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CHAPTER 7 STEADY-STATE RESPONSE ANALYSES

1. Introduction

The steady state error is a measure of system accuracy. These errors arise from the nature of the inputs, system type and from nonlinearities of system components such as static friction, backlash, etc. These are generally aggravated by amplifiers drifts, aging or deterioration. The steady state performance of a stable control system is generally judged by its steady state error to step, ramp and parabolic inputs.

Consider a unity feedback system as shown in the Fig. 1. The input is R(s) , the output is C(s), the feedback signal H(s) and the difference between input and output is the error signal E(s).

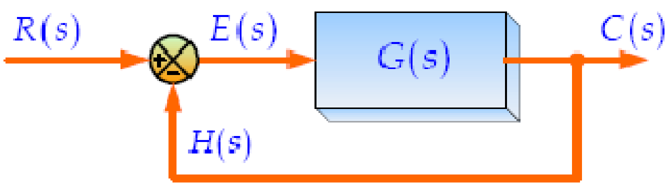


Fig.1 Closed-loop unity feedback control system

The closed loop transfer function (CLTF) is:

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)}$$

As we know $C(s)=E(s)G(s)$

Therefore,

$$E(s)=\frac{1}{1+G(s)}R(s)$$

Steady-state error e_{ss} may be found using the Final Value Theorem (FVT) as follows:

$$e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to 0}SE(s)=\lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

The above equation shows that the steady state error depends upon the input R(s) and the forward transfer function G(s) . The expression for steady-state errors for various types of standard test signals are derived in the following sections:

2. Steady state error and standard test input

2.1 Step input

Input $r(t)=1(t)$

or $R(s)=L[r(t)]=\frac{1}{s}$

$$e_{ss}=\lim_{s\to 0}\frac{sR(s)}{1+G(s)}=\lim_{s\to 0}\frac{s(1/s)}{1+G(s)}=\lim_{s\to 0}\frac{1}{1+G(s)}=\frac{1}{1+G(0)}=\frac{1}{1+K_p}$$

Where K_p is the ***position error constant*** and equals G(0) or $K_p=\lim_{S\to 0}G(S)$

2.2 Ramp input

Input $r(t)=t$ or $\dot{r}(t)=1$

or $R(s)=L[r(t)]=\frac{1}{s^2}$

$$e_{ss}=\lim_{s\to 0}\frac{sR(s)}{1+G(s)}=\lim_{s\to 0}\frac{s(1/s^2)}{1+G(s)}=\lim_{s\to 0}\frac{1}{s+sG(s)}=\lim_{s\to 0}\frac{1}{sG(s)}=\frac{1}{K_v}$$

Where K_v is the ***velocity error constant*** and equals $K_v=\lim_{S\to 0}S\,G(S)$

2.3 Parabolic input

Input $r(t)=\frac{1}{2}t^2$ or $\ddot{r}(t)=1$

or $R(s)=L[r(t)]=\frac{1}{s^3}$

$$e_{ss}=\lim_{s\to 0}\frac{sR(s)}{1+G(s)}=\lim_{s\to 0}\frac{s(1/s^3)}{1+G(s)}=\lim_{s\to 0}\frac{1}{s^2+s^2G(s)}=\lim_{s\to 0}\frac{1}{s^2G(s)}=\frac{1}{K_a}$$

Where K_a is the ***acceleration error constant*** and equals $K_a=\lim_{S\to 0}S^2\,G(S)$

3. Steady state error and system type

The system type refers to the how many poles of G(s) are located at the origin (s = 0).

For G(S) given below, the system is of ***n-type***.

$$G(s)=\frac{K'(s+z_1)(s+z_2)\cdots(s+z_j)}{s^n(s+p_1)(s+p_2)\cdots(s+p_k)}$$

3.1 Type zero system (n=0)

In that case $G(S)=\frac{K}{S^0}=K$

$$e_{ss}(\text{Position})=\frac{1}{1+G(0)}=\frac{1}{1+K}=\frac{1}{1+K_p}$$

$$e_{ss}(\text{Velocity})=\lim_{s\to 0}\frac{1}{sG(s)}=\lim_{s\to 0}\frac{1}{sK}=\infty$$

$$e_{ss}(\text{Acceleration})=\lim_{s\to 0}\frac{1}{s^2G(s)}=\lim_{s\to 0}\frac{1}{s^2K}=\infty$$

Thus a system with n = 0, or no integration in G(s) has

• A constant position error, • Infinite velocity error and • Infinite acceleration error

3.1 Type one system (n=1)

In that case $G(S)=\frac{K}{S^1}=\frac{K}{S}$

$$e_{ss}(\text{Position})=\frac{1}{1+G(0)}=\lim_{s\to 0}\frac{1}{1+\frac{K}{s}}=\frac{1}{1+\infty}=0$$

$$e_{ss}(\text{Velocity})=\lim_{s\to 0}\frac{1}{sG(s)}=\lim_{s\to 0}\frac{1}{s\frac{K}{s}}=\frac{1}{K}=\frac{1}{K_v}$$

$$e_{ss}(\text{Acceleration})=\lim_{s\to 0}\frac{1}{s^2G(s)}=\lim_{s\to 0}\frac{1}{s^2\frac{K}{s}}=\frac{1}{0}=\infty$$

Thus a system with n =1, or with one integration in G(s) has

• A zero position error, • A constant velocity error and • Infinite acceleration error

3.1 Type two system (n=2)

In that case $G(S) = \frac{K}{s^2}$

$$e_{ss}(\text{Position}) = \frac{1}{1+G(0)} = \lim_{s\to 0}\frac{1}{1+\frac{K}{s^2}} = \frac{1}{1+\infty} = 0$$

