4 Open Channel Hydraulics

The subject of open channel hydraulics is extensive enough to require a complete text. Obviously, an exhaustive coverage cannot be given in one short chapter. The treatment given here is intended only to cover certain basic principles and to give the details necessary to design stable, open channels; to do simple channel routings; and to compute simple backwater profiles. The interested reader can consult several excellent texts for additional details (Chow, 1959; Henderson, 1966).

BASIC RELATIONSHIPS

Continuity Equation

When dealing with the hydraulics of open channel flow, there are three basic relationships that must be kept in mind. These relationships are the continuity equation, the energy equation, and the momentum equation. If we consider a stream with a cross section as shown in Fig. 4.1, the continuity equation may be written as

$$\text{inflow} - \text{outflow} = \text{change in storage}, \quad (4.1)$$

where inflow represents the volume of flow across section 1 during a time interval, outflow represents the volume of flow across section 2 during this time interval, and change in storage represents the change in the volume of water stored within the section from 1 to 2.

The continuity equation may also be written in terms of flow rates as

$$\text{inflow rate} - \text{outflow rate} = \text{rate of change in storage}, \quad (4.2)$$

where inflow rate and outflow rate represent the rate of flow across sections 1 and 2, respectively, and the rate of change in storage is the rate at which the volume of water is accumulating or diminishing within the section.

The flow rate, Q, is generally expressed in cubic feet per second (cfs) or cubic meters per second (cms) and may be written

$$Q = vA, \quad (4.3)$$

where v is the average velocity of flow at a cross-section and A is the area of the cross section. v is generally given in feet per second (fps) or meters per second (m/sec) and A in square feet (ft^2) or square meters (m^2). Throughout this chapter, units on symbols appearing in equations will not be given unless needed for clarity. Standard units are feet and seconds or meters and seconds.

It should be kept in mind that v is the average velocity of the flow perpendicular to the cross section. The actual pattern of flow velocity can be quite com-

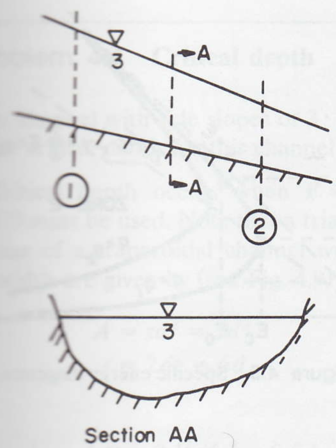


Figure 4.1 Typical channel sections.

Figure 4.2 shows typical distributions of flow velocity with various channel cross sections. Figure 4.3 shows a velocity profile for an idealized situation (Chow,

Both Figs. 4.2 and 4.3 show that the actual velocity in contact with the channel boundary is quite low.

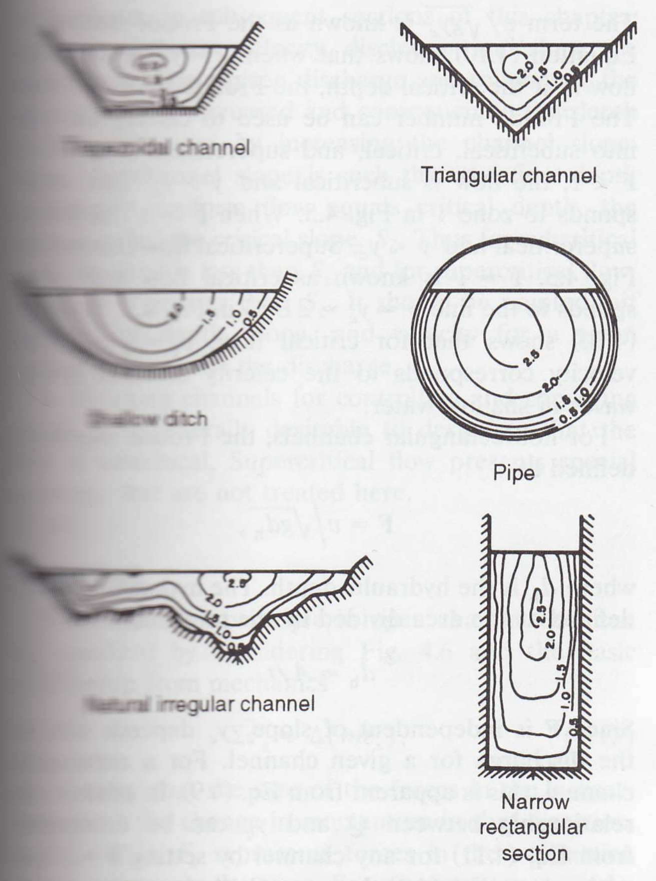


Figure 4.2 Typical velocity distributions.

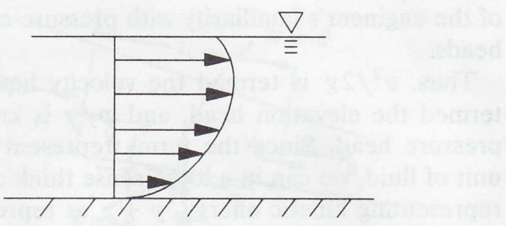


Figure 4.3 Typical velocity profile.

Theoretically, with a solid boundary, the flow velocity at the boundary is zero. Actually for natural channels it is difficult to determine precisely where the channel boundary is. The important point is that particles along the channel boundary are subjected to an actual velocity that is considerably lower than the average flow velocity of the cross-section.

Energy

In basic fluid mechanics, the energy equation is generally written in the form of Eq. (4.4). This relationship is known as Bernoulli's equation or Bernoulli's theorem:

$$\frac{v_1^2}{2g} + y_1 + z_1 + \frac{p_1}{\gamma} = \frac{v_2^2}{2g} + y_2 + z_2 + \frac{p_2}{\gamma} + h_{L,1-2}. \tag{4.4}$$

The terms in this equation are shown in Fig. 4.4. The Bernoulli equation represents an energy balance between two points along the channel. Again, v is the average flow velocity, g is the gravitational constant, y is the depth of flow, z is the elevation of the channel bottom, p is a pressure, γ is the unit weight of water, and $h_{L,1-2}$ represents the energy loss between sections 1 and 2.

Each complete term of Eq. (4.4) has the units of a length. Since the equation is an energy equation, one should consider that the terms represent energy per unit of flowing fluid. Since the units are a length, the terms are commonly associated with a "head" because

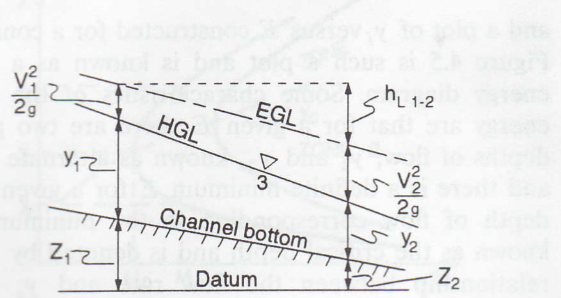


Figure 4.4 Terms in Bernoulli equation.

of the engineer's familiarity with pressure and pressure heads.

Thus, $v^2/2g$ is termed the velocity head, $y + z$ is termed the elevation head, and p/γ is known as the pressure head. Since the terms represent energy per unit of fluid, we can in a loose sense think of $v^2/2g$ as representing kinetic energy, $y + z$ as representing potential energy, and p/γ as representing stored energy.

The sum of the velocity head, elevation head, and pressure head represents the total energy. The line labeled EGL in Fig. 4.4 represents this sum and is known as the energy grade line. The sum of the elevation head and pressure head is known as the hydraulic grade line (HGL). The factor that distinguishes open channel flow from pipe flow is that in open channel flow, the free water surface is exposed to the atmosphere so that p/γ is zero. Thus, the pressure head term can generally be ignored for open channel problems, and hence, the HGL coincides with the water surface. A rather obvious fact is that the EGL must be sloping downward in the direction of flow. The EGL can only go up if external energy (through a pump for example) is supplied to the flow.

If we consider a channel section in which there is no energy loss, we can write

$$v^2/2g + y + z = \text{constant}. \tag{4.5}$$

If we take the datum elevation to be the channel bottom, we have

$$v^2/2g + y = \text{constant} = E, \tag{4.6}$$

where the constant E is known as the specific energy.

Consider now a wide rectangular channel so that the depth all across the channel cross section is y. We can then relate the flow rate on a per unit of width basis and the average flow velocity by

$$q = vy, \tag{4.7}$$

where q is the flow rate per unit of width.

Equation (4.6) can now be written as

$$q^2/2gy^2 + y = E \tag{4.8}$$

and a plot of y versus E constructed for a constant q. Figure 4.5 is such a plot and is known as a specific energy diagram. Some characteristics of the specific energy are that for a given E there are two possible depths of flow, y_1 and y_2, known as alternate depths, and there is a definite minimum E for a given q. The depth of flow corresponding to the minimum E is known as the critical depth and is denoted by y_c. The relationship between the flow rate and y_c can be determined by differentiating Eq. (4.8) and setting the

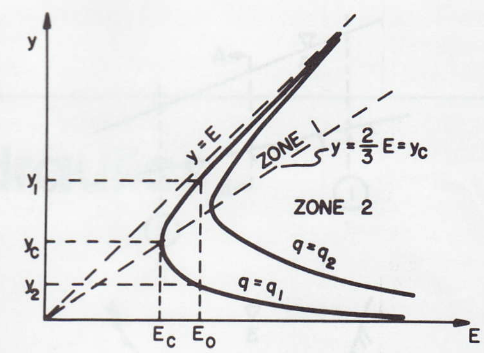


Figure 4.5 Specific energy diagrams.

differential to zero:

$$dE/dy = -2q^2/2gy^3 + 1 = 0$$

or

$$y_c = \sqrt[3]{q^2/g}. \tag{4.9}$$

Since $q = vy_c$, we can write Eq. (4.9) as

$$v/\sqrt{gy_c} = 1. \tag{4.10}$$

The term $v/\sqrt{gy_c}$ is known as the Froude number **F**. Equation (4.10) shows that when $y = y_c$ or when the flow is at the critical depth, the Froude number is one. The Froude number can be used to classify the flow into subcritical, critical, and supercritical flow. When $\mathbf{F} < 1$, the flow is subcritical and $y > y_c$. This corresponds to zone 1 in Fig. 4.5. When $\mathbf{F} > 1$, the flow is supercritical and $y < y_c$. Supercritical flow is zone 2 in Fig. 4.5. $\mathbf{F} = 1$ is known as critical flow and corresponds to the line $y = y_c = 2E/3$ in Fig. 4.5. Equation (4.10) shows that for critical flow, $v_c = \sqrt{gy}$. This velocity corresponds to the celerity of small gravity waves in shallow water.

