Table 4.7 *m* and *n* for Vegetation

Vegetation	Height (in.)	Retardance	Erosivity	m	n
Bermuda	12	B	Resistant	0.20	−0.60
good stand			Erodible	0.21	−0.51
	6	C	Resistant	0.13	−0.62
			Erodible	0.11	−0.59
	2.5	D	Resistant	0.10	−0.67
			Erodible	0.08	−0.63
	1.5	E	Resistant	0.072	−0.65
			Erodible	0.05	−0.67
Grass mix	uncut	B	Resistant	0.20	−0.51
good stand			Erodible	0.18	−0.48
	6–8	C	Resistant	0.12	−0.53
			Erodible	0.11	−0.50
	4–5	D	Resistant	0.084	−0.58
			Erodible	0.063	−0.60
Lespedeza	11	C	Resistant	0.12	−0.47
			Erodible	0.080	−0.53
	4.5	D	Resistant	0.13	−0.42
			Erodible	0.080	−0.47

and $n = -0.63$ from Table 4.7. Thus

$$R = 0.08(0.04)^{-0.63} = 0.61 \text{ ft.}$$

From Fig. 4.15d, v is determined as 5 fps. The solution follows the procedures of Example Problem 4.11 from this point.

Temporary Channel Linings

When vegetated linings are selected for a channel and flow can not be diverted from the channel during the establishment of vegetation, some form of temporary lining should be used to stabilize the channel during the period of vegetal establishment. This lining should be constructed of materials that will deteriorate as vegetation emerges and will not interfere with its growth.

A selected group of these linings is listed in Table 4.8 along with a brief description of the materials. Data on the effectiveness of these channel linings were collected by McWhorter *et al.* (1968) at Mississippi State University. The results of these tests indicated that the maximum allowable depth varies inversely with slope, with different relationships being given for erosion resistant and easily eroded soils. The maximum allowable depth, d_{max}, can be represented as a power function of S according to Eq. (4.28).

The data of McWhorter *et al.* (1968) as presented by Normann (1975) were analyzed, and values for m and n were determined. The results are presented in Table 4.9. McWhorter *et al.* (1968) also found that the temporary linings acted much like vegetation; hence Manning's n was not constant. An equation with a form similar to that of Manning's equation was used, but the exponents of R and S were not necessarily $\frac{2}{3}$ and $\frac{1}{2}$. The suggested form of the velocity equation is also given in Table 4.9.

In order to use the equations in Table 4.9 for design purposes, one would typically design the channel for the permanent vegetation and then select a temporary channel lining that would be stable in the channel. A lower-return period storm might be selected for the design of the temporary lining since the exposure time is short during vegetal establishment and since damages during this period can be easily repaired. Procedures for making the calculations are given in Example Problem 4.14.

Example Problem 4.14 Temporary channel liner

Select a temporary lining for use in the channel designed in Example Problem 4.11. Assume that a lower-return period storm is used for the flexible lining design since it needs to be

Table 4.8 Description of Temporary Liners (McWhorter *et al.*, 1968)

Excelsior mat

Excelsior mat is composed of 0.8 pound/yd^2 of excelsior (dried, shredded wood) covered with a fine paper net covering. The paper net, reinforced along the edges, has an opening size of approximately $\frac{1}{2} \times 2$ in. The mat is held in place by steel pins or staples at the rate of five staples per 6 linear ft of mat, with two staples along each side and one in the middle. At the start of each roll, four or five staples are spaced approximately 1 ft apart. Where more than one mat is required, the mats are butt-joined and securely stapled.

Straw and erosionet

This lining consists of straw applied at a rate of 3 tons per acre (1.25 lb/yd^2). The straw is covered with Erosionet 315 (See description following). This lining is pinned in the same manner as jute mesh, as described later in this table.

$\frac{3}{8}$ in. Fiberglass mat

This lining is fine, loosely woven glass fiber mat similar to furnace air filter material. It has a weight of 0.11 lb/yd^2. This material is not to be confused with more dense fiberglass mats used to eliminate plant growth. Steel pins or staples are placed at the rate of five staples per 6 linear ft of mat, with two staples along each side and one in the middle. At the start of each roll four or five staples are spaced approximately 1 ft apart. Where more than one mat is required, the mats are butt-joined and securely stapled.

$\frac{1}{2}$ in. Fiberglass mat

This lining is a fine, loosely woven glass fiber mat, similar to but denser than the $\frac{3}{8}$ in. fiberglass mat, as it weighs 0.35 lb/yd^2. The stapling procedure is the same as for the $\frac{3}{8}$ in. fiberglass mat.

Erosionet 315

Erosionet is a paper yarn approximately 0.05 in. in diameter, woven into a net with openings approximately $\frac{7}{8}$ in. $\times$ $\frac{1}{2}$ in. The material has little erosion prevention capability in itself and is generally used to hold other lining material in place. Erosionet weighs about 0.20 lb/yd^2 and is pinned in the same manner as jute mesh as described later in this table.

Fiberglass roving

Fiberglass roving is delivered as a lightly bound ribbon of continuous glass fibers. The material is applied to the channel bed using a special venturi nozzle driven by an air compressor, which separates the fibers and results in a web-like mat of glass fibers. The glass fibers are tacked with asphalt for adhesion to each other and to the soil. The single layer of fiberglass roving consists of one layer of blown fiberglass fibers applied at a minimum rate of 0.25 lb/yd^2 tacked with asphalt emulsion or asphalt cement at a minimum rate of 0.25 gal./yd^2. The double layer application consists of two alternating layers of fiberglass and asphalt, each layer consisting of fiberglass roving at 0.25 lb/yd^2.

Jute mesh

Jute mesh is a mat lining woven of jute yarn that varies from $\frac{1}{8}$ to $\frac{1}{4}$ in. in diameter. The mat weighs approximately 0.80 lb/yd^2, with openings about $\frac{3}{8}$ in. $\times$ $\frac{3}{4}$ in. Steel pins or staples are used to hold the jute mesh in place. The pins or staples should be spaced not more than 3 ft apart in three rows for each strip, with one row along each edge and one row alternately spaced in the center. At the overlapping edges of parallel strips, staples should be spaced at 2 ft or less. At all anchor slots, junction slots, and check slots, spacing should be 6 in. or less.

Table 4.9 Coefficients for Eqs. (4.28) and (4.29)[a]

Type lining	Erodible soil m	Erodible soil n	Erosion-resistant soil m	Erosion-resistant soil n	Velocity equation
Bare soil	0.0030	−0.687	0.0084	−0.687	$V = 22.81\,R^{0.591}S^{0.286}$
Fiberglass roving with asphalt tack (single layer)	0.0067	−0.960	0.0141	−0.960	$V = 42.45\,R^{0.667}S^{0.5}$
Fiberglass roving with asphalt tack (double layer)	0.0143	−1.01	0.027	−1.01	$V = 59.20\,R^{0.667}S^{0.5}$
Jute mesh	0.0076	−0.875	0.0202	−0.883	$V = 61.53\,R^{1.028}S^{0.431}$
Excelsior mat	0.0572	−0.585	0.101	−0.585	$V = 32.29\,R^{1.340}S^{0.351}$
Straw and erosionet	0.052	−0.652	0.082	−0.652	$V = 70.76\,R^{1.455}S^{0.529}$
Fiberglass mat $\frac{3}{8}$ in.	0.025	−0.670	0.046	−0.670	$V = 73.53\,R^{1.330}S^{0.512}$
Fiberglass mat $\frac{1}{2}$ in.	0.048	−0.646	0.083	−0.646	$V = 14.84\,R^{1.235}S^{0.086}$
Erosionet	0.049	−0.642	0.084	−0.642	$V = 41.45\,R^{0.855}S^{0.40}$

[a]Adapted from McWhorter *et al.* (1968).

Table 4.10 Initial Calculations for Example Problem 4.14

	Maximum depth[a] (ft)	Hydraulic radius[b] (ft)	Top width[b] (ft)	Area[b] (ft^2)	Velocity[c] (fps)	Flow (cfs)
Jute mesh	0.127	0.084	1.75	0.149	1.20	0.18
Excelsior	0.376	0.241	3.02	0.758	1.55	1.17
Straw and erosionet	0.427	0.270	3.21	0.907	1.92	1.74
Fiberglass (two layers)	0.369	0.237	3.00	0.737	4.53	3.34

[a] From Eq. (4.28) and coefficients in Table 4.9.
[b] From equation in Fig. 4.9.
[c] From velocity equations in Table 4.9.

effective for only a short period of time. The design flow is found to be 10 cfs.

Solution: From Problem 4.11, the soil is easily eroded. The slope is 4%, and a parabolic channel is used with 6.8 ft top width and a depth of 1.9 ft. To facilitate selection of the lining, the values shown in Table 4.10 were calculated from Eq. (4.28) and Table 4.9.

Obviously, none of the linings are acceptable since the discharge at d_{max} is less than the design discharge. The channel will have to be redesigned for stability during the period of temporary lining. This will require an increase in the top width without increasing the total depth, thus maintaining stability. The design is made using the trial and error procedure shown in Table 4.11

What has been shown thus far is that a channel having a shape defined by a parabola with $T = 20$ and $D = 1.9$ and lined with a double layer of fiberglass will be stable enough to carry 10 cfs at a depth of 0.37 ft. A quick calculation shows that the channel if unlined will be unstable if constructed in most soils. We have seen in Example Problem 4.11 that the channel, when grass lined only, had to have a T of 6.8 ft if the D was 1.9 to safely carry 25 cfs. It thus appears that the channel need not be constructed 1.9 ft deep since the top width exceeds the required top width.

Holding the basic channel shape the same, the actual depth of flow under the long grass condition when carrying 25 cfs can be recalculated. Using retardance class B, a trial and error procedure can be used to arrive at the flow depth. Try $d = 1.00$ ft:

$$t = T\left(\frac{d}{D}\right)^{0.5} = 20\left(\frac{1.00}{1.90}\right)^{0.5} = 14.51$$

$$R = \frac{t^2 d}{1.5t^2 + 4d^2} = 0.658$$

$$v = 1.2 \text{ fps} \qquad \text{(Fig. 4.15b)}$$

$$A = 2td/3 = 9.67$$

$$Q = vA = 11.60 \qquad \text{too small.}$$

Table 4.11 Final Calculations for Example Problem 4.14

Lining type	Maximum depth (ft)	Hydraulic radius at d_{max} (ft)	Top width at d_{max} (ft)	Area at d_{max} (ft^2)	Velocity at d_{max} (fps)	Discharge at d_{max} (cfs)
Top width[a], 12 ft fiberglass, two layers	0.369	0.243	5.28	1.30	4.61	5.99 too low
Top width[a], 15 ft fiberglass, two layers	0.369	0.244	6.61	1.62	4.62	7.49 too low
Top width[a], 18 ft fiberglass, two layers	0.369	0.245	7.93	1.95	4.63	9.04 too low
Top width[a], 20 ft fiberglass, two layers	0.369	0.245	8.81	2.16	4.63	10.00 OK

[a] T at a depth of 1.9 ft.

