Approximate Analysis Methods of ACI

A- Two-Way Slab on Stiff Supports: Stiff support (brick walls, reinforced concrete walls or stiff beams with $h_{beam} > 3h_{slab}$) on all four sides may be used for slab supports, so that Two-Way Slab action is obtained. Method 3 has been proposed for analysis and design of Two-Way Slab that supported on stiff supports. Direct design method and equivalent frame method can provide alternative approaches for analysis and design of Two-Way Slab system on stiff supports.

B-Two-Way Slabs on Flexible Supports: Two-Way Slab may be supported on flexible beams ($h_{beam} < 3h_{slab}$) on four sides. Method 3 cannot be used for analysis or design of these slab systems. Then the direct design method or equivalent frame method can be considered as the main design approaches for these systems.

4) Deflection Control of Two way slabs

Minimum thickness/span ratios enable the designer to avoid extremely complex deflection calculations in routine designs. Deflections of two-way slab systems need not be computed if the overall slab thickness meets the minimum requirements specified in ACI 9.5.3. Minimum slab thicknesses for flat plates, flat slabs (and waffle slabs), and two-way slabs, based on the provisions in ACI 9.5.3, are summarized in Table 4-1, where (l_n) is the clear span length in the long direction of a two-way slab panel. The tabulated values are the controlling minimum thicknesses governed by interior or exterior panels assuming a constant slab thickness for all panels making up a slab system. Practical spandrel beam sizes will usually provide beam-to-slab stiffness ratios, (α_f) greater than the minimum specified value of 0.8; if this is not the case, the spandrel beams must be ignored in computing minimum slab thickness. A standard size drop panel that would allow a 10% reduction in the minimum required thickness of a flat slab floor system is illustrated in Fig. 4-2. Note that a larger size and depth drop may be used if required for shear strength; however, a corresponding lesser slab thickness is not permitted unless deflections are computed.

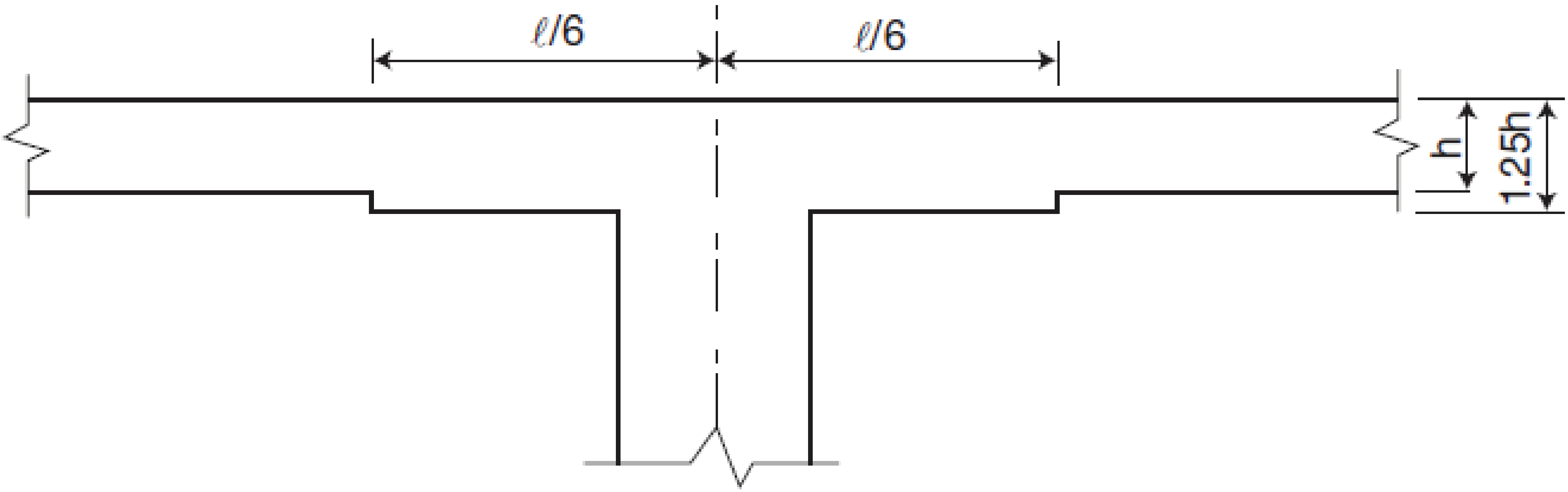


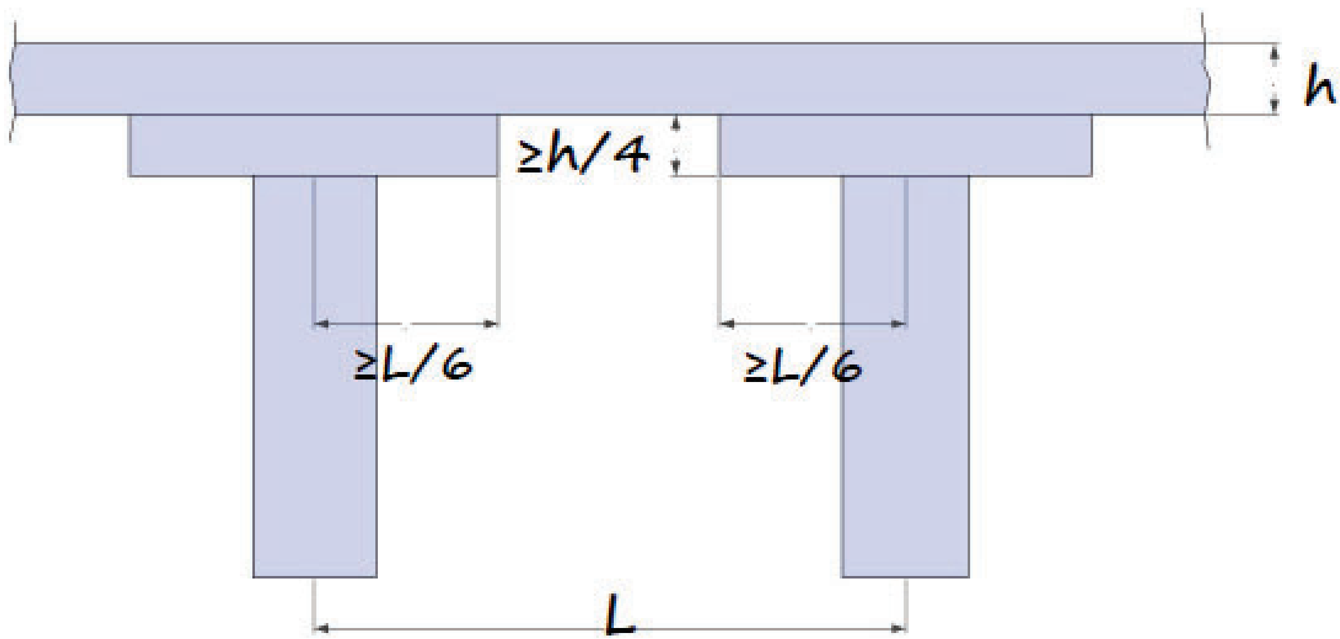
Figure 4-2 Drop Panel Details (ACI 13.3.7)

13.3.7 — When a drop panel is used to reduce the amount of negative moment reinforcement over the column of a flat slab, the dimensions of the drop panel shall be in accordance with 13.2.5. In computing required slab reinforcement, the thickness of the drop panel below the slab shall not be assumed to be greater than one-quarter the distance from the edge of drop panel to the face of column or column capital.

13.2.5 — When used to reduce the amount of negative moment reinforcement over a column or minimum required slab thickness, a drop panel shall:

(a) project below the slab at least one-quarter of the adjacent slab thickness; and

(b) extend in each direction from the centerline of support a distance not less than one-sixth the span length measured from center-to-center of supports in that direction.



