Professor in civil engineering Otto Christian Mohr (1835-1918), who in 1895 suggested its use in the stress analysis.

In this section, we will show how to apply the equations for plane-stress transformation using a graphical procedure that is often convenient to use and easy to remember.

If we write the earlier mentioned equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

in the form:

$$\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right) = \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

then the parameter θ can be eliminated by squaring each equation and adding them together. The result is:

$$\left[\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

Since σx, σy, Txy are known constants, then the above equation can be written in a more compact form as:

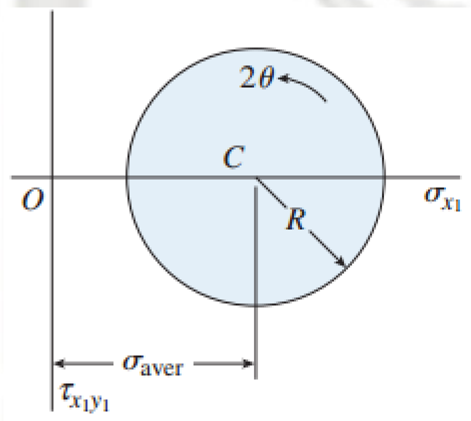
$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

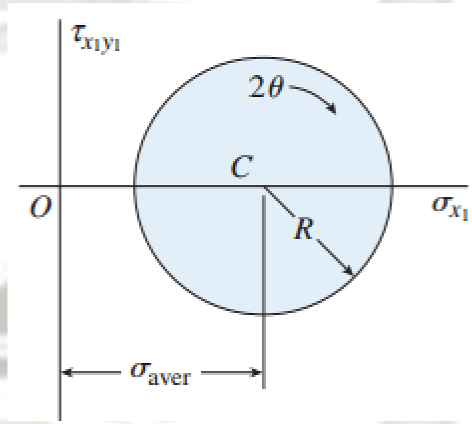
$$(\sigma_{x'} - \sigma_{\text{avg}})^2 + \tau_{x'y'}^2 = R^2 \qquad \text{or} \qquad (\sigma_{x_1} - \sigma_{\text{aver}})^2 + \tau_{x_1y_1}^2 = R^2 \Longrightarrow R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

If we establish coordinate axes, σ positive to the right and T positive downward, and then plot the above equation, it will be seen that this equation represents a circle having a radius R and center on the s axis at point C (σ avg, 0)

Mohr’s circle can be plotted from in either of two forms. In the first form of Mohr’s circle, we plot the normal stress σx1 positive to the right and the shear stress Ƭx1y1 positive downward. The advantage of plotting shear stresses positive downward is that the angle 2θ on Mohr’s circle will be positive when counterclockwise, which agrees with the positive direction of 2θ in the derivation of the transformation equations.

In the second form of Mohr’s circle, Ƭx1y1 is plotted positive upward but the angle 2θ is now positive clockwise which is opposite to its usual positive direction. Both forms of Mohr’s circle are mathematically correct, and either one can be used. However, it is easier to visualize the orientation of the stress element if the positive direction of the angle 2θ is the same in Mohr’s circle as it is for the element itself. Furthermore, a counterclockwise rotation agrees with the customary right-hand rule for rotation. Therefore, we will choose the first form of Mohr’s circle in which positive shear stress is plotted downward and a positive angle 2θ is plotted counterclockwise.



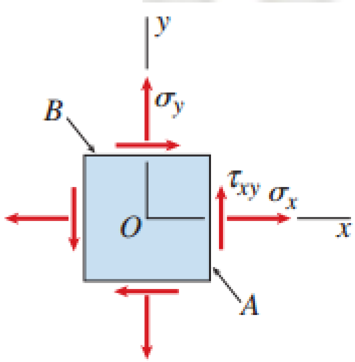


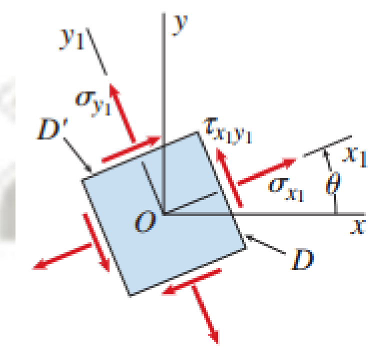
Construction of Mohr’s circle of stress:

