One Dimensional Steady State Conduction

PLANE WALL

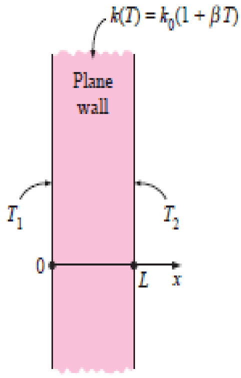
EX.1

Consider a 2-m-high and 0.7-m-wide bronze plate whose thickness is 0.1 m. One side of the plate is maintained at a constant temperature of 600 K while the other side is maintained at 400 K. The thermal conductivity of the bronze plate can be assumed to vary linearly in that temperature range as $k(T)$ _ $k_0(1+\beta T)$ where k_0 _ 38 W/m · K and β = 9.21 *10^-4 K^{-1}. Disregarding the edge effects and assuming steady one-dimensional heat transfer, determine the rate of heat conduction through the plate.

$$q = -K_O \frac{A}{L}\left[(T_2 - T_1) + \frac{\beta}{2}\left(T_2^{\,2} - T_1^{\,2}\right)\right]$$

$$= -38 * \frac{2*0.7}{0.1}\left[(400 - 600) + \frac{9.21*10^{-4}}{2}\left(400^2 - 600^2\right)\right]$$

$$=155.4$$



EX.2

A certain material 2.5 cm thick, with a cross-sectional area of 0.1 m^2, has one side maintained at 35°C and the other at 95°C. The temperature at the center plane of the material is 62°C, and the heat flow through the material is 1 kW. Obtain an expression for the thermal conductivity of the material as a function of temperature.

Assume Linear variation: $k = k_0(1+\beta T)$

$$q = -\frac{k_0 A}{\Delta x}\left[T_3 - T_1 + \frac{\beta}{2}(T_3^2 - T_1^2)\right] = -\frac{k_0 A}{\Delta x/2}\left[T_2 - T_1 + \frac{\beta}{2}(T_2^2 - T_1^2)\right]$$

$T_3 = 95°\text{C},\ T_2 = 62°\text{C},\ T_1 = 35°\text{C},\ \Delta x = 0.025$

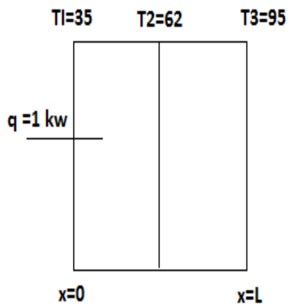
$$2\left[62 - 35 + \frac{\beta}{2}(62^2 - 35^2)\right] = \left[95 - 35 + \frac{\beta}{2}(95^2 - 35^2)\right]$$

$$\beta = -4.68\times10^{-3}$$

$$1000 = q = \frac{k_0(0.1)}{0.025}\left[95 - 35 - \frac{4.68\times10^{-3}}{2}(95^2 - 35^2)\right]$$

$$k_0 = 5.988$$

$$k = 5.988(1 - 4.68\times10^{-3}\ \mathrm{T})\ \frac{\mathrm{W}}{\mathrm{m}\cdot°\mathrm{C}}$$

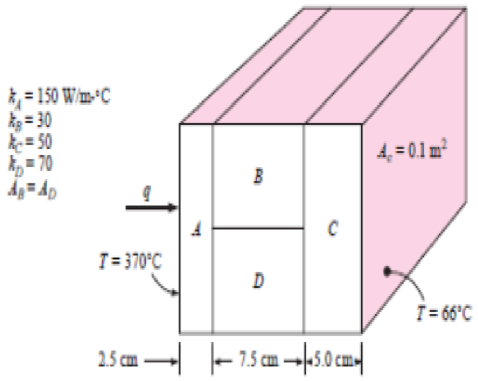


CLASS WORK

A plane wall is constructed of a material having a thermal conductivity that varies as the square of the temperature according to the relation $k = k_0(1+\beta T^2)$. Derive an expression for the heat transfer in such a wall.

EX.3

Find the heat transfer per unit area through the composite wall Assume one-dimensional heat flow.



$$R = \frac{\Delta x}{kA}$$

$$R_A = \frac{0.025}{(150)(0.1)} = 1.667 \times 10^{-3}$$

$$R_B = \frac{0.075}{(30)(0.05)} = 0.05$$

$$R_C = \frac{0.05}{(50)(0.1)} = 0.01$$

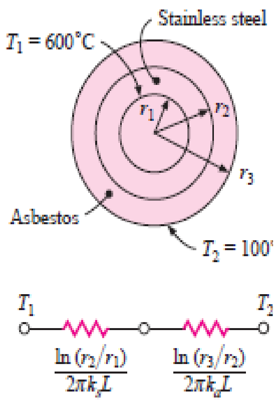
$$R_D = \frac{0.075}{(70)(0.05)} = 0.02143$$

$$R = R_A + R_C + \frac{1}{\frac{1}{R_B} + \frac{1}{R_D}} = 2.667 \times 10^{-2}$$

$$q = \frac{\Delta T}{R} = \frac{370 - 66}{2.667 \times 10^{-2}} = 11{,}400 \text{ W}$$

EX.4

A thick-walled tube of stainless steel [18% Cr, 8% Ni, k =19 W/m.°C] with 2-cm inner diameter(ID) and 4-cm outer diameter (OD) is covered with a 3-cm layer of asbestos insulation [k =0.2 W/m. ◦C]. If the inside wall temperature of the pipe is maintained at 600°C, calculate the heat loss per meter of length. Also calculate the tube–insulation interface temperature.



