EX1: Find the local extreme values of the function:

$$f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$$

$$f_x = y - 2x - 2 = 0$$
$$f_y = x - 2y - 2 = 0$$
$f_x = f_y = 0$

$y - 2x - \cancel{2} = x - 2y - \cancel{2}$

$3x = 3y \longrightarrow x = y$ sub in $f_x = 0$ or in $f_y = 0$

$y - 2y - 2 = 0 \longrightarrow -y = 2$

or $y = -2$ $\therefore$ $x = -2$

Therfore the point $(-2,-2)$ is the only point where f may take on an extreme value

$f_{xx} = -2$, $f_{yy} = -2$, $f_{xy} = 1$

The discriminant of f at $(a,b) = (-2,-2)$ is
$f_{xx}f_{yy} - f_{xy}^2 = (-2)(-2) - (1)^2 = 3$

$f_{xx} < 0$ and $f_{xx}f_{yy} > 0$
Tells us that f has a local maximum at $(-2,-2)$. The value of f at this point is 8

Ex2: Find the local extreme values of $f(x,y) = xy$

$$\left.\begin{array}{l} f_x = y = 0 \\ f_y = x = 0 \end{array}\right\} \quad f_x = f_y$$

$$x = y = 0$$

Thus, the $(0,0)$ is the only point wher f might have an extreme value.

$$f_{xx} = 0 \qquad f_{yy} = 0 \qquad f_{xy} = 1$$

$$f_{xx} f_{yy} - f_{xy}^2 = -1$$

is negative, therfor, the function has a saddle point at $(0,0)$.

EX3: Find the local extreme values of $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy$.

$$f_x = 12x - 6x^2 + 6y = 0$$

and

$$f_y = 6y + 6x = 0$$

$$12x - 6x^2 + 6y = 6y + 6x$$

$$6x - 6x^2 = 0$$

$$6x(1-x) = 0 \longrightarrow x = 0 \longrightarrow y = 0$$

$$x = 1 \longrightarrow y = -1$$

critical points are $(0,0)$ and $(1,-1)$

for $(0,0)$

$$f_{xx}(0,0) = 12 - 12x\Big|_{(0,0)} = 12 - 0 = 12$$

$$f_{yy}(0,0) = 6$$

$$f_{xy}(0,0) = 6$$

$$f_{xx}f_{yy} - f_{xy}^2 = 36 > 0 \text{ and}$$

$f_{xx} > 0 \ \therefore$ local minimum at $(0,0)$ of $f(0,0) = 0$

$f(1,-1)$

$f_{xx}(1,-1) = 0$

$f_{yy}(1,-1) = 6$

$f_{xy}(1,-1) = 6$

$f_{xx} f_{yy} - f_{xy}^2 = -36 < 0$

$\therefore$ saddle point.

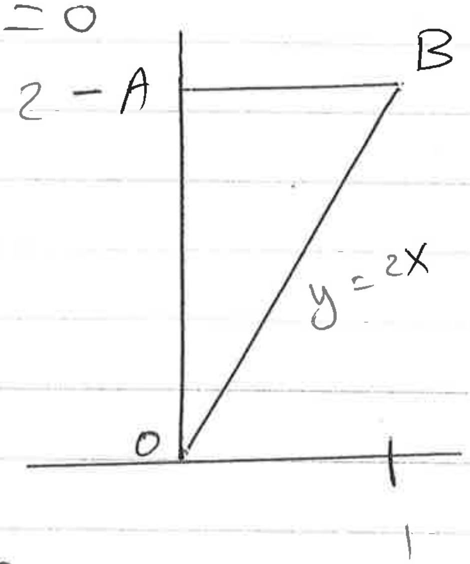
Ex4: Find the absolute maxima and minima of f

$$f(x,y) = 2x^2 - 4x + y^2 - 4y + 1 \quad \text{on}$$

the closed triangular plate bounded by the lines $x=0$, $y=2$, and $y=2x$ in the first quadrant.

① for interior points of the triangular region, $f_x(x,y) = 4x - 4 = 0 \qquad x = 1$

and $f_y(x,y) = 2y - 4 = 0$



$f_x = f_y = 0$

$$4x - 4 = 2y - 4$$

$$\therefore y = 2x = 2$$

but $(1,2)$ is not an interior point of the region.