For nonrectangular channels, the Froude number is defined as

$$\mathbf{F} = v/\sqrt{gd_h}, \tag{4.11}$$

where d_h is the hydraulic depth. The hydraulic depth is defined as the area divided by the top width

$$d_h = A/t. \tag{4.12}$$

Since **F** is independent of slope, y_c depends only on the discharge for a given channel. For a rectangular channel, this is apparent from Eq. (4.9). In general, the relationship between Q and y_c can be determined from Eq. (4.11) for any channel by setting $\mathbf{F} = 1$ and noting from Eq. (4.3) that $v = Q/A$.

Example Problem 4.1 Critical depth

A triangular channel with side slopes of 3 : 1 is carrying 20 cfs. What is the critical depth for this channel and flow rate?

Solution: Critical depth occurs when F = 1. Equations (4.11) and (4.12) must be used. Note that a triangular channel is a special case of a trapezoidal channel with $b = 0$. The area and top width are given by (see Fig. 4.9)

$$A = zd^2 = 3d^2$$
$$t = 2dz = 6d.$$

Therefore

$$d_h = A/t = 3d^2/6d = 0.5d.$$

From Eq. (4.11),

$$1 = \frac{v}{\sqrt{gd_h}} = \frac{Q/A}{\sqrt{0.5gd}}$$

$$1 = \frac{20/3d^2}{\sqrt{16.1d}} = \frac{1.66}{d^{5/2}}$$

$$d_c = 1.23 \text{ ft.}$$

As shown in subsequent sections of this chapter, channel roughness, velocity, discharge, and slope are interrelated. For a given discharge and roughness, the velocity can be increased and consequently, the depth of flow decreased by increasing the channel slope. When the channel slope is such that the flow depth resulting in uniform flow equals critical depth, the slope is called the critical slope, S_c. Thus for subcritical flow, the slope is less than S_c and for supercritical flow, the slope is greater than S_c. It should be pointed out that critical depth, slope, and velocity for a given section change with the discharge.

In designing channels for controlling and conveying runoff, it is generally desirable to design so that the flow is subcritical. Supercritical flow presents special problems that are not treated here.

Momentum

The momentum principle in open channel flow can be visualized by considering Fig. 4.6 and the basic relationship from mechanics

$$\Sigma F_s = \Delta(mv_s), \tag{4.13}$$

which states that the sum of the forces in the s-direction equals the change in momentum in that direction. In Eq. (4.13), F_s represents forces in the s-direction and m represents the mass. For a constant mass and a channel

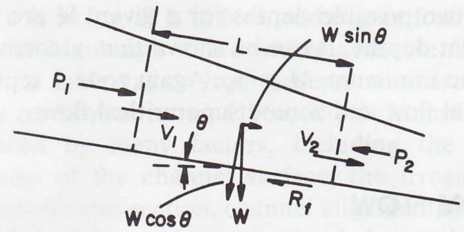


Figure 4.6 Sketch for momentum relationship.

per unit width consideration

$$\Delta(mv_s) = \rho q(v_2 - v_1).$$

The forces in the s-direction are

$$\Sigma F_s = P_1 + W \sin\theta - P_2 - R_f,$$

where P_1 and P_2 are pressure forces per unit width given by

$$P = \gamma y^2/2.$$

R_f is a frictional resistance, and $W \sin\theta$ is the s-direction component of the weight. Combining terms, we have

$$\frac{\gamma y_1^2}{2} - \frac{\gamma y_2^2}{2} + W\sin\theta - R_f = \rho q(v_2 - v_1). \tag{4.14}$$

If a short section is considered so that R_f is negligible and the channel slope is small so that $\sin\theta$ is near zero, Eq. (4.14) can be written as

$$\frac{\gamma y_1^2}{2} + \rho q v_1 = \frac{\gamma y_2^2}{2} + \rho q v_2$$

or

$$\frac{y_1^2}{2} + \frac{qv_1}{g} = \frac{y_2^2}{2} + \frac{qv_2}{g} = M, \tag{4.15}$$

where M is the specific force plus momentum and is a constant. Again it is possible to plot y versus M for a constant q in the form of a specific force plus momentum curve. Figure 4.7 is such a plot again showing two possible depths for a given M and a definite minimum

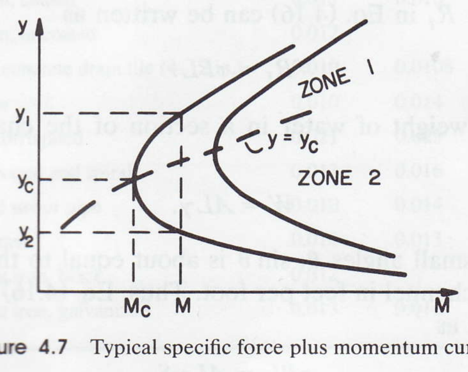


Figure 4.7 Typical specific force plus momentum curve.

M. The two possible depths for a given M are known as sequent depths. It can be shown that y corresponding to the minimum M is y_c. Again zone 1 represents subcritical flow and zone 2 supercritical flow.

UNIFORM FLOW

Open channel flow is generally classified with respect to changes in flow properties with time and with location along the channel. If the flow characteristics at a point are unchanging with time, the flow is said to be steady flow; otherwise the flow is unsteady. Similarly, if the flow properties are the same at every location along the channel, the flow is uniform. Flow with properties that change with channel location is nonuniform flow. In natural flow situations, the flow is generally nonsteady and nonuniform. However, in the design of most channels, steady, uniform flow is assumed with the channel design being based on some peak or maximum discharge.

When we speak of uniform flow, steady, uniform flow is generally what is considered. For uniform flow, y_1 and y_2 and v_1 and v_2 in Fig. 4.6 are equal. Thus, Eq. (4.14) reduces to

$$R_f = W \sin\theta \tag{4.16}$$

or the frictional forces are just equal to the downstream component of the weight. That is, the frictional resistance and gravitational forces are in equilibrium.

The frictional resistance to flow may be expressed as a shear, τ, per unit area times the resisting area. Neglecting the resistance generated at the surface of the flow between the water and air, the resisting area over which τ operates is the length, L, of a section times the wetted perimeter, P, of the channel. The wetted perimeter is simply the length of the boundary between the water and the channel sides and bottom at any cross section or the distance around the flow cross section starting at one edge of the channel and traveling along the sides and bottom of the channel to the other channel edge.

Thus R_f in Eq. (4.16) can be written as

$$R_f = \tau PL. \tag{4.17}$$

The weight of water in a section of the channel is simply

$$W = AL\gamma. \tag{4.18}$$

For small angles θ, $\sin\theta$ is about equal to the slope of the channel in feet per foot. Thus, Eq. (4.16) may be written as

$$\tau PL = AL\gamma S, \tag{4.19}$$

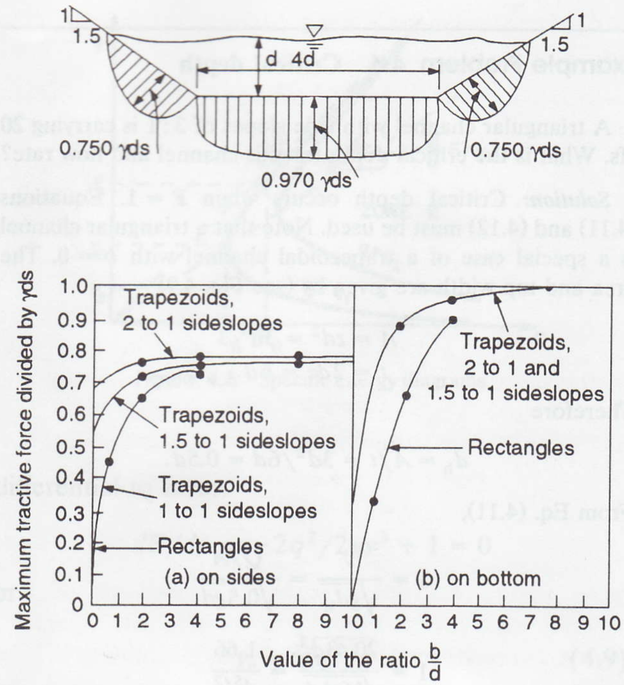


Figure 4.8 Tractive force distribution for trapezoidal channels (Lane and Carlson, 1953).

which upon rearrangement is

$$\tau = \gamma(A/P)S.$$

The term A/P represents the hydraulic radius, R, defined as the flow area divided by the wetted perimeter. Thus, we have

$$\tau = \gamma RS. \tag{4.20}$$

In this equation, τ represents the average shear around the periphery of the flow. At some points the actual shear will exceed τ and at other points it will be less than τ. Lane and Carlson (1953) found the shear on the periphery of a trapezoidal channel varied as shown in Fig. 4.8. The maximum shear is near γdS rather than γRS. In designing channels for stability using a critical tractive force approach as shown later, the maximum shear can be calculated as γdS.

Experimental studies on water flow in pipes have shown that τ is proportional to the Darcy–Weisbach friction factor, f, and the square of the flow velocity. That is

$$\tau = f\rho v^2/8 \tag{4.21}$$

or combining Eqs. (4.20) and (4.21),

$$v = \sqrt{8\gamma/f\rho}\sqrt{RS}.$$

By letting $\sqrt{8\gamma/f\rho} = C$, Chezy's equation for open channel flow is obtained as

$$v = C\sqrt{RS} = CR^{1/2}S^{1/2}, \tag{4.22}$$

where C is a factor related to the roughness of the channel.