Try $d = 1.3$, then $t = 16.54$, $R = 0.853$, $v = 3.0$, $A = 14.33$, and $Q = 43$. The channel is too large.
Try $d = 1.15$, then $t = 15.56$, $R = 0.756$, $v = 2.1$, $A = 11.93$, and $Q = 25$ cfs. This channel is OK.

Therefore, the final channel design with freeboard added would be a depth of 1.45 ft and a top width of 17.5 ft. The channel would be sprigged or seeded to Bermuda grass with a double layer of fiberglass roving with each layer tacked with asphalt to protect the channel during the establishment of the vegetation.

A similar procedure could be used to arrive at the channel design if other liners were used.

The decision to classify a soil as erodible or erosion resistant is somewhat subjective. Normann (1975) suggests that the erodibility of the soil, K in the Universal Soil Loss equation, can be used as an indicator of erosion resistance. He suggests the following classification:

$$K = 0.50 \text{ erodible}$$
$$K = 0.17 \text{ erosion resistant.}$$

For K values between 0.17 and 0.50, one would need to interpolate between the values of m and n in Table 4.9. Soil erodibility values are discussed in Chapter 8.

Riprap Linings

In situations where vegetation is not suitable, riprap is often used to stabilize channels. Riprap is generally rocks of various sizes arranged to prevent erosion of channel banks and bottom.

Rocks used for riprap should be dense and hard enough to resist deterioration due to exposure to air, water, and temperature extremes, including repeated freezing and thawing if necessary. Sometimes rock that is initially quarried may appear satisfactory but is not able to withstand weathering. If doubt exists as to the suitability of a rock source, a geologist should be consulted. Rough angular rocks are generally preferred as they interlock and resist overturning better than smooth, rounded rocks.

Surfaces on which riprap is placed should be well compacted and stable. It is especially important to ensure that the toe sections for channel bank riprap are safe from scour and sloughing, since failure of the toe may result in failure of the entire bank. Rocks should be placed in a manner that prevents segregation by size. Dumping in a manner that allows excessive rolling of the rocks in a downslope direction and spreading with a dozer potentially result in segregation. Generally front-end loaders or bucket elevators or draglines are satisfactory. Some hand work is usually required to ensure a stable and uniform riprap surface.

The design of a riprap-lined channel involves the selection of a rock size large enough that the force attempting to overturn individual rocks is less than the gravitational force holding the rocks in place. Since riprap is graded, the design procedures must also include a definition of an appropriate gradation of particle sizes such that erosion of the smaller particles on the surface will leave an armored channel that is stable. Finally, the design procedures must include a methodology for selecting appropriate underlying filters so that water flowing beneath the riprap will not erode the base material. Procedures for selecting these materials are included in this section.

Flow on a Plane Sloping Bed

At the present time, riprap design procedures are evolving. Three procedures are presented: (a) a procedure reported by the Federal Highway Administration (FHA procedure) (Norman, 1975); (b) a procedure in the Soil Conservation Service (1979) Engineering Field Manual (SCS procedure); and (c) a procedure developed at Colorado State University (CSU procedure) and reported by Stevens and Simons (1971) and Simons and Senturk (1977, 1992). The FHA and SCS procedures are similar in that a stone diameter is specified in terms of the depth of flow and channel slope. These two procedures are based on experiments and field observations. The CSU procedure includes a theoretical analysis plus laboratory and field studies. The CSU procedure is more complete and allows the specification of a safety factor. Presumably with a safety factor of 1.0, the rocks are in a state of incipient motion.

A complication in riprap design is the gradation of rock sizes. Rocks up to some particular size may be unstable in a flow, but larger rocks might tend to hold them in place. Experimental work with riprap is difficult and time consuming because of the size of the rocks involved, the many possible gradations of rocks, variation in rock shape, materials and handling costs, and the generally high flow rates required. These factors have tended to limit studies on the stability of riprap under controlled conditions.

The CSU procedure is the most theoretically complete and conservative of the three procedures. It should result in satisfactory designs. Channel sections lined with riprap should be carefully monitored, especially for the first few years after completion, to ensure that the selected riprap is stable. Any damage should be repaired immediately to prevent much more extensive damage from developing.

The FHA procedure uses a maximum stable depth of flow given by Eq. (4.28) with $n = -1.0$ and $m =$

$5D_{50}/\gamma$, where D_{50} is the riprap diameter in feet such that 50% of the stones have a diameter smaller than D_{50} and γ is the unit weight of water (62.4 lb/ft^3). Thus $d_{\max}$ is given by

$$d_{\max} = 5(D_{50}/\gamma S). \quad (4.31)$$

The velocity of flow is given by Manning's equation with a roughness, n, given by

$$n = 0.0395 D_{50}^{1/6} \quad (4.32)$$

so that

$$v = \frac{37.7}{D_{50}^{1/6}} R^{2/3} S^{1/2}. \quad (4.33)$$

This equation is known as the Manning–Strickler equation. Channel design is done by computing $d_{\max}$ and v for an assumed D_{50} and then determining the appropriate channel dimensions. The calculations are made easier by assuming $R = d_{\max}$.

A paper by Abt *et al.* (1988) suggests that Manning's n for riprap in steep channels can be approximated by

$$n = 0.0456(D_{50}S)^{0.159} \quad (4.34)$$

where D_{50} is in inches and S is in feet per foot. Although this relationship has not been officially adopted in any design procedures, the data presented by Abt *et al.* indicate that it better describes Manning's n than does Eq. 4.32 for the conditions they tested.

Example Problem 4.15 Riprap—FHA procedure

Determine the D_{50} riprap size required to convey 115 cfs down a 10% slope in a rectangular channel 18 ft wide. Riprap is for the bottom only. Use the FHA procedure.

Solution: Assume $R = d_{\max}$, $\gamma = 62.4$, $S = 0.10$. Then

$$d_{\max} = \frac{5D_{50}}{\gamma S} = 0.801 D_{50}$$

$$v = \frac{37.7}{D_{50}^{1/6}}(d_{\max})^{2/3} S^{1/2} = \frac{37.7}{D_{50}^{1/6}}(0.801 D_{50})^{2/3}(0.10)^{1/2}$$

$$v = 10.28 D_{50}^{1/2}$$

$$Q = vA = 10.28 D_{50}^{1/2} d_{\max} B = 10.28 D_{50}^{1/2}(0.801 D_{50})(18)$$

$$115 = 148.22 D_{50}^{3/2}$$

$$D_{50} = 0.84 \text{ ft}$$

Note:

$$d_{\max} = 0.68 \text{ ft}$$

$$R = \frac{db}{2d + b} = \frac{0.68(18)}{2(0.68) + 18} = 0.63 \text{ ft}.$$

Therefore, the assumption that $R = d$ is reasonable. If the Abt relationship for n is used, the result is $v = 8.4$ fps and $D_{50} = 0.95$ ft.

The SCS procedure is based on a chart that can be approximated by

$$D_{75} = 13.5 d^{1.1} S$$

for rock diameter, D_{75}, in feet, depth of flow, d, in feet, and S in feet per foot. If D_{75} is about $1.5D_{50}$, as recommended by Simons and Senturk (1977, 1992), then

$$D_{50} = 9d^{1.1}S$$

or

$$d_{\max} = (D_{50}/9S)^{0.91}.$$

The SCS also presented a chart based on the Isbash curves, which can be approximated by

$$v = 12.84 D_{50}^{0.51}$$

This relationship assumes $D_{100} = 2D_{50}$. An unattractive theoretical aspect of this procedure is that v is not expressed as a function of slope and thus the equation should not be considered a general result. If the expression $D_{50} = 9d^{1.1}S$ is substituted into the relationship, the result is $v = 39.4 d^{0.56} S^{0.51}$, which is analogous to Manning's equation.

Example Problem 4.16 Riprap—SCS procedure

Work Example Problem 4.15 using the SCS approximations.

Solution

$$Q = vA = 12.84 D_{50}^{0.51}(dB) = 12.84 D_{50}^{0.51}\left(\frac{D_{50}}{9S}\right)^{0.91} 18$$

$$115 = 254 D_{50}^{1.42}$$

$$D_{50} = 0.57 \text{ ft}$$

$$d_{\max} = \left(\frac{0.57}{9(0.1)}\right)^{0.91} = 0.66 \text{ ft}.$$

For this problem, the FHA and SCS criteria result in similar designs with the FHA procedure resulting in larger estimates for the required D_{50}. This will generally be the case.

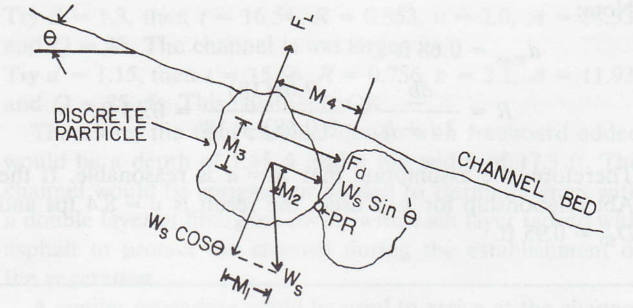

Figure 4.16 Forces on a particle in a channel bed. F_d, drag force; F_L, lift force; PR, point of rotation.

Simons and Senturk (1977, 1992) have analyzed several procedures for determining the required particle sizes for stable channel design. They present the CSU procedure, which encompasses a safety factor (SF) concept. A SF of one represents a point of incipient motion or the flow condition where forces holding particles and those tending to move particles are in exact balance. A SF of 1.5 would be preferred to add stability for particles smaller than D_{50} and to recognize statistical variability and thus prevent the initiation of localized movement, which might lead to a general failure of the riprap protection.

The FHA and SCS procedures are found to have safety factors of less than 1.0 using the CSU criteria (presented later). This indicates potential failure problems at design flows according to the CSU criteria.

The CSU procedures is developed by considering the forces on a particle on a channel bed sloping at an angle θ as shown in Fig. 4.16 along with the moment arms about the point of rotation, PR. Summing moments about PR:

$$F_L M_4 + F_d M_3 + W_s \sin\theta M_2 = W_s \cos\theta M_1. \quad (4.35)$$

These terms are defined in Fig. 4.16. The SF for a given flow situation is the ratio of the resisting moments to the overturning moments, or

$$\mathrm{SF}_b = \frac{W_s \cos\theta M_1}{W_s \sin\theta M_2 + F_L M_4 + F_d M_3}. \quad (4.36)$$

The key to a stable design is to make the safety factor greater than one. To calculate a safety factor, Eq. (4.36) must be manipulated so that it contains parameters that are readily measurable or can be determined from tables and graphs.

