5/28 The design characteristics of a gear-reduction unit are under review. Gear B is rotating clockwise with a speed of 300 rev/min when a torque is applied to gear A at time $t = 2$ s to give gear A a counterclockwise acceleration α which varies with time for a duration of 4 seconds as shown. Determine the speed N_B of gear B when $t = 6$ s.

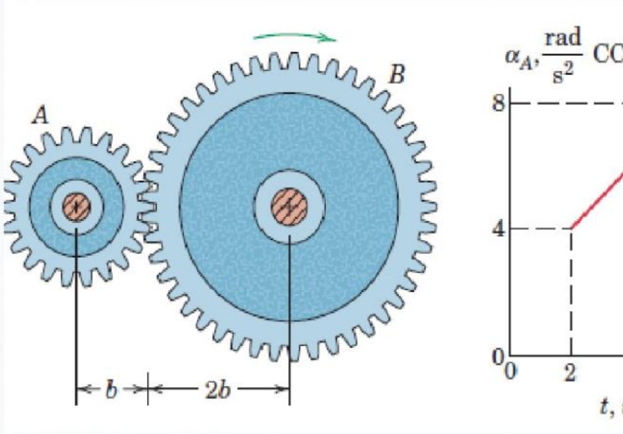
Solution

$N_B = 300$ r.p.m at $t = 2$ Sec

N_B ? at $t = 6$ Sec

$$\omega_B = \omega_A = \frac{N_B}{N_A} = \frac{R_A}{R_B} \Rightarrow N_B = 0.5 N_A \Rightarrow N_A = 2N_B = 2(300) = 600 \text{ r.p.m}$$

$$\alpha_A = \frac{d\omega}{dt} \Rightarrow \int d\omega = \alpha\, dt$$



$$\therefore \int_{\omega_0}^{\omega_A} d\omega = \int_2^6 \alpha_A\, dt = \text{Area under the curve}$$

$$\therefore \int_{600 \times \frac{2\pi}{60}}^{\omega_A} d\omega = \left[4(4) + \tfrac{1}{2} * 4(4)\right]$$

$\therefore \omega_A = 86.83$ rad/sec

$\therefore \omega_B = 0.5\,\omega_A = 0.5(86.83)$

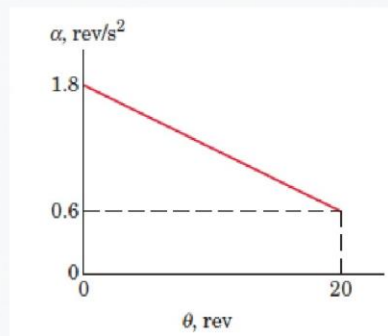
$= 43.41$ rad/sec

$\therefore N_B = 43.41 * \frac{60}{2\pi} \cong 415$ rev/min

5/27 A clockwise variable torque is applied to a flywheel at time $t = 0$ causing its clockwise angular acceleration to decrease linearly with angular displacement θ during 20 revolutions of the wheel as shown. If the clockwise speed of the flywheel was 300 rev/min at $t = 0$, determine its speed N after turning the 20 revolutions. (*Suggestion:* Use units of revolutions instead of radians.)

5/27) $t = 0 \Rightarrow N_o = 300$ r.p.m & ω at $\theta = 20$ rev

Solution



$$\omega = \frac{d\theta}{dt} \quad \& \quad \alpha = \frac{d\omega}{dt}$$

$$\therefore \alpha \, d\theta = \omega \, d\omega$$

$$\therefore \int_{\omega_o}^{\omega} \omega \, d\omega = \int_{\theta_o = 0}^{\theta = 20 \text{rev}} \alpha \, d\theta = \text{Area under the curve}$$

$$\therefore \frac{1}{2} \omega^2 \Big|_{\omega_o}^{\omega} = \left[(0.6)(2\pi) * (20)(2\pi) + \frac{1}{2} (1.2)(2\pi) * (20)(2\pi) \right]$$

$$\therefore \omega^2 - \left(300 * \frac{2\pi}{60}\right)^2 = 2 \, (1894.964)$$

$$\therefore \omega = 53.6833 \text{ rad}$$

$$\therefore N = 53.6833 * \frac{60}{2\pi}$$

$$= 512.64$$

5/11 The angular acceleration of a body which is rotating about a fixed axis is given by $\alpha = -k\omega^2$, where the constant $k = 0.1$ (no units). Determine the angular displacement and time elapsed when the angular velocity has been reduced to one-third its initial value $\omega_0 = 12$ rad/s.

