Al-Mustansiriyah University Faculty of Engineering Mech. Eng. Dept. Graduate studies Exam : Second Semester		Subject: Mathematics II Max. Time: Three Hours Class: 2nd Date: 1/4/2018

2017 – 2018 **Max.Mark:100%**

Anser only Three quastions

Q1/ Choose the correct answer to reverse the order of the integration:

$$\int_{\frac{1}{2}}^{1}\int_{x^3}^{x} f(x,y)\,dy\,dx$$

$a-\left(\int_{0}^{\frac{1}{2}}\int_{y}^{1} f(x,y)dx\,dy+\int_{\frac{1}{2}}^{3}\int_{y}^{y^3} f(x,y)dx\,dy+\right)$ $b-\left(\int_{\frac{1}{6}}^{1}\int_{y^3}^{y} f(x,y)dx\,dy\right)$

$c-\left(\int_{\frac{1}{4}}^{2}\int_{y^3}^{y} f(x,y)dx\,dy\right)$ $d-(none\ of\ them)$

…..(33mark)

Q2/ Choose the correct answer to evaluate the integration:

$$\int_{0}^{1}\int_{x^2}^{x} \frac{1}{\sqrt{x^2+y^2}\cos(\tan^{-1}(\frac{y}{x}))}\,dy\,dx$$

a-(8/6) **b(1/4)** **c(1)** **d(none of them)**

…..(33mark)

Q3/ Consider the lines:

L_1: $x = 1 + 2c$ L_2: $x = 2 + s$

$y = 2 + 3c$ $y = 4 + 2s$

$z = 3 + 4c$ $z = -1 - 4s$

1- Find the point of intersection of the two lines.
2- Find the plane determined by these two lines.
3- Find the distance from the point **P = (2, 1, 3)** to the line $\mathbf{L_1}$.

…..(33mark)

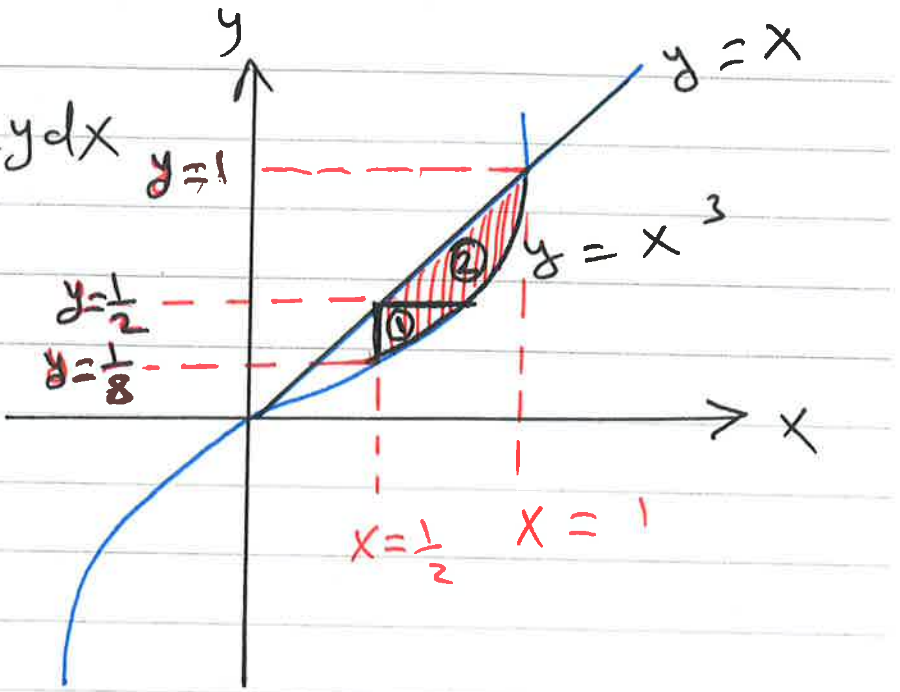
Q4/ Let the points **A=(2, 4, -3)**, **B=(3,-1, 1)** and C=(-3, 0,1). Find:1- Parametric equations for the line **L** containing **A** and **B**. 2- Determine where the line **L** intersects **xy –plane**. 3-Area of a triangle **ABC**. 4- An equation for the plane containing **A**, **B**, and **C**.