Drop panel with dimensions less than those specified in 13.2.5 may be used to increase shear strength. In computing required slab reinforcement, the thickness of drop panel below the slab shall not assumed greater than (1/4) the distance from edge of column or column capital (ACI 13.3.7)

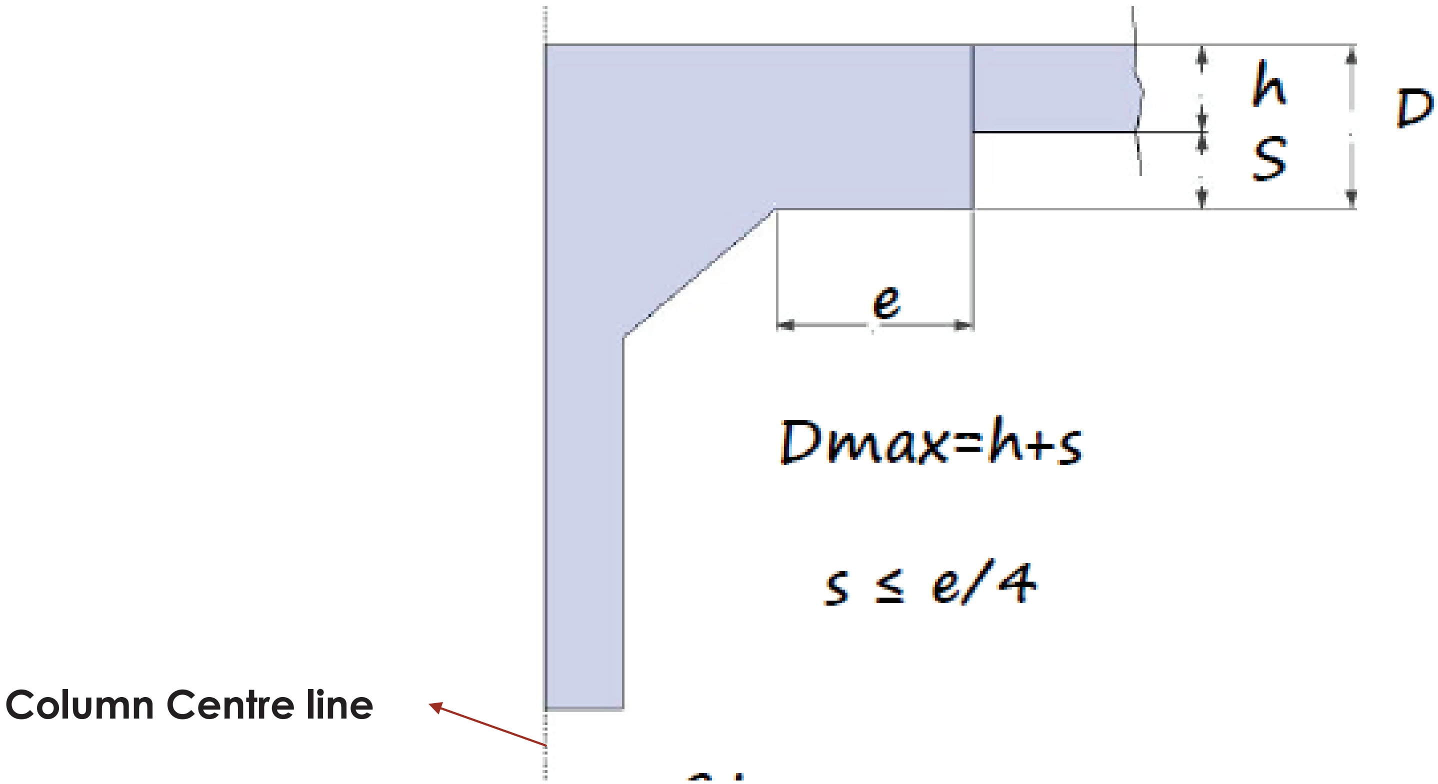


Table ***4-1*** *Minimum Thickness of Slabs without Interior Beams (Table 9.5(c))*

Yield strength, f_y **(Mpa.)** psi*	Without drop panels†			With drop panels†		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams††		Without edge beams	With edge beams††	
(280 Mpa.) 40,000	$\frac{\ell_n}{33}$ **	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{40}$	$\frac{\ell_n}{40}$
(420 Mpa.) 60,000	$\frac{\ell_n}{30}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$
(520 Mpa.) 75,000	$\frac{\ell_n}{28}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{34}$	$\frac{\ell_n}{34}$

* *For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.*

** *For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.*

† *Drop panel is defined in 13.2.5.*

†† *Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.*

Table 4-1 and Figure 4-3 gives the minimum slab thickness h based on the requirements given in ACI 9.5.3; α_{fm} is the average value of α_f (ratio of flexural stiffness of beam to flexural stiffness of slab) for all beams on the edges of a panel, and β is the ratio of clear spans in long to short direction.

Table 4-1 Minimum Thickness for Two-Way Slab Systems

Two-Way Slab System		α_{fm}	β	Minimum h
Flat Plate, exterior panel	**Min. h = 125 mm**	—	≤ 2	$\ell_n/30$
Flat Plate, interior panel and exterior panel with edge beam[1]	[Min. h = 5 in.]	—	≤ 2	$\ell_n/33$
Flat Slab[2]	**Min. h = 100 mm**	—	≤ 2	$\ell_n/33$
Flat Slab, interior panel and exterior panel with edge beam[1]	[Min. h = 4 in.]	—	≤ 2	$\ell_n/36$

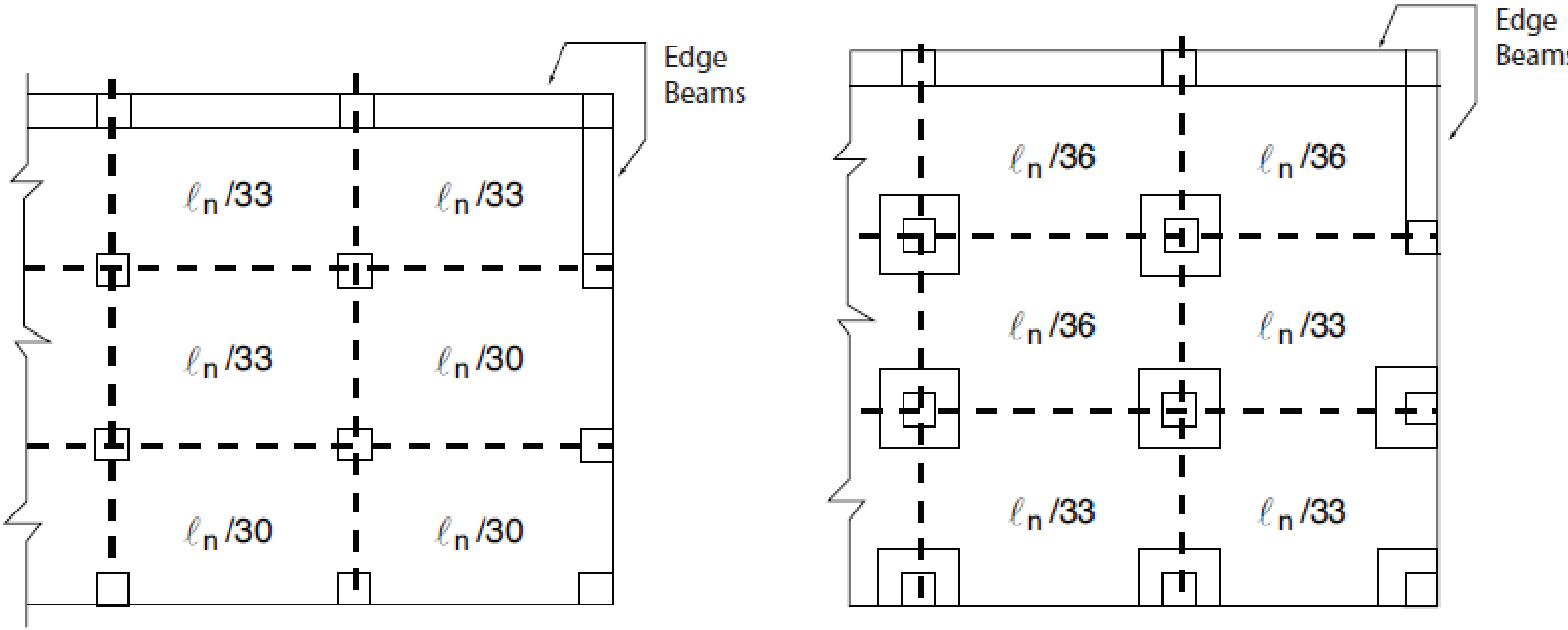
Continue Table (4 – 1)

Two-Way Beam-Supported Slab[3]	≤ 0.2 1.0 ≥ 2.0	≤ 2 1 2 1 2	$\ell_n/30$ $\ell_n/33$ $\ell_n/36$ $\ell_n/37$ $\ell_n/44$
Two-Way Beam-Supported Slab[1,3]	≤ 0.2 1.0 ≥ 2.0	≤ 2 1 2 1 2	$\ell_n/33$ $\ell_n/36$ $\ell_n/40$ $\ell_n/41$ $\ell_n/49$

[1] Spandrel beam-to-slab stiffness ratio $\alpha_f \geq 0.8$ (ACI 9.5.3.3)

[2] Drop panel length $\geq \ell/3$, depth $\geq 1.25h$ (ACI 13.3.7)

[3] Min. h = 5 in. for $\alpha_m \leq 2.0$; min. h = 3.5 in. for $\alpha_m > 2.0$ (ACI 9.5.3.3)

