CHAPTER 7

# Exergy Analysis

chapter objective ►

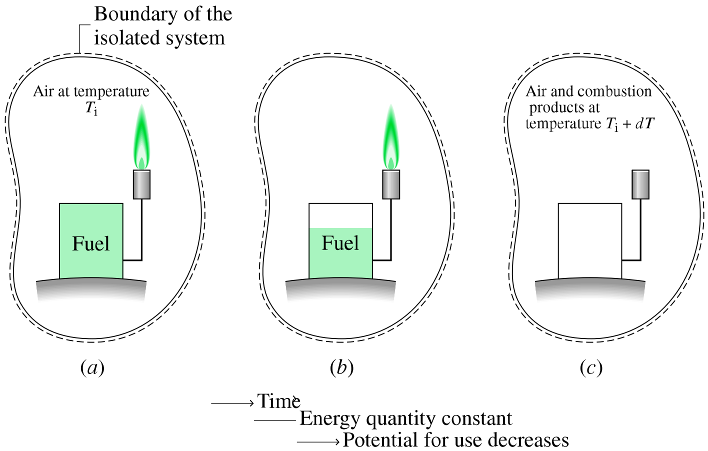
**ENGINEERING CONTEXT** The **objective** of this chapter is to introduce *exergy analysis*, a method that uses the conservation of mass and conservation of energy principles together with the second law of thermodynamics for the design and analysis of thermal systems. Another term frequently used to identify exergy analysis is *availability analysis*.

The importance of developing thermal systems that make effective use of nonrenewable resources such as oil, natural gas, and coal is apparent. The method of exergy analysis is particularly suited for furthering the goal of more efficient resource use, since it enables the locations, types, and true magnitudes of waste and loss to be determined. This information can be used to design thermal systems, guide efforts to reduce sources of inefficiency in existing systems, and evaluate system economics.

## 7.1 Introducing Exergy

Energy is conserved in every device or process. It cannot be destroyed. Energy entering a system with fuel, electricity, flowing streams of matter, and so on can be accounted for in the products and by-products. However, the energy conservation idea alone is inadequate for depicting some important aspects of resource utilization.

► ***for example...*** Figure 7.1*a* shows an isolated system consisting initially of a small container of fuel surrounded by air in abundance. Suppose the fuel burns (Fig. 7.1*b*) so that finally there is a slightly warm mixture of combustion products and air as shown in Fig. 7.1*c*. Although the total *quantity* of energy associated with the system would be unchanged, the initial fuel–air combination would have a greater economic value and be intrinsically more useful than the final warm mixture. For instance, the fuel might be used in some device to generate electricity or produce superheated steam, whereas the uses to which the slightly warm combustion products can be put would be far more limited in scope. We might say that the system has a greater *potential for use* initially than it has finally. Since nothing but a final warm mixture would be achieved in the process, this potential would be largely wasted. More precisely, the initial potential would be largely *destroyed* because of the irreversible nature of the process. ◄




▲ **Figure 7.1** Illustration used to introduce exergy.

Anticipating the main results of this chapter, we can read *exergy* as potential for use wherever it appears in the text. The foregoing example illustrates that, unlike energy, *exergy is not conserved*.

Subsequent discussion shows that exergy not only can be destroyed by irreversibilities but also can be transferred to a system or from a system, as in losses accompanying heat transfers to the surroundings. Improved resource utilization can be realized by reducing exergy destruction within a system and/or losses. An objective in exergy analysis is to identify sites where exergy destructions and losses occur and rank order them for significance. This allows attention to be centered on the aspects of system operation that offer the greatest opportunities for improvement.

## 7.2 Defining Exergy

The basis for the exergy concept is present in the introduction to the second law provided in Chap. 5. A principal conclusion of Sec. 5.1 is that an opportunity exists for doing work whenever two systems at different states are brought into communication. In principle, work can be developed as the systems are allowed to come into equilibrium. When one of the two systems is a suitably idealized system called an ***exergy reference environment*** or simply, an *environment,* and the other is some system of interest, ***exergy*** is the *maximum theoretical work* obtainable as they interact to equilibrium.