$$\frac{q}{L} = \frac{2\pi(T_1 - T_2)}{\ln(r_2/r_1)/k_s + \ln(r_3/r_2)/k_a} = \frac{2\pi(600-100)}{(\ln 2)/19 + (\ln \frac{5}{2})/0.2} = 680 \text{ W/m}$$

$$\frac{q}{L} = \frac{T_a - T_2}{\ln(r_3/r_2)/2\pi k_a} = 680 \text{ W/m}$$

where T_a is the interface temperature, which may be obtained as

$$T_a = 595.8^\circ\text{C}$$

EX.5

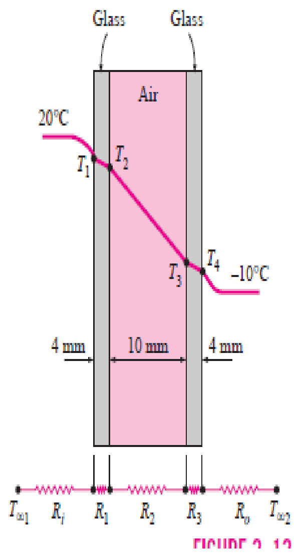
Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two4-mm-thick layers of glass (k _ 0.78 W/m · °C) separated by a 10-mm-wide stagnant air space (k _ 0.026 W/m · °C). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is _10°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h1$ _ 10 W/m2 · °C and $h2$ _40 W/m2 · °C, which includes the effects of radiation.

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_1 = R_3 = R_{glass} = \frac{L_1}{k_1 A} = \frac{0.004\ \text{m}}{(0.78\ \text{W/m} \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.00427^\circ\text{C/W}$$

$$R_2 = R_{air} = \frac{L_2}{k_2 A} = \frac{0.01\ \text{m}}{(0.026\ \text{W/m} \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.3205^\circ\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(40\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.02083^\circ\text{C/W}$$



Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{total} &= R_{conv,1} + R_{glass,1} + R_{air} + R_{glass,2} + R_{conv,2} \\ &= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083 \\ &= 0.4332^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

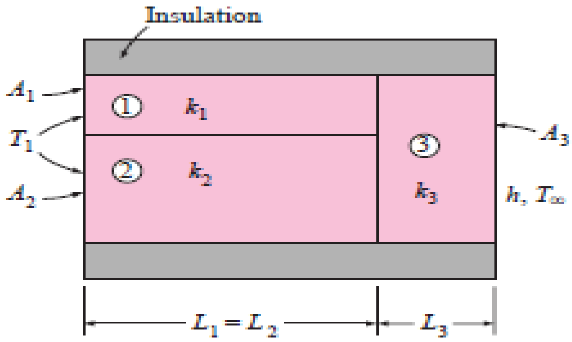
$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{[20 - (-10)]^\circ\text{C}}{0.4332^\circ\text{C/W}} = 69.2\ \text{W}$$

The inner surface temperature of the window in this case will be

$$T_1 = T_{\infty1} - \dot{Q}R_{conv,1} = 20^\circ\text{C} - (69.2\ \text{W})(0.08333^\circ\text{C/W}) = 14.2^\circ\text{C}$$

CLASS WORK

Draw the thermal resistance



EX.6

a)A hollow sphere is constructed of aluminum with an inner diameter of 4 cm and an outer diameter of 8 cm. The inside temperature is 100°C and the outer temperature is 50°C. Calculate the heat transfer.

b)if the sphere is covered with a 1-cm layer of an insulating material having k =50 mW/m. °C and the outside of the insulation is exposed to an environment with h=20 W/m^2 . °C and $T\infty$ =10°C. The inside of the sphere remains at 100°C. Calculate the heat transfer under these conditions.

a)

$$q = \frac{4\pi k(T_i - T_0)}{\frac{1}{r_i} - \frac{1}{r_0}} \qquad k = 204 \ \frac{\mathrm{W}}{\mathrm{m \cdot {}^\circ C}}$$

$$= \frac{(4)\pi(204)(100-50)}{\frac{1}{0.02} - \frac{1}{0.04}} = 5127 \ \mathrm{W}$$

b)

$$q = \frac{\Delta T}{\sum R}$$

$$R_{\mathrm{alum}} = \frac{\frac{1}{0.02} - \frac{1}{0.04}}{4\pi(204)} = 9.752 \times 10^{-3}$$

$$R_{\mathrm{ins}} = \frac{\frac{1}{0.04} - \frac{1}{0.05}}{4\pi(0.05)} = 7.958$$

$$R_{\mathrm{conv}} = \frac{1}{hA} = \frac{1}{(20)(4\pi)(0.05)^2} = 1.592$$

$$q = \frac{100-10}{0.00975 + 7.958 + 1.592} = 9.41 \ \mathrm{W}$$

CLASS WORK

A hot steam pipe having an inside surface temperature of 250◦C has an inside diameter of 8 cm and a wall thickness of 5.5 mm. It is covered with a 9-cm layer of insulation having *k* =0.5 W/m.°C, followed by a 4-cm layer of insulation having *k* =0.25W/m.°C. The outside temperature of the insulation is 20°C. Calculate the heat lost per meter of length. Assume *k* =47 W/m. °C for the pipe.

EX.7

A 2-mm-diameter and 10-m-long electric wire is tightly wrapped with a 1-mm-thick plastic cover whose thermal conductivity is k = 0.15 W/m · °C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at T= 30°C with a heat transfer coefficient of h = 24 W/m^2 · °C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine if doubling the thickness of the plastic cover will increase or decrease this interface temperature.

$$\dot{Q} = \dot{W}_e = VI = (8\text{ V})(10\text{ A}) = 80\text{ W}$$

The total thermal resistance is

$$R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(24\text{ W/m}^2.°\text{C})[\pi(0.004\text{ m})(10\text{ m})]} = 0.3316\text{ °C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(2/1)}{2\pi(0.15\text{ W/m.°C})(10\text{ m})} = 0.0735\text{ °C/W}$$

$$R_{\text{total}} = R_{\text{conv}} + R_{\text{plastic}} = 0.3316 + 0.0735 = 0.4051\text{ °C/W}$$

Then the interface temperature becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}} = 30°\text{C} + (80\text{ W})(0.4051\text{ °C/W}) = \mathbf{62.4°C}$$

The critical radius of plastic insulation is

$$r_{cr} = \frac{k}{h} = \frac{0.15\text{ W/m.°C}}{24\text{ W/m}^2.°\text{C}} = 0.00625\text{ m} = 6.25\text{ mm}$$

Doubling the thickness of the plastic cover will increase the outer radius of the wire to 3 mm, which is less than the critical radius of insulation. Therefore, doubling the thickness of plastic cover will increase the rate of heat loss and decrease the interface temperature.