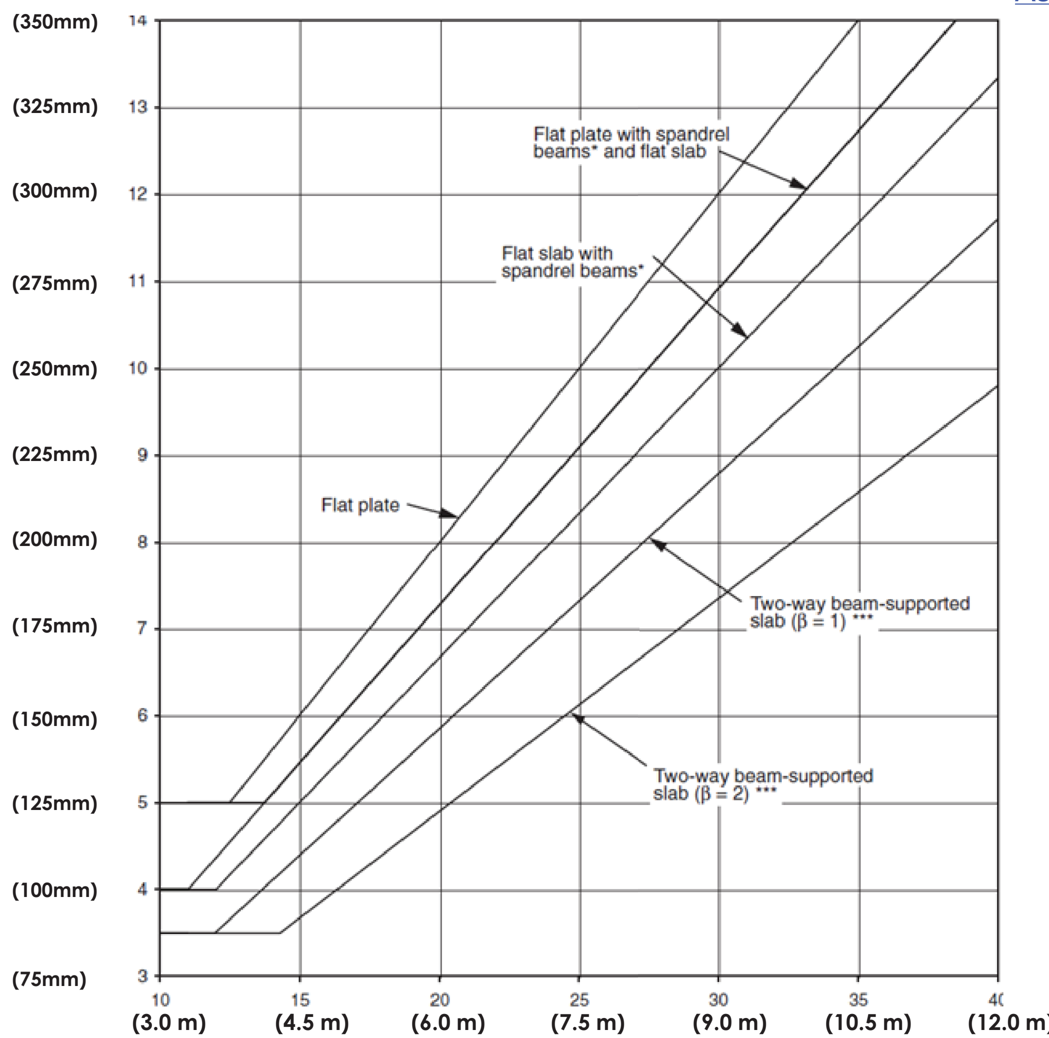


Flat Plates | Flat Slabs

α_f for edge beam ≥ 0.8

ℓ_n = the longer clear span

Figure 4-3 Minimum Thickness for Flat Plates and Flat Slab (Grade 60 Reinforcing Steel)



*Spandrel beam-to-slab stiffness ratio $\alpha_f \geq 0.8$
**$\alpha_{fm} > 2.0$

Figure 4-4 Minimum Slab Thickness for Two-Way Slab Systems

9.5.3.2 — For slabs without interior beams spanning between the supports and having a ratio of long to short span not greater than 2, the minimum thickness shall be in accordance with the provisions of Table 9.5(c) and shall not be less than the following values:

(a) Slabs without drop panels as defined in 13.2.5 ....125 mm;
(b) Slabs with drop panels as defined in 13.2.5 ....100 mm

9.5.3.3 — For slabs with beams spanning between the supports on all sides, the minimum thickness, h, shall be as follows:

(a) For α_{fm} equal to or less than 0.2, the provisions of 9.5.3.2 shall apply;

(b) For α_{fm} greater than 0.2 but not greater than 2.0, h shall not be less than

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \qquad (9\text{-}12)$$

and not less than 125 mm;

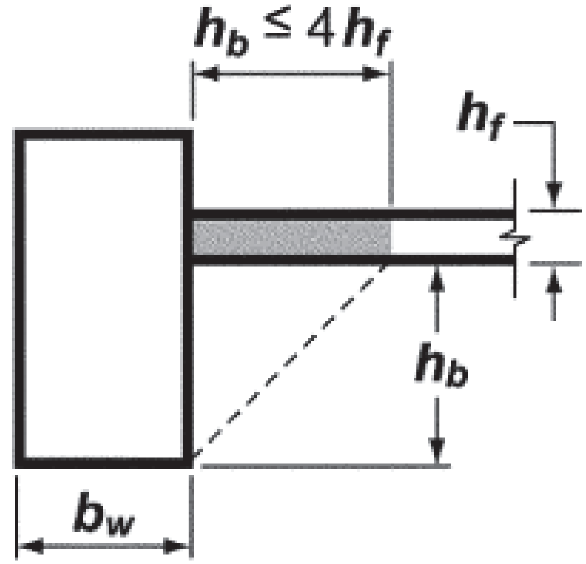
(c) For α_{fm} greater than 2.0, h shall not be less than

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} \qquad (9\text{-}13)$$

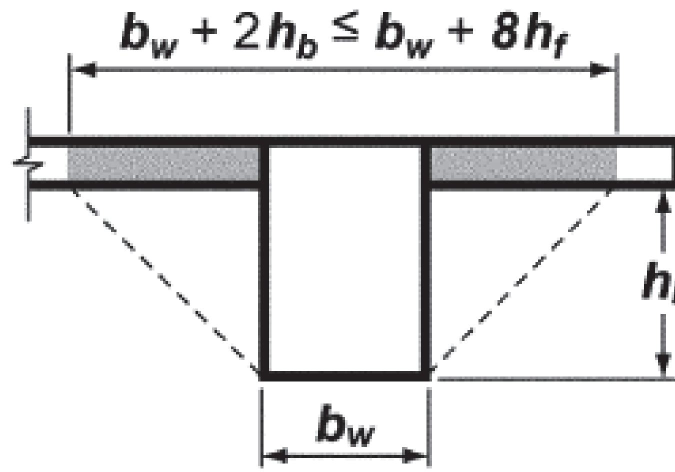
and not less than 90 mm;

(d) At discontinuous edges, an edge beam shall be provided with a stiffness ratio α_f not less than 0.80 or the minimum thickness required by Eq. (9-12) or (9-13) shall be increased by at least 10 percent in the panel with a discontinuous edge.

Term ℓ_n in (b) and (c) is length of clear span in long direction measured face-to-face of beams. Term β in (b) and (c) is ratio of clear spans in long to short direction of slab.



13.2.4 — For monolithic or fully composite construction, a beam includes that portion of slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness.



Ratio of Flexure Stiffness of Longitudinal Beams and Slabs (α_f)

When beams are used along the column lines in a two way floor system then the relative size of the beam to the thickness of slab can be considered as an important parameter that affecting the behavior and design of the floor system. This parameter (α_f) can be best measured by the ratio of the flexural stiffness of the beam to the flexural stiffness of the slab whose width equals the distance between the centerline of panels on each side of the beam.

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} \qquad (13\text{-}3)$$

Where E_{cb} , E_{cs} are modulus of elasticity of concrete for beams and slabs respectively. And calculated according to article 8.5.1 of ACI 318M - 11 .

8.5.1 — Modulus of elasticity, E_c, for concrete shall be permitted to be taken as $w_c^{1.5} 0.043 \sqrt{f_c'}$ (in MPa) for values of w_c between 1440 and 2560 kg/m^3. For normalweight concrete, E_c shall be permitted to be taken as $4700 \sqrt{f_c'}$.

$$E_c = 4700\sqrt{f_c'}$$

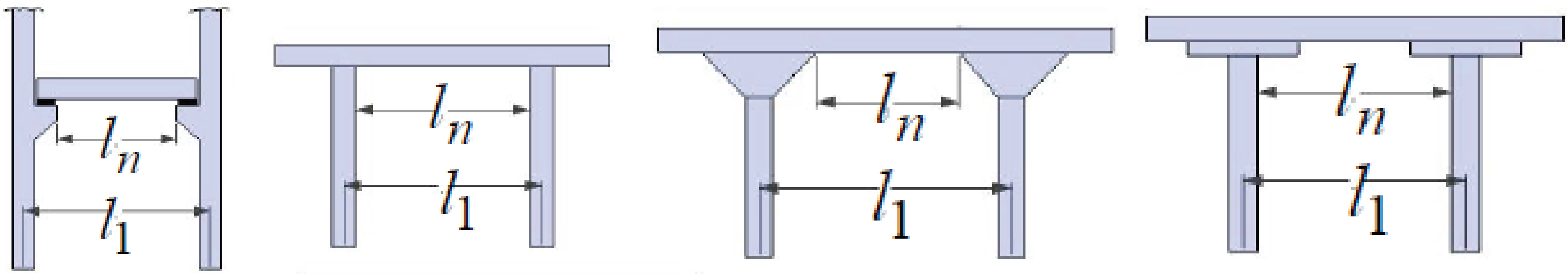
Where I_b , I_s are moment of inertia for beams and slabs respectively. And calculated according to prerequisite of engineering mechanics of gross section. Or using the following approximate method.