One readily measurable parameter is the angle of repose of a given riprap, given in Fig. 4.17. When there is no flow, the lift and drag forces are zero. Under these conditions, if the angle of the channel bottom, θ,

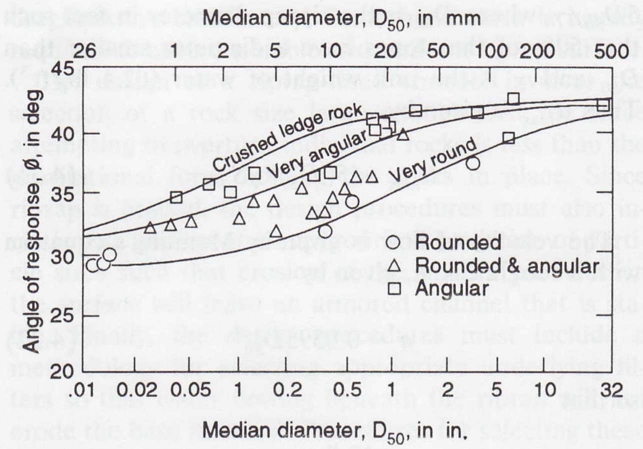

Figure 4.17 Angle of repose of dumped riprap (after Simons and Senturk, 1977, 1992).

is increased until the particles just begin to move, the particles are at their angle of repose ϕ, and the safety factor is 1.0; hence,

$$\tan\phi = M_1/M_2. \quad (4.37)$$

Using Eq. (4.37) in (4.36),

$$\mathrm{SF}_b = \frac{\cos\theta \tan\phi}{\sin\theta + (F_L M_4/W_s M_2) + (F_d M_3/W_s M_2)} \quad (4.38)$$

or

$$\mathrm{SF}_b = \frac{\cos\theta \tan\phi}{\sin\theta + \eta_b \tan\phi}, \quad (4.39)$$

where η_b is a stability parameter given by

$$\eta_b = \frac{F_L}{W_s}\frac{M_4}{M_1} + \frac{F_d}{W_s}\frac{M_3}{M_1} \quad (4.40)$$

The nature of η_b can be determined by looking at the safety factor for a plane horizontal bed, where θ is equal to zero. Under these conditions, the safety factor becomes

$$\mathrm{SF}_{\text{plane}} = \frac{W_s M_1}{F_L M_4 + F_d M_3} = \frac{1}{(F_L M_4/W_s M_1) + (F_d M_3/W_s M_1)}$$

or

$$\mathrm{SF}_{\text{plane}} = 1/\eta_b. \quad (4.41)$$

For $\mathrm{SF}_{\text{plane}}$ equal to 1.0, the bed is at the point of incipient motion, and the tractive force is equal to the critical tractive force. Under conditions other than incipient motion on a plane bed, it is reasonable to assume that the safety factor can be given by the ratio of critical to actual tractive force since there is no gravity component along the channel bed. Therefore

$$SF_b = \frac{1}{\eta_b} = \frac{\tau_c}{\tau} \tag{4.42}$$

and

$$\eta_b = \frac{\tau}{\tau_c}. \tag{4.43}$$

For fully turbulent flow, Gessler (1971) indicates that the Shield's diagram can be reanalyzed to give

$$\tau_c = 0.047\gamma(SG - 1)D,$$

where SG is the specific gravity of the particles and D is the representative particle diameter, typically the average diameter. Using Gessler's analysis,

$$\eta_b = \frac{21\tau}{\gamma(SG - 1)D}. \tag{4.44}$$

If τ is given by $\gamma\, dS$, these equations can be used to design a channel if the flow velocity is determined from Eq. (4.33). An illustration of the design procedure is given in Example Problem 4.17. It should be noted that these equations do not apply to channel banks, but only to the channel bottoms. Channel bank stability is considered in the following section.

Example Problem 4.17 Riprap—CSU procedure

A channel is being designed to convey a flow of 115 cfs down a 10% slope. The soil is collodial silt; hence the critical tractive force is so small that a lining is needed. Select an average diameter of riprap needed to stabilize the channel. For this example, neglect the stability problems associated with the side slopes. Assume a bottom width of 18 ft, a specific gravity of 2.65, and a rectangular cross section. Design for a safety factor of 1.5.

Solution: The solution procedure involves a trial and error approach of selecting a riprap size, calculating the depth of flow required to convey the flow, and checking the safety factor to ensure that the channel is stable. Assume a D_{50} of 2.5 ft, from Eq. (4.32).

$$n = 0.0395D_{50}^{1/6} = 0.046.$$

From Manning's equation, assuming a wide channel,

$$Q = Av = bd\frac{1.49}{n}d^{2/3}S^{1/2}$$

$$d = \left[\frac{nQ}{1.49bS^{1/2}}\right]^{3/5} = \left[\frac{0.046(115)}{1.49(18)(0.10)^{1/2}}\right]^{3/5}$$

$d = 0.75$ ft depth required to convey the flow.

Checking for stability using Eqs. (4.44) and (4.39),

$$\tau = \gamma dS = (62.4)(0.75)(0.10) = 4.68 \text{ lb/ft}^2$$

$$\eta_b = \frac{21\tau}{\gamma(SG - 1)D_{50}} = \frac{21(4.68)}{62.4(2.65 - 1)2.5} = 0.382.$$

Assuming an angular riprap, Fig. 4.17 gives $\phi = 42°$. For a 10% slope, $\theta = 5.71°$. Hence, from Eq. (4.39),

$$SF_b = \frac{\cos\theta\tan\phi}{\sin\theta + \eta_b\tan\phi} = \frac{(\cos 5.71)(\tan 42)}{\sin 5.71 + 0.382\tan 42}$$

$SF_b = 2.02$ over designed.

Calculations to select a better design are contained in Table 4.12. Use a riprap with a D_{50} of 1.7 ft on the channel bed. Obviously, there is a problem with stability of the side slopes. Also the gradation of riprap must be specified and a filter blanket selected. This is covered in subsequent sections and examples.

Example Problem 4.18 Riprap—safety factor

Calculate SF for Example Problems 4.15 and 4.16.

Solution

$$SF = \frac{\cos\theta\tan\phi}{\sin\theta + \eta_b\log\phi}$$

Table 4.12 Calculations for Example Problem 4.17

D_{50} (ft)	Manning's n	Angle of repose (°)	Depth to convey flow (ft)	Tractive force τ (lb/ft^2)	Stability factor (η_b)	Safety factor (SF_b)
2.5	0.046	42	0.751	4.68	0.382	2.02
2.0	0.044	42	0.734	4.58	0.467	1.72
1.5	0.042	42	0.713	4.45	0.605	1.39
1.7	0.043	42	0.722	4.49	0.541	1.53

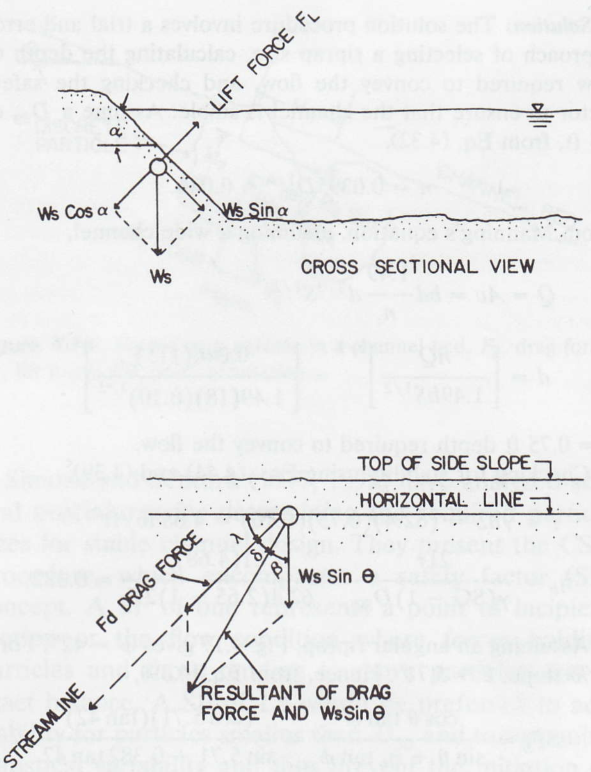

Figure 4.18 Forces on a particle on a stream channel wall.

From Problem 4.15, $d = 0.68$ ft, $D_{50} = 0.84$ ft, and $\phi = 42°$.

$$\tau = \gamma\, dS = 62.4(0.68)0.10 = 4.24 \text{ psf}$$

$$\eta_b = \frac{21\tau}{\gamma(\text{SG} - 1)D_{50}} = \frac{21(4.24)}{62.4(1.65)0.84} = 1.03$$

$$\text{SF} = \frac{\cos(5.71)\tan(42)}{\sin(5.71) + 1.03\tan(42)} = 0.87.$$

From Problem 4.16, $d = 0.66$ ft, $D_{50} = 0.57$ ft, and $\phi = 42°$:

$$\tau = 62.4(0.66)0.10 = 4.12$$

$$\eta_b = \frac{21(4.12)}{62.4(1.65)0.57} = 1.47$$

$$\text{SF} = \frac{\cos(5.71)\tan(42)}{\sin(5.71) + 1.47\tan(42)} = 0.63.$$

Based on the CSU criteria, both of these designs have SF < 1.

Channel Bank Stability

The forces on a channel bank are shown in Fig. 4.18. These forces are different from those in Fig. 4.16 for a channel bed since the drag forces are not aligned with the downslope gravitational forces. The solution of the equations describing the safety factor for this case have been given by Stevens and Simons (1971) and Simons and Senturk (1977, 1992) as

$$\text{SF} = \frac{\cos\alpha\tan\phi}{\eta'\tan\phi + \sin\alpha\cos\beta} \tag{4.45}$$

$$\beta = \tan^{-1}\left(\frac{\cos\lambda}{2\sin\alpha/\eta\tan\phi + \sin\lambda}\right) \tag{4.46}$$

$$\eta = \frac{21\tau_{max}}{\gamma(\text{SG} - 1)D_{50}} \tag{4.47}$$

and

$$\eta' = \eta\frac{1 + \sin(\lambda + \beta)}{2} \tag{4.48}$$

where τ_{max} is the maximum shear on the channel bank.

In order to derive Eqs. (4.45) through (4.48), it was assumed that the ratio of lift to drag forces was one-half. The use of the procedures is illustrated in Example Problem 4.19.

When calculating the shear forces on a channel bank, it is desirable to take into account variations in channel shear across the channel bed. Figure 4.8 shows that for a trapezoidal channel, the maximum tractive force on the channel walls in $K\gamma\, dS$, where K is 0.74 to 0.78 depending on the channel side slope.

Example Problem 4.19 Riprap size—channel bank

Based on construction considerations and machinery limitations, side slopes of 2.5 : 1 are selected for the channel in Example Problem 4.17. Select a riprap size that will be stable on the channel sideslopes.