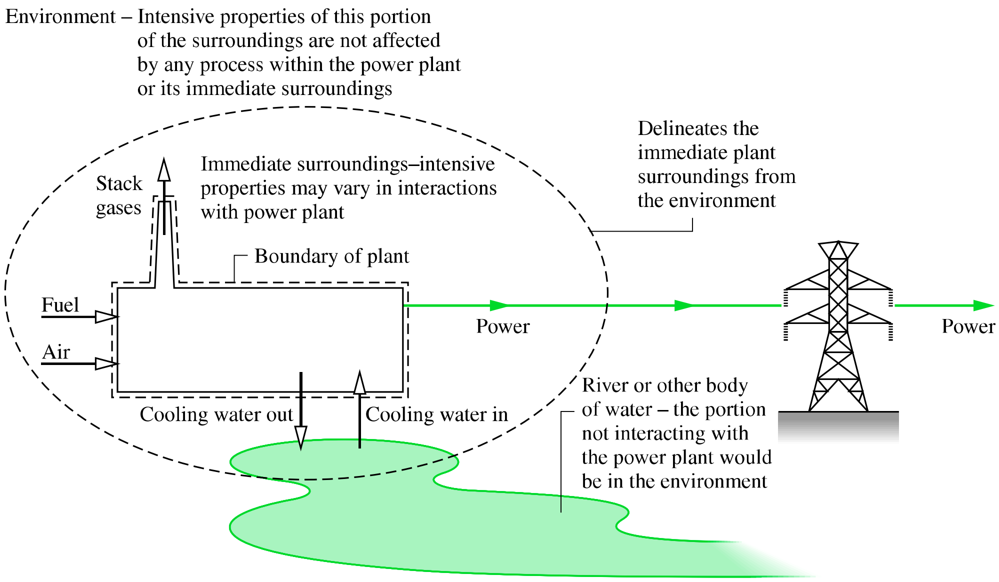
*exergy reference environment*

*exergy*

The definition of exergy will not be complete, however, until we define the reference environment and show how numerical values for exergy can be determined. These tasks are closely related because the numerical value of exergy depends on the state of a system of interest, as well as the condition of the environment.

### ▶ 7.2.1 Exergy Reference Environment

Any system, whether a component of a larger system such as a steam turbine in a power plant or the larger system itself (power plant), operates within surroundings of some kind. It is important to distinguish between the environment used for calculating exergy and a system's surroundings. Strictly speaking, the term surroundings refers to everything not included in the system. However, when considering the exergy concept, we distinguish between the




▲ **Figure 7.2** Schematic of a power plant and its surroundings.

*immediate* surroundings, where intensive properties may vary during interactions with the system, and the larger portion of the surroundings at a distance, where the intensive properties are unaffected by any process involving the system and its immediate surroundings. The term *environment* identifies this larger portion of the surroundings.

► ***for example...*** Fig. 7.2 illustrates the distinction between a system consisting of a power plant, its immediate surroundings, and the environment. In this case, the environment includes portions of the surrounding atmosphere and the river at a distance from the power plant. Interactions between the power plant and its immediate surroundings have no influence on the temperature, pressure, or other intensive properties of the environment. ◄

## MODELING THE ENVIRONMENT

The physical world is complicated, and to include every detail in an analysis is not practical. Accordingly, in describing the environment, simplifications are made and a model results. The validity and utility of an analysis using any model are, of course, restricted by the idealizations made in formulating the model. In this book the environment is regarded to be a simple compressible system that is *large* in extent and *uniform* in temperature, $T_0$, and pressure, $p_0$. In keeping with the idea that the environment represents a portion of the physical world, the values for both $p_0$ and $T_0$ used throughout a particular analysis are normally taken as typical environmental conditions, such as 1 atm and 25°C. The intensive properties of each phase of the environment are uniform and do not change significantly as a result of any process under consideration. The environment is also regarded as free of irreversibilities. All significant irreversibilities are located within the system and its immediate surroundings.

Although its intensive properties do not change, the environment can experience changes in its extensive properties as a result of interactions with other systems. Changes in the extensive properties internal energy $U_e$, entropy $S_e$, and volume $V_e$ of the environment are related through the *first T dS* equation, Eq. 6.10. Since $T_0$ and $p_0$ are constant, Eq. 6.10 takes the form

$$\Delta U_e = T_0 \, \Delta S_e - p_0 \, \Delta V_e \tag{7.1}$$

In this chapter kinetic and potential energies are evaluated relative to the environment, all parts of which are considered to be at rest with respect to one another. Accordingly, as indicated by the foregoing equation, a change in the energy of the environment can be a change in its internal energy only. Equation 7.1 is used below to develop an expression for evaluating exergy. In Chap. 13 the environment concept is extended to allow for the possibility of chemical reactions, which are excluded from the present considerations.

### ▶ 7.2.2 Dead State

Let us consider next the concept of the *dead state,* which is also important in completing our understanding of the property exergy.

If the state of a fixed quantity of matter, a closed system, departs from that of the environment, an opportunity exists for developing work. However, as the system changes state toward that of the environment, the opportunity diminishes, ceasing to exist when the two are in equilibrium with one another. This state of the system is called the ***dead state.*** At the dead state, the fixed quantity of matter under consideration is imagined to be sealed in an envelope impervious to mass flow, at rest relative to the environment, and internally in equilibrium at the temperature $T_0$ and pressure $p_0$ of the environment. At the dead state, both the system and environment possess energy, but the value of exergy is zero because there is no possibility of a spontaneous change within the system or the environment, nor can there be an interaction between them.