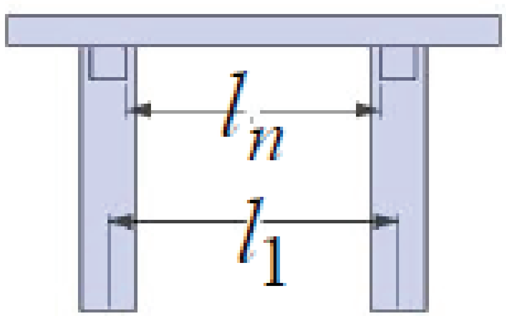
$$I_b \cong (b_w h^3 / 12) \times 2 \quad \text{for interior beam}$$

$$I_b \cong (b_w h^3 / 12) \times 1.5 \quad \text{for edge beam}$$

***** The effective length of the slab used in computing the moment of inertia is the distance between the centerlines of the adjacent panels for the slab above the interior beams, while the length of slab above the edge beam is the distance from the centerline of the panel to the end edge.***

***** l_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other support in other cases.***





Where l_1 length of span in direction of analysis measured center-to-center of supports, mm.

Note : $l_n \geq 0.65 l_1$

*** Compute (α_{fm}) for each panel where (α_{fm}) is the average value of (α_f) for all beams on edges of a panel (perpendicular beams).**

*** Compute (β) for each panel**

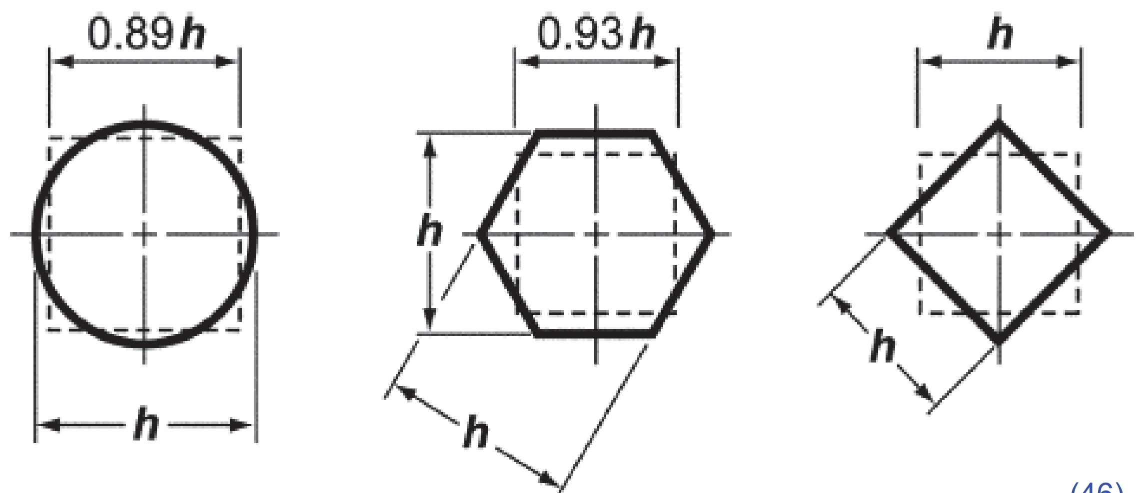
$$\beta = \frac{\text{clear span in long direction}}{\text{clear span in short direction}}$$

*** At discontinuous edge, if (α_f) of the edge beam < 0.8 then increase (h) with at least (10%) in the discontinuous edge.**

*** ACI 13.6.2.5: circular or regular polygon shaped support shall be treated as square support with the same area**

<u>Note:</u> (h) for any shape when be treated as a square is calculated as follow:

$$h = \sqrt{A}$$



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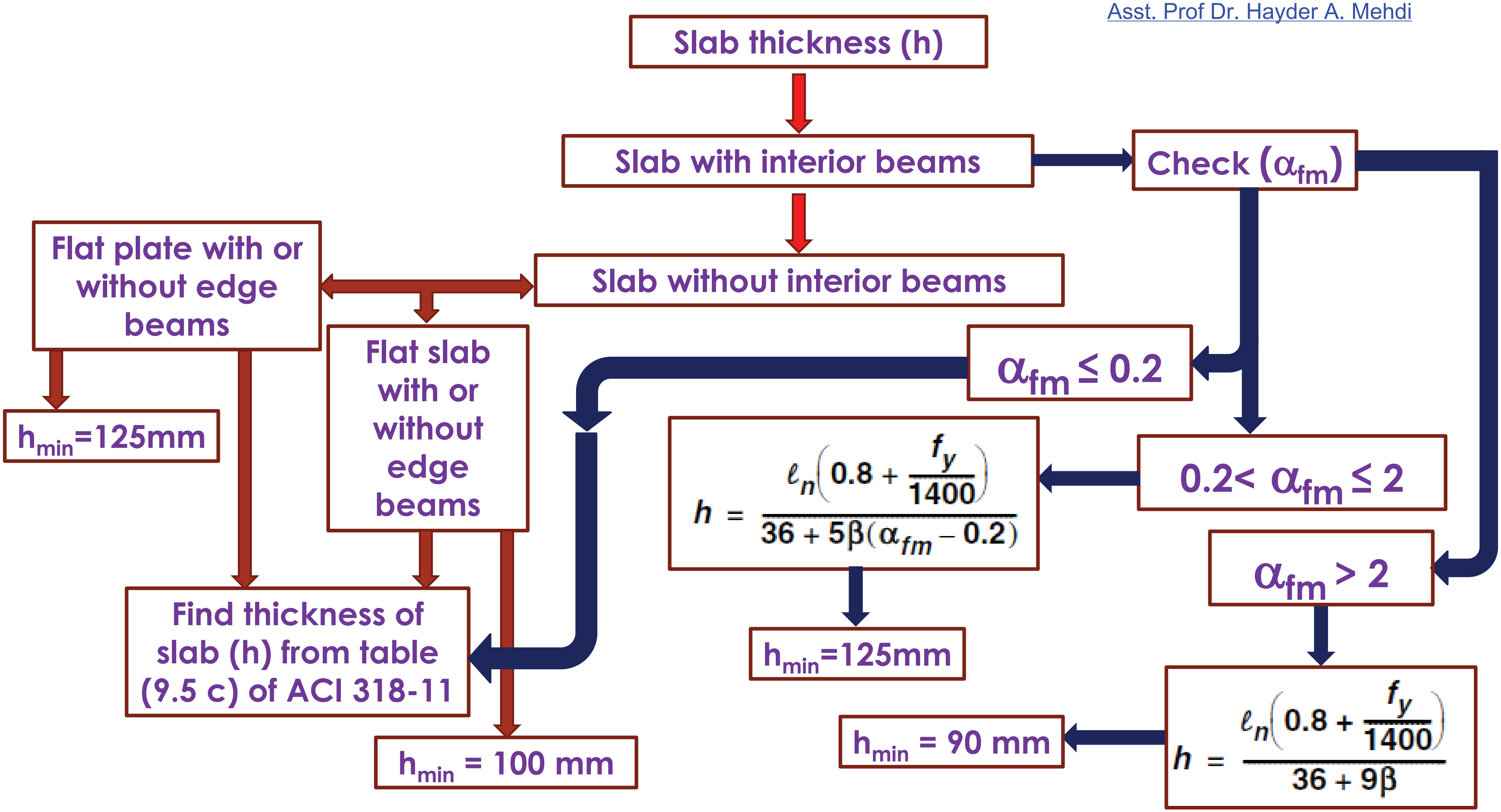
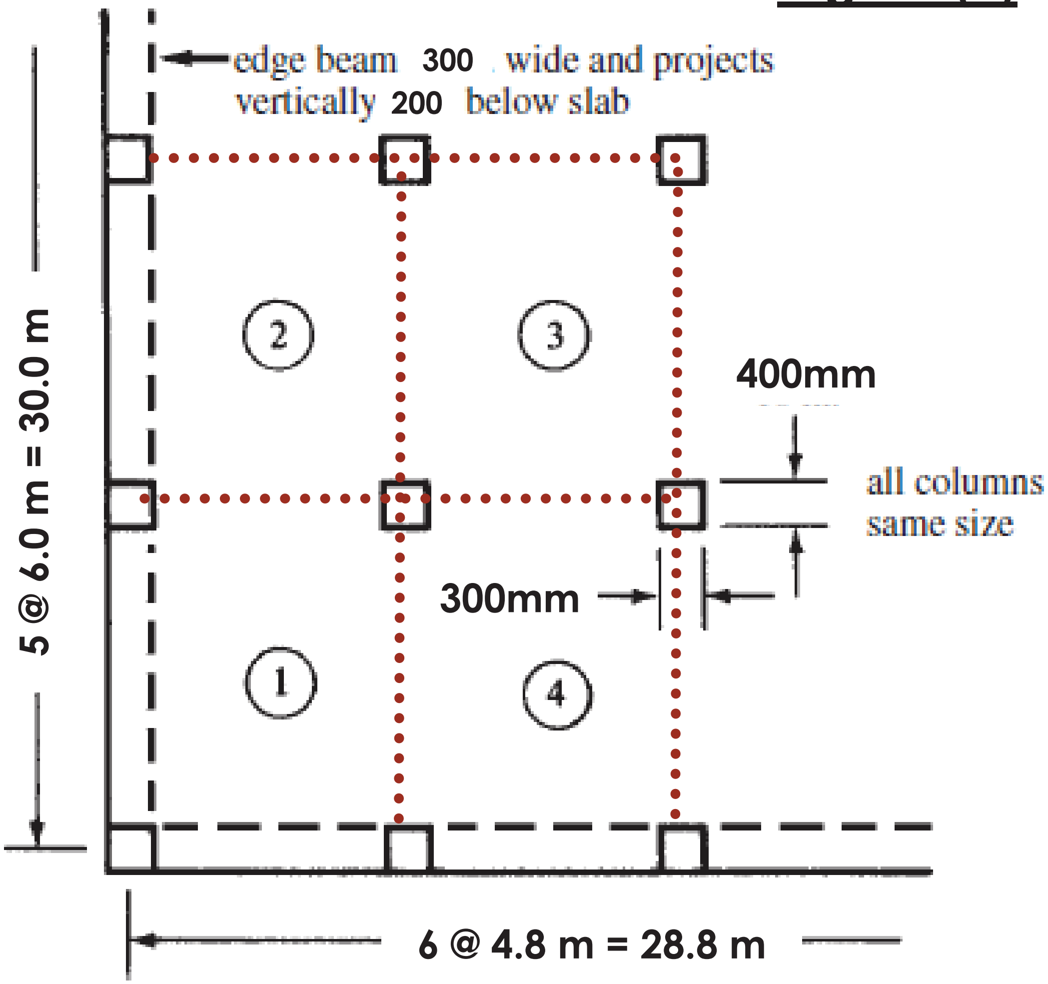