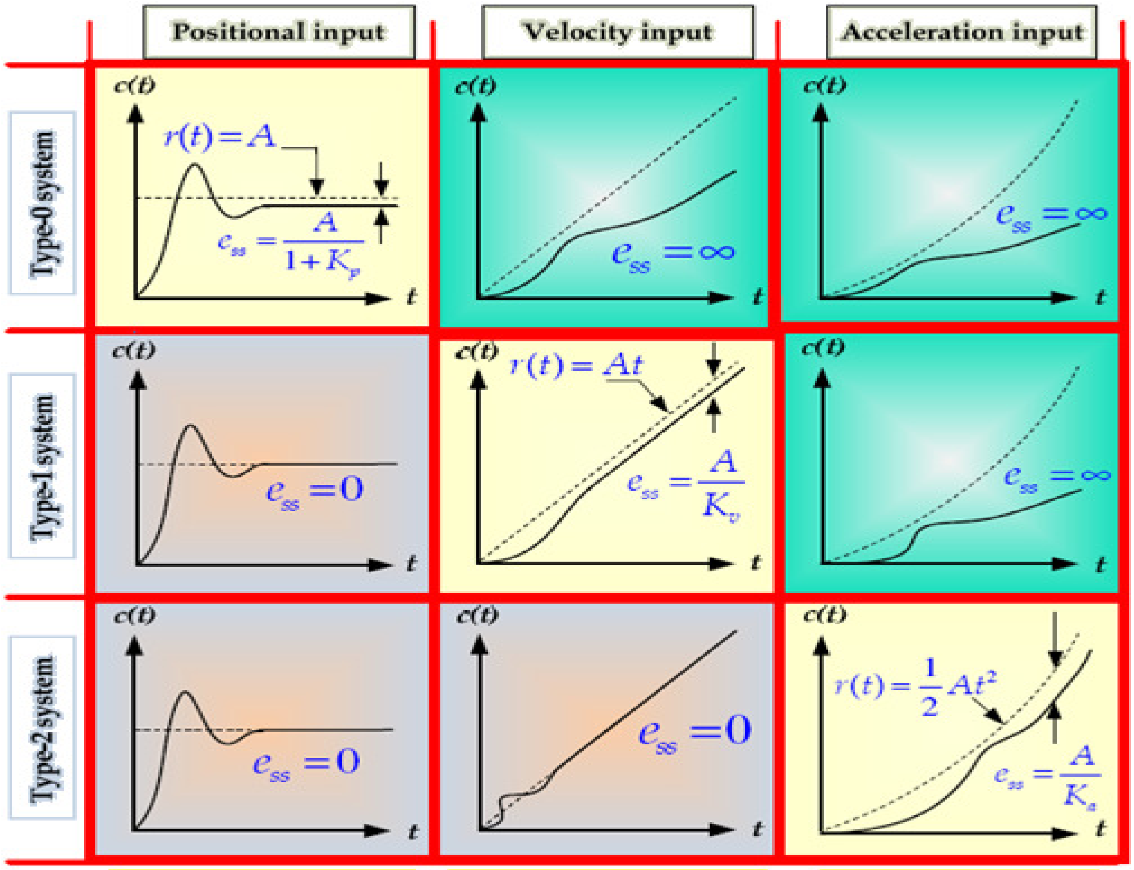
$$e_{ss}(\text{Velocity}) = \lim_{s\to 0}\frac{1}{sG(s)} = \lim_{s\to 0}\frac{1}{s\frac{K}{s^2}} = \frac{1}{\infty} = 0$$

$$e_{ss}(\text{Acceleration}) = \lim_{s\to 0}\frac{1}{s^2G(s)} = \lim_{s\to 0}\frac{1}{s^2\frac{K}{s^2}} = \frac{1}{K} = \frac{1}{K_a}$$

Thus a system with n = 2, or with one integration in G(s) has
• A zero position error, • A zero velocity error and • A constant acceleration error

Summary

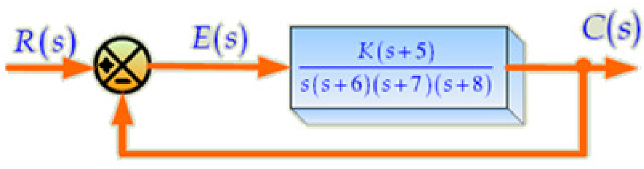
	Step Input $r(t) = 1$	Ramp Input $r(t) = t$	Acceleration Input $r(t) = \frac{1}{2}t^2$
Type 0 system	$\frac{1}{1+K}$	∞	∞
Type 1 system	0	$\frac{1}{K}$	∞
Type 2 system	0	0	$\frac{1}{K}$



$$K_p = \lim_{s\to 0} G(s) \qquad K_v = \lim_{s\to 0} sG(s) \qquad K_a = \lim_{s\to 0} s^2G(s)$$

Example #1

For the system shown below, find K so that there is 10% error in the steady state



Since the system type is 1, the error stated in the problem must apply to a ***ramp input***; only a ramp yields a finite error for in a type 1 system. Thus,

$$e(\infty)=\frac{1}{k_v}=0.1$$

Therefore,

$$k_v=10=\lim_{s\to 0} sG(s)=\frac{K\times 5}{6\times 7\times 8}=\frac{5K}{336} \quad\Rightarrow\quad K=\frac{336\times 10}{5}=672$$

4. Steady state error of non-unity feedback systems

Consider the non-unity feedback control system shown in Fig. 2.

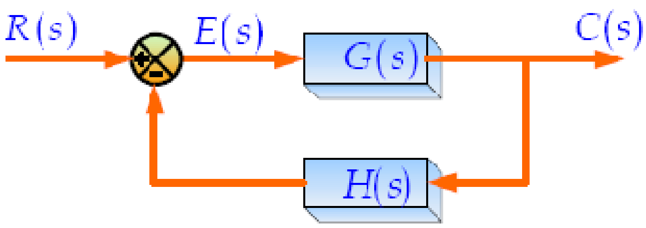


Fig. 2, non-unity feedback system

Add to the block shown in Fig. 2, two feedback blocks H(S) = +1 and H(S) = −1 as shown in Fig. 3.

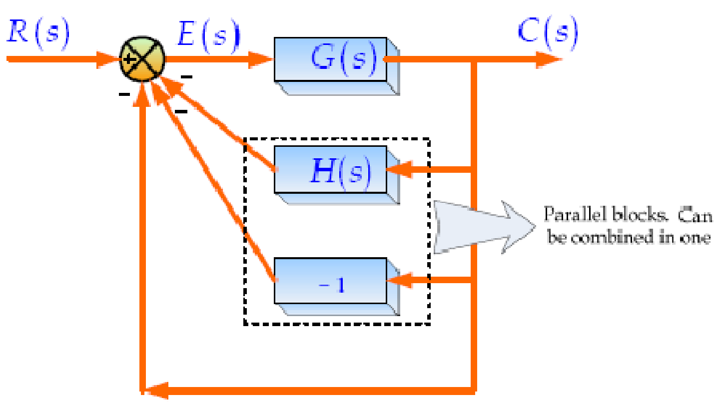
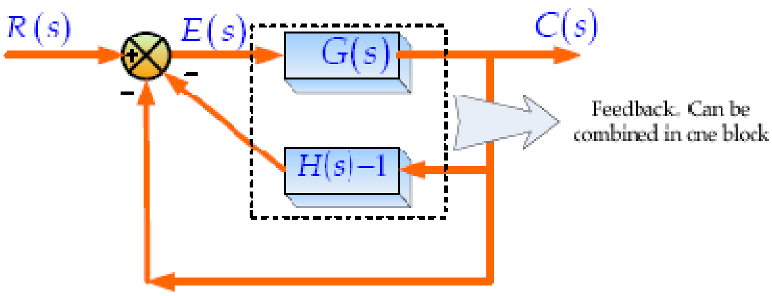
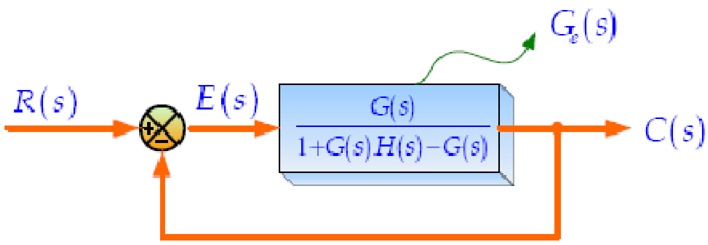


