2- Macaulay's Method (Discontinuity Functions or Singularity Functions):

The method of integration, used to find the equation of the elastic curve for a beam, is convenient if the load or internal moment can be expressed as a continuous function throughout the beam's entire length. If several different loadings act on the beam, however, this method can become tedious to apply, because separate loading or moment functions must be written for each region of the beam.

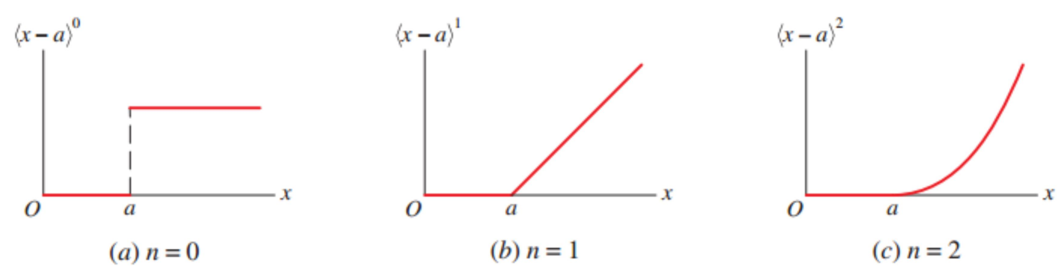
In this method, we will find the equation of the elastic curve using a single expression, either formulated directly from the loading on the beam, w = w(x), or from the beam's internal moment, M = M(x). Then when this expression for w is substituted into EI d4y/dx4 = w(x) and integrated four times, or if the expression for M is substituted into EI d2y/dx2 =M(x) and integrated twice, the constants of integration will only have to be determined from the boundary conditions. Macaulay functions, **can be written in general form as:**

$$\langle x-a\rangle^n = \begin{cases} 0 & \text{when } x < a \\ (x-a)^n & \text{when } x \geq a \end{cases} \quad \text{for } n \geq 0 \ (n = 0,\ 1,\ 2, \ldots)$$

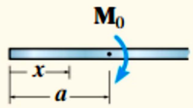
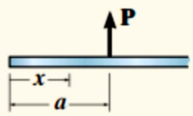
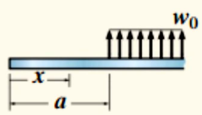
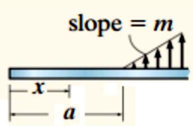
Here x represents the location of a point on the beam, and a is the location where the distributed loading begins. As stated by the equation, only when x ≥ a is <x-a>n=(x-a)n; otherwise it is zero. Integration of the Macaulay function follows the same rules as for ordinary functions, i.e.

$$\int \langle x-a\rangle^n dx = \ = \frac{\langle x-a\rangle^{n+1}}{n+1} + C$$

Three Macaulay functions corresponding, respectively, to n = 0, n = 1, and n = 2 are plotted in the figure shown.



Macaulay functions for a uniform and triangular load are shown in table below. Using integration, the Macaulay functions for shear, $V = \int w(x)\,dx$, and moment, $M = \int V\,dx$, are also shown in the table.

Loading	Loading Function $w = w(x)$	Shear $V = \int w(x)dx$	Moment $M = \int Vdx$
	$w = M_0\langle x-a\rangle^{-2}$	$V = M_0\langle x-a\rangle^{-1}$	$M = M_0\langle x-a\rangle^{0}$
	$w = P\langle x-a\rangle^{-1}$	$V = P\langle x-a\rangle^{0}$	$M = P\langle x-a\rangle^{1}$
	$w = w_0\langle x-a\rangle^{0}$	$V = w_0\langle x-a\rangle^{1}$	$M = \frac{w_0}{2}\langle x-a\rangle^{2}$
	$w = m\langle x-a\rangle^{1}$	$V = \frac{m}{2}\langle x-a\rangle^{2}$	$M = \frac{m}{6}\langle x-a\rangle^{3}$