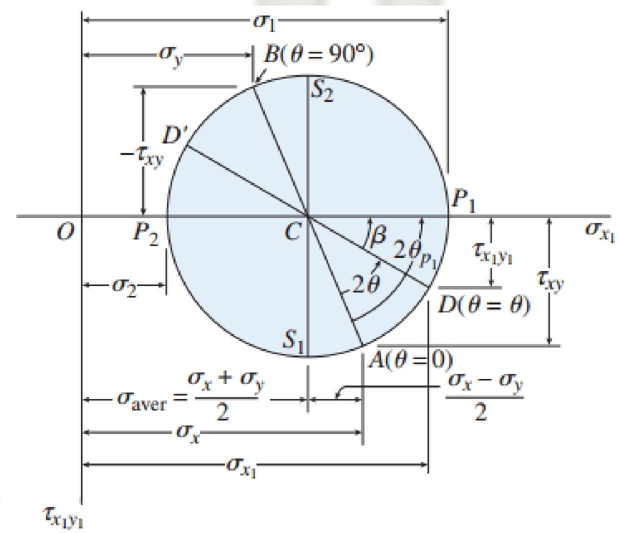
With σx, σy, and Ƭxy known, the **procedure for constructing Mohr’s circle** is as follows:

1. Draw a set of coordinate axes with σx1 (positive to the right) and Ƭx1y1 as ordinate (positive downward).
2. Locate the center C of the circle at the point having coordinates σx1= σ avg. and Ƭx1y1=0.
3. Locate point A, representing the stress conditions on the x face of the element, by plotting its coordinates σx1= σx and Ƭx1y1= Ƭxy. Note that point A on the circle corresponds to θ=0. Also, note that the x face of the element is labeled “A” to show its correspondence with point A on the circle.
4. Locate point B, representing the stress conditions on the y face of the element, by plotting its coordinates σx1= σy and Ƭx1y1= -Ƭxy. Note that point B on the circle corresponds to θ=90°. In addition, the y face of the element is labeled “B” to show its correspondence with point B on the circle.
5. Draw a line from point A to point B. This line is a diameter of the circle and passes through the center C. Points A and B, representing the stresses on planes at 90° to each other, are at opposite ends of the diameter (and therefore are 180° apart on the circle).
6. Using point C as the center, draw Mohr’s circle through points A and B. The circle drawn in this manner has radius R.

Then, from the geometry of the figure, we obtain the following expressions for the coordinates of point D:





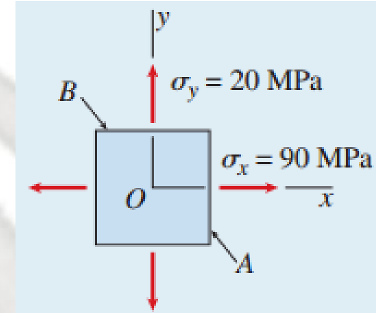


$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + R\cos\beta \qquad \tau_{x_1y_1} = R\sin\beta$$

EXAMPLE 3-7

Using Mohr's circle, determine the stresses acting on an element inclined at an angle θ=30°.

Solution:



$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = \frac{90\ \text{MPa} + 20\ \text{MPa}}{2} = 55\ \text{MPa}$$

Point *A*, representing the stresses on the *x* face of the element ($\theta = 0$), has coordinates

$$\sigma_{x_1} = 90\ \text{MPa} \qquad \tau_{x_1y_1} = 0$$

Similarly, the coordinates of point *B*, representing the stresses on the *y* face ($\theta = 90°$), are

$$\sigma_{x_1} = 20\ \text{MPa} \qquad \tau_{x_1y_1} = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{90\ \text{MPa} - 20\ \text{MPa}}{2}\right)^2 + 0} = 35\ \text{MPa}$$

Stresses on an element inclined at $\theta = 30°$. The stresses acting on a plane oriented at an angle $\theta = 30°$ are given by the coordinates of point *D*, which is at an angle $2\theta = 60°$ from point *A*