Fig.3

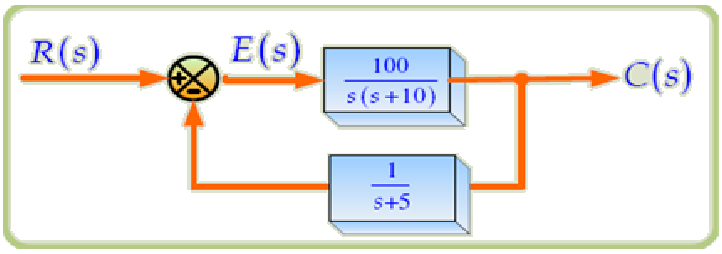




Example #2

For the system shown below, find

- The system type
- Appropriate error constant associated with the system type, and
- The steady state error for unit step input



The equivalent feed-forward transfer function is

$$G_e(s)=\frac{G(s)}{1+G(s)H(s)-G(s)}=\frac{G(s)}{1+G(s)\left[H(s)-1\right]}$$

$$=\frac{\dfrac{100}{s(s+10)}}{1+\dfrac{100}{s(s+10)}\left(\dfrac{1}{(s+5)}-1\right)}=\frac{\dfrac{100}{s(s+10)}}{s(s+10)(s+5)+\dfrac{100}{s(s+10)}\left(\dfrac{(1-s-5)}{(s+5)}\right)}$$

$$=\frac{100(s+5)}{s(s+10)(s+5)+100(1-(s-5))}=\frac{100(s+5)}{s^3+15s^2-50s-400}$$

System Type: 0

Appropriate error constant is K_p $\quad K_p=G_e(0)=\dfrac{100(0+5)}{0^3+15\times 0^2-50\times 0-400}=-\dfrac{5}{4}$

Steady State Error e_{ss}

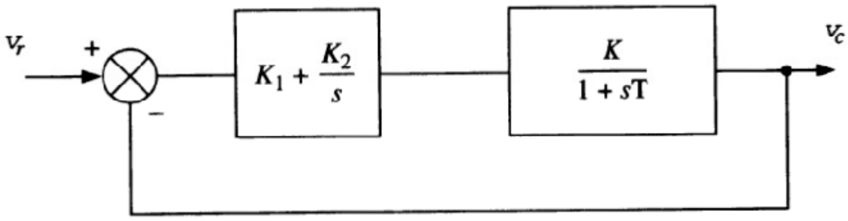
$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1-\frac{5}{4}} = \frac{1}{1-\frac{5}{4}} = -4$$

The negative value for E_{ss} implies that the output is larger than the input step.

Example #3

An engine speed control system is shown in Fig. . The engine itself is modeled as a first-order system with time constant T, while the electronic throttle controller may have the constants K_1 and K_2 set to arbitrary values.

1. What is the steady-state error for a step of magnitude A if $K_2 = 0$?
2. What is the steady-state error for a step of magnitude A when $K_2 \neq 0$?
3. Determine the steady-state error when the input is a ramp of slope A and (i) $K_2 = 0$, (ii) $K_2 \neq 0$.
4. Given $K_1 = 1.2$, $K_2 = 8.4$, and $T = 0.5$, what value of K gives a velocity error constant of 6 for a unit ramp input? Find the corresponding steady-state error, and sketch the input and output as functions of time for this case.



$$G(s) = \frac{K(K_1 s + K_2)}{s(1 + sT)}$$

1. When $K_2 = 0$, this transfer function reduces to

$$G(s) = \frac{KK_1}{1 + sT}$$

$$K_p = \lim_{s \to 0} G(s) = KK_1$$

$$e_{ss}(t) = \frac{1}{1 + K_p} = \frac{1}{1 + KK_1}$$

For a step of magnitude A, therefore

$$e_{ss}(t) = \frac{A}{1 + KK_1}$$

2. When $K_2 \neq 0$, the open-loop transfer function reverts to the form

$$G(s) = \frac{K(K_1 s + K_2)}{s(1 + sT)}$$

Which represent a type 1 system.

$$K_p = \lim_{S \to 0} G(S) = \infty$$

$e_{ss}(t) = 0$

3. When $K_2 = 0$ and the input is a ramp,

$$K_v = \lim_{S \to 0} S\,G(S) = 0$$

$$e_{ss} = \frac{A}{K_v} = \infty$$

When $K_2 \neq 0$,

$$K_v = \lim_{S \to 0} S\,G(S) = KK_2$$

$$e_{ss}(t) = \frac{A}{K_v} = \frac{A}{KK_2}$$

4. Given $K_1 = 1.2$, $K_2 = 8.4$, and $T = 0.5$, it is required that

$$K_v = 6 = 8.4K$$

Hence the result

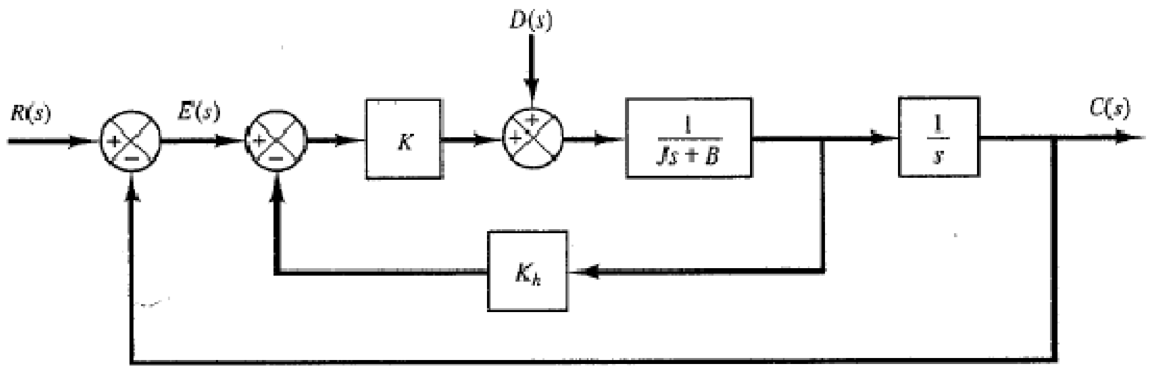
$$K = 0.714$$

The steady-state error of the closed-loop system becomes

$$e_{ss}(t) = \frac{1}{K_v} = 0.167$$

Example #4

Consider the servo system with tachometer feedback shown in Figure.
Obtain the error signal *E(s)* when both the reference input *R(s)* and disturbance input *D(s)* are present. Obtain also the steady-state error when the system is subjected to a reference input (unit-ramp input) and disturbance input (step input of magnitude d).