Solution: First the safety factor of the riprap selected in Example Problem 4.17 is calculated assuming the same material is used on the sides. From Example Problem 4.17,

$$D_{50} = 1.7 \text{ ft}; \quad n = 0.043; \quad \theta = 5.71°; \quad d = 0.722 \text{ ft}.$$

For a trapezoidal channel, the flow depth can be calculated to be 0.72 ft, which is insignificantly smaller than 0.722 ft for the rectangular channel in Example 4.17; hence we use 0.722 ft.

From Fig. 4.8 τ_{max} is given by $0.76\gamma\, dS$:

$$\tau_{max} = (0.76)(62.4)(0.722)(0.10) = 3.41 \text{ lb/ft}^2$$

$$\eta = \frac{21\tau_{max}}{\gamma(\text{SG} - 1)D_{50}} = \frac{21(3.41)}{62.4(2.65 - 1)1.7} = 0.408.$$

Assuming uniform flow, the streamlines are parallel to the channel bottom and

$$\lambda = \theta = 5.71°.$$

Also, for a 2.5 : 1 sideslope,

$$\alpha = \tan^{-1}\frac{1}{2.5} = 21.8°.$$

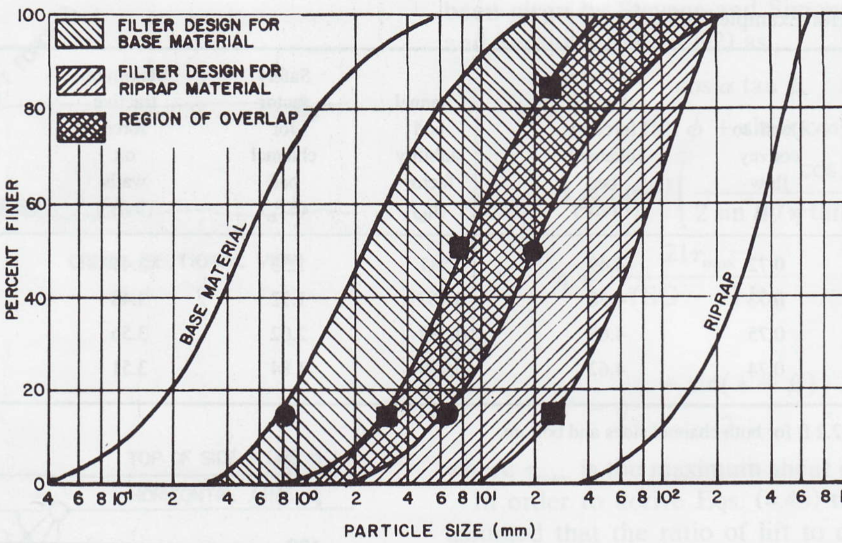

Figure 4.20 Size distribution determinations of filter material for Example Problem 4.20. The filter must have a size distribution within the region of overlap.

drain. A filter designed by the same criteria should prevent piping of the parent soil from beneath riprap. Filter thickness should be approximately one-half the thickness of the riprap, but in no case less than 6 to 9 in. An illustration of the use of these procedures is given in Example Problem 4.20. Plastic filter cloth is being used in some cases rather than granular filter materials. Normann (1975) should be consulted for details.

Example Problem 4.20 Riprap filter design

Select an appropriate riprap gradation for riprap with a D_{50} value of 1.0 ft. The base parent material on which the riprap is being placed has the properties D_{50} = 0.5 mm, D_{85} = 1.5 mm, and D_{15} = 0.17 mm. Select an appropriate filter blanket for the riprap.

Solution: Based on Fig. 4.19 with a D_{50} of 1.0 ft, the properties of the riprap are D_{100} = 2.0 ft = 610 mm, D_{50} = 1.0 ft = 305 mm, D_{85} = 1.7 ft = 520 mm, D_{15} = 0.42 ft = 130 mm, and D_0 = 0.10 ft = 30 mm.

These are plotted in Fig. 4.20 along with the size distribution of the parent material. Next the filter blanket must be sized. Look first at the requirements of the filter blanket with respect to the parent material:

Criterion (1)

$$\frac{D_{50}(\text{filter})}{D_{50}(\text{base})} < 40 \qquad \text{giving } D_{50}(\text{filter}) < 40 \times 0.5 = 20 \text{ mm}$$

Criterion (2)

$$\frac{D_{15}(\text{filter})}{D_{15}(\text{base})} > 5 \qquad \text{giving } D_{15}(\text{filter}) > 5 \times 0.17 = 0.85 \text{ mm}$$

and

$$\frac{D_{15}(\text{filter})}{D_{15}(\text{base})} < 40 \qquad \text{giving } D_{15}(\text{filter}) < 40 \times 0.17 = 6.18 \text{ mm}$$

Criterion (3)

$$\frac{D_{15}(\text{filter})}{D_{85}(\text{base})} < 5 \qquad \text{giving } D_{15}(\text{filter}) < 5 \times 1.5 = 7.5 \text{ mm.}$$

Therefore, with respect to the base parent material, the following criteria must be satisfied:

$$0.85 \text{ mm} < D_{15}(\text{filter}) < 6.8 \text{ mm}$$

and

$$D_{50}(\text{filter}) < 20 \text{ mm.}$$

These points are plotted as solid dots in Fig. 4.20 and curves approximating these conditions were drawn through the points.

Next, the filter must be sized relative to the riprap.

Criterion (1)

$$\frac{D_{50}(\text{riprap})}{D_{50}(\text{filter})} < 40 \qquad \text{giving } D_{50}(\text{filter}) > \frac{305}{40} = 7.6 \text{ mm}$$

Criterion (2)

$$\frac{D_{15}(\text{riprap})}{D_{15}(\text{filter})} > 5 \qquad \text{giving } D_{15}(\text{filter}) < \frac{130}{5} = 26 \text{ mm}$$

and

$$\frac{D_{15}(\text{riprap})}{D_{15}(\text{filter})} < 40 \qquad \text{giving } D_{15}(\text{filter}) > \frac{130}{40} = 3.3 \text{ mm}$$

Criterion (3)

$$\frac{D_{15}(\text{riprap})}{D_{85}(\text{filter})} < 5 \quad \text{giving } D_{85}(\text{filter}) > \frac{130}{5} = 26 \text{ mm}.$$

Therefore the filter must also meet these criteria, or

$$D_{50}(\text{filter}) > 7.6 \text{ mm}$$
$$3.3 \text{ mm} < D_{15}(\text{filter}) < 26 \text{ mm}$$
$$D_{85}(\text{filter}) > 26 \text{ mm}.$$

These points are also plotted in Fig. 4.20 as solid boxes and curves drawn through the points. The envelope of points satisfying both criteria are crosshatched. Any material selected with a size distribution falling within the crosshatched area will satisfy the design requirements.

Flow in Channel Bends

Because of the curvature in channel bends, the peak velocity typically occurs on the outside of the centerline, resulting in steeper velocity gradients and higher shear stress values on the outside banks than occur in straight channels. This extra shear must be considered when sizing riprap, vegetation, and temporary channel linings. A commonly used procedure in riprap-lined channels is to increase the riprap size in the channel bend, or in vegetated lined channels, to line sharp bends with riprap.

The location of the maximum shear varies so much within bends that it is not possible to determine the exact point at which protection is needed. Therefore, it is standard practice to protect the outside bank of the entire bend.

Data that can be used to predict shear in channel bends are not abundant. F. J. Watts [as reported by Norman (1975)] proposed that a correction factor for shear on the channels walls varying from 1.0 to 4.0 could be calculated on the basis of v^2/R_d, where v is the average flow velocity in a straight channel and R_d is the radius of the outside bank. A plot of the correction factor is given in Fig. 4.21 along with the limited verification data reported by Normann (1975). To use the relationship:

(1) Determine the velocity in a straight channel stretch.

(2) Determine the radius of curvature of the outside bank, R_d.

(3) Calculate v^2/R_d.

(4) Determine the correction factor, k_3, from Fig. 4.21.

(5) Calculate the corrected bank shear from

$$\tau = k_3 \gamma \, dS.$$

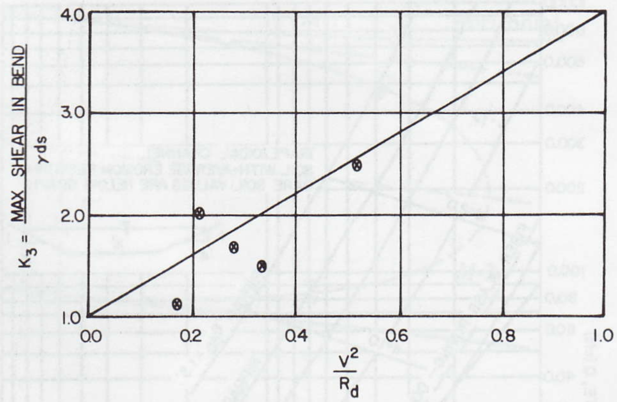

Figure 4.21 Correction factor for shear in flow in a bend (Normann, 1975).

(6) Use this shear from (5) in the stability parameters η and η' and determine the required riprap size using procedures previously discussed.

It must be pointed out that these procedures have very limited verification. Their use is still somewhat speculative at this point.

General Comments

The flow range over which differing channel linings offer protection depends on channel shape and slope. An example comparison made by Normann (1975) is given in Fig. 4.22. Although the procedures used to calculate the allowable discharge for the riprap have been shown in Simons and Senturk (1977, 1992) to sometimes yield slightly unstable design, the figure gives a reasonable guide to the type of channel lining required for varying flow rates and slopes. It should be pointed out that the ranges will change based on side slopes and erodibility of underlying material.

GRADUALLY VARIED FLOW

The relationships presented in this section are for wide, open channels where the hydraulic radius may be approximated by the depth of flow. Uniform flow requires a channel of constant cross section and sufficient length for the gravitational forces to achieve a balance with the frictional resistance. At changes in slope, cross section, or roughness, the two forces will not be balanced, and the flow conditions will adjust toward equilibrium. Within the channel reach where this adjustment occurs, the flow is said to be varied flow or nonuniform flow. If the change in flow conditions occurs gradually over relatively long channel reaches, the flow is said to be gradually varied flow.