Note: **In the slab with interior beams of discontinuous edge, the stiffness of edge beam (α_f) must be equal or greater than (0.8). If (α_f) less than (0.8), the minimum thickness of slab must be increase with 10%.**

Example (01)

Using the ACI Code, determine the minimum permissible total thicknesses required for the slabs in panels 3 and 2 for the floor system shown in Figure (1). Edge beams are used around the building perimeter, and they are 300mm wide and extend vertically for 200mm below the slab. No drop panels are used, and the concrete in the slab is the same as that used in the edge beams. *fy*= 420 Mpa.

Figure (1)



Flat-plate floor slab for Example

Solution

A) Panel (3) is interior panel

Step 1: the slab is flat plate use table 9.5c in ACI 318 – 11 as shown In the flow chart

From table 9.5c for interior panel

$$h_{min} = \frac{l_n}{33}$$

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} \qquad (13\text{-}3)$$

$$\alpha_f = 0$$

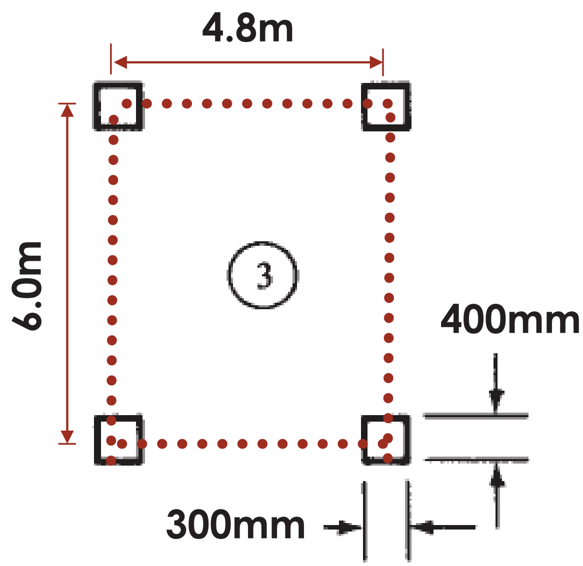
since the interior panels have no perimeter beams

Step 2: Fined clear span of largest length l_n

l_n= 6.0x1000 – 400
= 5600mm

$$h_{min} = \frac{5600}{33}$$

$$h_{min} = 169.7mm$$



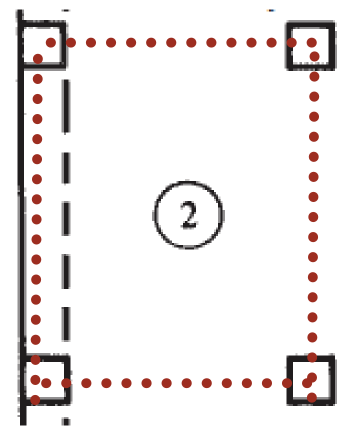
$$h_{min} \cong 170mm > 125mm\ ok$$

From above the minimum thickness required to The flat plate is **170mm** but must be not less than **125mm** according ACI, then using thickness of Slab equal to **175mm** is satisfy the above two Conditions.

A) Panel (2) is exterior panel

Step 1: there is an edge beam is found with width 300mm and clear depth 200mm

we must find α_f from equation (13-3).



$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} \qquad (13\text{-}3)$$

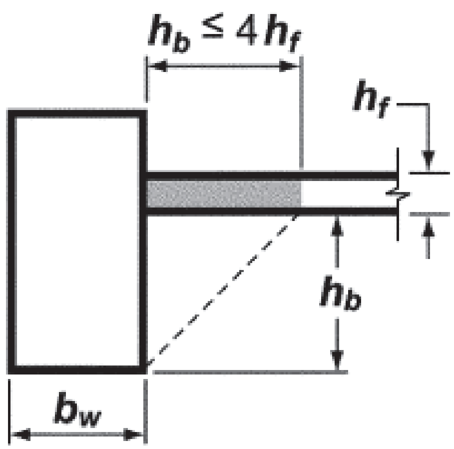
Step 2: from equation (13-3) we must find E_{cb}, E_{cs}, I_b, and I_s. And compressive strength of slab and beam is same.

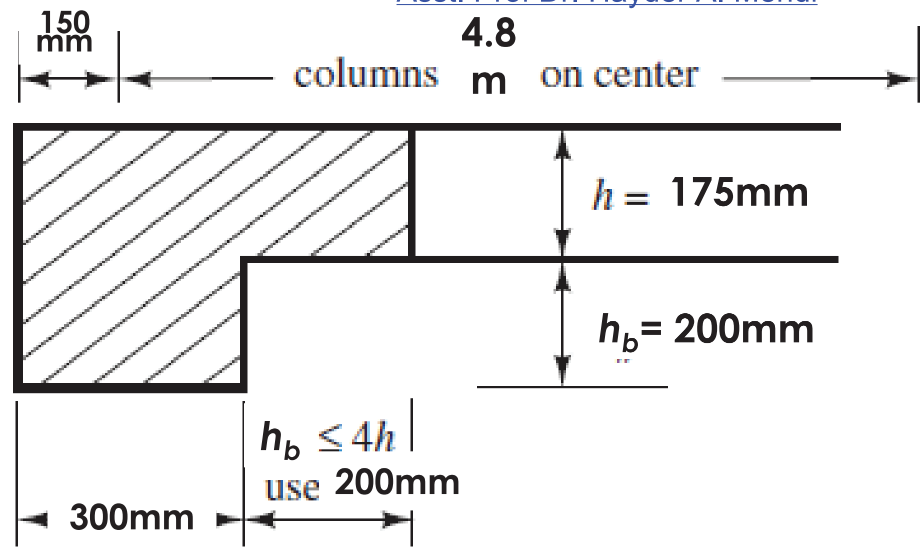
» $E_{cb} = E_{cs} = 4700\sqrt{f'_c}$

Step 3: fined moment of inertia of edge beam I_b

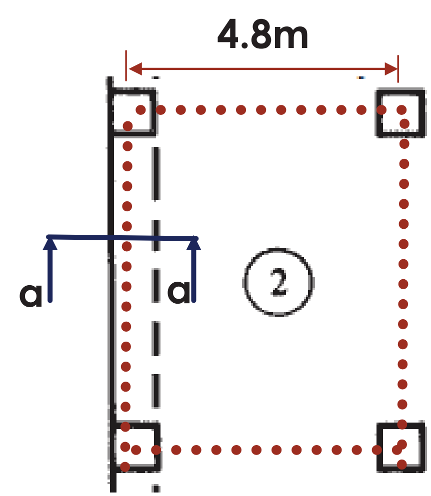
- fined dimensions of edge beam because of a part of slab is working as a flange with beam according ACI318-11 Code.