(Point *D*) $\quad \sigma_{x_1} = \sigma_{aver} + R\cos 60°$

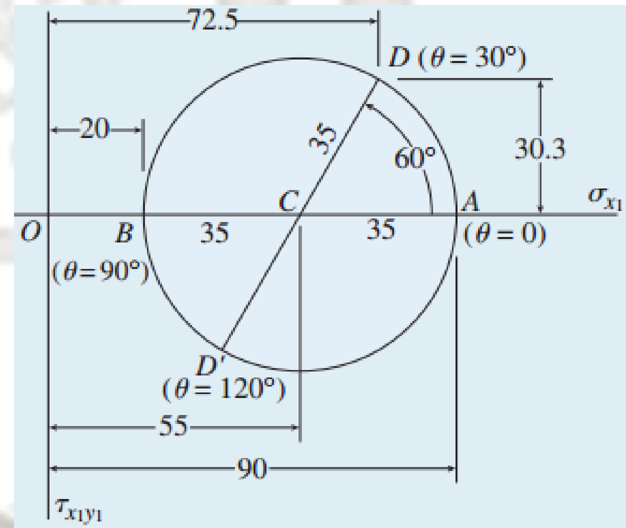
$$= 55\ \text{MPa} + (35\ \text{MPa})(\cos 60°) = 72.5\ \text{MPa}$$

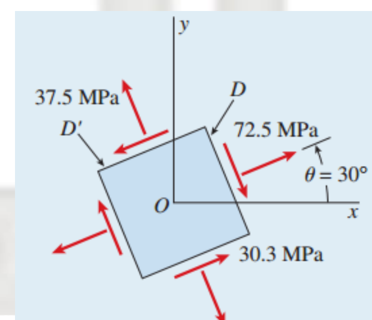
$$\tau_{x_1y_1} = -R\sin 60° = -(35\ \text{MPa})(\sin 60°) = -30.3\ \text{MPa}$$

(Point *D'*) $\quad \sigma_{x_1} = \sigma_{aver} - R\cos 60°$

$$= 55\ \text{MPa} - (35\ \text{MPa})(\cos 60°) = 37.5\ \text{MPa}$$

$$\tau_{x_1y_1} = R\sin 60° = (35\ \text{MPa})(\sin 60°) = 30.3\ \text{MPa}$$





EXAMPLE 3-8

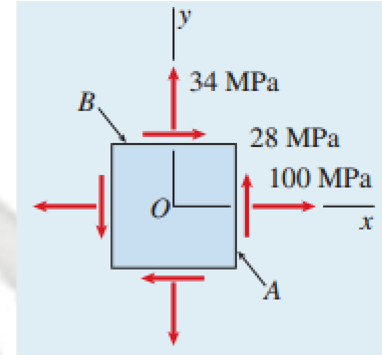
Using Mohr's circle, determine the following quantities:

(a) The stresses acting on an element inclined at an angle θ=40°

(b) The principal stresses

(c) The maximum shear stresses.

(Show all results on sketches of properly oriented elements).



Solution:

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = \frac{100\ \text{MPa} + 34\ \text{MPa}}{2} = 67\ \text{MPa}$$

Point *A*, representing the stresses on the *x* face of the element ($\theta = 0$), has coordinates

$$\sigma_{x_1} = 100\ \text{MPa} \qquad \tau_{x_1y_1} = 28\ \text{MPa}$$

Similarly, the coordinates of point *B*, representing the stresses on the *y* face ($\theta = 90°$) are

$$\sigma_{x_1} = 34\ \text{MPa} \qquad \tau_{x_1y_1} = -28\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{100\ \text{MPa} - 34\ \text{MPa}}{2}\right)^2 + (28\ \text{MPa})^2} = 43\ \text{MPa}$$

$$\tan \overline{ACP_1} = \frac{28\ \text{MPa}}{33\ \text{MPa}} = 0.848 \qquad \overline{ACP_1} = 40.3°$$

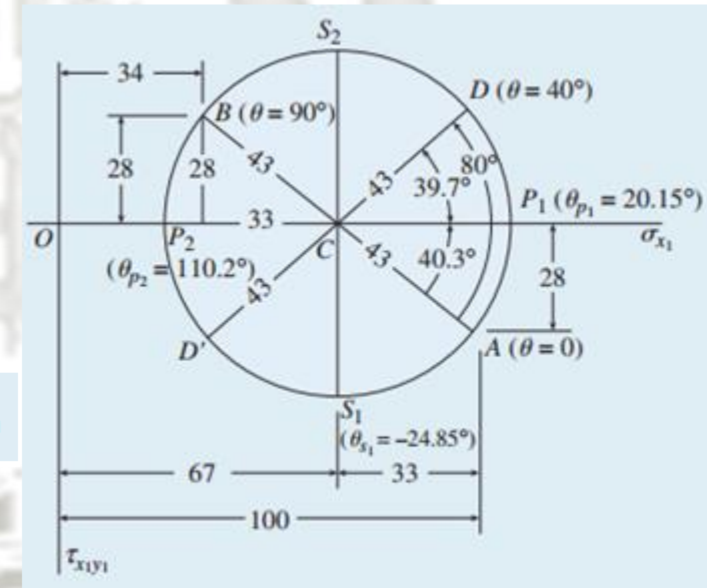
$$\overline{DCP_1} = 80° - \overline{ACP_1} = 80° - 40.3° = 39.7°$$