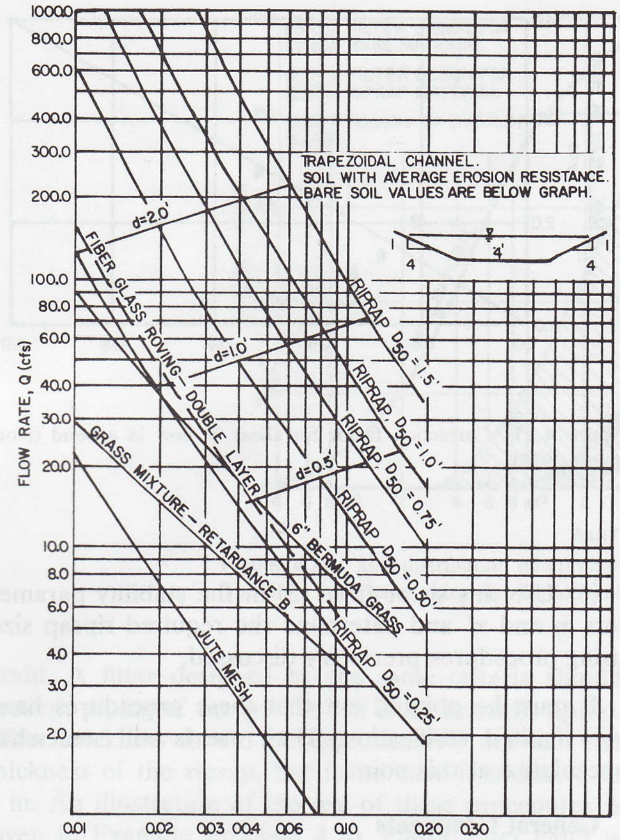

Figure 4.22 Comparison of maximum flow rate versus slope for various channel linings (Norman, 1975).

Equation (4.4) and Fig. 4.4 give the total energy, H, as

$$H = y + z + v^2/2g. \qquad (4.49)$$

This may be written as

$$H = y + z + Q^2/2gA^2. \qquad (4.50)$$

Differentiation with respect to x, the distance along the channel, yields

$$\frac{dH}{dx} = \frac{dy}{dx} + \frac{dz}{dx} - \frac{Q^2}{gA^3}\frac{dA}{dx}. \qquad (4.51)$$

If we consider a rectangular channel or a wide channel, the last term of this equation becomes

$$\frac{Q^2}{gA^3}\frac{dA}{dx} = \frac{q^2}{gy^3}\frac{dy}{dx}.$$

The term dH/dx represents the slope of the energy grade line, S, which is by convention taken as positive downward. Similarly dz/dx is the channel slope, S_0, also positive downward. Thus

$$-S = -S_0 + \left(1 - \frac{q^2}{gy^3}\right)\frac{dy}{dx} \qquad (4.52)$$

Noting that q^2/gy^3 is $\mathbf{F}^2$ and rearranging the equation results in

$$\frac{dy}{dx} = \frac{S_0 - S}{1 - \mathbf{F}^2}. \qquad (4.53)$$

This equation gives the slope of the water surface with respect to the channel bottom. If dy/dx is positive, the flow is getting deeper in the downstream direction. If dy/dx is negative, the flow is getting shallower in the downstream direction. A dy/dx of zero implies uniform flow.

A channel is said to have a mild slope if the normal depth, y_n, is greater than the critical depth, y_c. Similarly, if $y_n < y_c$, the slope is a steep slope, and if $y_n = y_c$, the slope is termed a critical slope. A slope that is negative or runs uphill in the downstream direction is known as an adverse slope. Finally a channel with no slope is said to be a horizontal channel.

In sketching gradually varied flow profiles, the profiles are conventionally labeled with the first letter of the slope type. Thus M denotes a mild slope, S a steep slope, etc.

If the flow depth exceeds both y_n and y_c, the flow is said to be in zone 1 and is denoted with the subscript 1. If the depth is between y_n and y_c (or between y_c and y_n), the zone designation is 2. A depth less than both y_n and y_c is in zone 3.

Figure 4.23 depicts possible flow profiles or backwater curves. The slope of the water surface for the various situations can be deduced from Eq. (4.53). To do this, one can approximate S as the slope calculated from Manning's equation using the actual depth of flow. S_0 is the slope in Manning's equation corresponding to y_n. The appropriate equations are

$$S = \frac{q^2 n^2}{2.22\, y^{10/3}}$$

and

$$S_0 = \frac{q^2 n^2}{2.22\, y_n^{10/3}}.$$

If $y_n > y$, $S_0 < S$ If $y > y_n$, $S_0 > S$. Also note that if S_0 is less than or equal to zero, y_n is not defined.

As an example of determining the slope of the water surface, consider an M_1 profile. In this situation $y_n > y_c$, $y > y_n$, and $y > y_c$. Thus $\mathbf{F} < 1$ and $S_0 > S$. This means that both the numerator and denominator of Eq. (4.53) are positive, so dy/dx is positive and the flow depth increases in the downstream direction.

As another example, consider the S_2 profile. Here $y_c > y_n$, $y > y_n$, and $y < y_c$. This means $\mathbf{F} > 1$ and $S_0 > S$. Thus, the numerator of Eq. (4.53) is positive and the denominator is negative. The S_2 curve has dy/dx negative or the depth decreases in the downstream direction.

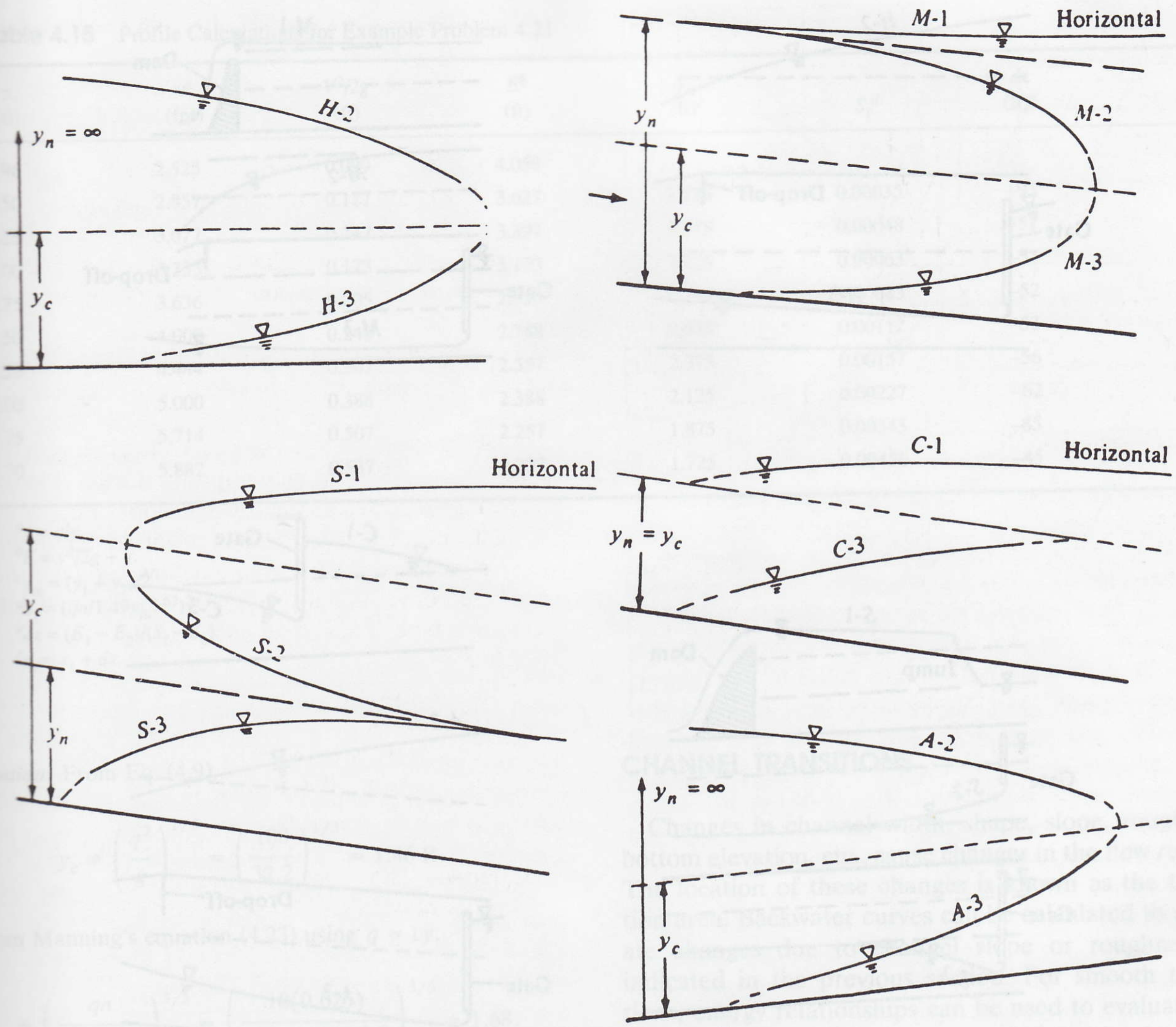

Figure 4.23 Possible flow profiles.

The above reasoning can be applied to each of the zones and profiles with the results shown in Table 4.14. Flow profiles develop at changes in channel slope, roughness, and cross section. Figure 4.24 shows some typical situations where profiles develop.

An approximate calculation of backwater profiles can be done by considering Fig. 4.4 and noting

$$E_1 + z_1 = E_2 + z_2 + h_L.$$

By definition

$$S_0 = (z_1 - z_2)/\Delta x,$$

where Δx is the length of the channel reach. Also

$$h_L = dE = S_f \Delta x,$$

where S_f is the friction slope or slope of the energy

Table 4.14 Slope of Water Surface Profiles with Respect to Channel Bottom

Type	Designation	Slope
Mild	M1	+
	M2	−
	M3	+
Steep	S1	+
	S2	−
	S3	+
Critical	C1	+
	C3	+
Horizontal	H2	−
	H3	+
Adverse	A2	−
	A3	+