An Irish engineer named Manning found that the equation

$$v = KR^{2/3}S^{1/2}$$

fit experimental data quite nicely. This equation is known as Manning's equation and differs from Chezy's equation only in the exponent on R. So that the factor related to the channel roughness would increase as roughness increased, Manning's equation is generally written as

$$v = (1/n)\,R^{2/3}S^{1/2}$$

in the metric system with v in meters per second and R in meters. The coefficient n is known as Manning's n. In the English system of units, Manning's equation is

$$v = \frac{1.49}{n}R^{2/3}S^{1/2}, \tag{4.23}$$

where v is in fps, R is in feet, and S is in feet per foot. Tables of Manning's n are widely available. Table 4.1 is such a table taken from several sources, drawing heavily on Schwab *et al.* (1966, 1971). Manning's n is influenced by many factors, including the physical roughness of the channel surface, the irregularity of the channel cross section, channel alignment and bends, vegetation, silting and scouring, and obstruction within the channel. Chow (1959) displays some photographs of typical channels and the associated values for Manning's n.

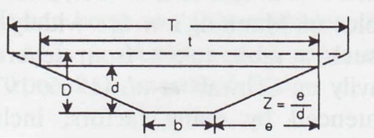
Figure 4.9 contains some useful relationships for calculating the hydraulic properties of A, P, R, and top width, T, for three common channels. For natural channels, these properties are best determined from measurements based on the actual cross sections of the channel.

Table 4.1 Typical Values for Manning's n

Type and description of conduits	Min. (n Values[a])	Design (n Values[a])	Max. (n Values[a])
Channels, lined			
Asphaltic concrete, machine placed		0.014	
Asphalt, exposed prefabricated		0.015	
Concrete	0.012	0.015	0.018
Concrete, rubble	0.016		0.029
Metal, smooth (flumes)	0.011		0.015
Metal, corrugated	0.021	0.024	0.026
Plastic	0.012		0.014
Shotcrete	0.016		0.017
Wood, planed (flumes)	0.009	0.012	0.016
Wood, unplaned (flumes)	0.011	0.013	0.015
Channels, earth			
Earth bottom, rubble sides	0.028	0.032	0.035
Drainage ditches, large, no vegetation			
(a) < 2.5 hydraulic radius	0.040		0.045
(b) 2.5–4.0 hydraulic radius	0.035		0.040
(c) 4.0–5.0 hydraulic radius	0.030		0.035
(d) > 5.0 hydraulic radius	0.025		0.030
Small drainage ditches	0.035	0.040	0.040
Stony bed, weeds on bank	0.025	0.035	0.040
Straight and uniform	0.017	0.0225	0.025
Winding, sluggish	0.0225	0.025	0.030
Channels, vegetated			
(See subsequent discussion)			
Natural Streams			
(a) Clean, straight bank, full stage, no rifts or deep pools	0.025		0.033
(b) Same as (a) but some weeds and stones	0.030		0.040
(c) Winding, some pools and shoals, clean	0.035		0.050
(d) Same as (c), lower stages, more ineffective slopes and sections	0.040		0.055
(e) Same as (c), some weeds and stones	0.033		0.045
(f) Same as (d), stony sections	0.045		0.060
(g) Sluggish river reaches, rather weedy or with very deep pools	0.050		0.080
(h) Very weedy reaches	0.075		0.150
Pipe			
Asbestos cement		0.009	
Cast iron, coated	0.011	0.013	0.014
Cast iron, uncoated	0.012		0.015
Clay or concrete drain tile (4–12 in.)	0.010	0.0108	0.020
Concrete	0.010	0.014	0.017
Metal, corrugated	0.021	0.025	0.0255
Steel, riveted and spiral	0.013	0.016	0.017
Vitrified sewer pipe	0.010	0.014	0.017
Wood stave	0.010	0.013	
Wrought iron, black	0.012		0.015
Wrought iron, galvanized	0.013	0.016	0.017

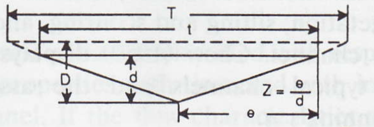
[a] Selected from numerous sources.

Note: Freeboard = D–d for all selection



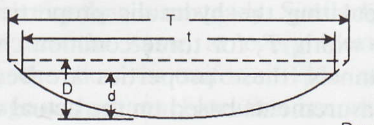
Cross-sectional area a	Wetted perimeter, p	Hydraulic radius $R = \frac{a}{p}$	Top width
$bd + Zd^2$	$b + 2d\sqrt{Z^2+1}$	$\frac{b + Zd^2}{b + 2d\sqrt{Z^2+1}}$	$t = b + 2dZ$ $T = b + 2DZ$

Trapezoidal cross section



Cross-sectional area a	Wetted perimeter, p	Hydraulic radius $R = \frac{a}{p}$	Top width
Zd^2	$2d\sqrt{Z^2+1}$	$\frac{Zd}{2\sqrt{Z^2+1}}$ or $\frac{d}{2}$ approx.	$t = 2dZ$ $T = \frac{D}{d}t$

Triangular cross section



Cross-sectional area a	Wetted perimeter, p	Hydraulic radius $R = \frac{a}{p}$	Top width
$\frac{2}{3}td$	$t + \frac{8d^2}{3t}$	$\frac{t^2d}{1.5t^2 + 4d^2}$ or $\frac{2d}{3}$ approx.	$t = \frac{a}{0.67d}$ $T = t\left(\frac{D}{d}\right)^{1/2}$

Parabolic cross section

Figure 4.9 Properties of typical channels.

The expression for the hydraulic radius for wide shallow channels can be simplified from that shown in Fig. 4.9. Consider the trapezoidal channel shown in Fig. 4.10. If the trapezoid is approximated by a rectangle, one can write

$$R = \frac{A}{P} = \frac{bd}{b + 2d}.$$

If $b \gg d$, then the $2d$ in the denominator can be ignored leaving

$$R \approx bd/b = d.$$

For a parabolic channel, if $t \gg d$, then $4d^2$ in the denominator of the expression for R can be ignored leaving

$$R \approx \frac{t^2 d}{1.5t^2} = \frac{2}{3}d.$$

These approximations can serve as initial estimates for d in trial and error solutions that often arise in open channel hydraulics.

The hydraulic elements of a circular conduit of diameter D can be calculated from

$$A = \frac{D^2}{8}(\theta - \sin\theta) \tag{4.24}$$

$$R = \frac{D}{4}\left(1 - \frac{\sin\theta}{\theta}\right) \tag{4.25}$$

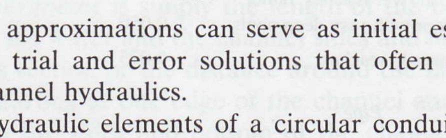


Figure 4.10 Approximation of trapezoidal channel with rectangular channel.

The angle θ is defined in Fig. 4.11 and measured in radians. Example Problems 4.2, 4.3, and 4.4 illustrate the use of Eqs. (4.24) and (4.25) to solve open channel flow problems dealing with circular conduits.

The maximum flow capacity of a circular conduit actually occurs at a depth equal to $0.938D$. Figure 4.12 shows how the hydraulic elements of a circular conduit change with depth. The subscript 0 refers to a depth

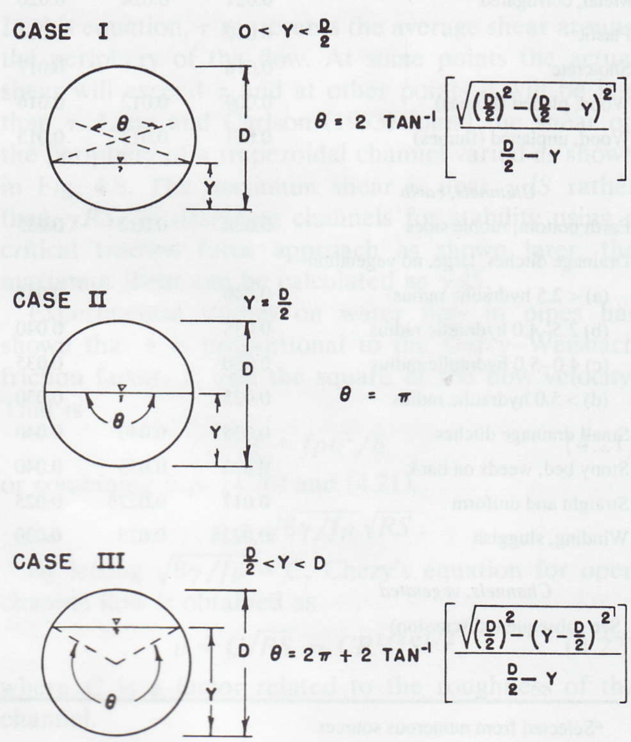


Figure 4.11 Definition of θ.

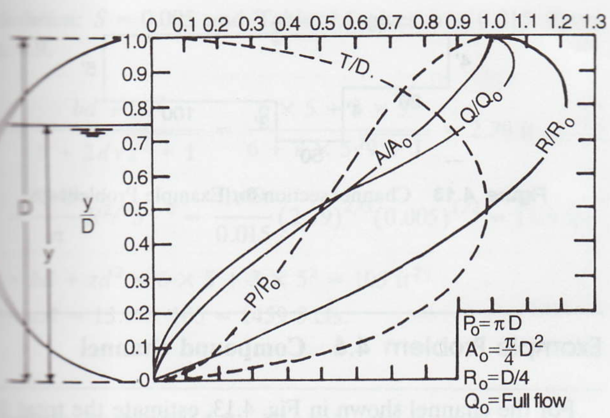


Figure 4.12 Hydraulic properties of a circular conduit.

equal to D. The line labeled Q/Q_0 assumes that n is constant with depth. Even though the maximum flow occurs at $0.938D$, it is common to design circular conduits to carry maximum flows when flowing full. Wave action and irregularities make it difficult to maintain flow at $0.938D$.

Example Problem 4.2 Flow in circular pipe 1

A circular corrugated metal pipe (CMP) that is 3 ft in diameter is flowing 1 ft deep. What is the discharge if the slope of the pipe is 4%?