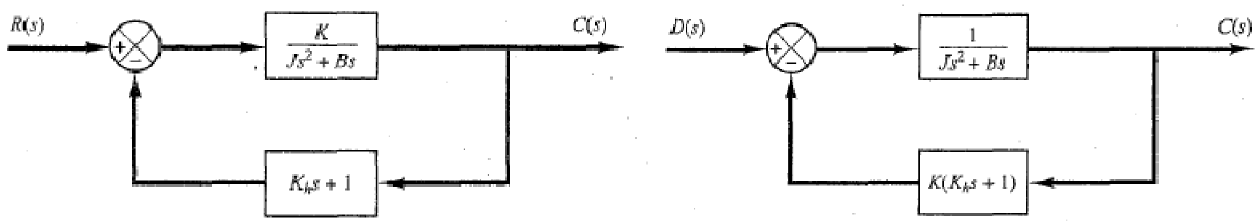
Using super position:

(when D(S) = 0)
$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + (B + KK_h)s + K}$$

(when R(S) = 0)
$$\frac{C(s)}{D(s)} = \frac{1}{Js^2 + (B + KK_h)s + K}$$

The overall output is

$$C(s) = \frac{1}{Js^2 + (B + KK_h)s + K}[KR(s) + D(s)]$$



Since

$$E(s) = R(s) - C(s)$$
$$= R(s) - \frac{KR(s) + D(s)}{Js^2 + (B + KK_h)s + K}$$

we obtain

$$E(s) = \frac{1}{Js^2 + (B + KK_h)s + K}\{[Js^2 + (B + KK_h)s]R(s) - D(s)\}$$

Hence the steady-state error can be obtained as follows:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$
$$= \lim_{s\to 0} \frac{s}{Js^2 + (B + KK_h)s + K}[s(Js + B + KK_h)R(s) - D(s)]$$
$$= \lim_{s\to 0}\left[\frac{B + KK_h}{K} s^2 R(s) - \frac{1}{K} sD(s)\right]$$

Since $R(s) = 1/s^2$ (unit-ramp input) and $D(s) = d/s$ (step input of magnitude d) the steady-state error is

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0}\left[\frac{B + KK_h}{K}\frac{s^2}{s^2} - \frac{1}{K}\frac{sd}{s}\right]$$
$$= \frac{B + KK_h}{K} - \frac{d}{K}$$

Example 5:

For the control system shown below in Fig A,

a) Determine the values of gain K and the time constant T so that the system response for unit-step input is as shown in Fig. B.

b) With these values of K and T, obtain in part (a), find the rise time and peak time and percentage overshoot.

c) Calculate the position error coefficient and the steady-state error.

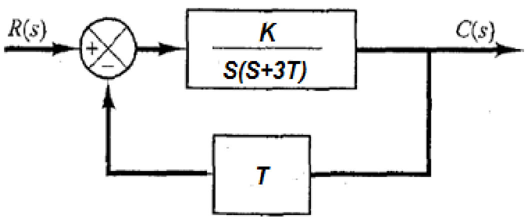


Fig. A

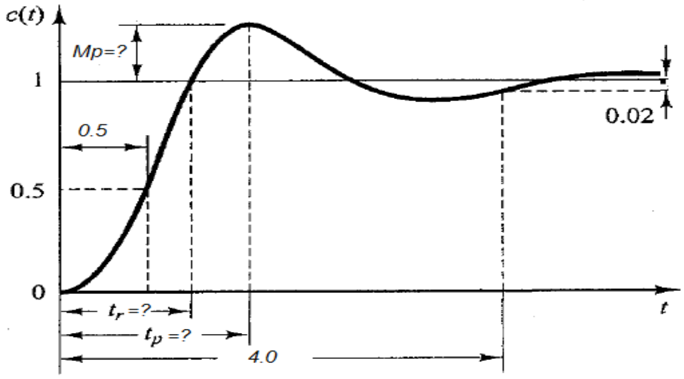


Fig. B

From system response (Fig. B),

$T_S = 4$ (based on 2% tolerance) and $T_d = 0.5$

since

$$T_s = \frac{4}{\xi\,\omega_n} = 4$$

Therefore,

$$\xi\,\omega_n = 1$$

Also, we have the delay time $T_d = 0.5$

$$T_d = \frac{1 + 0.7\xi}{\omega_n} = 0.5$$

Multiply both sides by ξ, then

$$T_d = \frac{\xi + 0.7\xi^2}{\xi\,\omega_n} = 0.5$$

But we get that, $\xi\,\omega_n = 1$, therefore

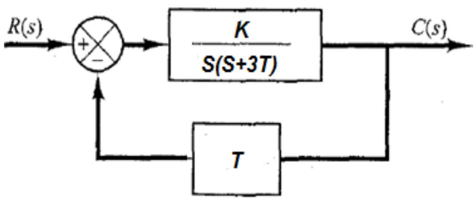
$$0.7\xi^2 + \xi - 0.5 = 0$$

Solving this equation

$$\xi = 0.392281\ (accepted)$$
$$\xi = -1.82085\ (rejected)$$

$\omega_n = \frac{1}{\xi} = 2.5492\ rad/s$

From the system block diagram



The system T.F.