EX.8

Water flows at 50°C inside a 2.5-cm-inside-diameter tube such that *hi* =3500 W/m^2 .°C. The tube has a wall thickness of 0.8 mm with a thermal conductivity of 16 W/m. °C. The outside of the tube loses heat by free convection with *ho* =7.6W/m^2 . °C. Calculate the overall heat-transfer coefficient and heat loss per unit length to surrounding air at 20°C.

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3500)\pi(0.025)(1.0)} = 0.00364\ ^\circ\mathrm{C/W}$$

$$R_t = \frac{\ln(d_o/d_i)}{2\pi k L}$$

$$= \frac{\ln(0.0266/0.025)}{2\pi(16)(1.0)} = 0.00062\ ^\circ\mathrm{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(7.6)\pi(0.0266)(1.0)} = 1.575\ ^\circ\mathrm{C/W}$$

$$q = \frac{\Delta T}{\sum R} = U A_o \Delta T$$

$$U_o = \frac{1}{A_o \sum R} = \frac{1}{[\pi(0.0266)(1.0)](0.00364 + 0.00062 + 1.575)}$$

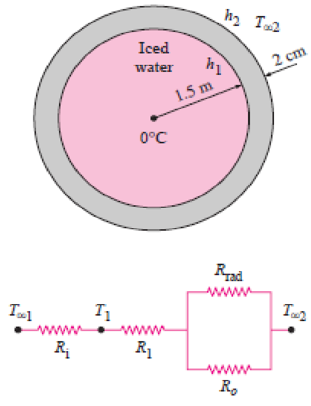
$$= 7.577\ \mathrm{W/m^2 \cdot {}^\circ C}$$

$$q = U A_o \Delta T = (7.577)\pi(0.0266)(1.0)(50-20) = 19\ \mathrm{W}\ \text{(for 1.0 m length)}$$

EX.9

A 3-m internal diameter spherical tank made of 2-cm-thick stainless steel(k = 15 W/m · °C) is used to store iced water at $T1$ = 0°C. The tank is located in a room whose temperature is $T2$ = 22°C. The walls of the room are also at22°C. The outer surface of the tank is black and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are $h1$= 80 W/m^2 · °C and $h2$ =10 W/m^2 · °C, respectively. Determine the rate of heat transfer to the iced water in the tank

$$A_1 = \pi D_1^2 = \pi(3\ \text{m})^2 = 28.3\ \text{m}^2$$
$$A_2 = \pi D_2^2 = \pi(3.04\ \text{m})^2 = 29.0\ \text{m}^2$$



Also, the radiation heat transfer coefficient is given by

$$h_{rad} = \varepsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$$

Then the individual thermal resistances become

$$R_i = R_{conv,1} = \frac{1}{h_1 A_1} = \frac{1}{(80\ \text{W/m}^2 \cdot {}^\circ\text{C})(28.3\ \text{m}^2)} = 0.000442^\circ\text{C/W}$$

$$R_1 = R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52 - 1.50)\ \text{m}}{4\pi\ (15\ \text{W/m} \cdot {}^\circ\text{C})(1.52\ \text{m})(1.50\ \text{m})}$$
$$= 0.000047^\circ\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A_2} = \frac{1}{(10\ \text{W/m}^2 \cdot {}^\circ\text{C})(29.0\ \text{m}^2)} = 0.00345^\circ\text{C/W}$$

$$R_{rad} = \frac{1}{h_{rad} A_2} = \frac{1}{(5.34\ \text{W/m}^2 \cdot {}^\circ\text{C})(29.0\ \text{m}^2)} = 0.00646^\circ\text{C/W}$$

The two parallel resistances R_o and R_{rad} can be replaced by an equivalent resistance R_{equiv} determined from

$$\frac{1}{R_{equiv}} = \frac{1}{R_o} + \frac{1}{R_{rad}} = \frac{1}{0.00345} + \frac{1}{0.00646} = 444.7\ \text{W/}^\circ\text{C}$$

which gives

$$R_{equiv} = 0.00225^\circ\text{C/W}$$

$$R_{total} = R_i + R_1 + R_{equiv} = 0.000442 + 0.000047 + 0.00225 = 0.00274^\circ\text{C/W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{total}} = \frac{(22 - 0)^\circ\text{C}}{0.00274^\circ\text{C/W}} = 8029\ \text{W} \qquad (\text{or } \dot{Q} = 8.027\ \text{kJ/s})$$

To check the validity of our original assumption, we now determine the oute surface temperature from

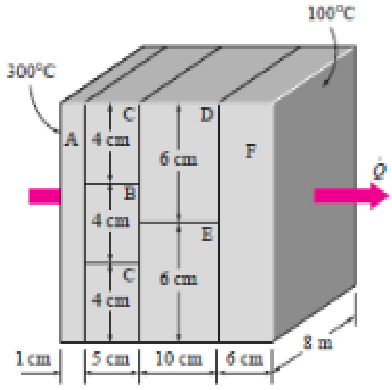
$$\dot{Q} = \frac{T_{\infty 2} - T_2}{R_{equiv}} \longrightarrow T_2 = T_{\infty 2} - \dot{Q}R_{equiv}$$
$$= 22^\circ\text{C} - (8029\ \text{W})(0.00225^\circ\text{C/W}) = 4^\circ\text{C}$$

H.W

1- A 1.0-mm-diameter wire is maintained at a temperature of 400°C and exposed to a convection environment at 40°C with h=120 W/m^2 . °C. Calculate the thermal conductivity that will just cause an insulation thickness of 0.2 mm to produce a “critical radius.” How much of this insulation must be added to reduce the heat transfer by 75 percent from that which would be experienced by the bare wire?