Solution: Refer to Fig. 4.11 with the pipe radius, r, equal to $D/2$. Since $y < D/2$,

$$\theta = 2\tan^{-1}\left[\frac{\left[r^2-(r-y)^2\right]^{1/2}}{r-y}\right]$$

$$= 2\tan^{-1}\left[\frac{\left[1.5^2-(1.5-1.0)^2\right]^{1/2}}{1.5-1.0}\right]$$

$$= 2\tan^{-1}(2.828) = 2.46.$$

From Eqs. (4.24) and (4.25),

$$A = \frac{D^2}{8}(\theta - \sin\theta) = \frac{9}{8}(2.46 - \sin 2.46) = 2.06$$

$$R = \frac{D}{4}\left(1 - \frac{\sin\theta}{\theta}\right) = \frac{3}{4}\left(1 - \frac{\sin 2.46}{2.46}\right) = 0.56$$

$$Q = \frac{1.49}{n}R^{2/3}S^{1/2}A.$$

From Table 4.1, $n = 0.024$,

$$Q = \frac{1.49}{0.024}(0.56)^{2/3}(0.04)^{1/2}(2.06) = 17.4 \text{ cfs}.$$

Example Problem 4.3 Flow in circular pipe 2

A circular corrugated metal pipe that is 3 ft in diameter is flowing 2 ft deep. What is the discharge if the slope of the pipe is 4%?

Solution: Refer to Fig. 4.11 Since $y > D/2$,

$$\theta = 2\pi + 2\tan^{-1}\left[\frac{\left[r^2-(y-r)^2\right]^{1/2}}{r-y}\right]$$

$$= 6.28 + 2\tan^{-1}\left[\frac{\left[1.5^2-(2.0-1.5)^2\right]^{1/2}}{1.5-2}\right] = 3.81.$$

From Eqs. (4.24) and (4.25),

$$A = \frac{D^2}{8}(\theta - \sin\theta) = 5.00$$

$$R = \frac{D}{4}\left(1 - \frac{\sin\theta}{\theta}\right) = 0.87$$

$$Q = \frac{1.49}{n}R^{2/3}S^{1/2}A.$$

From Table 4.1, $n = 0.024$,

$$Q = \frac{1.49}{0.024}(0.87)^{2/3}(0.04)^{1/2}(5.00) = 56.6 \text{ cfs}.$$

Example Problem 4.4 Flow in circular pipe 3

A circular corrugated metal pipe that is 3 ft in diameter is carrying 30 cfs. How deep is the water flowing if the slope of the pipe is 4%?

Solution:

$$Q = \frac{1.49}{n}R^{2/3}S^{1/2}A$$

$$30 = \frac{1.49}{n}\left[\frac{D}{4}\left(1 - \frac{\sin\theta}{\theta}\right)\right]^{2/3}S^{1/2}\frac{D^2}{8}(\theta - \sin\theta).$$

After substituting $D = 3$, $n = 0.024$, and $S = 0.04$, this equation can be rearranged as

$$2.604 = \left(1 - \frac{\sin\theta}{\theta}\right)^{2/3}(\theta - \sin\theta).$$

This relationship can be solved by trial by assuming values for θ, comparing the right-hand side of the equation to the left-hand side and continuing until a match is achieved.

Trial θ	Right-hand side	
3.14	3.14	
2.50	1.58	
2.90	2.51	
2.94	2.61	OK

$\theta = 2.94$ is a solution. Since $\theta < \pi$, y must be less than r and can be obtained from

$$\theta = 2\tan^{-1}\left[\frac{\left[r^2 - (r-y)^2\right]^{1/2}}{r-y}\right]$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\left[r^2 - (r-y)^2\right]^{1/2}}{r-y}$$

$$\tan^2\left(\frac{\theta}{2}\right) = \frac{\left[r^2 - (r-y)^2\right]}{(r-y)^2}$$

$$\tan^2\left(\frac{2.94}{2}\right) = \frac{\left[2.25 - (1.5-y)^2\right]}{(1.5-y)^2}.$$

When this equation is solved for y, the result is $y = 1.35$ ft.

Example Problem 4.5 Flow in circular pipe 4

Use Fig. 4.12 to solve Example Problems 4.2, 4.3, and 4.4.

Solution:

$$Q_0 = \frac{1.49}{n} R_0^{2/3} S^{1/2} A_0.$$

$R_0 = D/4$ and $A_0 = \pi D^2/4$; therefore

$$Q_0 = \frac{1.49}{0.024}\left(\frac{3}{4}\right)^{2/3}(0.04)^{1/2}\frac{\pi 3^2}{4} = 72.4 \text{ cfs}.$$

When $y = 1$, $y/D = 0.33$. From Fig. 4.12, $Q/Q_0 = 0.23$. Therefore $Q = 0.23(72.4) = 16.7$ cfs. When $y = 2$, $y/D = 0.67$. From Fig. 4.12, $Q/Q_0 = 0.78$. Therefore $Q = 0.78(72.4) = 56.5$ cfs. When $Q = 30$, $Q/Q_0 = 0.41$. From Fig. 4.12, $y/D = 0.44$. Therefore $y = 0.44(3) = 1.32$ ft.

Natural channels often have a main channel section and an overbank section. Most flow occurs in the main channel; however, during flood events overbank flows may occur. The usual procedure for calculating such flows is to break the channel into cross-sectional parts and sum the flow calculated for the various parts. In determining the hydraulic radius for the various parts, only that part of the wetted perimeter in contact with an actual channel boundary is used. Thus

$$V_i = \frac{1.49}{n_i} S^{1/2}\left(\frac{A_i}{P_i}\right)^{2/3} \tag{4.26}$$

and

$$Q = \sum_{i=1}^{n} V_i A_i.$$

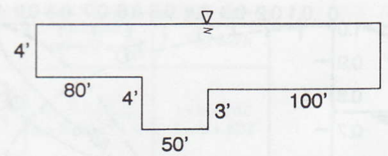


Figure 4.13 Channel section for Example Problem 4.6.

Example Problem 4.6 Compound channel

For the channel shown in Fig. 4.13, estimate the total flow for a depth of 8 ft. The channel has a slope of 0.5%. Manning's n is 0.06 for the overbank area and 0.03 for the main channel.

Solution: Use Eq. (4.26).

$A_1 = 80 \times 4 = 320$, $A_2 = 50 \times 8 = 400$,

$A_3 = 100 \times 5 = 500$

$P_1 = 80 + 4 = 84$, $P_2 = 4 + 50 + 3 = 57$,

$P_3 = 100 + 5 = 105$

$$Q = 1.49(0.005)^{1/2}\left[\frac{(320/84)^{2/3}320}{0.06} + \frac{(400/57)^{2/3}400}{0.03} + \frac{(500/105)^{2/3}500}{0.06}\right]$$

$$= 9010 \text{ cfs}.$$

DESIGN OF OPEN CHANNELS

Nonerodible Channels

The design of nonerodible open channels can be done by using Manning's equation [Eq. (4.23)]. Manning's n should be chosen carefully. Adequate consideration should be given to adding a freeboard or extra depth to the channel as a safety measure to protect against underestimates of flow or roughness and wave action. Generally a freeboard of around 20% of the depth or 0.3 to 0.5 ft, whichever is greater, should be added to the channel depth. Thus, the major consideration in the design of channels in nonerodible material is to ensure adequate capacity.

Example Problem 4.7 Flow rate concrete channel 1

Consider a concrete channel that is trapezoidal with 3:1 side slopes and a 6-ft bottom width. The channel is on a 0.5% slope and is flowing at a depth of 5 ft. What is the flow rate?

Solution: $S = 0.005$ and Table 4.1 gives $n = 0.015$. From Fig. 4.9,

$$R = \frac{bd + zd^2}{b + 2d\sqrt{z^2 + 1}} = \frac{6 \times 5 + 3 \times 5^2}{6 + 2 \times 5\sqrt{9 + 1}} = 2.79 \text{ ft}$$

$$v = \frac{1.49}{n} R^{2/3} S^{1/2} = \frac{1.49}{0.015} (2.79)^{2/3} (0.005)^{1/2} = 13.9 \text{ fps}$$

$$A = bd + zd^2 = 6 \times 5 + 3 \times 5^2 = 105 \text{ ft}^2$$

$$Q = vA = 13.9 \times 105 = 1459.5 \text{ cfs.}$$

Example Problem 4.8 Flow depth concrete channel 2

The channel of Example Problem 4.7 is carrying 75 cfs. How deep is the water flowing?

Solution:

$$Q = vA = \frac{1.49}{n} R^{2/3} S^{1/2} A$$

$$= \frac{1.49}{n} \left[\frac{bd + zd^2}{b + 2d\sqrt{z^2 + 1}} \right]^{2/3} S^{1/2}(bd + zd^2)$$

$$75 = \frac{1.49}{0.015} \left[\frac{6d + 3d^2}{6 + 6.32d} \right]^{2/3} (0.005)^{1/2} (6d + 3d^2)$$

$$10.68 = \left[\frac{6d + 3d^2}{6 + 6.32d} \right]^{2/3} (6d + 3d^2).$$

This last relationship must be solved by trial for a d such that the right-hand side of the equation is equal to 10.68.

Trial d	Right-hand side	
1	7.30	
1.5	15.93	
1.2	10.32	
1.22	10.65	OK

The channel is flowing 1.22 ft deep.

Example Problem 4.9 Froude number

Calculate the Froude number of the flow in example problem 4.7.

Solution:

$$F = \frac{v}{(gd_h)^{1/2}}.$$

Example Problem 4.7 gives $A = 105 \text{ ft}^2$; therefore

$$d_h = \frac{A}{t} = \frac{A}{b + 2dz} = \frac{105}{6 + 2 \times 5 \times 3} = 2.92$$

$$F = \frac{13.9}{(32.2 \times 2.92)^{1/2}} = 1.43.$$

Thus the flow is supercritical. The high flow velocity is an early indicator of the possibility of supercritical flow.

Erodible Channels

In designing channels to be constructed in erodible materials there are two major considerations. The channel must have adequate capacity to carry the flow and it must have adequate stability to resist the erosive action of the flowing water. Erodible channels may be either vegetated or nonvegetated. Vegetation tends to protect the channel from erosion, thus permitting higher flow velocities. On the other hand, vegetation increases the roughness of the channel. The design of nonvegetated channels is considered next followed by the design of vegetated channels. Flexible linings and riprap linings are discussed in subsequent sections.

Nonvegetated Channels

Two main design procedures are used for ensuring the stability of erodible channels. One procedure is based on a limiting velocity concept and the other on a limiting tractive force (boundary shear) concept. Table 4.2 shows allowable velocities and tractive force values for several kinds of channels. This table is taken from Lane (1955) based on the work by Fortier and Scobey (1926). The values are for aged, stable channels. For newly constructed channels, the values shown in Table 4.6 should be used.