$$\frac{C(s)}{R(s)} = \frac{K}{S^2 + 3Ts + KT}$$

The system characteristic equation is

$$S^2 + 3Ts + KT = 0$$

The standard form of 2nd order system characteristic equation is

$$S^2 + 2\xi\omega_n s + {\omega_n}^2 = 0$$

By comparing

$$2\xi\omega_n = 3T$$
$$T = \frac{2}{3}$$

Also,

$${\omega_n}^2 = KT$$
$$K = \frac{{\omega_n}^2}{T} = 9.7476$$

The % maximum over shoot M_p

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 = 26.19\,\%$$

Rise Time:

$$T_r = \frac{\pi - \beta}{\omega_n\sqrt{1-\xi^2}}$$

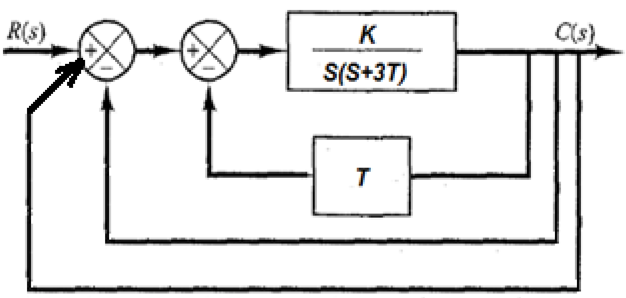
B = $\cos^{-1}(\xi)$ = 66.90349° = 1.167686 rad

$$T_r = \frac{\pi - 1.167686}{2.5492\sqrt{1-0.392281^2}} = 0.842\ sec.$$

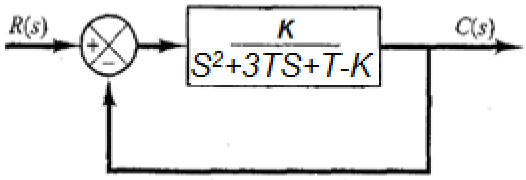
Peak Time:

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$
$$T_p = \frac{\pi}{2.5492\sqrt{1-0.392281^2}} = 1.34\ sec.$$

To get the steady-state error and position error coefficient, the system must be unity feedback, so we will add +ve and –ve feedback as shown in fig



Then the unity feedback system will be



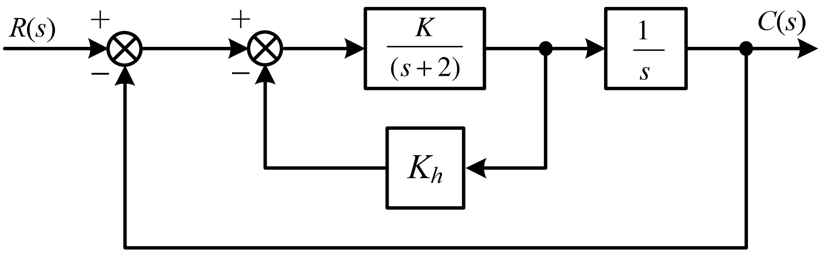
$$K_p = \lim_{S\to 0} G(s)H(s) = \frac{K}{T-K} = \frac{9.7476}{0.6667-9.7476} = -1.07342$$

$$E_{ss} = \frac{1}{1+K_p} = \frac{1}{1-1.07342} = -13.62$$

Example 6:

The control system, shown in Fig. below, is subjected to a unit ramp function,

a) Determine the value of K and K_h such that the system has an overshoot of 16.303% and a damped natural frequency of 3.4641 rad/sec.
b) Calculate the rise time, peak time and settling time based on ±2% tolerance
c) Define the system type
d) Calculate the steady-state error of that system
e) If an integrator is added in the forward path after the 1st summing point directly, calculate the new steady-state error.



Maximum overshoot = 0.16303 = $e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$ taking (ln) for both sides

$-1.813821 = -\frac{\pi\zeta}{\sqrt{1-\zeta^2}}$

$3.29 = \frac{\pi^2\zeta^2}{1-\zeta^2}$

$0.333347 - 0.33347\,\zeta^2 = \zeta^2$

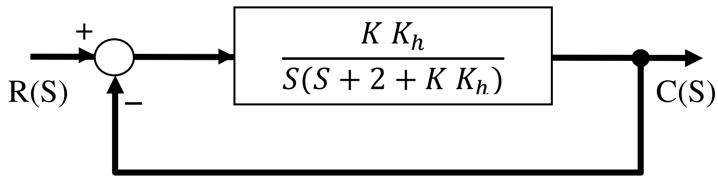
$\zeta^2 = \frac{0.333347}{1.333347} = 0.25$

$\zeta = \sqrt{0.25} = 0.5$

Since the damped natural frequency (ω_d) = $\omega_n\sqrt{1-\zeta^2}$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{3.4641}{\sqrt{1-0.25}} = 4\ rad/sec.$$

The block diagram can be simplified as follows



The closed loop transfer function $\frac{C(S)}{R(S)} = \frac{K}{S^2 + (2+KK_h)S + K}$

But the general form T.F. of the 2nd order system is $\frac{C(S)}{R(S)} = \frac{\omega_n{}^2}{S^2 + (2\zeta\omega_n)S + \omega_n{}^2}$

By comparing the coefficient of both T.F.

$K = \omega_n{}^2 = 16$ ##

$2 + KK_h = 2\zeta\omega_n$

$K_h = 0.125$ ##

The angle $\beta = cos^{-1}(0.5) * \frac{\pi}{180} = 1.0472\, rad$

Rise time (T_r) = $\frac{\pi - \beta}{\omega_d} = \frac{\pi - 1.0472}{3.4641} = 0.605\, sec.$ ##

Peak time (T_p) = $\frac{\pi}{\omega_d} = \frac{\pi}{3.4641} = 0.907\, sec.$ ##

Settling time (T_s) based on ±2% tolerance = $\frac{4}{\zeta\omega_n} = \frac{4}{0.5*4} = 2.0\, sec.$ ##

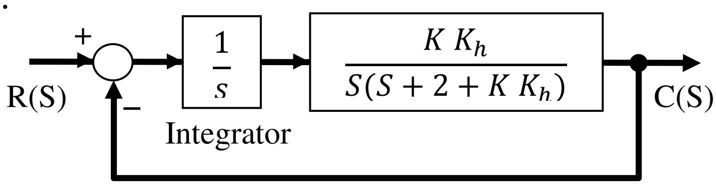
The system type = 1

The velocity error coefficient $K_V = \lim_{S\to 0} S\,G(s)H(s) = \lim_{S\to 0} S\,\frac{K}{S(S+2+K\,K_h)} =$

$\frac{K}{2+K\,K_h} = \frac{16}{2+2} = 4$

The steady-state error (E_{ss}) = $\frac{1}{K_V} = 0.25$ ##

When an integrator is added in the forward path after the 1st summing point directly as shown in Fig.



The new G(s)H(s) = $\frac{K\,K_h}{S^2\,(S+2+K\,K_h)}$

The new velocity error coefficient $K_V = \lim_{S\to 0} S\,G(s)H(s) = \lim_{S\to 0} S\,\frac{K}{S^2(S+2+K\,K_h)} =$

$\frac{K}{0} = \infty$

The new steady-state error (E_{ss}) = $\frac{1}{K_V} = \frac{1}{\infty} = 0$ ##

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Sheet 5 (Performance of feedback control systems)

1) A thermometer requires 1 min to indicate 98% of the response to a step input. Assuming the thermometer to be a first-order system, find the time constant.