$\alpha = -K\omega^2$, $K = 0.1 = \text{const.}$, $\omega_o = 12$ rad/s

Solution $\quad \omega = \frac{1}{3}\omega_o$, $\Delta\theta$ & t

$$\alpha = \frac{d\omega}{dt} = -K\omega^2 \Rightarrow d\omega = -K\omega^2 dt$$

$$\therefore \int_{\omega_o}^{\omega} \frac{d\omega}{\omega^2} = -K\int_0^t dt \Rightarrow \left.\frac{\omega^{-2+1}}{-2+1}\right|_{\omega_o}^{\omega} = -Kt$$

$$\therefore \left. -\frac{1}{\omega}\right|_{\omega_o}^{\omega} = -Kt \Rightarrow -\frac{1}{\omega} + \frac{1}{\omega_o} = -Kt \quad \text{—} \ (1)$$

$$\therefore -\frac{1}{\frac{12}{3}} + \frac{1}{12} = -0.1\,t \Rightarrow t = 1.667 \text{ Sec}$$

from eq (1) $\Rightarrow -\frac{1}{\omega} = -\left(Kt + \frac{1}{\omega_o}\right) \Rightarrow$

$$\omega = \frac{1}{Kt + \frac{1}{\omega_o}} \Rightarrow \frac{d\theta}{dt} = \frac{1}{Kt + \frac{1}{\omega_o}}$$

$$\therefore \int_{\theta_o}^{\theta} d\theta = \int_0^t \frac{dt}{Kt + \frac{1}{\omega_o}}$$

$$\therefore \theta - \theta_o = \left.\frac{1}{K}\ln\left(Kt + \frac{1}{\omega_o}\right)\right|_0^t$$

5/12 The angular position of a radial line in a rotating disk is given by the clockwise angle $\theta = 2t^3 - 3t^2 + 4$, where θ is in radians and t is in seconds. Calculate the angular displacement $\Delta\theta$ of the disk during the interval in which its angular acceleration increases from 42 rad/s² to 66 rad/s².

$\theta = 2t^3 - 3t^2 + 4$, $\Delta\theta$? , $\alpha_1 = 42$ rad/s² & $\alpha_2 = 66$ rad/s²

Solution

$$\theta = 2t^3 - 3t^2 + 4$$

$$\omega = \frac{d\theta}{dt} = 6t^2 - 6t$$

$$\alpha = \frac{\partial w}{dt} = 12t - 6$$

$\therefore$ at $\alpha = 42 \Rightarrow 42 = 12t - 6 \Rightarrow t_1 = 4$ Sec

at $\alpha = 66 \Rightarrow 66 = 12t - 6 \Rightarrow t_2 = 6$ Sec

$\therefore \theta_1 = 2(4)^3 - 3(4)^2 + 4 = 84$ rad

$\therefore \theta_2 = 2(6)^3 - 3(6)^2 + 4 = 328$ rad

$\therefore \Delta\theta = \theta_2 - \theta_1 = 328 - 84 = 244$ rad

Ans.

5/20 Point A of the circular disk is at the angular position $\theta = 0$ at time $t = 0$. The disk has angular velocity $\omega_0 = 0.1$ rad/s at $t = 0$ and subsequently experiences a constant angular acceleration $\alpha = 2$ rad/s^2. Determine the velocity and acceleration of point A in terms of fixed **i** and **j** unit vectors at time $t = 1$ s.