2-

Consider a 5-m-high, 8-m-long, and 0.22-m-thick wall whose representative cross section is as given in the figure. The thermal conductivities of various materials used, in W/m · °C, are $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, and $k_E = 35$. The left and right surfaces of the wall are maintained at uniform temperatures of 300°C and 100°C, respectively. Assuming heat transfer through the wall to be one-dimensional, determine (*a*) the rate of heat transfer through the wall; (*b*) the temperature at the point where the sections *B*, *D*, and *E* meet; and (*c*) the temperature drop across the section *F*. Disregard any contact resistances at the interfaces.



3- Steam at 320°C flows in a stainless steel pipe (k =15 W/m · °C) whose inner and outer diameters are 5 cm and5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation (k = 0.038 W/m · °C). Heat is lost to the surroundings at 5°C by natural convection and radiation, with a combined natural convection and radiation heat transfer coefficient of 15 W/m^2 .°C. Taking the heat transfer coefficient inside the pipe to be 80 W/m^2 · °C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

4- Derive a relation for the critical radius of insulation for a sphere.

HEAT-SOURCE SYSTEMS

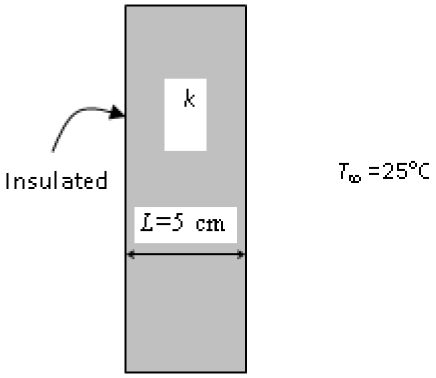
Plane Wall with Heat Sources

EX.1

Consider a large 5-cm-thick brass plate (k = 111 W/m · °C) in which heat is generated uniformly at a rate of 2 * 10^5 W/m^3. One side of the plate is **insulated** while the other side is exposed to an environment at 25°C with a heat transfer coefficient of 44 W/m^2 · °C. Explain where in the plate the highest and the lowest temperatures will occur, and determine their values.

$$T_s = T_\infty + \frac{\dot{g}L}{h} = 25°\text{C} + \frac{(2\times10^5 \text{ W/m}^3)(0.05\text{ m})}{44 \text{ W/m}^2.°\text{C}} = \mathbf{252.3\,°C}$$

$$T_o = T_s + \frac{\dot{g}L^2}{2k} = 252.3°\text{C} + \frac{(2\times10^5 \text{ W/m}^3)(0.05\text{ m})^2}{2(111\text{ W/m.°C})} = \mathbf{254.5\,°C}$$

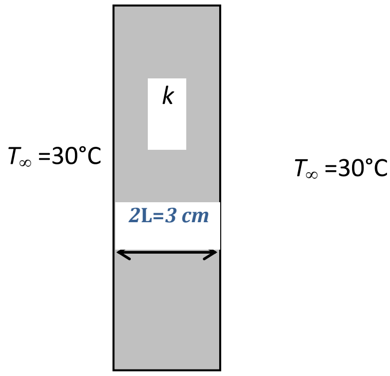


EX.2

Consider a large 3-cm-thick stainless steel plate (k _15.1 W/m · $^{\circ}$C) in which heat is generated uniformly at a rate of 5 * 10^5 W/m^3. Both sides of the plate are exposed to an environment at 30°C with a heat transfer coefficient of 60 W/m^2· $^{\circ}$C. Explain where in the plate the highest and the lowest temperatures will occur, and determine their values.

$$T_s = T_\infty + \frac{\dot{g}L}{h} = 30°\text{C} + \frac{(5\times10^5\ \text{W/m}^3)(0.015\,\text{m})}{60\,\text{W/m}^2.°\text{C}} = \mathbf{155°C}$$

$$T_o = T_s + \frac{\dot{g}L^2}{2k} = 155°\text{C} + \frac{(5\times10^5\ \text{W/m}^3)(0.015\,\text{m})^2}{2(15.1\,\text{W/m.°C})} = \mathbf{158.7°C}$$



EX.3

A certain semiconductor material has a conductivity of 0.0124 W/cm .°C. A rectangular bar of the material has a cross-sectional area of 1 cm^2 and a length of 3 cm. One end is maintained at300°C and the other end at 100°C, and the bar carries a current of 50 A. Assuming the longitudinal surface is insulated, calculate the midpoint temperature in the bar. Take the resistivity as 1.5×10−3_Ω/ cm.