- b_w = 300mm
- h_b = 200mm
- h_f = 175mm
- $4h_f$ = 700mm
- $4h_f \geq h_b$ o.k





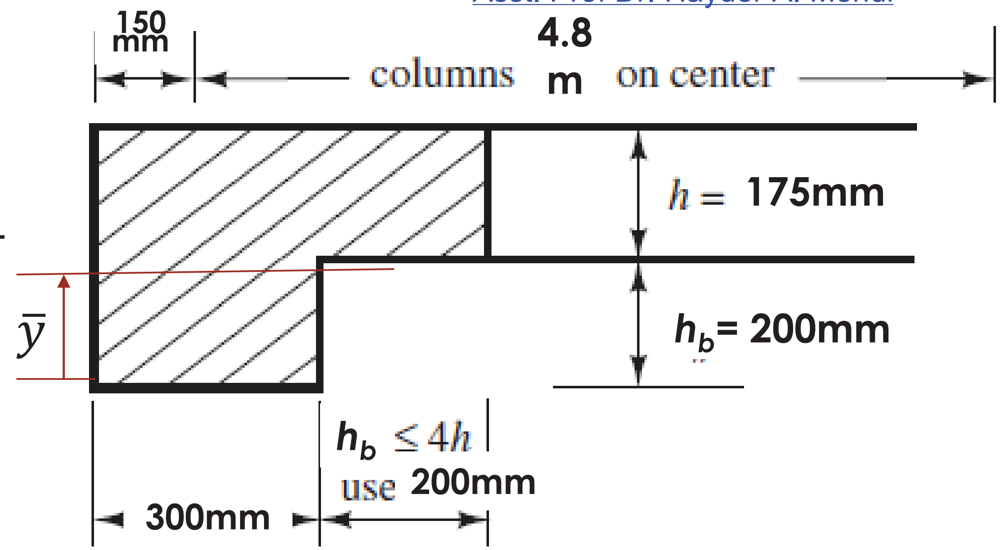
(a) Edge beam dimensions



» calculate moment of inertia by using accurate method about neutral axis of edge beam.

$$\bar{y} = \frac{(500)(175)(287.5) + (300)(200)(100)}{(500)(175) + (300)(200)}$$

$$\bar{y} = \frac{31156250}{147500} = 211.23mm$$



(a) Edge beam dimensions

$$I_b = \frac{(300)(375)^3}{12} + (300)(375)(211.23 - 187.5)^2 + \frac{(200)(175)^3}{12} + (200)(175)(211.23 - 287.5)^2$$

I_b =1381709576+292921868.2=1674631444mm^4

» calculate moment of inertia by using accurate method about neutral axis of slab.

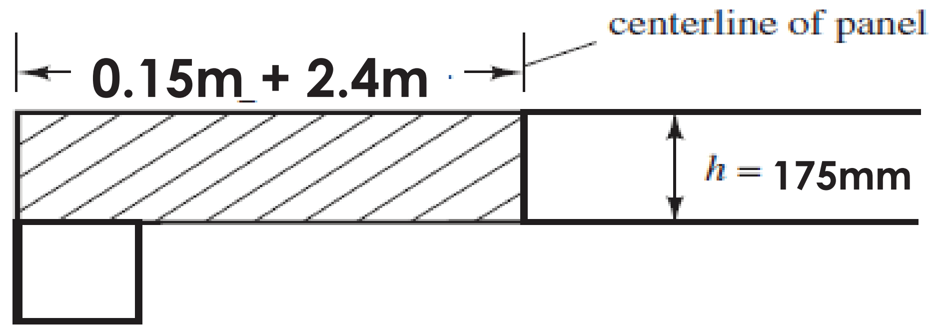
$$I_s = \frac{(2550)(175)^3}{12} = 1138867188mm^4$$

$$\propto_f = \frac{E_{cb} I_b}{E_{cs} I_s}$$ For this example $E_{cb}=E_{cs}$

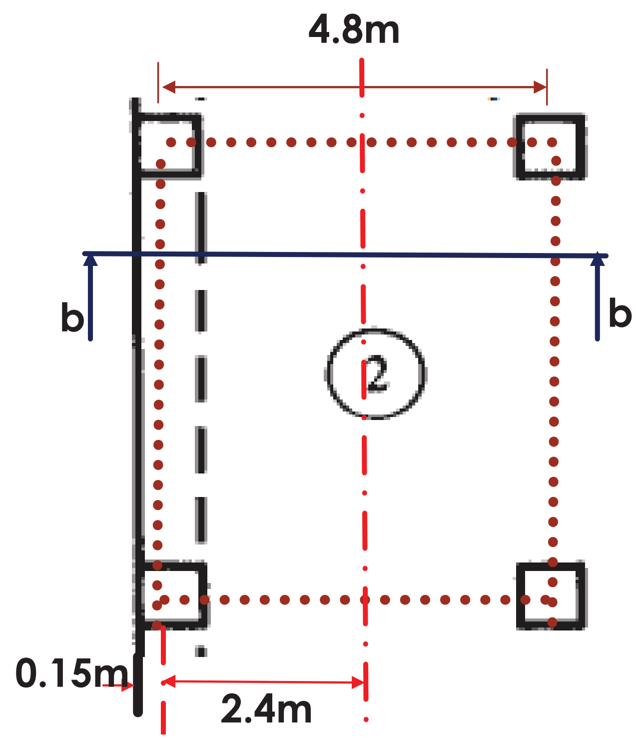
$$\propto_f = \frac{I_b}{I_s} = \frac{1674631444}{1138867188} = 1.47 > 0.8\ o.k$$

» because of α_f > 0.8 there is no need to increase the minimum thickness by 10%

» thickness h = 175mm is adequate for this slab



(b) Slab dimensions



» there is an approximate method for calculate the moment of inertia of beam as shown in the past lecture

$$I_b \cong (b_w h^3 / 12) \times 2$$ for interior beam

$$I_b \cong (b_w h^3 / 12) \times 1.5$$ for edge beam

$$I_b = \frac{(300)(375)^3}{12} x1.5 = 1977539063 mm^4$$

$$I_s = \frac{(2550)(175)^3}{12} = 1138867188 mm^4$$

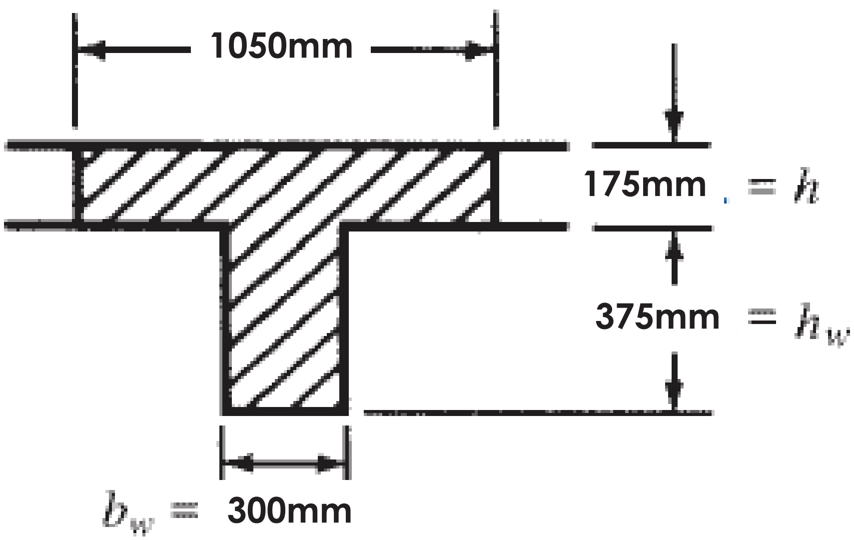
$$\propto_f = \frac{I_b}{I_s} = \frac{1977539063}{1138867188} = 1.74 > 0.8\ o.k$$

» thickness h = 175mm is adequate for this slab

Example (02)

The two-way slab shown in Figure (2) has been assumed to have a thickness of (175mm). Section A–A in the figure shows the beam cross section. Check the ACI equations to determine if the slab thickness is satisfactory for an interior panel. $\overline{fc}$ = 21Mpa., fy = 420Mpa., and normal-weight concrete.