$\theta = 0$ at $t = 0$ & $\omega_0 = 0.1$ rad/s at $t = 0$

<u>Solution</u> $\alpha = \text{const} = 2$ rad/s^2 & v_A & a_A <u>at</u> $t = 1$ sec

At $t = 0 \Rightarrow \theta = 0$

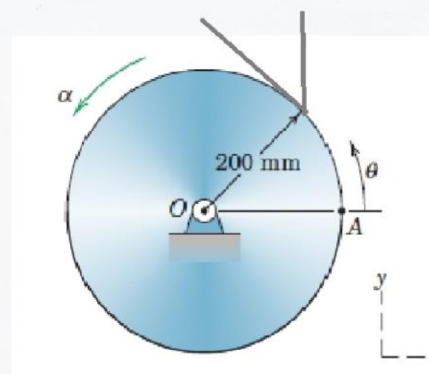
At $t = 1$ sec $\Rightarrow \theta = ?$

$\therefore \omega = \omega_0 + \alpha t \Rightarrow \omega = 0.1 + 2(1) \Rightarrow \omega = 2.1$ r

$\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow 2.1^2 = 0.1^2 + 2(2)\theta$

$\therefore \theta = 1.1$ rad $\Rightarrow \theta = 1.1 \times \frac{180}{\pi} = 63^\circ$

$\therefore$ at $t = 1$ sec $\Rightarrow \theta_A = 63^\circ$



$v_A = \omega r_A = 2.1 \times 0.2 = 0.42$ m/s

$a_A^t = \alpha r_A = 2 \times 0.2 = 0.4$ m/s^2

$a_A^n = \frac{v_A^2}{r_A} = \omega^2 r_A = \frac{0.42^2}{0.2} = 0.88$ m/s^2

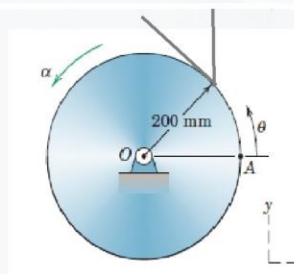
$\therefore v_A = v_A)_x\, i + v_A)_y\, j$

$= (-0.42 \sin(63))\, i + (0.42 \cos(63))\, j$

$= -0.3742\, i + 0.19\, j$

$a_A = (-a_A^n \cos\theta - a_A^t \sin\theta)\, i + (-a_A^n \sin\theta + a_A^t \cos\theta)\, j$

5/22 Repeat Prob. 5/20, except now the angular acceleration of the disk is given by $\alpha = 2\omega$, where ω is in radians per second and α is in radians per second squared. Determine the velocity and acceleration of point A in terms of fixed **i** and **j** unit vectors at time $t = 1$ s.



$\alpha = 2\omega$ & at $t=0 \Rightarrow \theta_0 = 0 \Rightarrow \omega_0 = 0.1$ rad/s

Solution v_A & a_A at $t = 1$ sec

$$\alpha = 2\omega \Rightarrow \frac{\partial \omega}{\partial t} = 2\omega \Rightarrow \frac{\partial \omega}{\omega} = 2dt$$

$$\therefore \int_{\omega_0}^{\omega} \frac{\partial \omega}{\omega} = \int_0^t 2dt \Rightarrow \ln\omega - \ln\omega_0 = 2t$$

$$\therefore \ln\omega = 2t + \ln(\omega_0) \Rightarrow \omega = e^{2t+\ln\omega_0}$$

$$\therefore \omega \Rightarrow \text{at } t = 1\text{sec} \Rightarrow \omega = e^{2(1)+\ln 0.1} = 0.739 \text{ rad/s}$$

$$\therefore \alpha_1 = 2\omega = 2(0.739) = 1.478 \text{ rad/s}^2$$

$$\omega = \frac{\partial \theta}{\partial t} = e^{2t+\ln 0.1} \Rightarrow \int_{\theta_0}^{\theta} \partial\theta = \int_0^{t=1} e^{2t+\ln 0.1}$$

$$\therefore \theta - 0 = \frac{1}{2}\left[e^{2(1)+\ln 0.1} - e^{\ln 0.1}\right]$$

$$\therefore \theta = 0.31945 \text{ rad} \Rightarrow \theta_A = 18.3^\circ$$

$$\therefore v_A = \omega \cdot r_A = 0.739(0.2) = 0.1478 \text{ m/s}$$

$$v_A = (-0.1478\sin\theta)\,i + (0.1478\cos\theta)\,j$$
$$= -0.0464\,i + 0.14\,j$$