2) Consider the unit-step response of a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{1}{S(S+1)}$$

Obtain the rise time, peak time, maximum overshoot, and settling time.

3) Consider the closed-loop system given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

Determine the values of ζ and ω_n, so that the system responds to a step input with approximately *5%* overshoot and with a settling time of 2 sec. (Use the 2% criterion.)

4) Figure 1 is a block diagram of a space-vehicle attitude-control system. Assuming the time constant *T* of the controller to be 3 sec and the ratio K/J = 2/9 rad^2/sec^2 Find the damping ratio of the system, rise time, maximum overshoot and peak time.

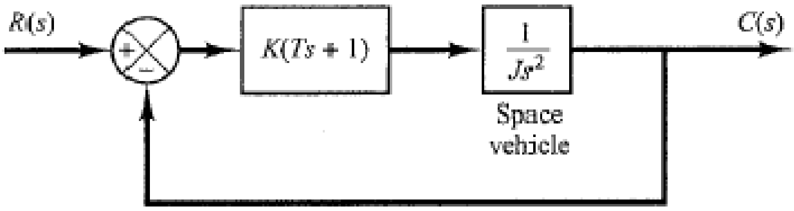


Fig. 1

5) Consider the system shown in Fig. 2(a). The damping ratio of this system is 0.158 and the undamped natural frequency is 3.16 rad/sec. To improve the relative stability, we employ tachometer feedback. Fig. 2(b) shows such a tachometer-feedback system. Determine the value of K_h, so that the damping ratio of the system is 0.5. Then find the rise time, maximum overshoot and settling time and compare them with those obtained from the original system.

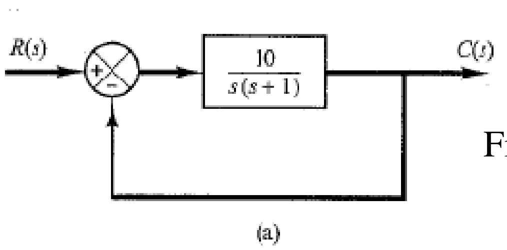
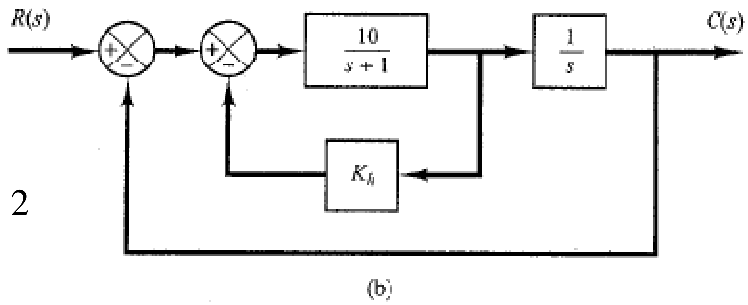


Fig. 2



6) Referring to the system shown in Fig. 3, determine the values of K and k such that the system has a damping ratio 0.7 and an undamped natural of 4 rad/sec.

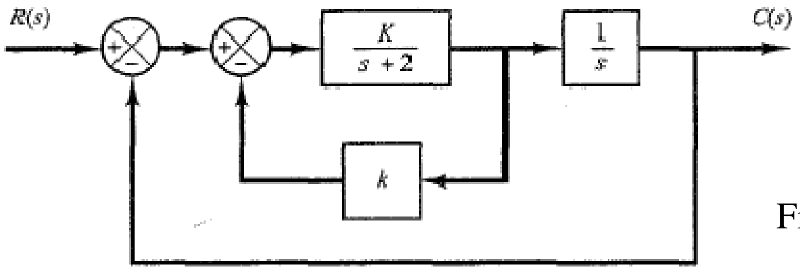


Fig. 3

7) For the system shown in Fig. 4, determine the values of gain K and velocity feedback constant K_h so that the maximum overshoot in the unit-step response is 0.2 and the peak time is 1 sec. With these values of K and K_h, obtain the rise time and settling time. Assume that $J = 1$ kg-m^2 and B = 1 N-m/rad/sec.

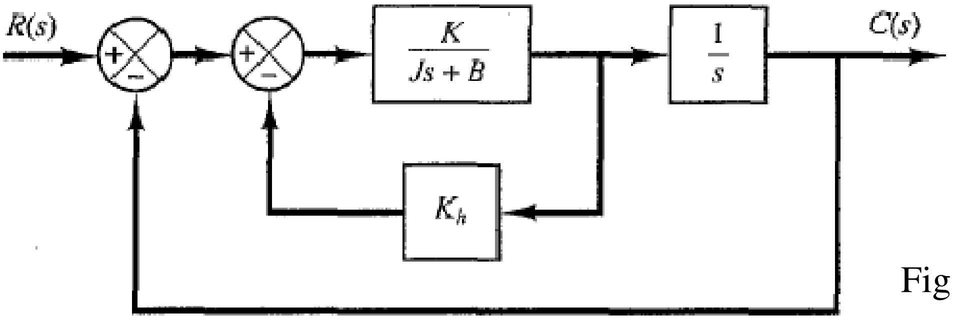
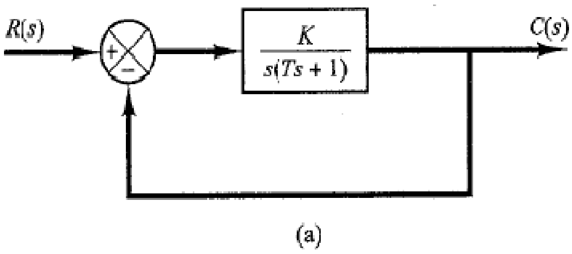


Fig. 4

8) When the control system shown in Fig. 5 (a) is subjected to a unit-step input, the system output responds as shown in Fig. 5 (b). Determine the values of K and T from the response curve.



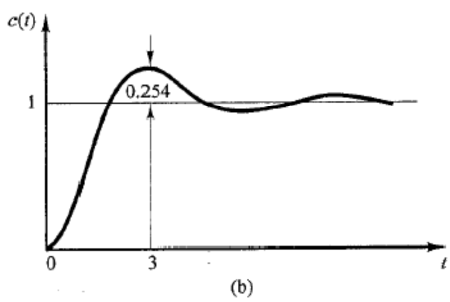


Fig. 5

9) For the closed-loop system given by

$$\frac{C(s)}{R(s)} = \frac{36}{S^2 + 2S + 36}$$

Calculate the rise time, peak time, maximum overshoot, and settling time when R(s) is considered as unit step input.