$$b_w + 2h_w \le b + 8h$$



Section A–A

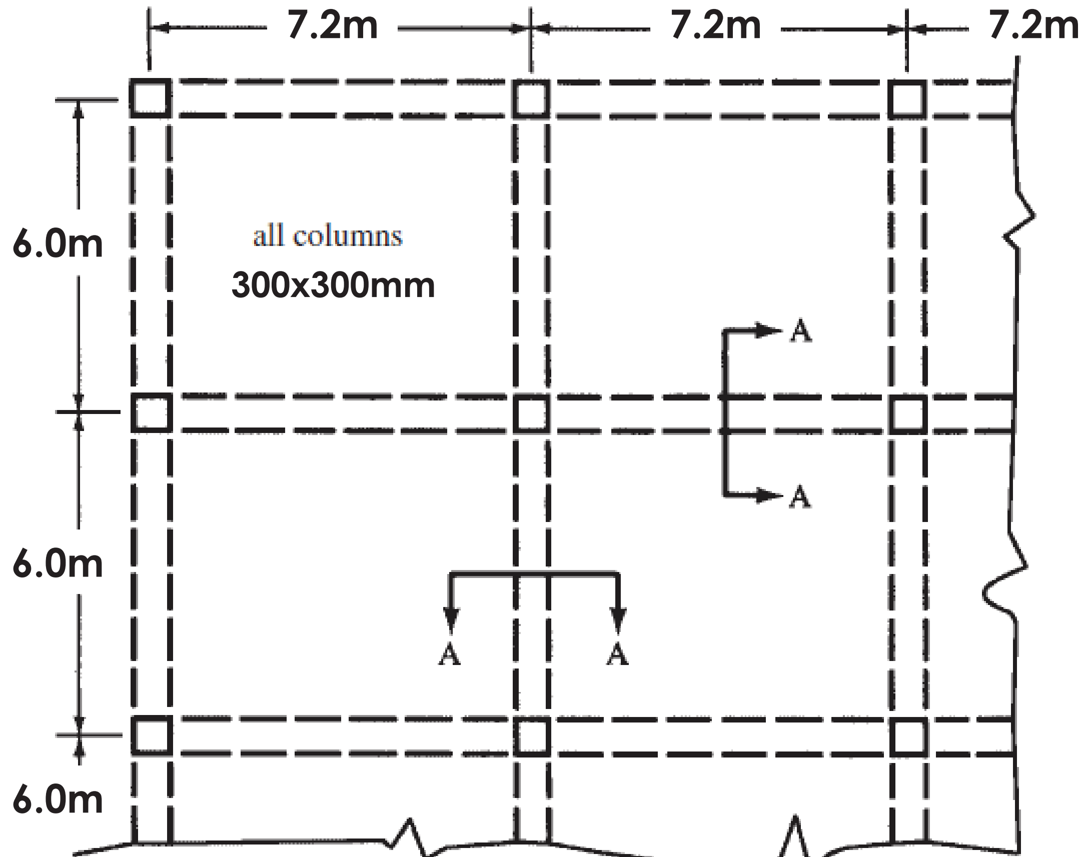


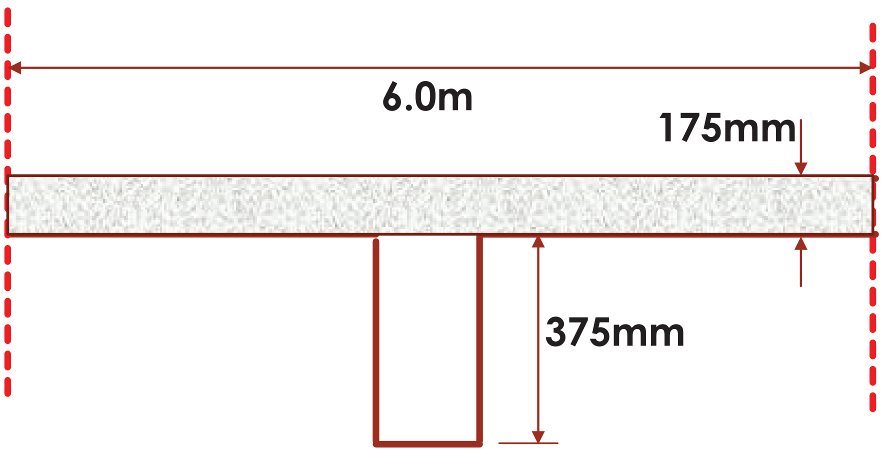
Figure (02)

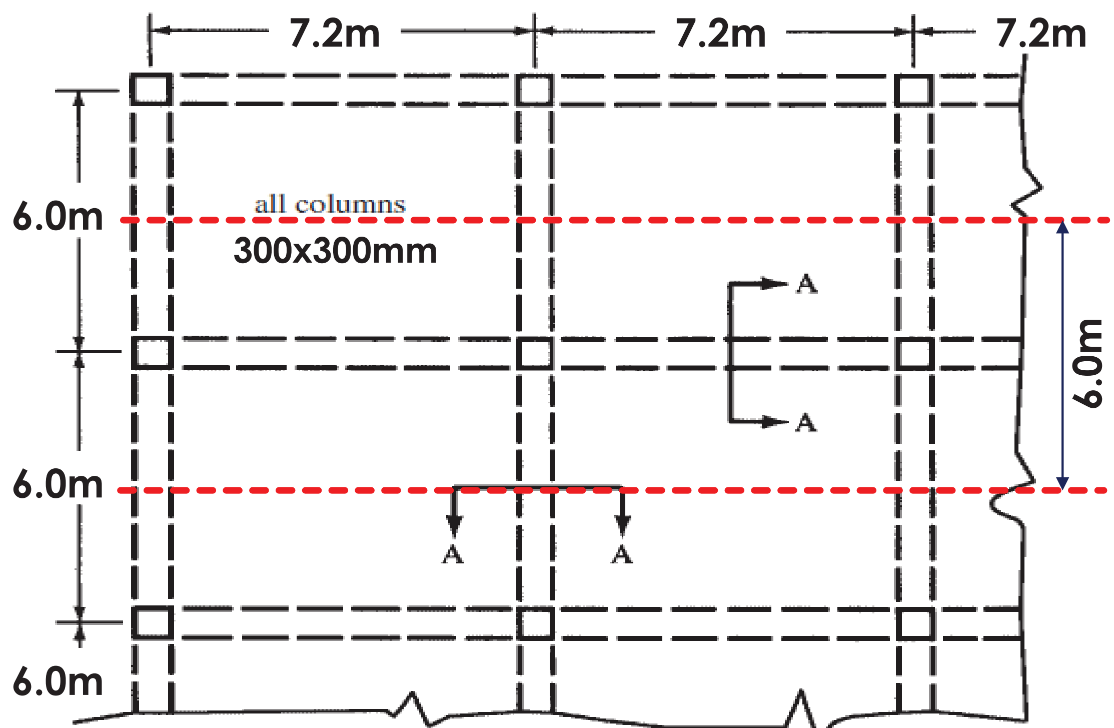
Solution:

Using the Same Concrete for Beams and Slabs, then $E_{cb}=E_{cs}$

» $\propto_f = \dfrac{E_{cb} I_b}{E_{cs} I_s}$ Then α_f becomes $\propto_f = \dfrac{I_b}{I_s}$

1) Computing α_1 for Long (Horizontal) Span for Interior Beams





I_s = gross moment of inertia of slab 6.0m wide

$$I_s = \frac{(6000)(175)^3}{12} = 2679687500 mm^4$$

I_b = gross I of T-beam cross section shown in Figure (2) about centroidal axis

» fined dimensions of interior beam because of a part of slab is working as a flange with beam according ACI318-11 Code.