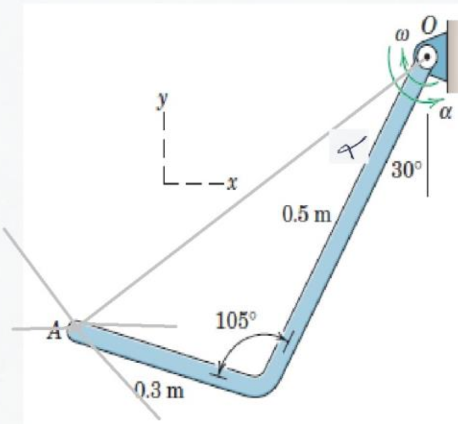
$$a_A^t = \alpha \cdot r_A = 1.478(0.2) = 0.2956 \text{ m/s}^2$$

$$a_A^n = \frac{v_A^2}{r_A} = \frac{0.1478^2}{0.2} = 0.10922 \text{ m/s}^2$$

5/10 The bent flat bar rotates about a fixed axis through point O. At the instant depicted, its angular properties are $\omega = 5$ rad/s and $\alpha = 8$ rad/s^2 with directions as indicated in the figure. Determine the instantaneous velocity and acceleration of point A.

solution $\omega = 5$ rad/sec & $\alpha = 8$ rad/s^2 & V_A & a_A ?

$OA = r_A = \sqrt{0.3^2 + 0.5^2 - 2(0.3)(0.5)\cos 105} = 0.646$ m



$\dfrac{0.646}{\sin 105} = \dfrac{0.3}{\sin\alpha} \Rightarrow \alpha = 26.65°$

$\therefore \theta = \alpha + 30 = 56.65°$

$v_A = \omega \cdot r_A = 5 * 0.646 = 3.23$ m/s

$V_A = V_A)_x\, i + V_A)_y\, j$

$V_A = (-3.23\cos 56.65)\, i + 3.23 \sin 56.65)\, j$

$\therefore V_A = -1.776\, i + 2.7\, j$

$a_A^t = \alpha \cdot R_A = 8 * 0.646 = 5.168$ m/s^2

$a_A^n = \dfrac{v_A^2}{R_A} = \omega \cdot R_A = \dfrac{3.23^2}{0.646} = 16.15$ m/s^2 $\angle 90-\theta$

$\therefore a_A)_x = (a_t \cos\theta + a_n \cos(90-\theta))\, i$

$= (5.168\cos 56.65 + 16.15\cos 33.35)\, i$

$a_A)_y = (-a_t \sin\theta + a_n \sin(90-\theta))\, j$

$= (-5.168\sin 56.65 + 16.15\sin 33.35)\, j$

5/9 A shaft is accelerated from rest at a constant rate to a speed of 3600 rev/min and then is immediately decelerated to rest at a constant rate within a total time of 10 seconds. How many revolutions N has the shaft turned during this interval?

Solution

$\alpha = \text{Const.}$, $\omega_0 = 0$, $\omega_1 = 3600$ r.p.m , $\omega_2 = 0$

$t_{total} = t_1 + t_2 = 10$ Sec

Case ①

$$\omega_1 = \cancel{\omega_0}^{0} + \alpha_1 t_1 \Rightarrow t_1 = \frac{\omega_1}{\alpha_1}$$

$$\omega_1^2 = \cancel{\omega_0^2}^{0} + 2\alpha_1 \theta_1 \Rightarrow \alpha_1 = \frac{\omega_1^2}{2\theta_1}$$

$$\therefore t_1 = \frac{\omega_1}{\alpha_1} = \frac{\omega_1}{\frac{\omega_1^2}{2\theta_1}} = \frac{2\theta_1}{\omega_1}$$

Case ②

$$\cancel{\omega_2}^{0} = \cancel{\omega_1} + \alpha_2 t_2 \Rightarrow t_2 = -\frac{\omega_1}{\alpha_2}$$

$$\cancel{\omega_2^2}^{0} = \omega_1^2 + 2\alpha_2 \theta_2 \Rightarrow \alpha_2 = -\frac{\omega_1^2}{2\theta_2}$$

$$\therefore t_2 = -\frac{\omega_1}{\alpha_2} = \frac{\omega_1}{\frac{\omega_1^2}{2\theta_2}} = \frac{2\theta_2}{\omega_1}$$