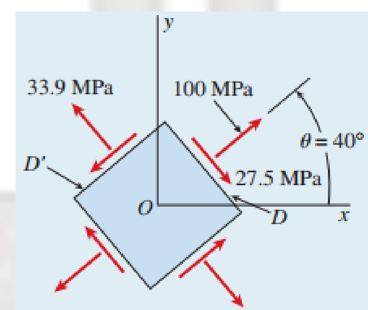
(Point *D*) $\sigma_{x_1} = 67\ \text{MPa} + (43\ \text{MPa})(\cos 39.7°) = 100\ \text{MPa}$

$\tau_{x_1y_1} = -(43\ \text{MPa})(\sin 39.7°) = -27.5\ \text{MPa}$

(Point *D'*) $\sigma_{x_1} = 67\ \text{MPa} - (43\ \text{MPa})(\cos 39.7°) = 33.9\ \text{MPa}$

$\tau_{x_1y_1} = (43\ \text{MPa})(\sin 39.7°) = 27.5\ \text{MPa}$





(b) *Principal stresses*. The principal stresses are represented by points P_1 and P_2 on Mohr's circle . The algebraically larger principal stress (point P_1) is

$$\sigma_1 = 67 \text{ MPa} + 43 \text{ MPa} = 110 \text{ MPa}$$

as seen by inspection of the circle. The angle $2\theta_{p_1}$ to point P_1 from point A is the angle ACP_1 on the circle, that is,

$$\overline{ACP_1} = 2\theta_{p_1} = 40.3° \qquad \theta_{p_1} = 20.15°$$

The algebraically smaller principal stress (represented by point P_2) is obtained from the circle in a similar manner:

$$\sigma_2 = 67 \text{ MPa} - 43 \text{ MPa} = 24 \text{ MPa}$$

The angle $2\theta_{p_2}$ to point P_2 on the circle is $40.3° + 180° = 220.3°$; thus, the second principal plane is defined by the angle $\theta_{p_2} = 110.2°$.

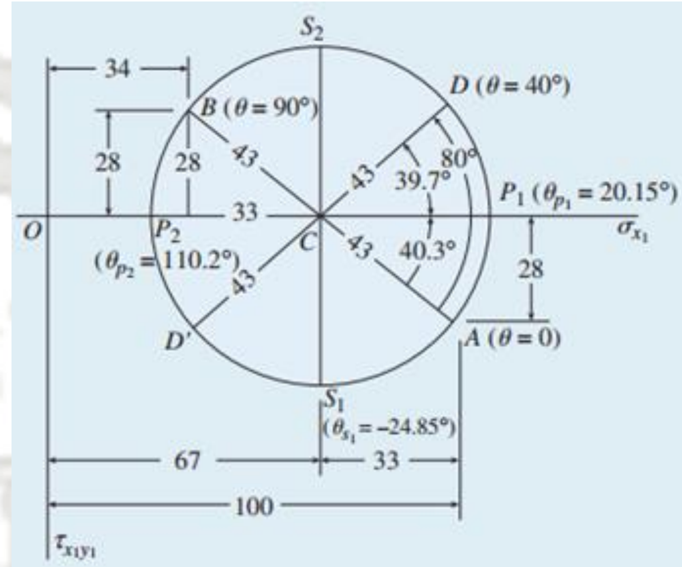
(c) *Maximum shear stresses*. The maximum shear stresses are represented by points S_1 and S_2 on Mohr's circle; therefore, the maximum in-plane shear stress (equal to the radius of the circle) is