When using the limiting velocity concept, one simply sizes the channel so that it has adequate capacity and so that the average velocity does not exceed the permissible velocity.

When using the limiting tractive force concept, a channel with adequate capacity and having an average shear stress given by Eq. (4.20) that is less than the values tabulated in Table 4.2 is sought. For channels in noncohesive materials, the particles on the channel sides may move due to the combined force exerted by the flowing water and the weight component of the particles down the side of the channel. Chow (1959) should be referred to for a treatment of tractive force considerations in noncohesive materials. In cohesive materials, the cohesion generally is much greater than the gravity component so that average shear based on Eq. (4.20) can be used in design.

An alternative approach to designing stable, unlined channels is to use regime relationships. These relationships define equilibrium conditions between flow and the channel boundaries. Chapter 10 discusses this approach.

Example Problem 4.10 Erodible channel design

Design a channel to carry 20 cfs down a 0.5% slope. The channel material is to be an ordinary firm loan. The water will be transporting colloidal silts. The channel is to be trapezoidal with 3 : 1 side slopes. Use (a) the limiting velocity approach and (b) the limiting tractive force approach.

Solution:

(a) Limiting velocity approach. From Table 4.2, $v_p = 3.5$ fps, $n = 0.020$,

$$v_p = \frac{1.49}{n} R^{2/3} S^{1/2}$$

$$R = \left[\frac{v_p n}{1.49 S^{1/2}} \right]^{3/2} = \left[\frac{3.5(0.020)}{1.49(0.005)^{1/2}} \right]^{3/2} = 0.54$$

$$A = \frac{Q}{v_p} = \frac{20.00}{3.5} = 5.71$$

$$R = \frac{bd + zd^2}{b + 2d\sqrt{z^2 + 1}} = \frac{bd + 3d^2}{b + 6.32d} = 0.54 \qquad \text{(a)}$$

$$A = bd + zd^2 = bd + 3d^2 = 5.71. \qquad \text{(b)}$$

Substituting Eq. (b) into Eq. (a) yields

$$\frac{5.71}{b + 6.32d} = 0.54$$

or

$$b = 10.58 - 6.32d. \qquad \text{(c)}$$

Substituting this into Eq. (b) yields

$$(10.58 - 6.32d)d + 3d^2 = 5.71$$

$$-3.32d^2 + 10.58d - 5.71 = 0.00.$$

This is a quadratic equation of the form

$$ax^2 + bx + c = 0,$$

which has as a solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Therefore

$$d = \frac{-10.58 \pm \sqrt{10.58^2 - 4(-3.32)(-5.71)}}{2(-3.32)}$$

$$d = \frac{-10.58 + 6.00}{-6.64} = 2.50;\ 0.69.$$

Table 4.2 Limiting Velocities and Tractive Forces for Open Channels (Straight after Aging)[a]

Material	n	For Clear Water: Velocity (fps)	For Clear Water: Tractive force (psf)	Water transporting colloidal silts: Velocity (fps)	Water transporting colloidal silts: Tractive force (psf)
Fine sand colloidal	0.020	1.50	0.027	2.50	0.075
Sandy loam noncolloidal	0.020	1.75	0.037	2.50	0.075
Silt loam noncolloidal	0.020	2.00	0.048	3.00	0.110
Alluvial silts noncolloidal	0.020	2.00	0.048	3.50	0.150
Ordinary firm loam	0.020	2.50	0.075	3.50	0.150
Volcanic ash	0.020	2.50	0.075	3.50	0.150
Stiff clay very colloidal	0.025	3.75	0.260	5.00	0.460
Alluvial silts colloidal	0.025	3.75	0.260	5.00	0.460
Shales and hardpans	0.025	6.00	0.670	6.00	0.670
Fine gravel	0.020	2.50	0.075	5.00	0.320
Graded loam to cobbles when noncolloidal	0.030	3.75	0.380	5.00	0.660
Graded silts to cobbles when collodial	0.030	4.00	0.430	5.50	0.800
Coarse gravel noncolloidal	0.025	4.00	0.300	6.00	0.670
Cobbles and shingles	0.035	5.00	0.910	5.50	1.100

[a]From Lane (1955).

If $d = 2.50$, then from Eq. (c) we get

$$b = 10.58 - 6.32(2.50) = -5.22,$$

which is clearly not possible. If $d = 0.69$, we get

$$b = 10.58 - 6.32(0.69) = 6.22.$$

Therefore the channel dimensions must be

$$b = 6.22 \text{ ft}, \quad d = 0.69 \text{ ft}, \quad z = 3.0.$$

Check:

$$R = \frac{bd + zd^2}{b + 2d\sqrt{z^2 + 1}} = \frac{6.22(0.69) + 3(0.69)^2}{6.22 + 2(0.69)\sqrt{10}} = 0.54$$

$$v = \frac{1.49}{n} R^{2/3} S^{1/2} = \frac{1.49}{0.02}(0.54)^{2/3}(0.005)^{1/2} = 3.5.$$

The velocity is OK.

$$A = bd + zd^2 = 6.22(0.69) + 3(0.69)^2 = 5.72$$

$$Q = vA = 3.50(5.72) = 20.00.$$

The capacity is OK.

Add 0.3 ft of freeboard to get the final design of $b = 6.2$ ft and $d = 1.0$ ft.

(b) Critical tractive force approach. From Table 4.2, $\tau_c = 0.15$, $n = 0.020$. Figure 4.8 shows that for shallow and wide ($b/d > 8$) trapezoidal channels, the maximum bottom shear is γdS. Therefore

$$\tau_c = \gamma dS$$

$$d = \frac{\tau_c}{\gamma S} = \frac{0.15}{62.4(0.005)} = 0.48$$

$$Q = \frac{1.49}{n} R^{2/3} S^{1/2} A$$

$$R = \frac{bd + zd^2}{b + 2d\sqrt{z^2 + 1}} = \frac{0.48b + 0.69}{b + 3.03}$$

$$A = bd + zd^2 = 0.48b + 0.69$$

$$20 = \frac{1.49}{0.02}\left(\frac{0.48b + 0.69}{b + 3.03}\right)^{0.667} (0.005)^{1/2}(0.48b + 0.69)$$

$$3.797 = \left(\frac{0.48b + 0.69}{b + 3.03}\right)^{0.667} (0.48b + 0.69).$$

Solving by trial and error, b is found to be 12.4 ft. The b/d ratio is $12.4/0.48 = 26$. Thus the assumption that the maximum bottom shear is γdS is verified. If b/d had been less than 8, τ_c would have been $K\gamma dS$, where K would be approximated from Fig. 4.8.

Upon verifying that a channel with a bottom width of 12.4 ft, a depth of 0.48 ft, and 3:1 side slopes will have an allowable velocity and adequate capacity, a freeboard of 0.3 ft is added giving a final design of $b = 12.4$ ft, $d = 0.8$ ft, and $z = 3$.

Vegetated Channels

From the previous section it can be seen that the allowable velocities and tractive forces for nonvegetated, erodible channels are quite small, thus requiring wide shallow channels. Regime theory relationships in Chapter 10 also predict wide shallow channels for these conditions. If the channel can be protected from erosion, the allowable velocities can be increased, resulting in deeper and more narrow channels. An inexpensive and permanent form of protection is vegetation—specifically grasses. Vegetation protects the channel material from the erosive action of the flow and binds the channel material together.

Vegetated waterways generally can be used to carry intermittent flows such as storm water runoff. They are not recommended for channels having sustained base flows as most vegetation cannot survive continual submergence or continual saturation in the root zone. This means that vegetated waterways would not be used as the channel carrying the discharge from a pipe spillway in a detention basin, as this flow is likely to be a sustained one. A compound channel with a small, lined channel in the center to carry base flows and a vegetated portion to carry storm flows may be used in these situations.

Vegetated waterways are somewhat more complex to design and require more care in their establishment than nonvegetated waterways. They carry high flows at high velocities and require a minimum of maintenance if properly constructed.

The additional design consideration for vegetated waterways is the variation in roughness (Manning's n) with the height of the vegetation and with the type of vegetation. Typically a tall grass presents a great deal of flow resistance to shallow flow. As the flow depth increases, the resistance may decrease. Often the grass will lay over in the direction of flow when the flow reaches sufficient depth. With the grass in this condition, the resistance is considerably reduced as compared to the shallow flow situation.

Experimental work has shown that Manning's n can be related to the product of the flow velocity and the hydraulic radius, vR. This experimental work has also shown that different grasses have different n–vR relationships. As a matter of fact, the same grass may have a different n–vR relationship depending on the height of the grass.

Grasses have been divided into five retardance classes, designated by A, B, C, D, and E. Table 4.3 lists the retardance class for a number of grasses that are commonly used. If the grass will be mowed part of the time and long part of the time, both conditions and retardance classes must be considered. If a particular vegetation is not listed in Table 4.3, a similar vegetation might be used as a guide in selecting the retardance. In comparing vegetation, density, stem diameter, stiffness, and other physical characteristics should be considered. Information in Table 4.4 may be used to estimate the vegetal retardance if specific information on the type of vegetation is not known.

The maximum permissible velocities shown in Table 4.5 should be used for established sod in good condition. The soil erodibility factor discussed in Chapter 8 can be used to classify soils as erosion resistant or easily eroded (see pp. 126). Flow at these maximum velocities may require channel maintenance operations. If poor vegetation exists due to shade, climate, soils, or other factors, the design velocity should be about 50% of the values of Table 4.5. Data in Table 4.6 may be used to select permissible velocities when specific vegetation and erosion characteristics of soils are not known.