$$k = 0.0124\ \frac{\mathrm{W}}{\mathrm{cm}\cdot{}^\circ\mathrm{C}} = 1.24\ \frac{\mathrm{W}}{\mathrm{m}\cdot{}^\circ\mathrm{C}}$$

$$\rho = 1.5\times10^{-3}\ \Omega\cdot\mathrm{cm}$$

$$R = (1.5\times10^{-3})\left(\frac{3}{1}\right) = 4.5\times10^{-3}$$

$$q = I^2R = (50)^2(4.5\times10^{-3}) = 11.25\ \mathrm{W}$$

$$q = \frac{q}{V} = \frac{11.25}{3\times10^{-6}} = 3.75\ \frac{\mathrm{MW}}{\mathrm{m}^3} \qquad T = -\frac{\dot{q}}{2k}x^2 + c_1x + c_2$$

$$L = 1.5\ \mathrm{cm} = 0.015\ \mathrm{m} \qquad T = 300\ \text{at}\ x = -0.015$$

$$T = 100\ \text{at}\ x = +0.015$$

$$300 - 100 = c_1(-0.015 - 0.015) \qquad c_1 = -6667$$

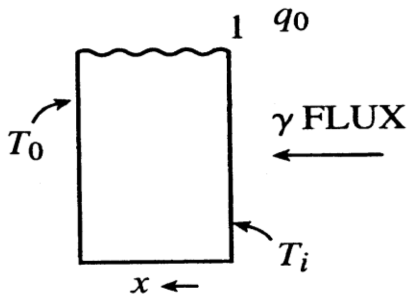
$$300 = \frac{(-3.75\times10^6)(0.015)^2}{(2)(1.24)} - (6667)(-0.015) + c_2 \qquad c_2 = 540.2$$

$$\text{at}\ x = 0 \qquad T = c_2 = 540.2^\circ\mathrm{C}$$

EX.4

Consider a shielding wall for a nuclear reactor. The wall receives a gamma-ray flux such that heat is generated within the wall according to the relation $\dot{q} = \dot{q}_0 e^{-ax}$

where $\dot{q}_0$ is the heat generation at the inner face of the wall exposed to the gamma-ray flux and a is a constant. Using this relation for heat generation, derive an expression for the temperature distribution in a wall of thickness L, where the inside and outside temperatures are maintained at T_i and T_0, respectively. Also obtain an expression for the maximum temperature in the wall.



$$\dot{q}_x = \dot{q}_0 e^{-ax} \qquad \frac{d^2T}{dx^2} = \frac{-\dot{q}_0 e^{-ax}}{k}$$

$$T = c_1 + c_2 x - \frac{\dot{q}_0}{a^2 k} e^{-ax}$$

Boundary conditions:

(1) $T = T_i$ at $x = 0$

(2) $T = T_0$ at $x = L$

$$c_1 = T_i + \frac{\dot{q}_0}{a^2 k} \qquad c_2 = \frac{T_0 - T_i - \frac{\dot{q}_0}{a^2 k}(1 - e^{-aL})}{L}$$

$$T = T_i + \frac{\dot{q}_0}{a^2 k} + \frac{T_0 - T_i - \frac{\dot{q}_0}{a^2 k}(1 - e^{-aL})x}{L} + \frac{-\dot{q}_0}{a^2 k} e^{-ax}$$

EX.5

Electric heater wires are installed in a solid wall having a thickness of 8 cm and k =2.5W/m. °C. The right face is exposed to an environment with h=50W/m^2 .°C and $T\infty$ =30°C, while the left face is exposed to h=75W/m^2 .°C and $T\infty$ =50°C. What is the maximum allowable heat-generation rate such that the maximum temperature in the solid does not exceed 300°C?

$k = 2.5$ $\quad h_1 = 75$ (left) $\quad h_2 = 50$ (right)

$T_{1\infty} = 50°C \quad T_{2\infty} = 30°C$

$$T = -\frac{\dot{q}x^2}{2k} + c_1 x + c_2$$

$$T = T_1 \text{ at } x = -0.04; \; T = T_2 \text{ at } x = +0.04$$

$$\frac{\partial T}{\partial x} = -\frac{\dot{q}x}{k} + c_1$$

$$T = T_{max} = 300 \text{ at } x = c_1 \frac{k}{\dot{q}} \tag{1}$$

$$h_1(T_{1\infty} - T_1) = -k\frac{\partial T}{\partial x}\bigg|_{x=-0.04} \tag{2}$$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=+0.04} = h_2(T_2 - T_{2\infty}) \tag{3}$$

$$300 = -\frac{\dot{q}}{2k}\left[c_1 \frac{k}{\dot{q}}\right]^2 + c_1\left[c_1 \frac{k}{\dot{q}}\right] + c_2 \tag{1}$$

$$75\left[50 + \frac{\dot{q}}{2k}(0.04)^2 + c_1(0.04) - c_2\right] = -k\left(\frac{+\dot{q}(0.04)}{2k}\right) \tag{2}$$

$$-k\left[\frac{-\dot{q}(0.04)}{2k}\right] = 50\left[\frac{-\dot{q}(0.04)^2}{2k} + c_1(0.04) + c_2 - 30\right] \tag{3}$$

3 Equations, 3 Unknowns, c_1, c_2, $\dot{q}$

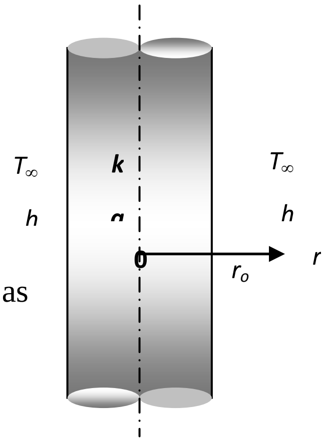
Solve for $\dot{q} = 2.46 \times 10^5$ W/m^3

Class Work

A 3.0-cm-thick plate has heat generated uniformly at the rate of 5×10^5 W/m^3. One side of the plate is maintained at 200°C and the other side at 45°C. Calculate the temperature at the center of the plate for k =16 W/m.°C.