$\because$ $t_1 + t_2 = 10$ Sec

$$\therefore \frac{2\theta_1}{\omega_1} + \frac{2\theta_2}{\omega_1} = 10 \Rightarrow 2(\theta_1 + \theta_2) = 10\,\omega_1$$

$$\therefore \theta_1 + \theta_2 = 5\left(3600 \times \frac{2\pi}{60}\right) = 600\pi \text{ rad}$$

5/8 If the rectangular plate of Prob. 5/7 starts from rest and point B has an initial acceleration of 5.5 m/s^2, determine the distance b if the plate reaches an angular speed of 300 rev/min in 2 seconds with a constant angular acceleration.

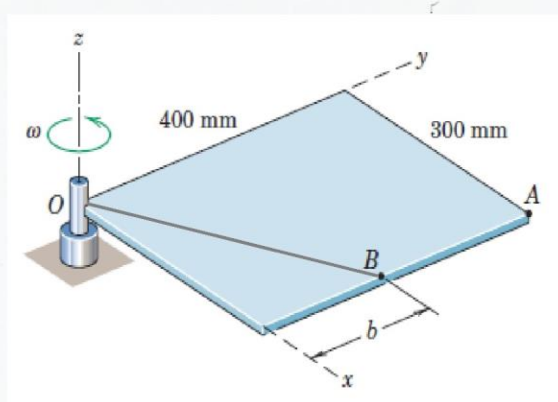
$V_o = 0$ at B , $a_o^B = 5.5\ m/s^2$, b? , $N = 300$ r.p.m

at $t = 2$ Sec , α = const.

Solution

at $\omega_o = 0 \Rightarrow a_o^n = 0$

$\therefore a_B = a_t = 5.5\ m/s^2$ (في بداية الحركة ⇐ السرعة = صفر ⇐ $a_n = 0$)



$$\omega = \omega_o + \alpha t$$

$$\therefore \left(300 \times \frac{2\pi}{60}\right) = 0 + \alpha(2)$$

$$\therefore \alpha = 15.708\ rad/s^2$$

$$a_t^B = \alpha \cdot R_B$$

$$R_B = \sqrt{0.3^2 + b^2}$$

$$\therefore 5.5 = 15.708 \times \sqrt{0.3^2 + b^2}$$

$$\therefore b = 180.55\ mm$$

Ans.

5/4 A torque applied to a flywheel causes it to accelerate uniformly from a speed of 200 rev/min to a speed of 800 rev/min in 4 seconds. Determine the number of revolutions N through which the wheel turns during this interval. (*Suggestion:* Use revolutions and minutes for units in your calculations.)

$\alpha = \text{const.}$, $N_1 = 200$ r.p.m , $N_2 = 800$ r.p.m

$\Delta t = 4$ Sec

Solution

$$\omega = \omega_0 + \alpha t$$

$$\left(\frac{800 \times 2\pi}{60}\right) = \left(\frac{200 \times 2\pi}{60}\right) + \alpha(4)$$

$$\therefore \alpha = 15.7 \text{ rad/s}^2$$

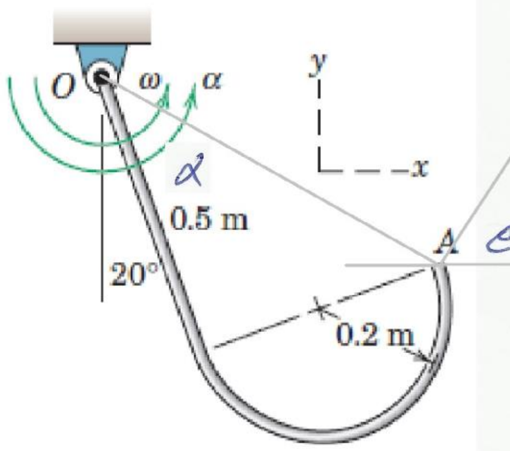
$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\left(\frac{800 \times 2\pi}{60}\right)^2 = \left(\frac{200 \times 2\pi}{60}\right)^2 + 2(15.7)\theta$$

$$\therefore \theta = 209.54 \text{ rad}$$

$\therefore$ No. of Revolutions $= \dfrac{209.54}{2\pi} = 33.35$ rev

5/3 The body is formed of slender rod and rotates about a fixed axis through point O with the indicated angular properties. If $\omega = 4$ rad/s and $\alpha = 7$ rad/s^2, determine the instantaneous velocity and acceleration of point A.