Figure 4.14 shows the n–vR relationship for the five retardance classes. The design procedure is to select

Table 4.3 Vegetal Retardance Classes (Soil Conservation Service, 1969)

Retardance	Cover	Condition
A		
	Reed canary grass	Excellent stand, tall (average 36 in.)
	Yellow bluestem Ischaemum	Excellent stand, tall (average 36 in.)
B		
	Smooth bromegrass	Good stand, mowed (average 12 to 15 in.)
	Bermuda grass	Good stand, tall (average 12 in.)
	Native grass mixture (little bluestem, blue grams, and other long and short midwest grasses)	Good stand, unmowed
	Tall fescue	Good stand, unmowed (average 18 in.)
	Lespedeza sericea	Good stand, not woody, tall (average 19 in.)
	Grass–legume mixture — Timothy, smooth bromegrass, or orchard grass	Good stand, uncut (average 20 in.)
	Reed canary grass	Good stand, mowed (average 12 to 15 in.)
	Tall fescue, with bird's foot trefoil or lodino	Good stand, uncut (average 18 in.)
	Blue grama	Good stand, uncut (average 13 in.)
C		
	Bahia	Good stand, uncut (6 to 8 in.)
	Bermuda grass	Good stand, mowed (average 6 in.)
	Redtop	Good stand, headed (15 to 20 in.)
	Grass–legume mixture — summer (Orchard grass, redtop, Italian ryegrass, and common lespedeza)	Good stand, uncut (6 to 8 in.)
	Centipedegrass	Very dense cover (average 6 in.)
	Kentucky bluegrass	Good stand, headed (6 to 12 in.)
D		
	Bermuda grass	Good stand, cut to 2.5 in. height
	Red fescue	Good stand, headed (12 to 18 in.)
	Buffalograss	Good stand, uncut (3 to 6 in.)
	Grass–legume mixture — fall, spring (Orchard grass, redtop, Italian ryegrass, and common lespedeza)	Good stand, uncut (4 to 5 in.)
	Lespedeza sericea	After cutting to 2 in. height, very good stand before cutting
E		
	Bermuda grass	Good stand, cut to 1.5 in. height
	Bermuda grass	Burned stubble

the vegetation, determine its retardance class and permissible velocity, and then design the channel based on the curves of Fig. 4.14. For situations where two retardance classes are applicable (for example mowed and unmowed grass), the channel should first be designed for stability based on the lower retardance and then additional depth added to the channel to accommodate the flow when the retardance increases. This procedure ensures a stable channel with adequate capacity regardless of the condition of the vegetation.

Temple *et al.* (1987) have developed the following approximation for the n–vR curves of Fig. 4.14,

$$n = \exp\left[I\left(0.01329\ln(vR)^2 - 0.09543\ln(vR) + 0.2971\right) - 4.16\right], \quad (4.27)$$

where the value of I is

Retardance	I
A	10.000
B	7.643
C	5.601
D	4.436
E	2.876

This relationship can be used in computer programs to make hydraulic computations for vegetated waterways. The relationships should not be used outside the range of the curves shown in Fig. 4.14.

The graphs of Fig. 4.15 are solutions to Manning's equation using the curves in Fig. 4.14. They can be used as a design aid for solving Manning's equation for all retardance classes.

Example Problem 4.11 Vegetated channel 1

Design a channel to carry 25 cfs on a 4% slope. Use a parabolic channel. The soil is easily eroded, and the grass may be mowed to 2.5 in. or it may be uncut.

Solution: Select Bermuda grass. Bermuda grass is in retardance B if unmowed and retardance D if mowed. The permissible velocity is selected from Table 4.5 as 6 fps. First design for the mowed condition

$$A = Q/v = 25/6 = 4.17\ \text{ft}^2.$$

Table 4.4 Guide to Selection of Vegetal Retardance[a]

Stand	Length of vegetation (in.)	Retardance class
Good	>30	A
	11–24	B
	6–10	C
	2–6	D
	<2	E
Fair	>30	B
	11–24	C
	6–10	D
	2–6	D
	<2	E

[a]Soil Conservation Service (1979) engineering field manual.

Table 4.5 Permissible velocities for Vegetated Channels (Ree, 1949)

	Permissible velocity (fps)					
	Erosion-resistant soils (% slope)			Easily eroded soils (% slope)		
Cover	0–5	5–10	Over 10	0–5	5–10	Over 10
Bermuda grass	8	7	6	6	5	4
Buffalo grass Kentucky bluegrass Smooth brome Blue grama Tall fescue	7	6	5	5	4	3
Lespedeza sericea Weeping lovegrass Kudzu Alfalfa Crabgrass	3.5	NR[a]	NR	2.5	NR	NR
Grass mixture	5	4	NR	4	3	NR
Annuals for temporary protection	3.5	NR	NR	2.5	NR	NR

[a]Not recommended.

Table 4.6 Permissible Velocities (fps)[a]

Soil texture	Bare channel	Retardance	Channel vegetation Poor	Fair	Good
Sand, silt	1.5	B	1.5	3	4
Sandy loam	1.5	C	1.5	2.5	3.5
Silty loam	1.5	D	1.5	2	3
Silty clay loam	2	B	2.5	4	5
Sandy clay loam	2	C	2.5	3.5	4.5
	2	D	2.5	3	4
Clay	2.5	B	3	5	6
	2.5	C	3	4.5	5.5
	2.5	D	3	4	5

[a]Soil Conservation (1979) engineering field manual.

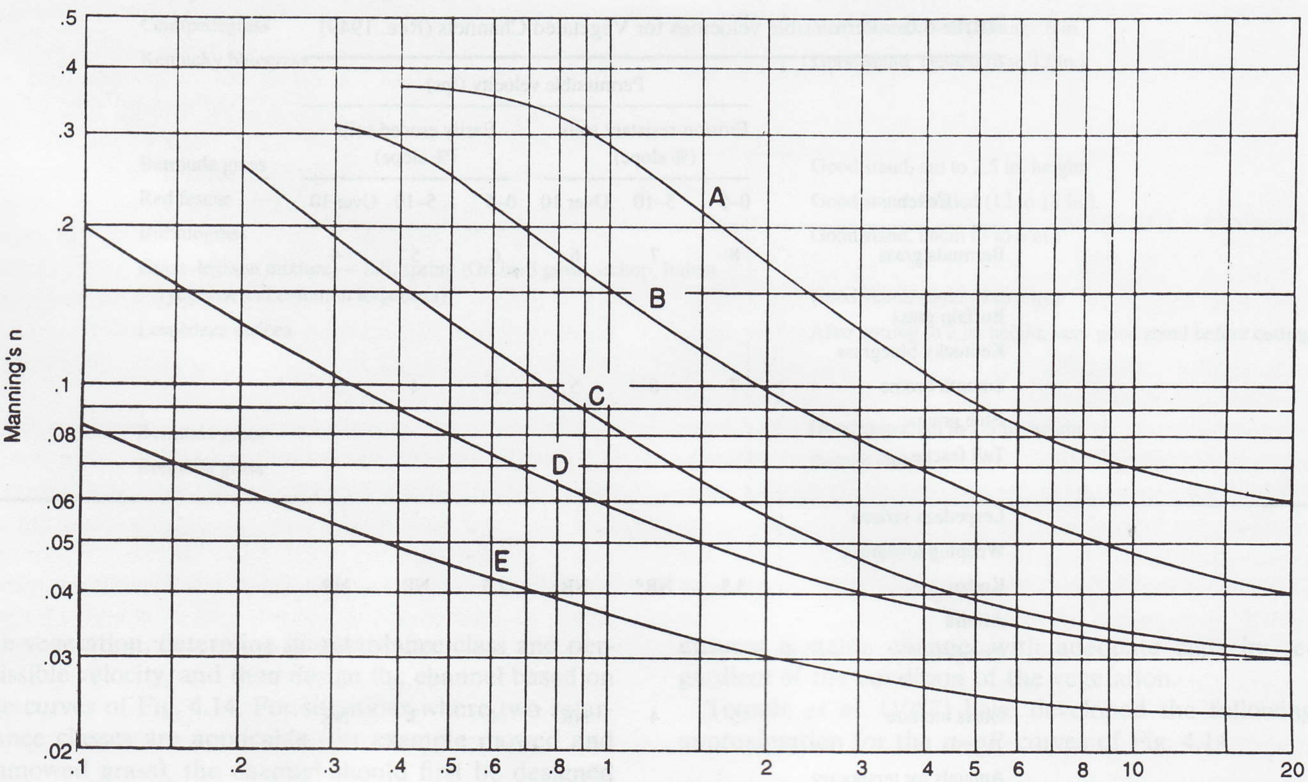


Figure 4.14 n–vR for various retardance classes.

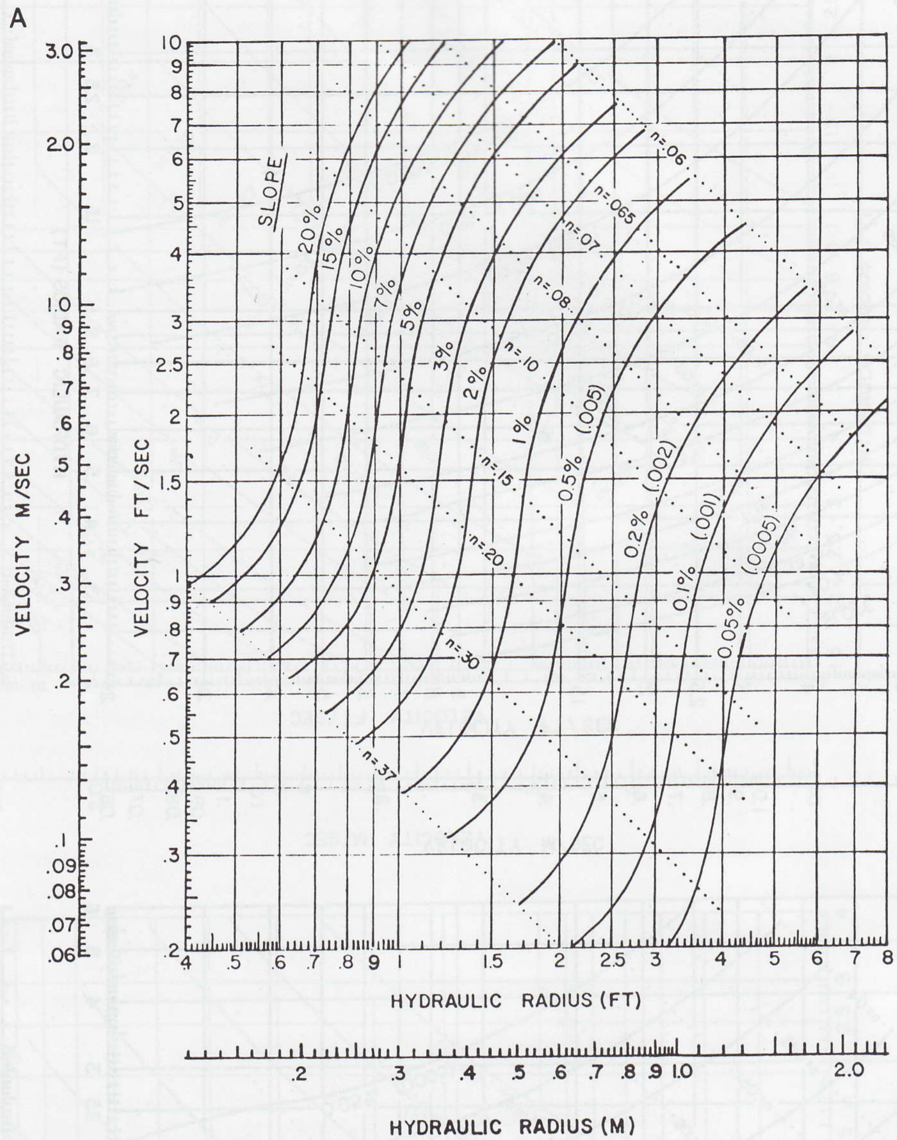


Figure 4.15a Solution to Manning's equation retardance class A.