CYLINDER WITH HEAT SOURCES

A 6-m-long 2-kW electrical resistance wire is made of 0.2-cm-diameter stainless steel (k _ 15.1 W/m .°C). The resistance wire operates in an environment at 30°C with a heat transfer coefficient of 140 W/m² .°C at the outer surface. Determine the surface temperature of the wire (a) by using the applicable relation and (b) by setting up the proper differential equation and solving it



$$\dot{g} = \frac{\dot{Q}_{gen}}{V_{\text{wire}}} = \frac{\dot{Q}_{gen}}{\pi r_o{}^2 L} = \frac{2000\ \text{W}}{\pi(0.001\ \text{m})^2(6\ \text{m})} = 1.061\times10^8\ \text{W/m}^3$$

The surface temperature of the wire is

$$T_s = T_\infty + \frac{\dot{g}r_o}{2h} = 30°\text{C} + \frac{(1.061\times10^8\ \text{W/m}^3)(0.001\ \text{m})}{2(140\ \text{W/m}^2.°\text{C})} = \mathbf{409°C}$$

(b) The mathematical formulation of this problem can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$$

and $\quad -k\dfrac{dT(r_0)}{dr} = h[T(r_0) - T_\infty]$ (convection at the outer surface)

$$\frac{dT(0)}{dr} = 0 \quad \text{(thermal symmetry about the centerline)}$$

Multiplying both sides of the differential equation by r and integrating gives

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{g}}{k}r \quad \rightarrow \quad r\frac{dT}{dr} = -\frac{\dot{g}}{k}\frac{r^2}{2} + C_1 \qquad (a)$$

Applying the boundary condition at the center line,

B.C. at $r = 0$: $\quad 0\times\dfrac{dT(0)}{dr} = -\dfrac{\dot{g}}{2k}\times 0 + C_1 \quad \rightarrow \quad C_1 = 0$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr} = -\frac{\dot{g}}{2k}r \qquad \rightarrow \qquad T(r) = -\frac{\dot{g}}{4k}r^2 + C_2 \qquad (b)$$

Applying the boundary condition at $r = r_0$,

B. C. at $\quad r = r_0$: $\quad -k\dfrac{\dot{g}r_0}{2k} = h\left(-\dfrac{\dot{g}}{4k}r_0^2 + C_2 - T_\infty\right) \quad \rightarrow \quad C_2 = T_\infty + \dfrac{\dot{g}r_0}{2h} + \dfrac{\dot{g}}{4k}r_0^2$

Substituting this C_2 relation into Eq. (b) and rearranging give

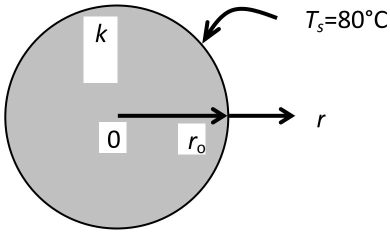
$$T(r) = T_\infty + \frac{\dot{g}}{4k}(r_0^2 - r^2) + \frac{\dot{g}r_0}{2h}$$

which is the temperature distribution in the wire as a function of r. Then the temperature of the wire at the surface ($r = r_0$) is determined by substituting the known quantities to be

$$T(r_0) = T_\infty + \frac{\dot{g}}{4k}(r_0^2 - r_0^2) + \frac{\dot{g}r_0}{2h} = T_\infty + \frac{\dot{g}r_o}{2h} = 30°\text{C} + \frac{(1.061\times10^8\ \text{W/m}^3)(0.001\ \text{m})}{2(140\ \text{W/m}^2.°\text{C})} = \mathbf{409°C}$$

SPHER WITH HEAT SOURCES

Consider a homogeneous spherical piece of radioactive material of radius r_0 =0.04 m that is generating heat at a constant rate of $g\cdot$ = 4 * 10^7 W/m^3. The heat generated is dissipated to the environment steadily. The outer surface of the sphere is maintained at a uniform temperature of 80°C and the thermal conductivity of the sphere is k = 15 W/m · °C. Assuming steady one-dimensional heat transfer, (*a*) express the differential equation and the boundary conditions for heat conduction through the sphere, (*b*) obtain a relation for the variation of temperature in the sphere by solving the differential equation, and (*c*) determine the temperature at the center of the sphere.



$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{g}}{k}=0 \qquad \text{with } \dot{g}=\text{constant}$$

and $\quad T(r_0)=T_s=80°\text{C}$ (specified surface temperature)

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the mid point)}$$

(*b*) Multiplying both sides of the differential equation by r^2 and rearranging gives

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=-\frac{\dot{g}}{k}r^2$$

Integrating with respect to r gives

$$r^2\frac{dT}{dr}=-\frac{\dot{g}}{k}\frac{r^3}{3}+C_1 \qquad (a)$$

Applying the boundary condition at the mid point,

B.C. at r = 0: $\quad 0\times\frac{dT(0)}{dr}=-\frac{\dot{g}}{3k}\times 0+C_1 \quad \rightarrow \quad C_1=0$

Dividing both sides of Eq. (*a*) by r^2 to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{g}}{3k}r$$

and $\quad T(r)=-\frac{\dot{g}}{6k}r^2+C_2 \qquad (b)$

Applying the other boundary condition at $r=r_0$,

B. C. at $\quad r=r_0$: $\quad T_s=-\frac{\dot{g}}{6k}r_0^2+C_2 \quad \rightarrow \quad C_2=T_s+\frac{\dot{g}}{6k}r_0^2$

Substituting this C_2 relation into Eq. (*b*) and rearranging give

$$T(r)=T_s+\frac{\dot{g}}{6k}(r_0^2-r^2)$$

which is the desired solution for the temperature distribution in the wire as a function of r.

(*c*) The temperature at the center of the sphere (r = 0) is determined by substituting the known quantities to be

$$T(0)=T_s+\frac{\dot{g}}{6k}(r_0^2-0^2)=T_s+\frac{\dot{g}r_0^2}{6k}=80°\text{C}+\frac{(4\times10^7\ \text{W/m}^3)(0.04\ \text{m})^2}{6\times(15\ \text{W/m.°C})}=\mathbf{791°C}$$

Thus the temperature at center will be about 711°C above the temperature of the outer surface of the sphere.