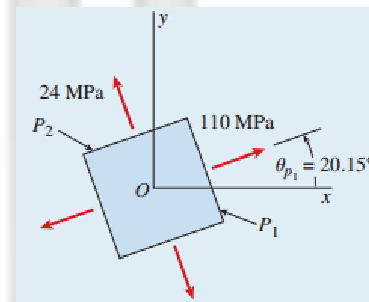
$$\tau_{max} = 43 \text{ MPa}$$

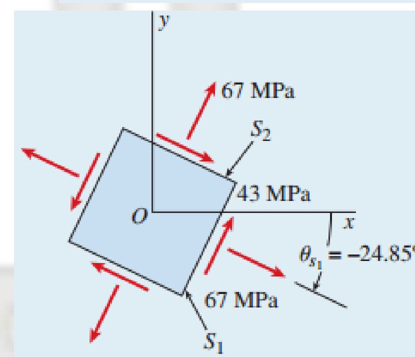
The angle ACS_1 from point A to point S_1 is $90° - 40.3° = 49.7°$, and therefore the angle $2\theta_{s_1}$ for point S_1 is

$$2\theta_{s_1} = -49.7°$$

This angle is negative because it is measured clockwise on the circle. The corresponding angle θ_{s_1} to the plane of the maximum positive shear stress is one-half that value, or $\theta_{s_1} = -24.85°$. maximum negative shear stress (point S_2 on the circle) has the same numerical value as the maximum positive stress (43 MPa).







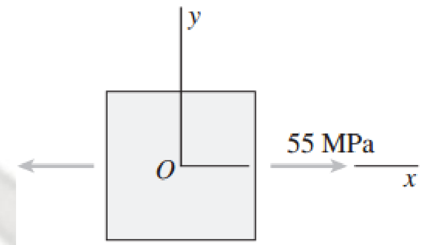
EXAMPLE 3-9

Using Mohr’s circle, determine:

(a) The stresses acting on an element oriented at an angle θ=-30° from the x axis (minus means clockwise).

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.



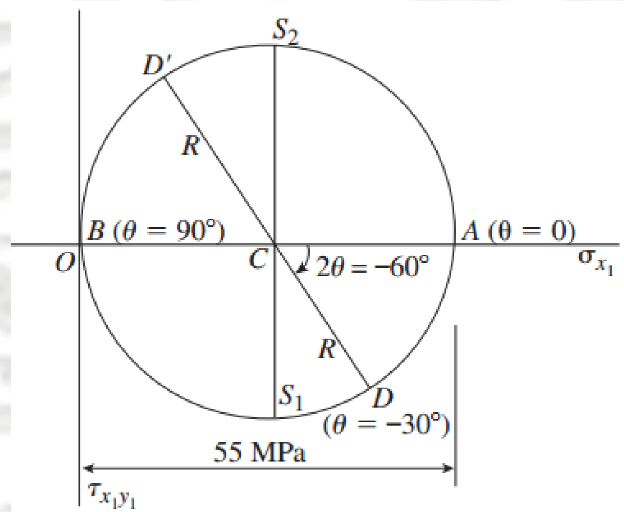
Solution:

$\sigma_x = 55$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

(a) ELEMENT AT $\theta = -30°$ (All stresses in MPa)

$2\theta = -60° \qquad \theta = -30° \qquad R = 27.5$ MPa

Point C: $\sigma_{x_1} = 27.5$ MPa



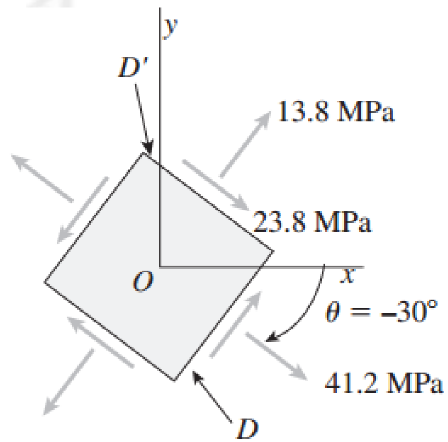
Point D: $\sigma_{x_1} = R + R\cos|2\theta|$

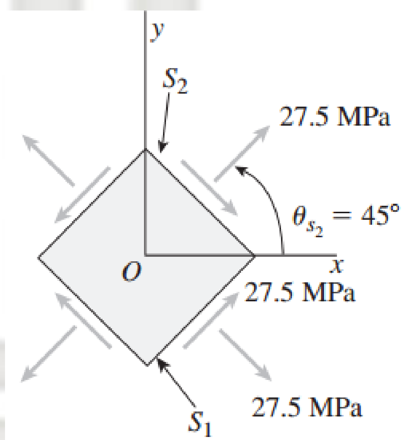
$$= R(1 + \cos 60°) = 41.2 \text{ MPa}$$