…..(33mark)

Examiner: ***Asst. Prof. Dr. Hayder M. Jaffal*** **Head of Department:** ***Prof. Dr. Mohammed H. Alhamdo***

Q1

$$\int_{1/2}^{1}\int_{x^3}^{x} f(x,y)\,dy\,dx$$



① and ②

$y = x^3$

$y = x$

$x^3 = x$

$x^2 = 1$

$x = \mp 1$

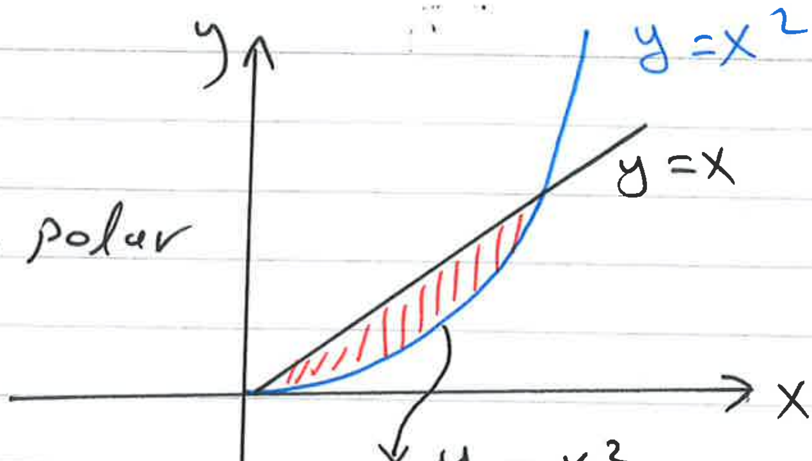
$(y = x)$ $x = \frac{1}{2} \longrightarrow y = \frac{1}{8}$

$(y = x^3)$ $x = \frac{1}{2} \longrightarrow y = \frac{1}{2}$

$$\int_{y=1/8}^{y=1/2}\int_{x=1/2}^{\sqrt[3]{y}} f(x,y)\,dx\,dy + \int_{y=1/2}^{y=1}\int_{x=y}^{\sqrt[3]{y}} f(x,y)\,dx\,dy$$

d- Non of them

Q2/ $$\int_0^1\int_{x^2}^{x} \frac{1}{\sqrt{x^2+y^2}\,\cos\left(\tan^{-1}\left(\frac{y}{x}\right)\right)}\,dy\,dx$$



by convert to polar coordinates

$y = x^2$

$\cancel{r}\sin\theta = r^{\cancel{2}}\cos^2\theta$

$r = \frac{\sin\theta}{\cos\theta}\,\frac{1}{\cos\theta}$

$r = \tan\theta \cdot \sec\theta$

$$\int_0^{\pi/4}\int_0^{\tan\theta\sec\theta} \frac{1}{\cancel{r}\cos\theta}\cdot \cancel{r}\,dr\,d\theta$$

$$= \int_0^{\pi/4}\int_0^{\tan\theta\sec\theta} \sec\theta\,dr\,d\theta = \int_0^{\pi/4} \sec\theta\, r\Big|_0^{\tan\theta\sec\theta} d\theta$$

$$= \int_0^{\pi/4} \tan\theta\,\sec^2\theta\,d\theta = \frac{\tan^2\theta}{2}\Big|_0^{45} = \frac{1}{2}$$

d- Non of them

Q3/

1- $1+2C = 2+S$ (1)

$2+3C = 4+2S$ (2)

$3+4C = -1-4S$ (3)

by solving (1) and (2)

$(1+2C = 2+S) \times 2$

$2+3C = 4+2S$

$\boxed{C=0}$ sub in (1)

$1+2(0) = 2+S$

$\boxed{S=-1}$

sub C and S in (3)

$3+4(0) = -1-4(-1)$

$3 = 3$ $\therefore$ intersect

sub $C=0$ into L_1

$x = 1+2(0) = 1$

$y = 2+3(0) = 2$

$z = 3+4(0) = 3$

intersection point (1, 2, 3)