10) Figure 6 shows three systems. System I is a positional servo system. System II is a positional servo system with PD control action. System III is a positional servo system with velocity feedback. Compare the unit-step response of the three systems and obtain the best one with respect to the speed of response and maximum overshoot.

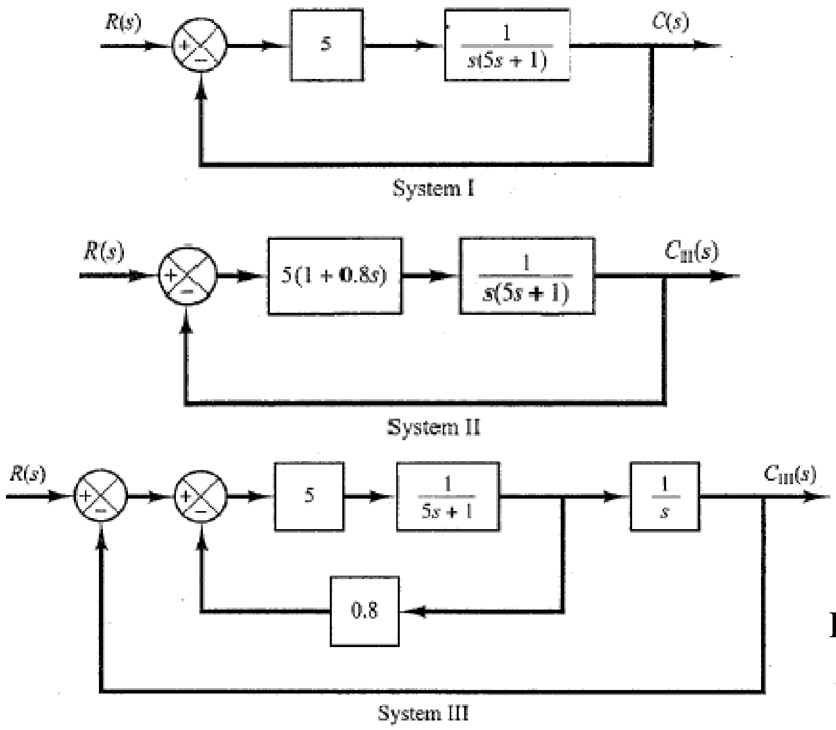


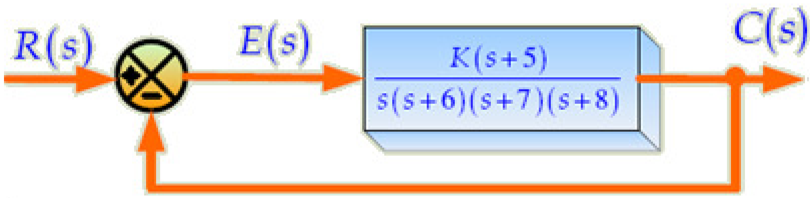
Fig. 6

11) For a unity feedback control systems given below, find the position (K_p), velocity (K_v) and acceleration (K_a) error coefficients.

$$G(s) = \frac{10}{(s+1)(2s+1)} \qquad \text{(c)} \quad G(s) = \frac{K}{s^2(0.5s+1)(s+1)}$$

$$G(s) = \frac{K}{s(s+1)(2s+1)} \qquad \text{(d)} \quad G(s) = \frac{K(s+4)}{s^2(s^2+6s+2)}$$

12) For the system shown in Fig. 7, find K so that there is 10% E_{ss}



13) For the system shown in Fig. 8, find

- The system type
- Appropriate error constant associated with the system type, and
- The steady state error for unit step input

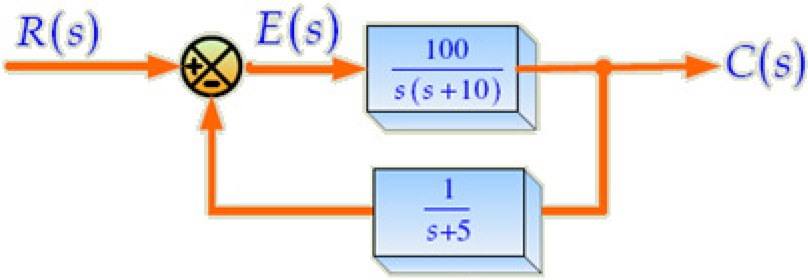


Fig. 8

14) Consider the system shown in Fig. 9. Prove that the steady-state error for a unit ramp input is $\frac{2\zeta}{\omega_n}$. Also show that the damping ratio is $\frac{B}{2\sqrt{KJ}}$ and the undamped natural frequency is $\sqrt{\frac{K}{J}\,(1-\zeta^2)}$

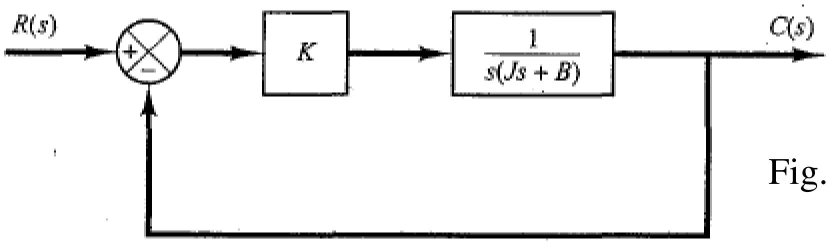


Fig. 9

15) Consider the system shown in Fig. 10(a). The steady-state error to a unit-ramp input is $e_{ss} = \frac{2\zeta}{\omega_n}$. Show that the steady-state error for following a ramp input may be eliminated if the input is introduced to the system through a proportional-plus-derivative filter, as shown in Fig. 10(b), and the value of k is properly set.

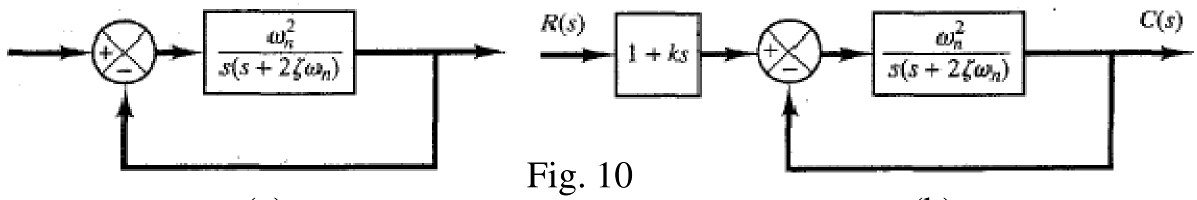


Fig. 10

(a) (b)

16) Consider servo system with tachometer feedback shown in Fig. 11. Obtain the error signal *E(s)* when both the reference input *R(s)* and disturbance input *D(s)* are present. Obtain also the steady-state error when the system is subjected to a reference input (unit-ramp input) and disturbance input (step input of magnitude d).