② (a) on OA

$$f(x,y) = f(0,y) = y^2 - 4y + 1$$

$f'(0,y) = 2y - 4 = 0 \rightarrow y = 2$

$f(0,0) = 1$

$f(0,2) = -3$

(b) on AB

$f(x,2) = 2x^2 - 4x - 3$

$f'(x,2) = 4x - 4 = 0 \rightarrow x = 1$

$f(0,2) = -3$

$f(1,2) = -5$

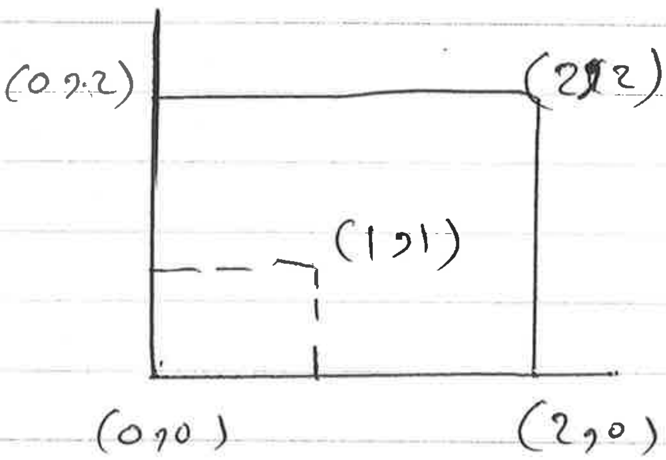
(c) on OB

$f(x,y) = f(x,2x) = 6x^2 - 12x + 1$

$f'(x,2x) = 12x - 12 = 0 \rightarrow x = 1$ and
$y = 2$
but $(1,2)$ is not an interior point of OB

Therfor, the absolute maximum is 1 at (0,0) and the absolute minum is (−5) at (1,2).

EX5: Find the absolute extrema of function $f(x,y) = x^2 - 3y^2 - 2x + 6y$ on the sequare region vertices (0,0), (0,2), (2,2) and (2,0) ?



$f(x,y) = x^2 - 3y^2 - 2x + 6y$ ①

$f_x = 2x - 2$ & $f_x = 0 \rightarrow 2x - 2 = 0 \rightarrow x = 1$

$f_y = -6y + 6$ & $f_y = 0 \rightarrow -6y + 6 = 0 \rightarrow y = 1$

(1,1) is the only critical point, this critical point is in the interior of R

The boundary of R consits of four line segments;

① The line segment between (0,0) and (2,0), one this line segment we have $y=0$ sub in ①

$f(x,0) = x^2 - 2x \qquad 0 \leq x \leq 2$

$f'(x,0) = 2x - 2 = 0$

$x = 1$

$y = 0$

the extrema occur either at the critical point $x=1$ or at the end points $x=0$ and $x=2$

The end points are (0,0), (2,0) and critical point is (1,0).

② The line segment between (0,0) and (0,2), on this line segment we have $x=0$, sub in ①

$f(0,y) = -3y^2 + 6y \qquad 0 \le y \le 2$

$\acute{f}(0,y) = -6y + 6 = 0 \longrightarrow y = 1$
$x = 0$

The extrema occur either at the critical point $y = 1$ or at the end points $y = 0$ and $y = 2$.

The end points are $(0,0)$, $(0,2)$ and critical point is $(0,1)$.

(3) The line segment between $(0,2)$ and $(2,2)$, on this line segment we have $y = 2$, sub in (1)

$f(x,2) = x^2 - 2x \qquad 0 \le x \le 2$

$\acute{f}(x,2) = 2x - 2 = 0 \longrightarrow x = 1$
$y = 2$

The extrema occur either at the critical point $x = 1$ or at the end points $x = 0$ and $x = 2$.

The end points are $(0,2)$, $(2,2)$ and critical point is $(1,2)$

④ The line segment between $(2,0)$ and $(2,2)$, on this segment we have $x=2$ sub in ①

$$f(2,y) = -3y^2 + 6y \qquad 0 \le y \le 2$$

$$f'(2,y) = -6y + 6 = 0 \longrightarrow y = 1$$
$$x = 2$$

The extrema occur either at the critical point $y=1$ or at the end points at $y=0$ and $y=2$.

The end points are $(2,0)$ $(2,2)$ and critical point is $(2,1)$

(x,y)	$(0,0)$	$(0,2)$	$(2,0)$	$(2,2)$	$(1,1)$	$(1,0)$	$(0,1)$	$(1,2)$	$(2,1)$
$f(x,y)$	0	0	0	0	2	-1	3	-1	3
						min	max	min	max

The absolute maximum occur at $(0,1)$, $(2,1)$ and absolute minimum occure at $(1,0)$, $(1,2)$.