H.W

1- Derive an expression for the temperature distribution in a hollow cylinder with heat sources that vary according to the linear relation

$\dot{q}=a+br$

with $\dot{q}_i$ the generation rate per unit volume at $r=r_i$. The inside and outside temperatures are $T=T_i$ at $r=r_i$ and $T=T_o$ at $r=r_o$.

2- The temperature distribution in a certain plane wall is

$\frac{T-T1}{T2-T1}=C_1+C_2x^2+C_3x^3$

where T_1 and T_2 are the temperatures on each side of the wall. If the thermal conductivity of the wall is constant and the wall thickness is L, derive an expression for the heat generation per unit volume as a function of x, the distance from the plane where $T=T_1$. Let the heat-generation rate be $\dot{q}_0$ at $x=0$.

HEAT TRANSFER FROM FINNED SURFACES

EX.1

A long stainless-steel rod [k =16 W/m.C] has a square cross section 12.5 by12.5 mm and has one end maintained at 250◦C. The heat-transfer coefficient is40 W/m^2 . ˚C, and the environment temperature is 90˚C. Calculate the heat lost by the rod.

SOL.

$$k = 16 \qquad h = 40 \qquad T_0 = 250°\text{C} \qquad T_\infty = 90°\text{C}$$

$$P = (4)(0.0125) = 0.05 \text{ m} \qquad A = (0.0125)^2 = 1.565 \times 10^{-4} \text{ m}^2$$

$$q = \sqrt{hPkA}\,\theta_0 = [(40)(0.05)(16)(1.565 \times 10^{-4})]^{1/2}(250 - 90) = 11.31 \text{ W}$$

EX.2

An aluminum fin [$k = 200$ W/m · °C] 3.0 mm thick and 7.5 cm long protrudes from a wall, as in Figure 2-9. The base is maintained at 300°C, and the ambient temperature is 50°C with $h = 10$ W/m^2 · °C. Calculate the heat loss from the fin per unit depth of material.

Solution

We may use the approximate method of solution by extending the fin a fictitious length $t/2$ and then computing the heat transfer from a fin with insulated tip as given by Equation (2-36). We have

$$L_c = L + t/2 = 7.5 + 0.15 = 7.65 \text{ cm [3.01 in]}$$

$$m = \sqrt{\frac{hP}{kA}} = \left[\frac{h(2z + 2t)}{ktz}\right]^{1/2} \approx \sqrt{\frac{2h}{kt}}$$

when the fin depth $z \gg t$. So,

$$m = \left[\frac{(2)(10)}{(200)(3 \times 10^{-3})}\right]^{1/2} = 5.774$$

From Equation (2-36), for an insulated-tip fin

$$q = (\tanh mL_c)\sqrt{hPkA}\,\theta_0$$

For a 1 m depth

$$A = (1)(3 \times 10^{-3}) = 3 \times 10^{-3} \text{ m}^2 \text{ [4.65 in}^2\text{]}$$

and

$$q = (5.774)(200)(3 \times 10^{-3})(300 - 50) \tanh[(5.774)(0.0765)]$$

$$= 359 \text{ W/m}$$

EX.3

Aluminum fins 1.5 cm wide and 1.0 mm thick are placed on a 2.5-cm-diameter tube to dissipate the heat. The tube surface temperature is 170°, and the ambient-fluid temperature is 25°C. Calculate the heat loss per fin for $h = 130$ W/m^2 · °C. Assume $k = 200$ W/m · °C for aluminum.

Solution

For this example we can compute the heat transfer by using the fin-efficiency curves in Figure 2-12. The parameters needed are

$$L_c = L + t/2 = 1.5 + 0.05 = 1.55 \text{ cm}$$
$$r_1 = 2.5/2 = 1.25 \text{ cm}$$
$$r_{2c} = r_1 + L_c = 1.25 + 1.55 = 2.80 \text{ cm}$$

$$r_{2c}/r_1 = 2.80/1.25 = 2.24$$
$$A_m = t(r_{2c} - r_1) = (0.001)(2.8 - 1.25)(10^{-2}) = 1.55 \times 10^{-5} \text{ m}^2$$
$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0155)^{3/2}\left[\frac{130}{(200)(1.55 \times 10^{-5})}\right]^{1/2} = 0.396$$

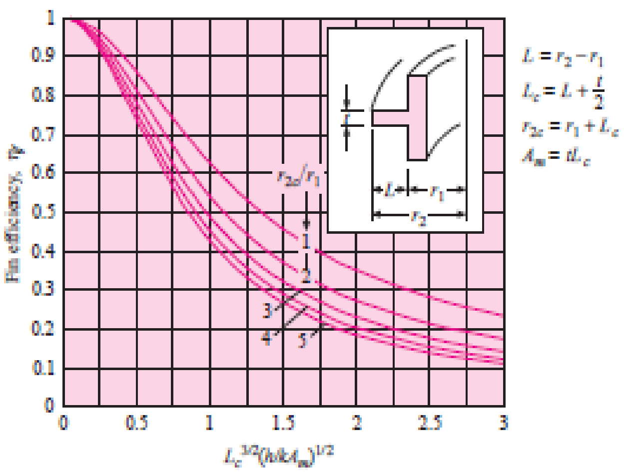
From Figure 2-12, $\eta_f = 82$ percent. The heat that would be transferred if the entire fin were at the base temperature is (both sides of fin exchanging heat)

$$q_{max} = 2\pi(r_{2c}^2 - r_1^2)h(T_0 - T_\infty)$$
$$= 2\pi(2.8^2 - 1.25^2)(10^{-4})(130)(170 - 25)$$
$$= 74.35 \text{ W } [253.7 \text{ Btu/h}]$$

The actual heat transfer is then the product of the heat flow and the fin efficiency:

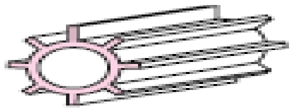
$$q_{act} = (0.82)(74.35) = 60.97 \text{ W } [208 \text{ Btu/h}]$$

Figure 2-12 | Efficiencies of circumferential fins of rectangular profile, according to Reference 3.