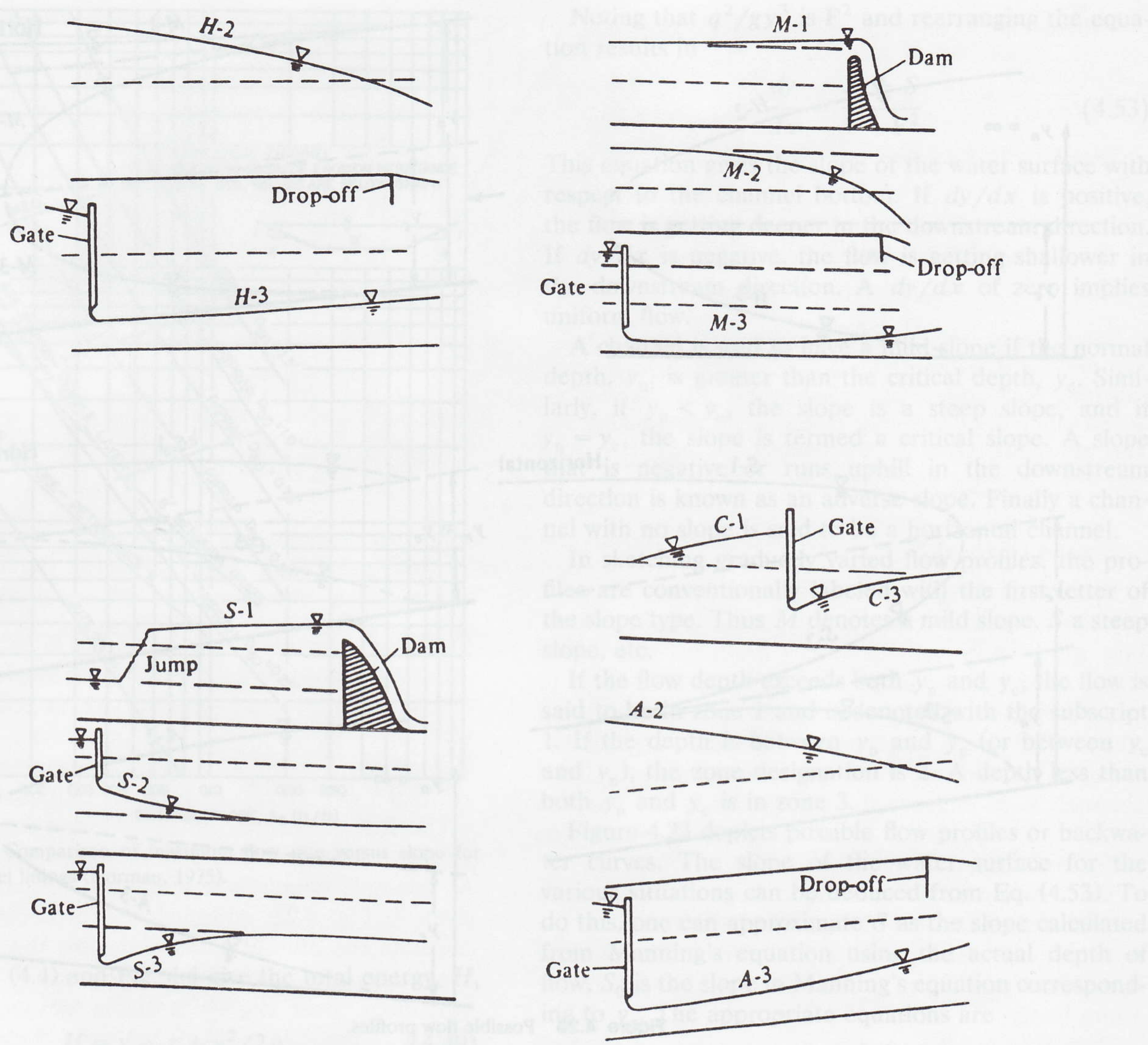

Figure 4.24 Typical flow profiles.

grade line. Combining these three equations results in

$$\Delta x = \frac{E_1 - E_2}{S_f - S_0}. \tag{4.54}$$

S_f can be approximated from Manning's equation by assuming an average flow depth for the reach. Example Problem 4.21 illustrates the computation of a backwater curve. Note that for subcritical flow, backwater curves should be computed in the upstream direction and for supercritical flow in the downstream direction. Profile calculations are started at points of known water surface elevations such as overfalls from a mild channel ($y = y_c$) or other types of control sections. Application of Eq. (4.54) is known as the direct step method.

Example Problem 4.21 Flow profile

A wide, rectangular channel is carrying 10 cfs/ft down a 0.5% slope. The channel has a Manning's n of 0.025. A 2.5-ft barrier in the channel causes flow to pass over the barrier at critical depth. Compute the flow profile upstream from the barrier to a point where the depth is within 10% of normal depth. Figure 4.25 illustrates the physical situation.

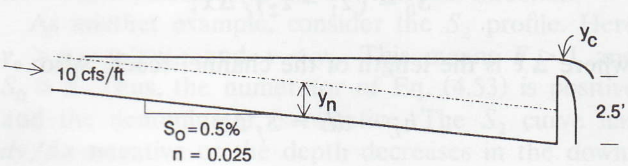

Figure 4.25 Sketch for Example Problem 4.21.

Table 4.15 Profile Calculations for Example Problem 4.21

y (ft)	v^a (fps)	$V^2/2g$ (ft)	E^b (ft)	y_m (ft)^c	$S_f{}^d$	dx (ft)^e	x^f (ft)
3.96	2.525	0.099	4.059				0
3.50	2.857	0.127	3.627	3.730	0.00035	–93	–93
3.25	3.077	0.147	3.397	3.375	0.00048	–51	–143
3.00	3.333	0.173	3.173	3.125	0.00063	–51	–195
2.75	3.636	0.205	2.955	2.875	0.00083	–52	–247
2.50	4.000	0.248	2.748	2.625	0.00112	–53	–300
2.25	4.444	0.307	2.557	2.375	0.00157	–56	–356
2.00	5.000	0.388	2.388	2.125	0.00227	–62	–418
1.75	5.714	0.507	2.257	1.875	0.00345	–85	–503
1.70	5.882	0.537	2.237	1.725	0.00456	–45	–548

[a] $v = q/y$.
[b] $E = v^2/2g + y$.
[c] $y_m = (y_1 + y_2)/2$.
[d] $S_f = (qn/1.49 y_m{}^{1.67})^2$.
[e] $dx = (E_1 - E_2)/(S_f - S_o)$.
[f] $x_2 = x_1 + dx$.

Solution: From Eq. (4.9),

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{100}{32.2}\right)^{1/3} = 1.46 \text{ ft}.$$

From Manning's equation (4.23) using $q = vy$,

$$y_n = \left(\frac{qn}{1.49 S^{1/2}}\right)^{3/5} = \left(\frac{10(0.025)}{1.49(0.005)^{1/2}}\right)^{3/5} = 1.68.$$

The depth of flow over the brink in Fig. 4.25 is

$$y = 2.5 + y_c = 3.96 \text{ ft}.$$

The solution is carried out by assuming depths and computing Δx. Table 4.15 shows the computations.

Equation (4.54) can be used for channels where the approximation that $y = R$ is not appropriate. The calculations are somewhat more cumbersome than those illustrated in Example Problem 4.21. Fortunately, extensive computer programs are available for calculating flow profiles in natural channels. Computers are generally needed because of irregularities in natural channels and the presence of flow obstructions in the form of bridges, culverts, low dams, etc. The most widely used program in the U.S. is HEC-2, a program developed by the U.S. Army Corps of Engineers (1982) Hydrologic Engineering Center in Davis, California.

CHANNEL TRANSITIONS

Changes in channel width, shape, slope, roughness, bottom elevation, etc., cause changes in the flow regime. The location of these changes is known as the transition area. Backwater curves can be calculated to evaluate changes due to channel slope or roughness as indicated in the previous section. For smooth transitions, energy relationships can be used to evaluate the impact of the transitions. A smooth transition is one in which energy losses are minimal.

Consider the channel transition shown in Fig. 4.26. Assuming no energy loss through the transition,, Eq. (4.4) becomes

$$\frac{v_1^2}{2g} + y_1 = \frac{v_2^2}{2g} + y_2 + \Delta z, \qquad (4.55)$$

showing that with a constant total energy there is a specific energy loss of Δz. A specific energy diagram can be used to visualize the flow change that occurs.

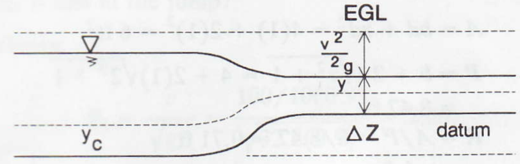

Figure 4.26 A channel transition.

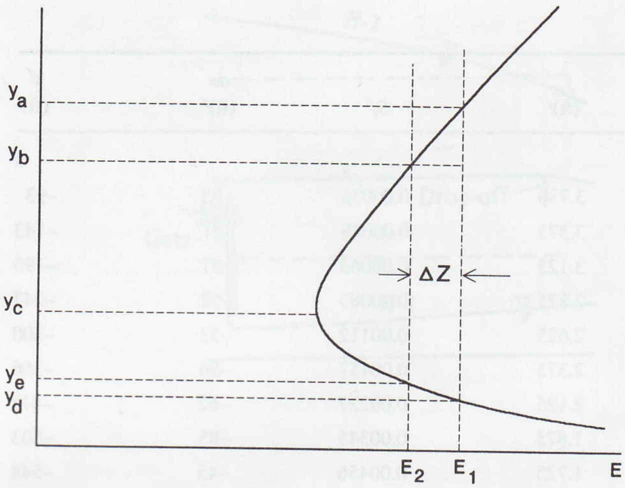

Figure 4.27 Specific energy representation in a transition.

Consider Fig. 4.27. If y_1 is subcritical and represented by y_a, y_2 must correspond to y_b so that the depth of flow due to a channel bottom rise is decreased. Conversely, if y_1 is supercritical and equal to y_d, then y_2 must correspond to y_e. It must be kept in mind that the specific energy diagram corresponds to a constant unit discharge or is based on a rectangular channel. For nonrectangular channels, Eq. (4.55) is still valid, but the specific energy representation of Fig. 4.27 can only be used conceptually, not analytically.

If the flow must pass through critical depth, the assumption of no energy loss may not be valid. This is especially true if the transition is from supercritical to subcritical flow. In such a situation, a hydraulic jump accompanied by considerable energy loss occurs. Hydraulic jumps are considered in the next section.

Example Problem 4.22 Channel transition 1

A trapezoidal channel with 2 : 1 side slopes and a 4-ft bottom width is flowing at a depth of 1 ft. The channel is concrete and on a slope of 0.1%. If the channel bottom is raised smoothly by 0.1 ft over a short distance, what will be the depth of flow at the exit of the transition?

Solution

$$n = 0.015 \quad \text{for concrete}$$

$$v_1 = \frac{1.5}{n} R^{2/3} S^{1/2}$$

$$A = bd + zd^2 = 4(1) + 2(1)^2 = 6 \text{ ft}^2$$

$$P = b + 2d\sqrt{z^2+1} = 4 + 2(1)\sqrt{2^2+1} = 8.47 \text{ ft}$$

$$R = A/P = 6/8.47 = 0.71 \text{ ft}$$

$$v_1 = \frac{1.5}{0.015}(0.71)^{2/3}(0.001)^{1/2} = 2.52 \text{ fps}$$

$$\mathrm{F} = \frac{v}{\sqrt{g d_\mathrm{h}}}$$

$$d_\mathrm{h} = \frac{A}{t} = \frac{6}{b + 2zd} = \frac{6}{4 + 2(2)(1)} = 0.75 \text{ ft}$$

$$\mathrm{F} = \frac{2.52}{\sqrt{32.3(0.75)}} = 0.51 \quad \text{subcritical}$$

$$\frac{v_1^2}{2g} + y_1 = \frac{v_2^2}{2g} + y_2 + \Delta z$$

$$\frac{(2.52)^2}{64.4} + 1.0 = \frac{v_2^2}{64.4} + y_2 + 0.1$$

$$0.9986 = \frac{v_2^2}{64.4} + y_2$$

$$v_2 = \frac{Q}{A} = \frac{v_1 A_1}{A_2} = \frac{2.52(6)}{4y_2 + 2y_2^2}$$

$$0.9986 = \frac{3.54}{(4y_2 + 2y_2^2)^2} + y_2.$$

Solve by trial

y_2	Right-hand side	
0.90	1.03	
0.75	0.958	
0.84	0.996	OK

$$y_2 = 0.84 \text{ ft}$$

Check the Froude number:

$$v_2 = \frac{Q}{A_2} = \frac{2.52(6)}{4(0.84) + 2(0.84)^2} = \frac{15.1}{4.77} = 3.16 \text{ ft}$$

$$d_\mathrm{h} = \frac{A}{t} = \frac{4.77}{4 + 2(2)(0.84)} = \frac{4.77}{7.36} = 0.65 \text{ ft}$$

$$\mathrm{F} = \frac{v}{\sqrt{g d_\mathrm{h}}} = 0.69 \quad \text{still subcritical.}$$

Solution OK.