From Fig. 4.15d for retardance class D, $R = 0.7$ ft.

$$R = 0.7 = \frac{t^2 d}{1.5t^2 + 4d^2}$$

(see Fig. 4.9)

$$A = 4.17 = \tfrac{2}{3}td.$$

For small parabolic channels, $d \approx 1.5R$. Using this approximation,

$$d = 1.05 \text{ ft}$$

$$t = \frac{3A}{2d} = \frac{3(4.17)}{2(1.05)} = 5.96 \text{ ft.}$$

Check:

$$R = \frac{(5.96)^2(1.05)}{1.5(5.96)^2 + 4(1.05)^2} = 0.646,$$

This is too small. Increase d to 1.25 feet, then

$$t = 3A/2d = 5.00 \quad \text{and} \quad R = 0.714.$$

Try $d = 1.17$ ft. Now $t = 3A/2d = 5.35$ and $R = 0.70$, which is OK.

The design for the short grass condition is

$$t = 5.35 \text{ ft}, \quad d = 1.17 \text{ ft}, \quad R = 0.7 \text{ ft}.$$

Now we must add depth using the same basic shape to get adequate capacity when the grass is long. When grass is long the retardance class is B. Try $D = 1.40$ ft. New top width

$$T = 5.35\left(\frac{1.40}{1.17}\right)^{1/2} = 5.85$$

and

$$R = \frac{t^2 d}{1.5t^2 + 4d^2} = 0.81.$$

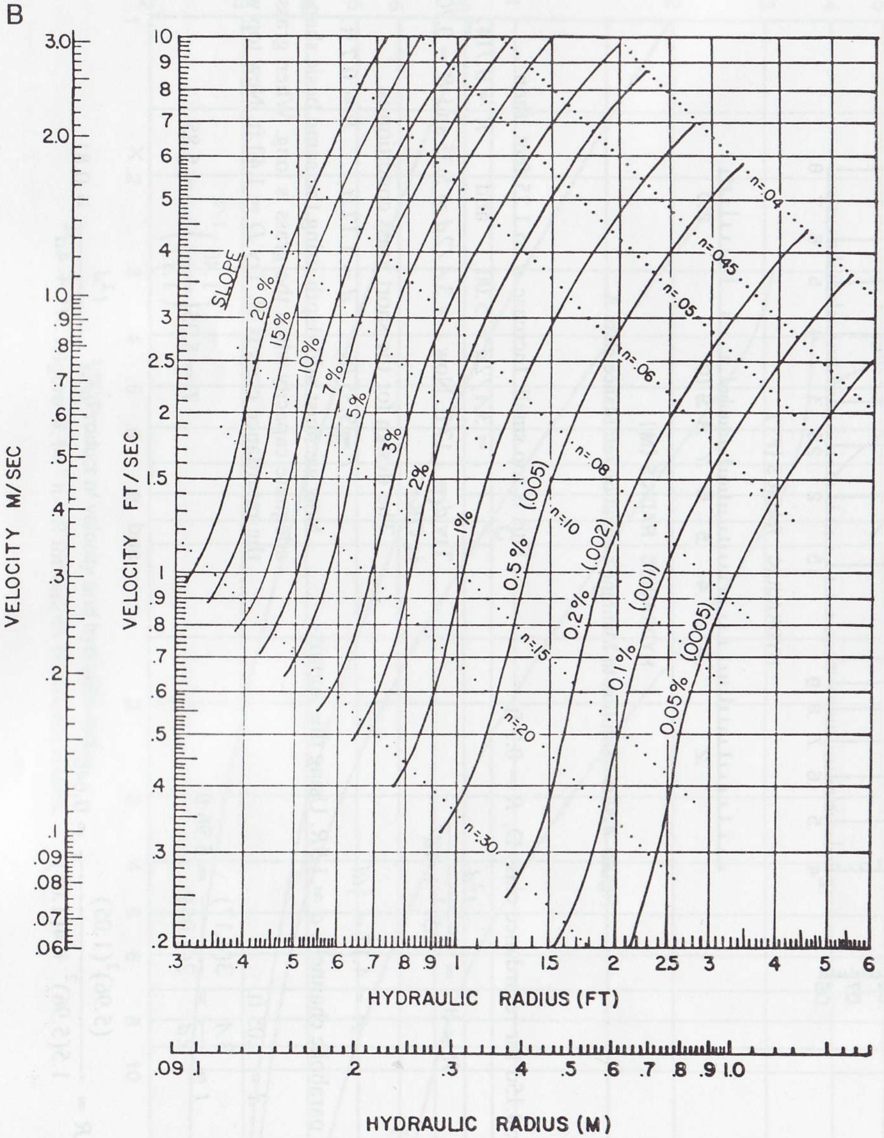


Figure 4.15b Solution to Manning's equation retardance class B.

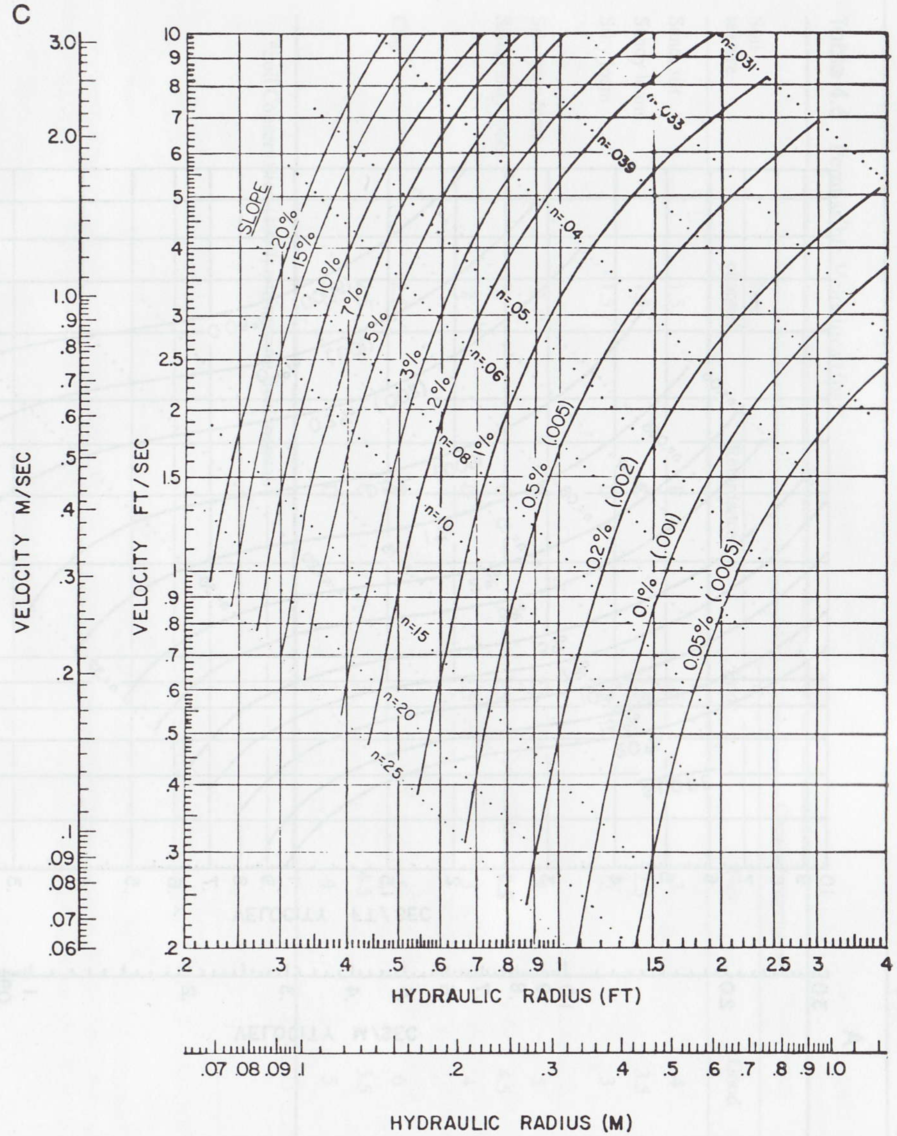


Figure 4.15c Solution to Manning's equation retardance class C.