EX.4

A finned tube is constructed as shown in Figure. Eight fins are installed as shown and the construction material is aluminum. The base temperature of the fins may be assumed to be 100°C and they are subjected to a convection environment at 30°C with h=15 W/m^2 .°C. The longitudinal length of the fins is 15 cm and the peripheral length is 2 cm. The fin thickness is 2 mm. Calculate the total heat dissipated by the finned tube. Consider only the surface area of the fins.



$k = 204;\ N = 8;\quad T_0 = 100°C;\quad T_\infty = 30°C;\ h = 15;\ L = 0.02;\ t = 0.002$

$P = (2)(0.15 + 0.002) = 0.304$

$A = (0.002)(0.15) = 0.0003$

$m = [(15)(0.304)/(294)(0.0003)]^{1/2} = 8.632$

$L_c = 0.02 + 0.001 = 0.021$

$q/\text{fin} = (hPkA)^{1/2}\,\theta_0 \tanh(mL_c)$

$= [(15)(0.304)(204)(0.0003)]^{1/2}(100 - 30) \tanh[(8.632)(0.021)]$

$= 6.62$ W/fin

Total = (8)(6.62) = 53 W

EX.5

Steam in a heating system flows through tubes whose outer diameter is $D1$ = 3 cm and whose walls are maintained at a temperature of 120°C. Circular aluminum fins (k = 180 W/m · °C) of outer diameter $D2$ = 6 cm and constant thickness t = 2 mm are attached to the tube, as shown in Figure. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at T= 25°C, with a combined heat transfer coefficient of h= 60 W/m^2 · °C.

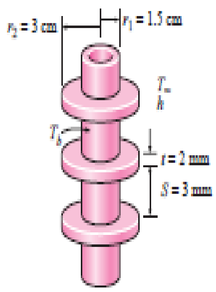
Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

$$A_{\text{no fin}} = \pi D_1 L = \pi(0.03\ \text{m})(1\ \text{m}) = 0.0942\ \text{m}^2$$

$$\begin{aligned}\dot{Q}_{\text{no fin}} &= hA_{\text{no fin}}(T_b - T_\infty)\\ &= (60\ \text{W/m}^2\cdot{}^\circ\text{C})(0.0942\ \text{m}^2)(120 - 25)^\circ\text{C}\\ &= 537\ \text{W}\end{aligned}$$

The efficiency of the circular fins attached to a circular tube is plotted in Fig. 3-43. Noting that $L = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.06 - 0.03) = 0.015$ m in this case, we have

$$\left.\begin{aligned}\frac{r_2+\frac{1}{2}t}{r_1} &= \frac{(0.03+\frac{1}{2}\times 0.002)\text{ m}}{0.015\text{ m}} = 2.07 \\ (L+\tfrac{1}{2}t)\sqrt{\frac{h}{kt}} &= (0.015+\tfrac{1}{2}\times 0.002)\text{ m}\times\sqrt{\frac{60\text{ W/m}^2\cdot{}^\circ\text{C}}{(180\text{ W/m}\cdot{}^\circ\text{C})(0.002\text{ m})}} = 0.207\end{aligned}\right\}\eta_{\text{fin}} = 0.95$$



$$\begin{aligned}A_{\text{fin}} &= 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t \\ &= 2\pi[(0.03\text{ m})^2 - (0.015\text{ m})^2] + 2\pi(0.03\text{ m})(0.002\text{ m}) \\ &= 0.00462\text{ m}^2\end{aligned}$$

$$\begin{aligned}\dot{Q}_{\text{fin}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} h A_{\text{fin}}(T_b - T_\infty) \\ &= 0.95(60\text{ W/m}^2\cdot{}^\circ\text{C})(0.00462\text{ m}^2)(120-25)^\circ\text{C} \\ &= 25.0\text{ W}\end{aligned}$$

Heat transfer from the unfinned portion of the tube is

$$\begin{aligned}A_{\text{unfin}} &= \pi D_1 S = \pi(0.03\text{ m})(0.003\text{ m}) = 0.000283\text{ m}^2 \\ \dot{Q}_{\text{unfin}} &= hA_{\text{unfin}}(T_b - T_\infty) \\ &= (60\text{ W/m}^2\cdot{}^\circ\text{C})(0.000283\text{ m}^2)(120-25)^\circ\text{C} \\ &= 1.60\text{ W}\end{aligned}$$

Noting that there are 200 fins and thus 200 interfin spacings per meter length of the tube, the total heat transfer from the finned tube becomes

$$\dot{Q}_{\text{total, fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 200(25.0 + 1.6)\text{ W} = 5320\text{ W}$$

Therefore, the increase in heat transfer from the tube per meter of its length ar a result of the addition of fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total, fin}} - \dot{Q}_{\text{no fin}} = 5320 - 537 = 4783\text{ W} \qquad \text{(per m tube length)}$$

Discussion The overall effectiveness of the finned tube is

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{no fin}}} = \frac{5320\text{ W}}{537\text{ W}} = 9.9$$

H.W

Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180°C. Circular aluminum alloy 2024-T6 fins (k = 186 W/m · °C) of outer diameter 6 cm and constant thickness1 mm are attached to the tube. The space between the fins is 3 mm, and thus there are 250 fins per meter length of the tube. Heat is transferred to the surrounding air at T= 25°C, with a heat transfer coefficient of 40 W/m^2 · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.