Transitions that consist of changes in channel width can be treated similar to changes in channel bottom elevation. Again specific energy curves cannot be used directly since they are based on a constant flow per unit width, q. When the channel width changes, q must change as well.

Example Problem 4.23 Channel transition 2

A rectangular channel 10 ft wide is carrying 75 cfs. The channel smoothly narrows to 8 ft in width. The flow depth in the 10-ft section is 2.5 ft. What is the depth in the 8 ft section assuming no energy losses?

Solution

$$\frac{v_1^2}{2g}+y_1=\frac{v_2^2}{2g}+y_2$$

$$v_1=\frac{Q}{A}=\frac{75}{10\times 2.5}=3\text{ fps}$$

$$v_2=\frac{75}{8y_2}$$

$$\frac{3^2}{64.4}+2.5=\frac{(75/8y_2)^2}{64.4}+y_2=\frac{1.36}{y_2^2}+y_2.$$

The solution may be found by trial to be $y_2 = 2.40$ ft. Thus the depth in the 8-ft section is 2.40 ft.

HYDRAULIC JUMP

An example of a flow transition that is abrupt and involves considerable energy loss is a hydraulic jump that involves a sudden transition from supercritical to subcritical flow. In looking at the flow profiles of Fig. 4.23, it can be seen that the profiles approach critical depth nearly vertically. This is also apparent from Eq. (4.53), where as y approaches y_c, F approaches 1 and dy/dx approaches infinity. When y approaches y_c as supercritical flow from below, a hydraulic jump may occur as shown in Figure 4.28.

A hydraulic jump cannot be analyzed using the energy equation because there is a large and unknown energy loss in the jump. By assuming that the specific force plus momentum is the same before and after a jump, Eq. (4.15) can be used:

$$\frac{y_1^2}{2}+\frac{q^2}{gy_1}=\frac{y_2^2}{2}+\frac{q^2}{gy_2}.$$

Through algebraic manipulations, it may be shown that

$$\frac{y_2}{y_1}=\frac{1}{2}\left(\sqrt{1+8F_1^2}-1\right) \qquad (4.56)$$

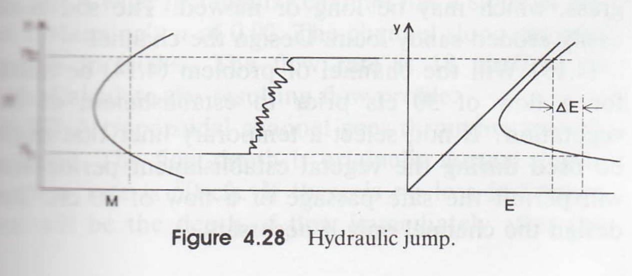

Figure 4.28 Hydraulic jump.

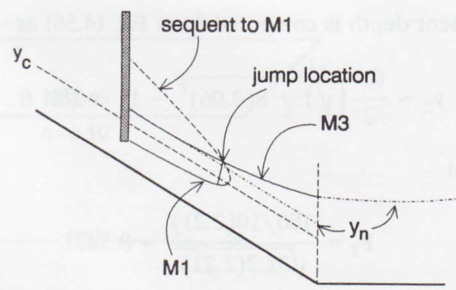

Figure 4.29 Location of a hydraulic jump.

and

$$\frac{y_1}{y_2}=\frac{1}{2}\left(\sqrt{1+8F_2^2}-1\right).$$

y_1 is known as the initial depth, and y_2 is the sequent depth. A hydraulic jump from $y_1 < y_c$ to $y_2 > y_c$ occurs whenever flow conditions are such that y_1 and y_2 are related by Eq. (4.15), that is momentum is conserved.

The location of a hydraulic jump can be found by plotting flow profiles and superimposing a plot of the possible sequent depth above the supercritical part of the flow. The jump occurs whenever the sequent depth line intersects the downstream flow profile (assuming the jump has zero length). Figure 4.29 illustrates the procedure.

The energy loss in a hydraulic jump can be computed directly from Bernoulli's equation as

$$E_l=\frac{q^2}{2g}\left(\frac{1}{y_1^2}-\frac{1}{y_2^2}\right)+(y_1-y_2).$$

Through algebraic manipulations, this relationship becomes

$$E_l=\frac{(y_2-y_1)^3}{4y_1y_2} \qquad (4.57)$$

Example Problem 4.24 Hydraulic jump

A rectangular channel is carrying 100 cfs. The channel is 10 ft wide and flowing 0.90 ft deep. Is a hydraulic jump possible? If so, what will be the sequent depth? How much energy is lost in the jump?

Solution

$$F_1=\frac{v}{\sqrt{gy}}=\frac{100/10(0.9)}{\sqrt{32.2(0.9)}}=2.06.$$

Since the flow is supercritical, a hydraulic jump is possible.

The sequent depth is computed from Eq. (4.56) as

$$y_2 = \frac{0.9}{2}\left(\sqrt{1 + 8(2.06)^2} - 1\right) = 2.21 \text{ ft}.$$

Note that

$$\mathbf{F}_2 = \frac{100/10(2.21)}{\sqrt{32.2(2.21)}} = 0.54.$$

The depth after the jump is subcritical as it must be

$$E_1 = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(2.21 - 0.9)^3}{4(2.21)(0.9)} = 0.29 \text{ ft}.$$

This loss can also be determined directly from Bernoulli's equation as

$$E_1 = \frac{v_1^2}{2g} + y_1 - \frac{v_2^2}{2g} - y_2$$

$$= \frac{[100/10(0.9)]^2}{64.4} + 0.9 - \frac{[100/10(2.21)]^2}{64.4} - 2.21$$

$$= 0.29 \text{ ft}.$$

Hydraulic jumps are accompanied by a great deal of turbulence and energy dissipation. If a hydraulic jump occurs in an erodible area of a channel, considerable degradation of the channel may occur. Hydraulic jumps are often used to provide energy dissipation below spillways and channel drop structures. To ensure that the jump occurs at a controlled location, generally on a reinforced concrete apron, stabilizing blocks are used to add drag forces to the flow. In this case,the momentum equation is modified to

$$\frac{y_1^2}{2} + \frac{q^2}{gy_1} = \frac{y_2^2}{2} + \frac{q^2}{gy_2} + \frac{F_B}{\gamma},$$

where F_B represents the drag force per unit width. The design of energy dissipation devices such as stilling basins is a special area of hydraulics and is discussed in the next chapter. Extensive model studies are often employed with the results presented in the form of dimensionless designs. These designs are then adapted to particular applications by using appropriate scaling factors. The St. Anthony Falls (SAF) stilling basin is an example.

Problems

(4.1) A trapezoidal concrete-lined ditch has a bottom width of 3 ft, a depth of 2 ft, and side slopes of 2 : 1. Estimate the discharge if the channel slope is 1.0%. What is the velocity? What is the Froude number?

(4.2) What will the depth of flow in the channel in problem (4.1) be if the flow rate is 50 cfs? What should be the freeboard?

(4.3) A channel is being designed to carry 30 cfs through a very colloidal stiff clay soil on a slope of 1%. Determine the design dimensions if the side slopes are 1 : 1 using both the tractive force and permissible velocity methods.

(4.4) The 10-year peak flow from a watershed is to be channeled through a grassed waterway of bluegrass on a slope of 4% over erosion resistant soil. The grass may be moved (2 to 5 in.) or unmowed (18 in.). The 10-year peak flow is 100 cfs. Design a grassed waterway to convey the flow.

(4.5) If a straw and erosionet liner is used in the channel of example problem (4.4), will the channel be stable before the vegetation is established under a flow of 10 cfs?

(4.6) Design a trapezoidal channel with 2 : 1 side slopes to carry 70 cfs down a 10% slope. The channel bottom width must be limited to 10 ft because of site considerations.

(4.7) A trapezoidal channel with 2 : 1 side slopes, an 8-ft bottom width, and a slope of 0.15% is flowing 1.4 ft deep. The channel is unlined and constructed in an erodible sandy loam soil. What is the flow rate? Is the channel stable at this flow rate?

(4.8) The channel of problem (4.7) is vegetated with Bermuda grass. What is the flow rate? Would there likely be any problems with this channel?

(4.9) Calculate the critical depth for the channel described in problem (4.7) if it carries 150 cfs.

(4.10) If the channel of problem 4.7 is concrete lined, what is the critical slope for the channel at a flow rate of 150 cfs?

(4.11) An elevated rectangular canal is flowing 3 ft deep. What is the horizontal force per unit length exerted by the water on the canal side walls?

(4.12) What size riprap should line the bottom of a trapezoidal channel with 4 : 1 side slopes, 10-ft bottom width, and a 7% slope? The channel is to carry 130 cfs.

(4.13) What size riprap should be used on the side slopes of the channel of problem (4.12)?

(4.14) A vegetated channel is to be used to carry 50 cfs down a 4% slope. The vegetation is to be Bermuda grass, which may be long or mowed. The soil is an easily eroded sandy loam. Design the channel.

(4.15) Will the channel of problem (4.14) be stable for a flow of 30 cfs prior to establishment of the vegetation? If not, select a temporary liner that might be used during the vegetal establishment period that will permit the safe passage of a flow of 30 cfs. Redesign the channel only if necessary.