$r_A = \sqrt{0.5^2 + 0.4^2} = 0.64$ m

$\alpha = \tan^{-1}\frac{0.4}{0.5} = 38.65^\circ$

$\theta = 38.65 + 20 = 58.66^\circ$

$v_A = \omega \cdot r_A = 4 \times 0.64 = 2.56$ m/s

$v_A)_x = 2.56 \cos\theta\, i$ & $v_A)_y = 2.56 \sin\theta\, j$

$\therefore v_A = 2.56 \cos 58.66\, i + 2.56 \sin 58.66\, j$

$= (1.33\, i + 2.18\, j)$ m/sec

Ans.

$a_t^A = \alpha \cdot r_A = 7 \times 0.64 = 4.48$ m/s^2

$a_n^A = \frac{v_A^2}{r_A} = \omega^2 r_A = \frac{2.56^2}{0.64} = 10.24$ m/s^2

$a^A = \sqrt{a_t^2 + a_n^2} = 11.18$ m/s^2

$a_A)_x = (a_t \cos\theta - a_n \cos(90-\theta))\, i$

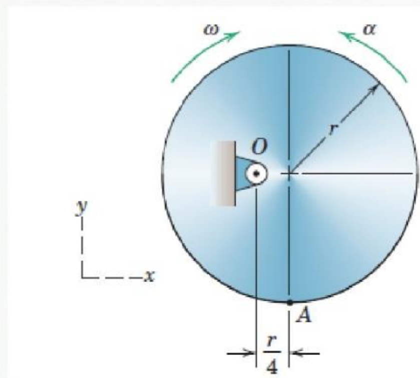
$a_A)_y = (a_t \sin\theta + a_n \sin(90-\theta))\, j$

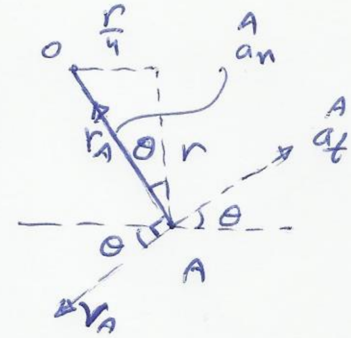
$\therefore a_A)_x = (4.48 \cos 58.66 - 10.24 \cos 31.34)\, i$

$a_A)_y = (4.48 \sin$ ...

5/1 The circular disk of radius $r = 0.16$ m rotates about a fixed axis through point O with the angular properties $\omega = 2$ rad/s and $\alpha = 3$ rad/s^2 with directions as shown in the figure. Determine the instantaneous values of the velocity and acceleration of point A.

Solution





$r_A = \sqrt{\left(\frac{r}{4}\right)^2 + r^2}$

$= \sqrt{\left(\frac{0.16}{4}\right)^2 + 0.16^2}$

$= 0.165$ m

$v_A = \omega\, r_A = 2 * 0.165$

$= 0.33$ m/sec

$\therefore\ \theta = \tan^{-1}\frac{\left(\frac{0.16}{4}\right)}{0.16} = 14^\circ$

$\therefore\ v_A)_x = -v_A \cos\theta\, i \quad \& \quad v_A)_y = -v_A \sin\theta\, j$

$\therefore\ v_A = -0.33 \cos 14\, i - 0.33 \sin 14\, j$

$v_A = -0.32\, i - 0.08\, j$ Ans.

$a_A^t = \alpha \cdot r_A = 3 * 0.165 = 0.495$ m/s^2

$a_A^n = \frac{v_A^2}{r_A} = \omega^2 \cdot r_A = \frac{0.33^2}{0.16} = 0.66$ m/s^2

$a_A = \sqrt{(a_A^t)^2 + (a_A^n)^2} = \sqrt{0.66^2 + 0.495^2} = 0.825$ m/s^2

$a_A)_x = (a_t \cos\theta - a_n \sin\theta)\, i \quad \& \quad a_A)_y = (a_t \sin\theta + a_n \cos\theta)\, j$