$$\tau_{x_1y_1} = R\sin|2\theta| = R\sin 60° = 23.8 \text{ MPa}$$

Point D': $\sigma_{x_1} = R - R\cos|2\theta| = 13.8$ MPa

$$\tau_{x_1y_1} = -R\sin|2\theta| = -23.8 \text{ MPa}$$





(b) MAXIMUM SHEAR STRESSES

Point S_1: $2\theta_{s_1} = -90° \qquad \theta_{s_1} = -45°$

$$\tau_{max} = R = 27.5 \text{ MPa}$$

Point S_2: $2\theta_{s_2} = 90° \qquad \theta_{s_2} = 45°$

$$\tau_{min} = -R = -27.5 \text{ MPa}$$

$\sigma_{aver} = R = 27.5$ MPa

EXAMPLE 3-10

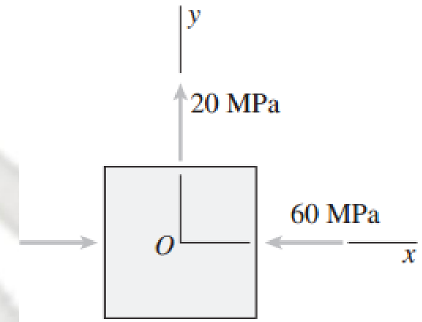
Using Mohr's circle, determine:

(a) The stresses acting on an element oriented at a counterclockwise angle θ=22.5° from the x axis.

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.

Solution:



$\sigma_x = -60$ MPa $\quad \sigma_y = 20$ MPa $\quad \tau_{xy} = 0$

(a) ELEMENT AT $\theta = 22.5°$

(All stresses in MPa)

$2\theta = 45° \qquad \theta = 22.5°$

$2R = 60 + 20 = 80$ MPa $\qquad R = 40$ MPa

Point C: $\sigma_{x_1} = -20$ MPa

(b) MAXIMUM SHEAR STRESSES

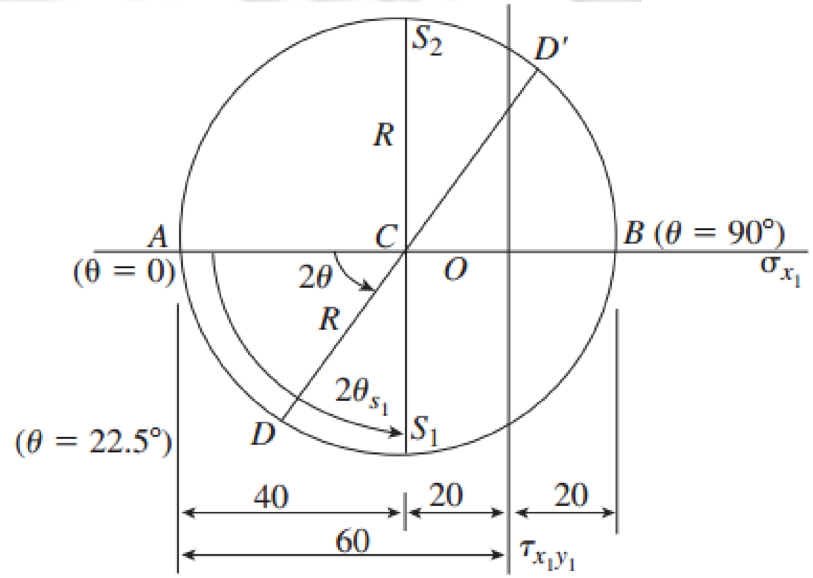
Point S_1: $2\theta_{s_1} = 90° \qquad \theta_{s_1} = 45°$

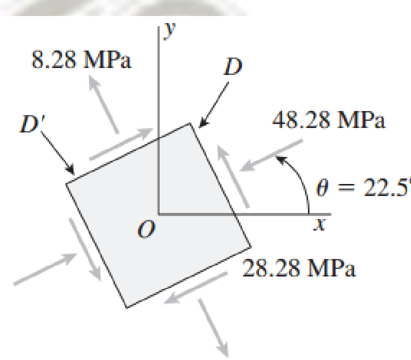
$\tau_{max} = R = 40$ MPa

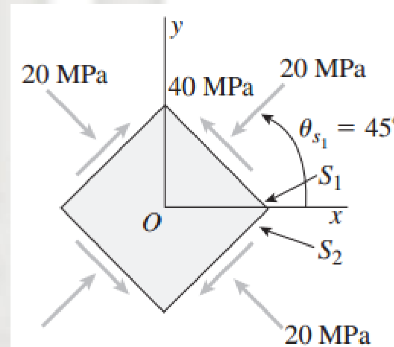
Point S_2: $2\theta_{s_2} = -90° \qquad \theta_{s_2} = -45°$