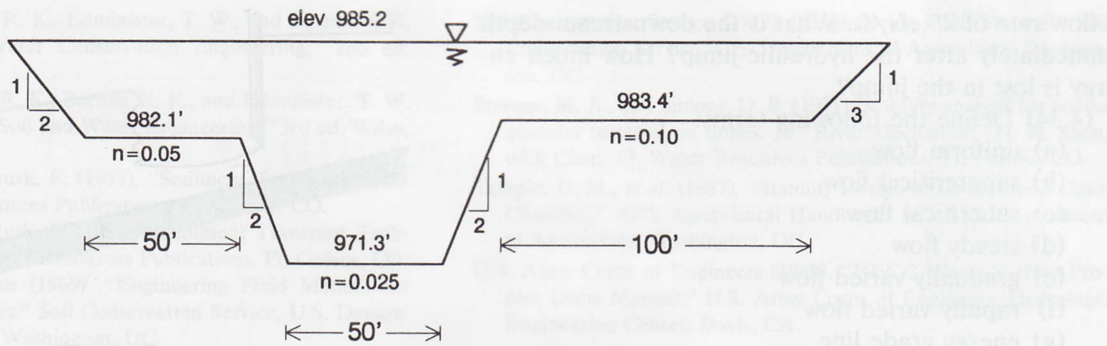

Figure 4A

(4.16) A circular, concrete storm sewer 3 ft in diameter is flowing at a depth of 2.1 ft. The sewer is on a 2% grade. What is the flow rate?

(4.17) What is the flow depth in the drain of problem (4.16) if it carries 25 cfs?

(4.18) What size circular, concrete storm drain would be required to carry 75 cfs down a 3% slope without surcharging the drain (i.e., always flowing as open channel flow)?

(4.19) Work problem (4.18) for circular corrugated metal pipe.

(4.20) Calculate the flow in the channel shown in Fig. 4A. The slope of the channel is 0.05%.

(4.21) At what depth would the channel of problem (4.20) be flowing if it were carrying 6000 cfs?

(4.22) Design a riprap-lined channel to carry 75 cfs down a 7% slope. Specify the required riprap size as well as the specifications of the filter material.

(4.23) What type of temporary lining should be used in a road ditch channel required to carry 10 cfs down a 7% slope?

(4.24) A 25 foot wide rectangular channel with a Manning's n of 0.025 is carrying 5000 cfs. The slope of the channel is 0.05%. At station 22 + 50 the slope of the channel changed abruptly to 5%. Calculate the flow profile in the upper channel from the channel break to a point where the depth is equal to 95% of normal depth.

(4.25) Calculate the flow profile in the lower channel to a point where the depth is equal to 95% of normal depth for the situation of problem 4.24.

(4.26) A wide rectangular channel has a slope of 5% and a Manning's n of 0.02. The channel slope changes abruptly to 0.04%. The flow rate is 12 cfs/foot of width. Calculate the resulting flow profile.

(4.27) A trapezoidal channel goes through a smooth transition. The flow depth is originally normal depth. The flow rate is 50 cfs. If there is no loss in energy, what will be the depth of flow immediately after the transition? The channel properties are:

	Upstream	Downstream
b	10 ft	8 ft
z	3 : 1	2 : 1
s	0.05%	0.05%

(4.28) Solve problem (4.27) as if the two channels are reversed so that the upstream channel becomes the downstream channel.

(4.29) A rectangular channel narrows from 20 ft to 15 ft. The bottom elevation simultaneously drops 2 ft. Both changes are smooth with little loss in energy. The flow rate is 400 cfs. What is the depth of flow downstream from the transition if the upstream depth is 4 ft?

(4.30) Work problem (4.29) as if the channel widens from 15 to 20 ft and the bottom elevation is raised by 2 ft.

(4.31) A hydraulic jump occurs in a wide channel where the flow is initially at a depth of 1 ft and a flow velocity of 14 fps. What is the depth after the jump? What is the energy loss within the jump?

(4.32) A rectangular channel has a Manning's n of 0.02, a slope of 0.1%, and a flow rate of 10 cfs/ft of width. Water enters the channel as supercritical flow. A hydraulic jump occurs. What must be the depth before and after the jump? How much energy is lost?

(4.33) Supercritical flow encounters some stabilizing blocks within a stilling basin. The drag force introduced by the blocks is given by $C_D \rho A v^2/2$, where C_D is a drag coefficient (use $C_D = 1$), ρ is the density of water (1.94 slugs/ft^3), and A is the cross-sectional area of the block perpendicular to the flow. The blocks are 1 ft high and occupy 75% of the flow cross section at a depth of 1 ft. Water enters the stilling well and strikes the blocks. The depth of flow is initially 2 ft with a flow rate of 25 cfs/ft. What is the downstream depth immediately after the hydraulic jump? How much energy is lost in the jump?

(4.34) Define the following terms:

(a) uniform flow
(b) supercritical flow
(c) subcritical flow
(d) steady flow
(e) gradually varied flow
(f) rapidly varied flow
(g) energy grade line
(h) velocity head
(i) pressure head
(j) flow profiles
(k) Froude number
(l) head loss.

(4.35) Flow in a wide rectangular channel encounters a barrier and an overflow spillway as shown in Fig. 4.B. Calculate the flow profile from the spillway back up to the channel to a point where the flow is within 0.2 ft of normal depth. The flow rate is 25 cfs/ft, the channel slope is 0.1%, and Manning's n is 0.015. The barrier is 3 ft high.

(4.36) Calculate the flow profiles resulting from the flow situation shown in Fig. 4.C. The underflow gate is 1000 feet upstream from the barrier. The barrier is 3 ft high. The channel slope is 0.1% slope, Manning's n is 0.015, and the flow rate is 25 cfs/ft. If a hydraulic jump will occur, locate the jump neglecting the length of the jump. The underflow gate clearance is 1.0 ft.

(4.37) What are surface water profiles used for?

(4.38) What impact might levees used to protect a particular region have on flood peaks upstream and downstream from the protected area?

(4.39) Water is flowing at 13 cfs/ft in a wide rectangular channel. What is the critical depth?

(4.40) A stream has a slope of 0.03%, a hydraulic radius of 2.2 m, and an average velocity of 1.2 m/sec. Estimate Manning's n. If the channel is 50 m wide, estimate the discharge in m^3/sec.

(4.41) A rectangular channel is carrying 10 cfs/ft of width. (a) Construct a specific energy diagram, and (b) construct a specific force and momentum diagram.

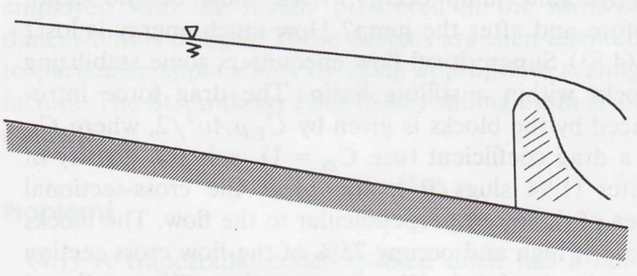

Figure 4B

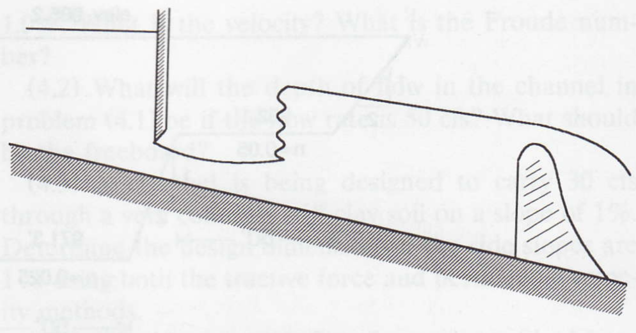

Figure 4C

(4.42) A hydraulic jump occurs in the channel of problem (4.41) with $y = 1.0$ ft. Use the diagram constructed for problem (4.41) to determine y_2 and the energy loss.

(4.43) Show that a generalized Froude number may be written as $F = \sqrt{Q^2 t/gA^3}$, where t is the top width.

References

Abt, S. R., *et al.* (1988). Resistance to flow over riprap in steep channels. *Water Resources Bull.* **24**(6):1193–1200

Anderson, A. G. (1973). Tentative design procedure for riprap lined channels—Field evaluation, Project report 146. Prepared for Highway Research Board. [Original not seen, cited in Normann (1975)]

Anderson, A. G., Paintal, A. A., and Davenport, J. T. Tentative design procedure for riprap lined channels, NCHRP Report 108. Highway Research Board, National Academy of Sciences, Washington, DC. [Original not seen, cited in Normann (1975).]

Chow, V. T. (1959). "Open Channel Hydraulics." McGraw-Hill, New York.

Fortier, S., and Scobey, F. S. (1926). Permissible canal velocities. *Trans. Am. Soc. Civil Eng.* **89**:940–984.

Gessler, J. (1971). Beginning and ceasing of sediment motion. *In* "River Mechanics" (H. W. Shen, ed.), Chap. 7. Water Resources Publications, Ft. Collins, CO.

Henderson, F. M. (1966). "Open Channel Flow." MacMillan, New York.

Lane, E. W. (1955). Design of stable channels. *Trans. Am. Soc. Civil Eng.* **120**: 1234–1260.

Lane, E. W. and Carlson, E. J. (1953). Some factors affecting the stability of canals constructed in course granular materials. *In* "Proceedings, Minnesota International Hydraulics Convention, Joint meeting IAHR and Hydraulics Division, ASCE."

McWhorter, J. C., Carpenter, T. G., and Clark, R. N. (1968). Erosion control criteria for drainage channels, Study conducted for the Mississippi State Highway Department and the Federal Highway Administration. Department of Agricultural Engineering, Mississippi State University, State College, MS.

Normann, J. M. (1975). Design of stable channels with flexible linings, Highway engineering circular No. 15. Federal Highway Administration, U.S. Department of Transportation, Washington, DC.

Ree, W. O. (1949). Hydraulic characteristics of vegetation for vegetated waterways. *Agric. Eng.* **30**:184–189.

Schwab, G. O., Frevert, R. K., Edminister, T. W., and Barnes, J. K. (1966). "Soil and Water Conservation Engineering," 2nd ed. Wiley, New York.

Schwab, G. O., Frevert, R. K., Barnes, K. K., and Edminister, T. W. (1971). "Elementary Soil and Water Engineering," 3rd ed. Wiley, New York.

Simons, D. B., and Senturk, F. (1977). "Sediment Transport Technology." Water Resources Publications, Ft. Collins, CO.

Simons, D. B., and Senturk, F. (1992). "Sediment Transport Technology" (2nd ed.). Water Resources Publications, Ft. Collins, CO.

Soil Conservation Service (1969). "Engineering Field Manual for Conservation Practices." Soil Conservation Service, U.S. Department of Agriculture, Washington, DC.

Soil Conservation Service (1979). "Engineering Field Manual." Soil Conservation Service, U.S. Department of Agriculture, Washington, DC.

Stevens, M. A., and Simons, D. B. (1971). Stability analysis for coarse granular material on slopes. *In* "River Mechanics" (H. W. Shen, ed.), Chap. 17. Water Resources Publications, Ft. Collins, CO.

Temple, D. M., *et al.* (1987). "Stability Design of Grass-Lined Open Channels," ARS Agricultural Handbooks 667. U.S. Department of Agriculture, Washington, DC.

U.S. Army Corps of Engineers (1982). "HEC-2 Water Surface Profiles Users Manual." U.S. Army Corps of Engineers, Hydrologic Engineering Center, Davis, CA.