4.3 Root Locus for Feedback Systems

4.3.1 Angle Criterion

The root locus for a feedback system is the path traced by the roots of the characteristic polynomial (the poles of the transfer function) as some system parameter is varied. Most control systems can be expressed in the form of Figure 4.4(a), where the transfer function $T(s)$ is

$$T(s) = \frac{KG(s)}{1 + KG(s)H(s)}$$

Closed- and open-loop transfer functions are defined.

The system parameter that is to be varied is the forward path gain K. It is normal practice to identify $T(s)$ as the *closed-loop transfer function* to distinguish it from another transfer function, which will now be introduced. In Figure 4.4(b), the feedback loop is broken at the variable called $Y_1(s)$. We can define a second transfer function, *the open- loop transfer function*, relating $Y_1(s)$ to $R(s)$. That second transfer function is

$$T_1(s) = KG(s)H(s) = \frac{Y_1(s)}{R(s)}$$

There are open-loop poles and zeros and closed-loop poles and zeros.

As always, each transfer function is defined with initial conditions equal to zero. Notice that the closed-loop transfer function depends heavily on the open-loop transfer function $KG(s)H(s)$. It is standard practice to define the zeros and poles of $G(s)H(s)$ as being *open-loop zeros* and *open-loop poles*, while the zeros and poles of $T(s)$ are called the *closed-loop zeros* and *closed-loop poles*. For convenience, the open-loop zeros and open-loop poles may be simply called *GH zeros* and *GH poles*. The term *roots* generally applies to the poles of $T(s)$ which are the roots of the characteristic polynomial.

Since the closed-loop poles are the roots of the characteristic polynomial, then those closed-loop poles satisfy

$$1 + KG(s)H(s) = 0 \quad [4.1]$$

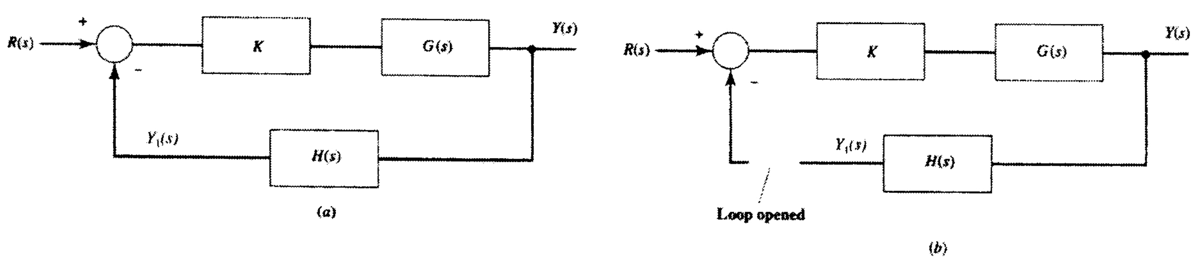

Figure 4.4 Feedback system with a variable gain K. (a) Closed-loop system. (b) Open-loop system with loop broken at $Y_1(s)$.

Table 4.2 *Some Root Locus Plots*

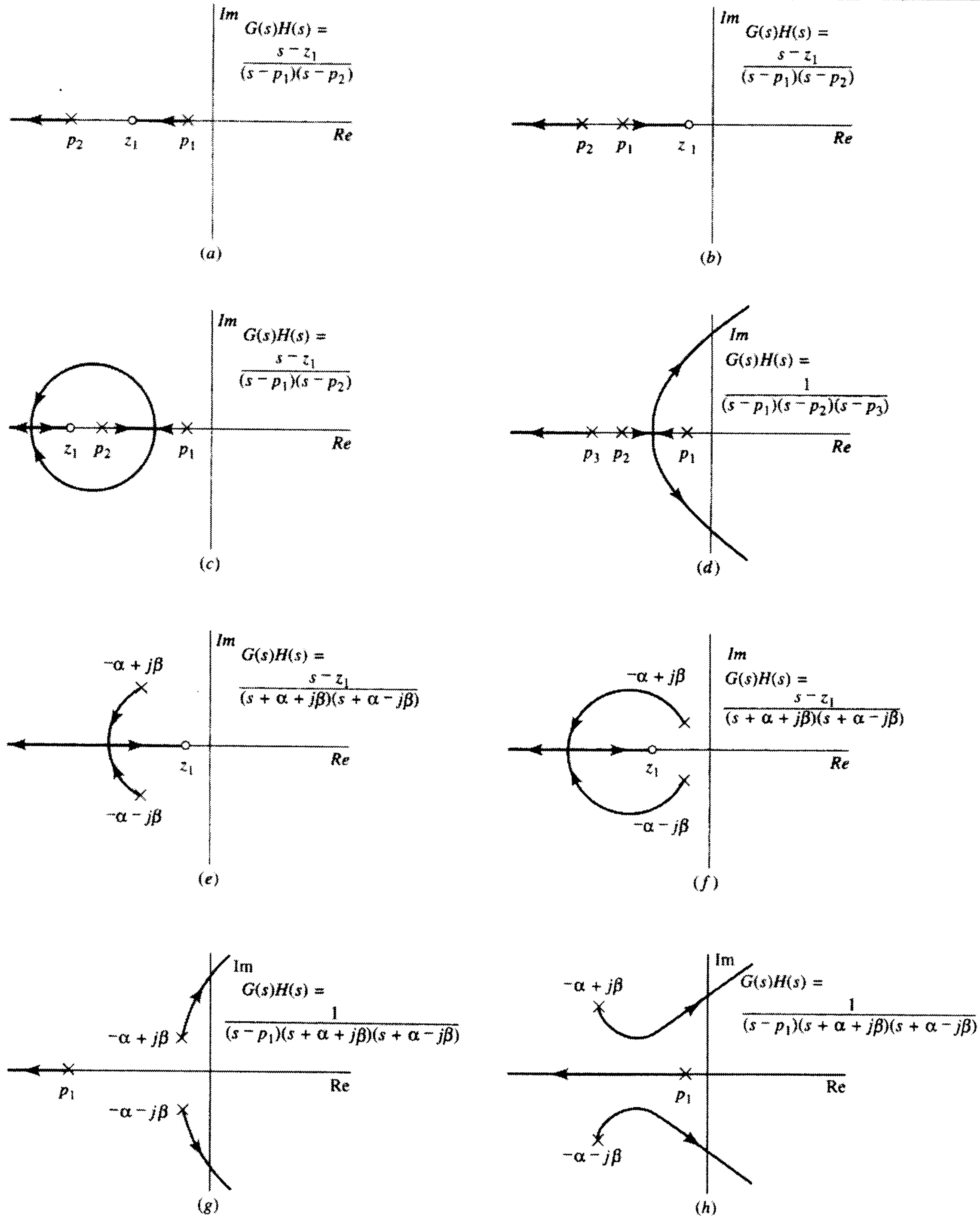

Table 4.2 *Some Root Locus Plots (Continued)*