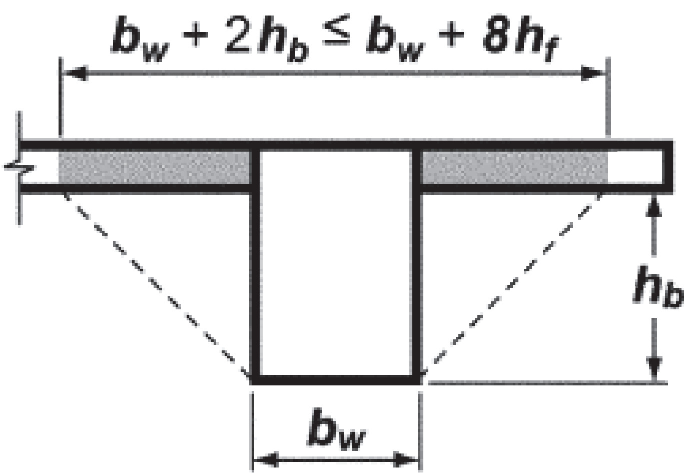
» b_w = 300mm

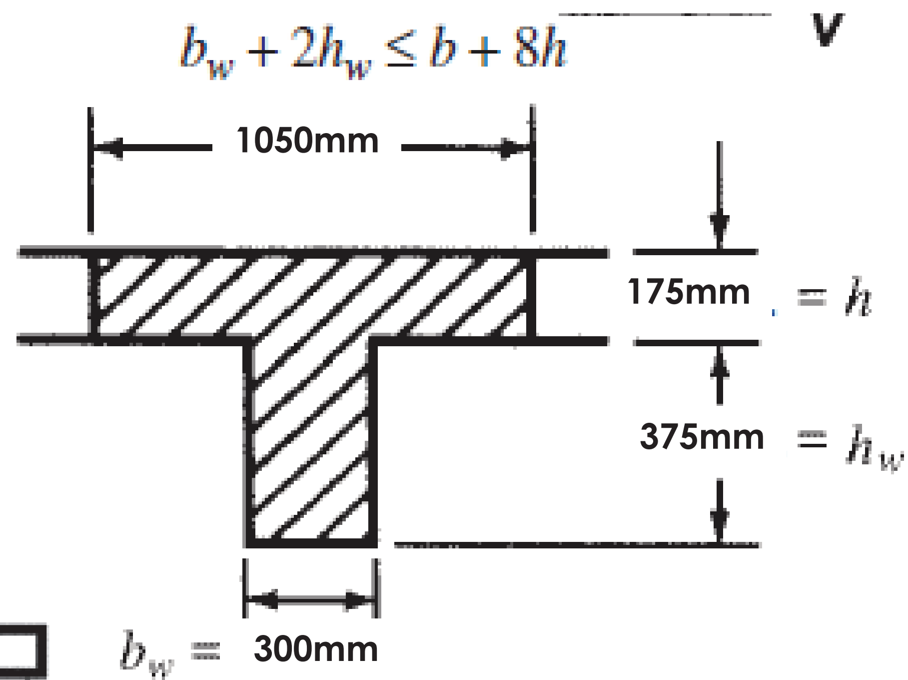
» h_w = 375mm

» h_f = 175mm

» $4h_f$ = 700mm

» $4h_f \geq h_w$ o.k

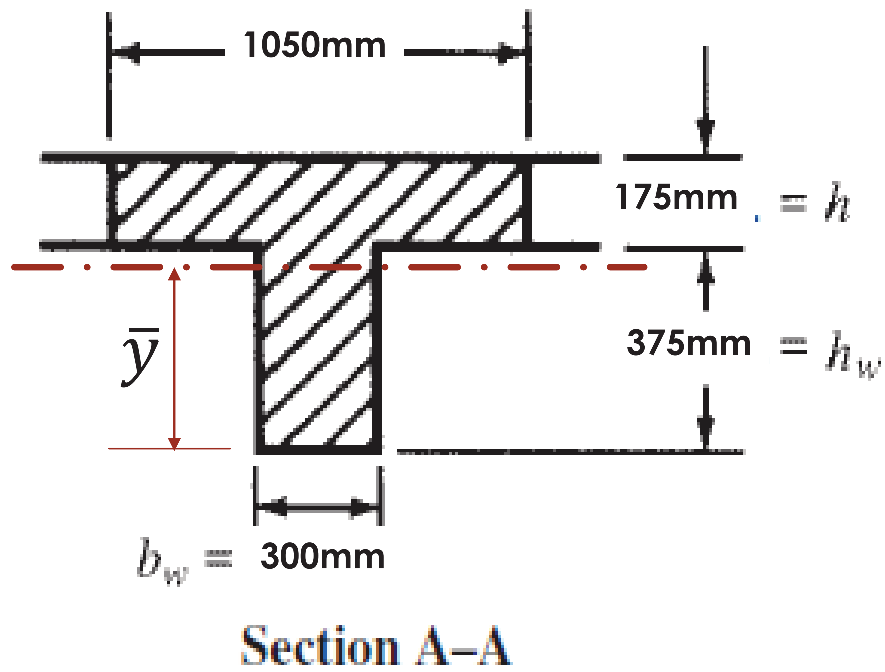




Section A–A

$$b_w + 2h_w \le b + 8h$$

- calculate moment of inertia by using accurate method about neutral axis of edge beam.



$$\bar{y} = \frac{(1050)(175)(462.5) + (300)(375)(187.5)}{(1050)(175) + (300)(375)}$$

$$\bar{y} = 358mm$$

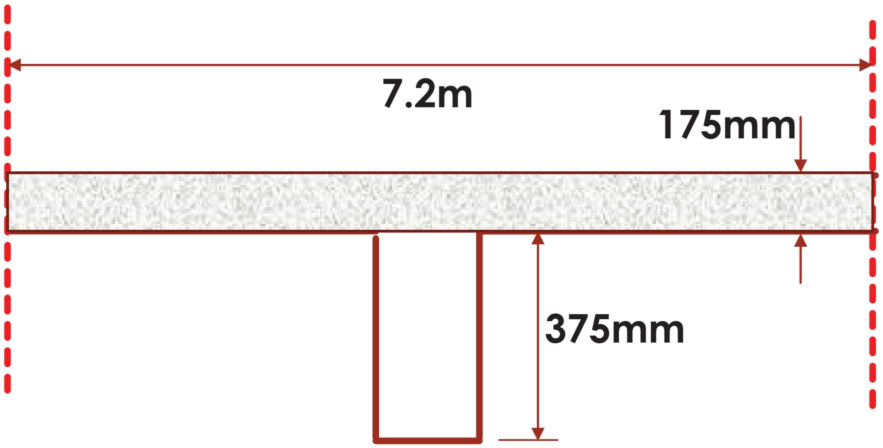
$$I_b = \frac{(300)(375)^3}{12} + (300)(375)(358 - 187.5)^2$$

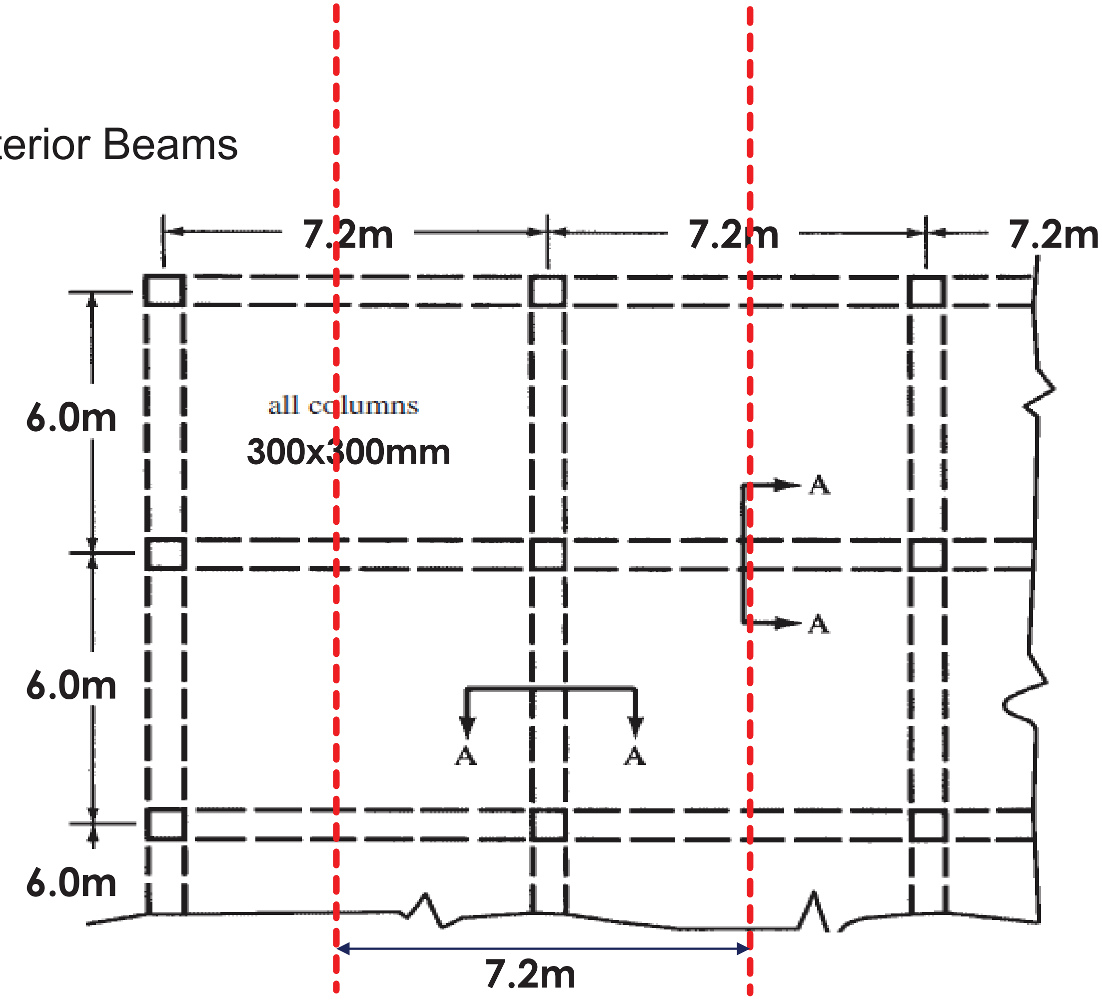
$$+ \frac{(1050)(175)^3}{12} + (1050)(175)(375 - 358 + 87.5)^2$$

$$I_b = 4588762500 + 7633939700 = 12222702200mm^4$$

$$\alpha_{f1} = \frac{I_b}{I_s} = \frac{12222702200}{2679687500} = 2.63$$

2) Computing α_2 for Short (Vertical) Span for Interior Beams





I_s = gross moment of inertia of slab 7.2 m wide

$$I_s = \frac{(7200)(175)^3}{12} = 3215625000 mm^4$$

$$\propto_{f2} = \frac{I_b}{I_s} = \frac{12222702200}{3215625000} = 2.19$$

$$\propto_{fm} = \frac{\propto_{f1} + \propto_{f2}}{2} = \frac{2.63 + 2.19}{2} = 2.41 > 2.0$$

Determining Slab Thickness per ACI Section 9.5.3.3

∴ Use ACI Equation 9-13

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$$

$l_{n\ long}$ = 7200 mm – 300 mm = 6900 mm

$l_{n\ short}$ = 6000 mm – 300 mm = 5700 mm

$$\beta = \frac{l_{n\ long}}{l_{n\ short}} = \frac{6900}{5700} = 1.21$$

$$h = \frac{6900\left(0.8 + \frac{420}{1400}\right)}{36 + 9(1.21)} = 162mm$$

$$h = 162mm > 90mm \ \therefore \ h_{min} = 162$$

$$\therefore \ assumed\ h = 175\ mm > h_{min} = 162$$

» thickness h = 175mm is adequate for this slab