$\tau_{min} = -R = -40$ MPa

$\sigma_{aver} = -20$ MPa







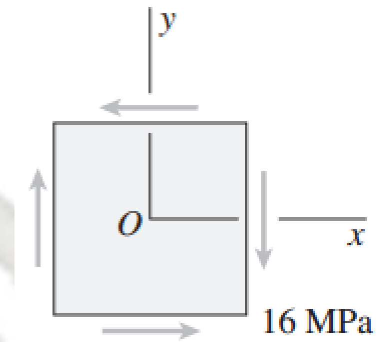
EXAMPLE 3-11

Using Mohr's circle, determine:

(a) The stresses acting on an element oriented at a counterclockwise angle θ=20° from the x axis.

(b) The principal stresses.

Show all results on sketches of properly oriented elements.



Solution:

$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = -16\ \text{MPa}$

(a) Element at $\theta = 20°$

(All stresses in MPa)

$2\theta = 40° \qquad \theta = 20° \qquad R = 16\ \text{MPa}$

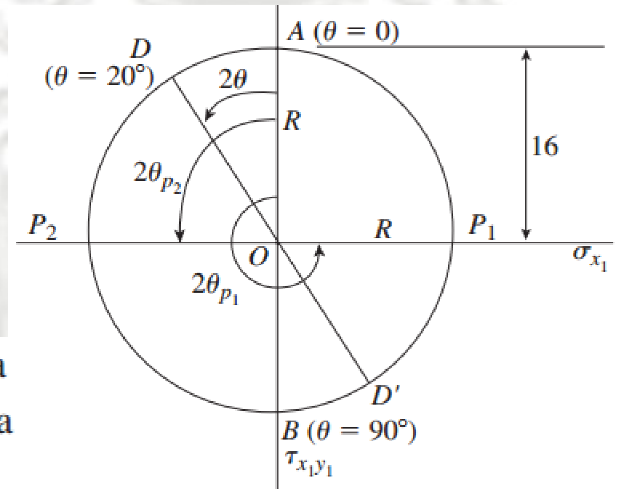
Origin O is at center of circle.

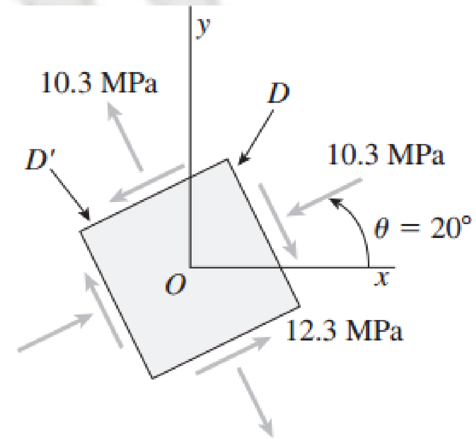
Point D: $\sigma_{x_1} = -R\sin 2\theta = -10.28\ \text{MPa}$

$\tau_{x_1y_1} = -R\cos 2\theta = -12.26\ \text{MPa}$

Point D': $\sigma_{x_1} = R\sin 2\theta = 10.28\ \text{MPa}$

$\tau_{x_1y_1} = R\cos 2\theta = 12.26\ \text{MPa}$





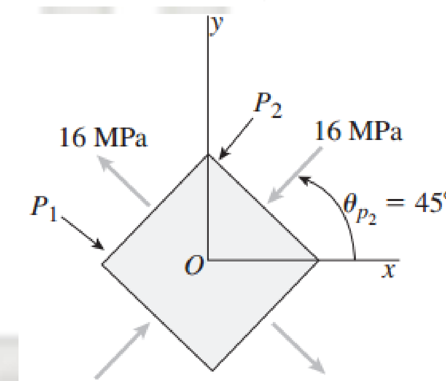
(b) Principal stresses

Point P_1: $2\theta_{p_1} = 270° \qquad \theta_{p_1} = 135°$

$\sigma_1 = R = 16\ \text{MPa}$

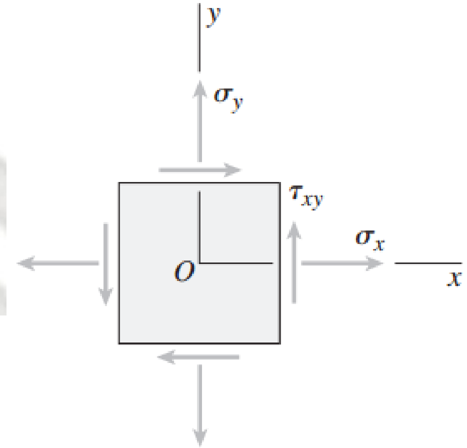
Point P_2: $2\theta_{p_2} = 90° \qquad \theta_{p_2} = 45°$