2-

$$\vec{N} = \vec{v}_1 \times \vec{v}_2$$

$$\vec{v}_1 = 2i + 3j + 4K$$

$$\vec{v}_2 = 2i + 4j - K$$

$$\vec{N} = \begin{vmatrix} i & -j & K \\ 2 & 3 & 4 \\ 1 & 2 & -4 \end{vmatrix}$$

$$\vec{N} = \begin{vmatrix} 3 & 4 \\ 2 & -4 \end{vmatrix} i - \begin{vmatrix} 2 & 4 \\ 1 & -4 \end{vmatrix} j + \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} K$$

$$\vec{N} = -20i + 12j + K$$

$$n_1 x + n_2 y + n_3 z = D$$

$$-20x + 12y + K = -20(1) + 12(2) + 1(3)$$

$$-20x + 12y + K = 7$$

3 -

$$\left.\begin{array}{l} P = (2, 1, 3) \\ P_1 = (1, 2, 3) \end{array}\right\} \quad \vec{P_1P} = i - j$$

$$\vec{v}_1 = 2i + 3j + 4k$$

$$\vec{P_1P} \times \vec{v}_1 = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 2 & 3 & 4 \end{vmatrix}$$

$$\vec{P_1P} \times \vec{v}_1 = \begin{vmatrix} -1 & 0 \\ 3 & 4 \end{vmatrix} i - \begin{vmatrix} 1 & 0 \\ 2 & 4 \end{vmatrix} j + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} k$$

$$\vec{P_1P} \times \vec{v}_1 = -4i - 4j + 5k$$

$$D = \frac{|\vec{P_1P} \times \vec{v}_1|}{|\vec{v}_1|} = \frac{\sqrt{(-4)^2 + (-4)^2 + (5)^2}}{\sqrt{(2)^2 + (3)^2 + (4)^2}}$$

$$D = \frac{\sqrt{57}}{\sqrt{29}} = 1.4019$$

Q4/

1 $\vec{AB} = i - 5j + 4k \qquad A(2,4,-3)$

$$\frac{x - x_1}{v_1} = \frac{y - y_1}{v_2} = \frac{z - z_1}{v_3} = c$$

$$\frac{x-2}{1} = \frac{y-4}{-5} = \frac{z+3}{4} = c$$

$x = 2 + c$
$y = 4 - 5c$
$z = -3 + 4c$

2- the line intersect xy-plane at
$z = 0$

$$\therefore \frac{(0)+3}{4} = c \longrightarrow c = \frac{3}{4}$$

$$x = 2 + \frac{3}{4} = \frac{11}{4}$$

$$y = 4 - 5\frac{3}{4} = \frac{1}{4}$$

intersection point is $\left(\frac{11}{4}, \frac{1}{4}, 0\right)$

3- $\text{Area} = \dfrac{|\vec{N}|}{2}$

$\vec{AB} = i - 5j + 4k$
$\vec{AC} = -5i - 4j + 4k$

$$\vec{N} = \vec{AB} \times \vec{AC} = \begin{vmatrix} i & -j & k \\ 1 & -5 & 4 \\ -5 & -4 & 4 \end{vmatrix}$$

$$\vec{N} = \begin{vmatrix} -5 & 4 \\ -4 & 4 \end{vmatrix} i - \begin{vmatrix} 1 & 4 \\ -5 & 4 \end{vmatrix} j + \begin{vmatrix} 1 & -5 \\ -5 & -4 \end{vmatrix} k$$

$$\vec{N} = -4i - 24j - 29k$$

$$Area = \frac{|\vec{N}|}{2} = \frac{\sqrt{(-4)^2 + (-24)^2 + (-29)^2}}{2} = 18.03$$

4- $n_1 x + n_2 y + n_3 z = D$

$$\vec{N} = 4i - 24j - 29k$$

$A(2, 4, -3)$

$$-4x - 24y - 29z = -4(2) - 24(4) - 29(-3)$$

$$-4x - 24y - 29z = -17$$