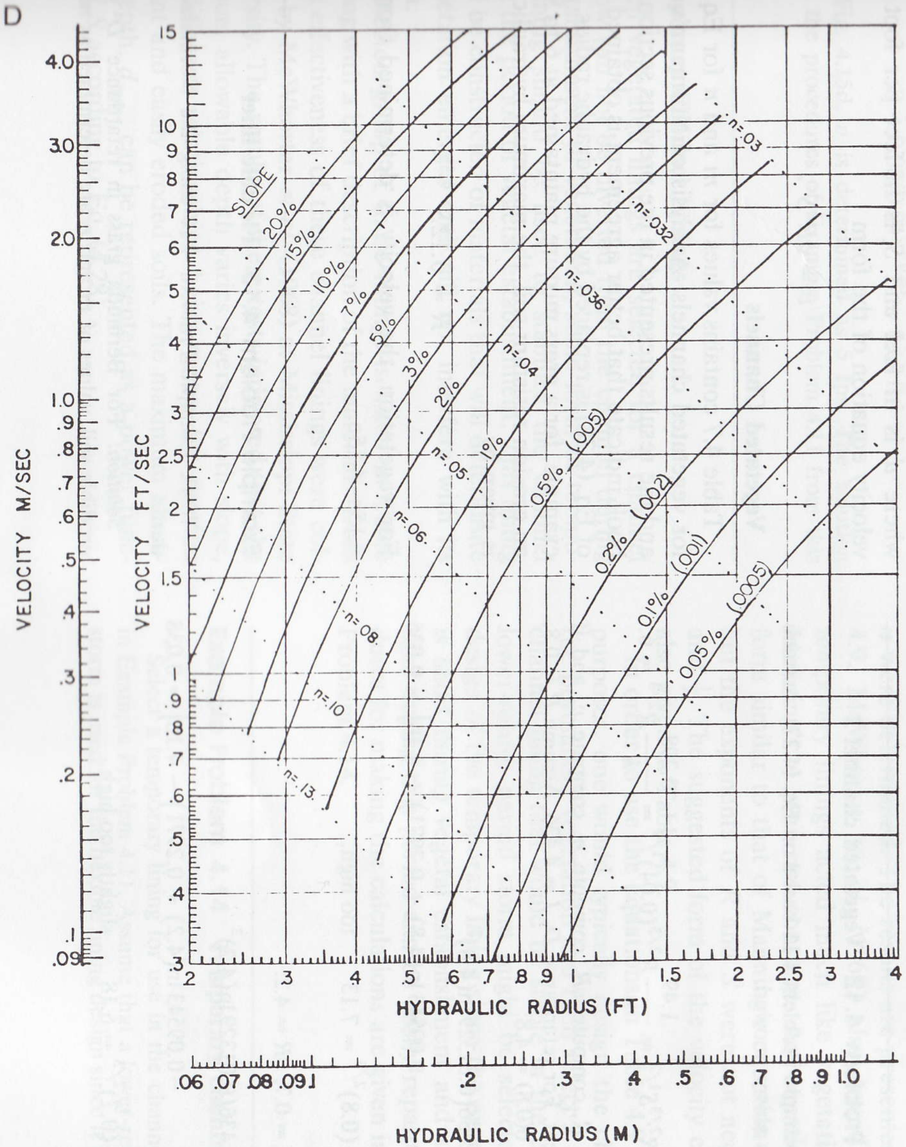


Figure 4.15d Solution to Manning's equation retardance class D.

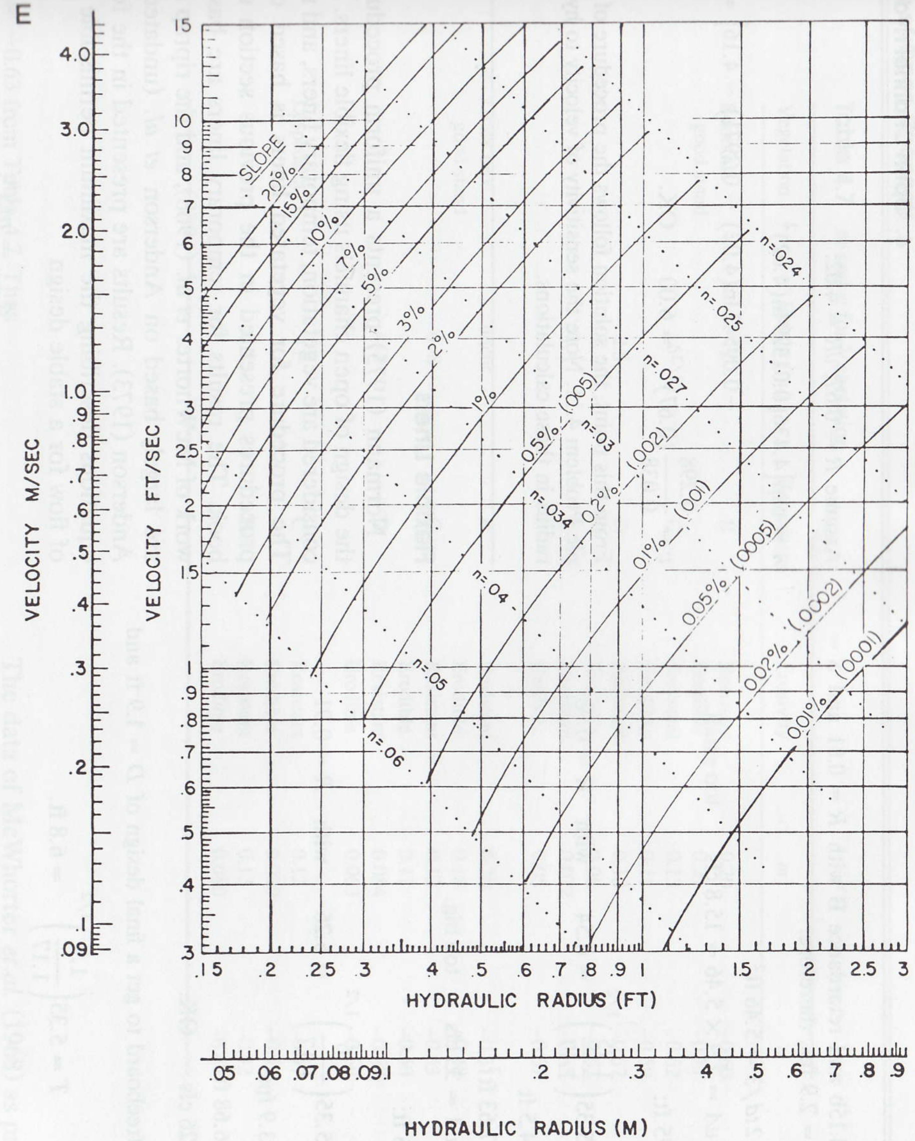


Figure 4.15e Solution to Manning's equation retardance class E.

From Fig. 4.15b and retardance B with $R = 0.81$ and $S = 0.04$, find $v = 2.9$ fps, therefore:

$$A = 2td/3 = 5.46 \text{ ft}^2$$
$$Q = vA = 2.9 \times 5.46 = 15.8 \text{ cfs} \qquad \text{too small.}$$

Try $D = 1.75$ ft:

$$T = 5.35\left(\frac{1.75}{1.17}\right)^{1/2} = 6.54 \qquad \text{with} \quad R = 0.98$$
$$v = 4.5 \text{ ft}$$
$$A = 7.63 \text{ ft}^2$$
$$Q = vA = 35 \text{ cfs} \qquad \text{too big.}$$

Try $D = 1.6$ ft:

$$T = 5.35\left(\frac{1.6}{1.17}\right)^{1/2} = 6.26 \qquad \text{with} \quad R = 0.91$$
$$v = 3.9 \text{ fps}$$
$$A = 6.68 \text{ ft}^2$$
$$Q = 26 \text{ cfs} \qquad \text{OK.}$$

Add 0.3 freeboard to get a final design of $D = 1.9$ ft and

$$T = 5.35\left(\frac{1.9}{1.17}\right)^{1/2} = 6.8 \text{ ft.}$$

Example Problem 4.12 Vegetated channel 2

Work Example Problem 4.11 based on Eq. (4.27). Assume the grass is always mowed.

Solution:

$$v = \frac{1.49}{n}R^{2/3}S^{1/2} = \frac{1.49}{n}R^{2/3}(0.04)^{1/2} = \frac{0.298}{n}R^{2/3} = 6.$$

Assume R, compute vR, compute n, compute v, and if $v \neq 6$, repeat. For retardance D, $I = 4.436$. Assume $R = 0.8$, then $vR = 6(0.8) = 4.8$:

$$n = \exp\left[4.436\left(0.01329\ln(4.8)^2 - 0.09543\ln(4.8) + 0.2971\right) - 4.16\right] = 0.036$$

$$v = \frac{0.298}{0.036}(0.8)^{2/3} = 7.13 \qquad \text{too high.}$$

Assume $R = 0.7$, $vR = 4.2$:

$$n = \exp\left[4.436\left(0.01329\ln(4.2)^2 - 0.09543\ln(4.2) + 0.2971\right) - 4.16\right] = 0.038$$

$$v = \frac{0.298}{0.038}(0.7)^{2/3} = 6.18 \qquad \text{slightly too high.}$$

Assume $R = 0.67$, $vR = 4.02$:

$$n = \exp\left[4.436\left(0.01329\ln(4.02)^2 - 0.09543\ln(4.02) + 0.2971\right) - 4.16\right] = 0.038$$

$$v = \frac{0.298}{0.038}(0.67)^{2/3} = 6.00 \qquad \text{OK.}$$

From this point, the solution follows the procedure of Example Problem 4.11. Note the sensitivity of velocity to hydraulic radius in these calculations.

Flexible Liners

Normann (1975) presents a uniform procedure for the design of open channels using flexible liners. Liners considered are vegetation, temporary liners, and riprap. The procedure for vegetated liners is based on the procedures presented in the previous section of this book. The results for temporary liners are based on work of McWhorter *et al.* (1968), and the riprap results are largely based on Anderson *et al.* (undated) and Anderson (1973). Results are presented in the form of equations describing the maximum permissible depth of flow for a stable design

$$d_{max} = mS^n, \tag{4.28}$$

where d is in feet and S is in feet per foot and a velocity equation of the form

$$v = aR^bS^c. \tag{4.29}$$

Vegetated Channels

Table 4.7 contains values for m and n for Eq. (4.28) for vegetated channels. Analysis of Normann's results and the results presented in the previous section of this book indicate that better agreement is obtained if d_{max} of Eq. (4.28) is replaced by the hydraulic radius, R. For example, for a grass mixture maintained at 6 to 8 in. on an erosion-resistant soil, the maximum hydraulic radius is given by

$$R = 0.12S^{-0.53}. \tag{4.30}$$

For vegetation, the velocity is determined from Figs. 4.15a–4.15e.

Example Problem 4.13 Flexible liner

Work Example Problem 4.11 using the Normann procedure.

Solution: For Bermuda grass in retardance D with an erodible soil, values of m and n are determined as $m = 0.08$