$\sigma_2 = -R = -16\ \text{MPa}$



EXAMPLE 3-12

Using Mohr’s circle, determine the stresses acting on an element oriented at an angle θ from the x axis. Show these stresses on a sketch of an element oriented at the angle θ. ($\sigma_x = 21$ MPa, $\sigma_y = 11$ MPa, $\tau_{xy} = 8$ MPa, $\theta = 50°$)



Solution:

$\sigma_x = 21$ MPa $\quad \sigma_y = 11$ MPa

$\tau_{xy} = 8$ MPa $\quad \theta = 50°$

(All stresses in MPa)

$$R = \sqrt{(5)^2 + (8)^2} = 9.4340 \text{ MPa}$$

$$\alpha = \arctan\frac{8}{5} = 57.99°$$

$$\beta = 2\theta - \alpha = 100° - \alpha = 42.01°$$

Point D ($\theta = 50°$):

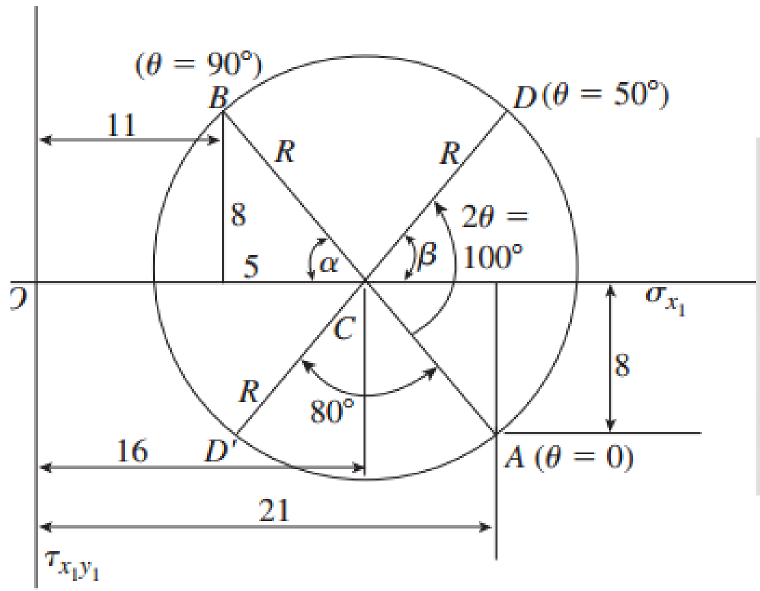
$$\sigma_{x_1} = 16 + R\cos\beta = 23.01 \text{ MPa}$$

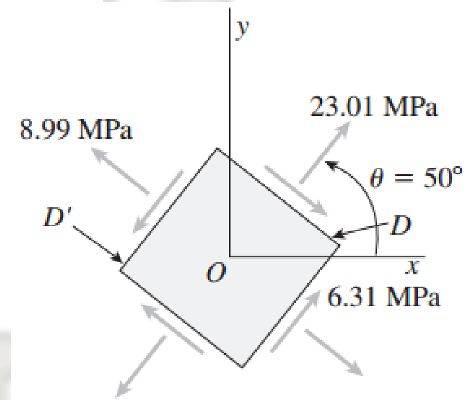
$$\tau_{x_1y_1} = -R\sin\beta = -6.31 \text{ MPa}$$

Point D' ($\theta = -40°$):

$$\sigma_{x_1} = 16 - R\cos\beta = 8.99 \text{ MPa}$$

$$\tau_{x_1y_1} = R\sin\beta = 6.31 \text{ MPa}$$





H.W

Using Mohr's circle, determine the stresses acting on an element oriented at an angle θ from the x axis. Show these stresses on a sketch of an element oriented at the angle θ.

1- $\sigma_x = -44 \text{ MPa} \quad \sigma_y = -194 \text{ MPa}$
$\tau_{xy} = -36 \text{ MPa} \quad \theta = -35°$

2- $\sigma_x = 31 \text{ MPa} \quad \sigma_y = -5 \text{ MPa}$
$\tau_{xy} = 33 \text{ MPa} \quad \theta = 45°$