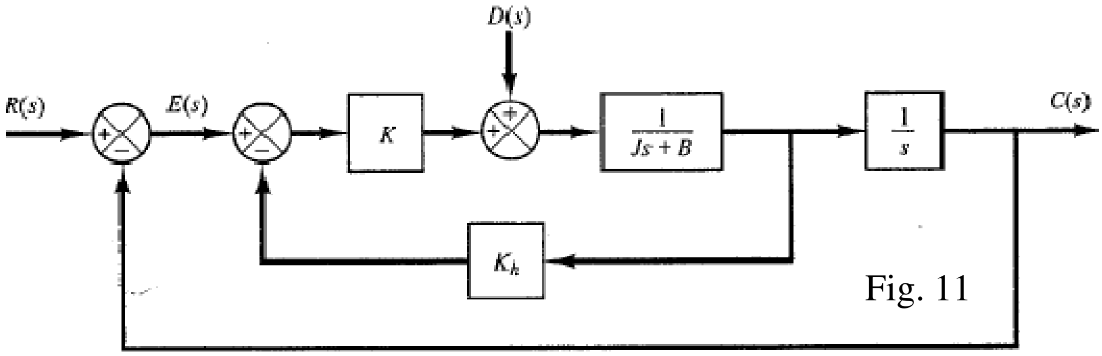


Fig. 11

17. For the control system shown in Fig. 12, Determine the steady-state error for a unit step when $K = 0.4$ and $G_p(s) = 1$. Select an appropriate value for $G_p(s)$ so that the steady-state error is equal to zero for the unit step input.

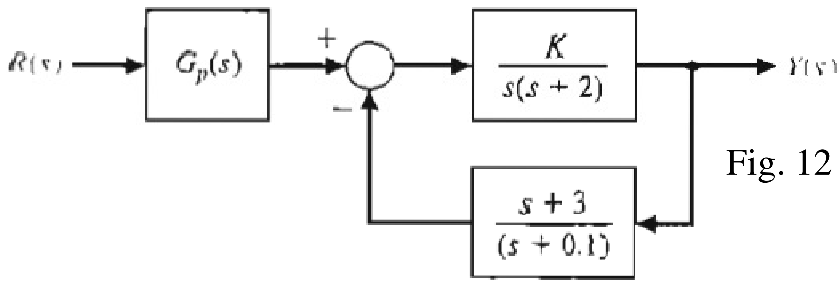


Fig. 12

18. Consider the closed-loop system in Fig. 13. Determine values of the parameters *k* and *a* so that the following specifications are satisfied:

(a) The steady-state error to a unit step input is zero.

(b) The closed-loop system has a percent overshoot of less than 5%.

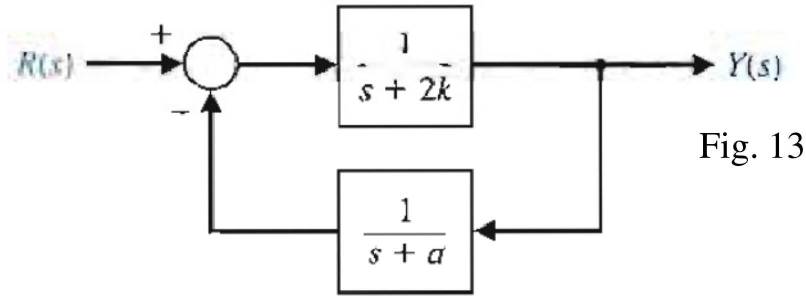


Fig. 13

19.

An engine speed control system is shown in Fig. 14. The engine itself is modeled as a first-order system with time constant T, while the electronic throttle controller may have the constants K_1 and K_2 set to arbitrary values.

1. What is the steady-state error for a step of magnitude A if $K_2 = 0$?
2. What is the steady-state error for a step of magnitude A when $K_2 \neq 0$?
3. Determine the steady-state error when the input is a ramp of slope A and (i) $K_2 = 0$, (ii) $K_2 \neq 0$.
4. Given $K_1 = 1.2$, $K_2 = 8.4$, and $T = 0.5$, what value of K gives a velocity error constant of 6 for a unit ramp input? Find the corresponding steady-state error, and sketch the input and output as functions of time for this case.

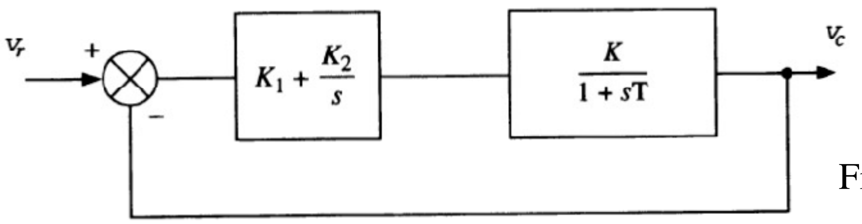


Fig. 14

20. For the control system shown in Fig. 15, the system is subjected to an input r(t)=2t. where K_1 and K_2 are constants.

(a) Obtain the system type,

(b) Calculate the value of K_2 if the steady-state error $E_{SS} = 0.085$ and $K_1 = 3.0$,

(c) If K_1 is set to zero and K_2 is reduced to its half value obtained in (b), calculate the steady-state error (E_{SS}).

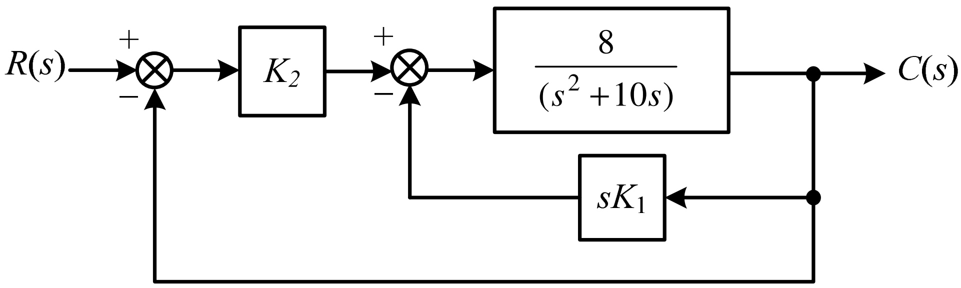


Fig. 15