*dead state*

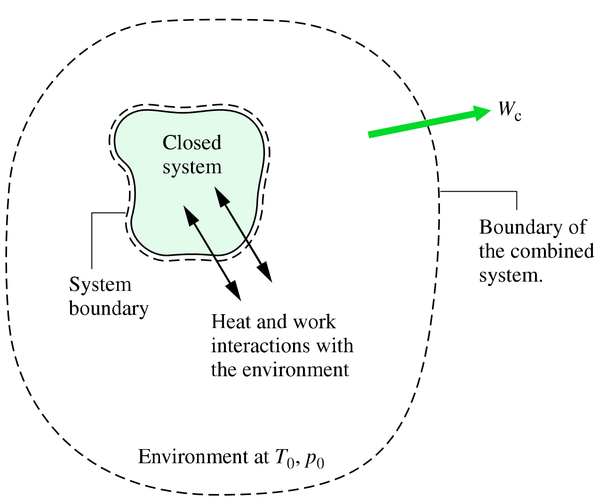
With the introduction of the concepts of environment and dead state, we are in a position to show how a numerical value can be determined for exergy. This is considered next.

### ▶ 7.2.3 Evaluating Exergy

The ***exergy of a system,*** E, at a specified state is given by the expression

$$\mathsf{E} = (E - U_0) + p_0(V - V_0) - T_0(S - S_0) \tag{7.2}$$

◀ *exergy of a system*




◀ **Figure 7.3** Combined system of closed system and environment.

**METHODOLOGY UPDATE**

In this book, E and e are used for exergy and specific exergy, respectively, while $E$ and $e$ denote energy and specific energy, respectively. Such notation is in keeping with standard practice. The appropriate concept, exergy or energy, will be clear in context. Still, care is required to avoid mistaking the symbols for these concepts.

where $E(= U + \text{KE} + \text{PE})$, $V$, and $S$ denote, respectively, the energy, volume, and entropy of the system, and $U_0$, $V_0$, and $S_0$ are the values of the same properties if the system were at the dead state. By inspection of Eq. 7.2, the units of exergy are seen to be the same as those of energy. Equation 7.2 can be derived by applying energy and entropy balances to the *combined system* shown in Fig. 7.3, which consists of a closed system and an environment. (See box).

## EVALUATING THE EXERGY OF A SYSTEM

Referring to Fig. 7.3, exergy is the maximum theoretical work that could be done by the combined system if the closed system were to come into equilibrium with the environment—that is, if the closed system passed to the dead state. Since the objective is to evaluate the maximum work that could be developed by the combined system, the boundary of the combined system is located so that the only energy transfers across it are work transfers of energy. This ensures that the work developed by the combined system is not affected by heat transfer to or from it. Moreover, although the volumes of the closed system and the environment can vary, the boundary of the combined system is located so that the total volume of the combined system remains constant. This ensures that the work developed by the combined system is fully available for lifting a weight in its surroundings, say, and is not expended in merely displacing the surroundings of the combined system. Let us now apply an energy balance to evaluate the work developed by the combined system.

**ENERGY BALANCE.** An energy balance for the combined system reduces to

$$\Delta E_c = \overset{0}{\cancel{Q_c}} - W_c \tag{7.3}$$

where $W_c$ is the work developed by the combined system, and $\Delta E_c$ is the energy change of the combined system, equal to the sum of the energy changes of the closed system and the environment. The energy of the closed system initially is denoted by $E$, which includes the kinetic energy, potential energy, and internal energy of the system. Since the kinetic energy and potential energy are evaluated relative to the environment, the energy of the closed system when at the dead state would be just its internal energy, $U_0$. Accordingly, $\Delta E_c$ can be expressed as

$$\Delta E_c = (U_0 - E) + \Delta U_e$$

Using Eq. 7.1 to replace $\Delta U_e$, the expression becomes

$$\Delta E_c = (U_0 - E) + (T_0 \, \Delta S_e - p_0 \, \Delta V_e) \tag{7.4}$$

Substituting Eq. 7.4 into Eq. 7.3 and solving for $W_c$ gives

$$W_c = (E - U_0) - (T_0 \, \Delta S_e - p_0 \, \Delta V_e)$$

As noted previously, the total volume of the combined system is constant. Hence, the change in volume of the environment is equal in magnitude but opposite in sign to the volume change of the closed system: $\Delta V_e = -(V_0 - V)$. With this substitution, the above expression for work becomes

$$W_c = (E - U_0) + p_0(V - V_0) - T_0 \, \Delta S_e \tag{7.5}$$

This equation gives the work developed by the combined system as the closed system passes to the dead state while interacting only with the environment. The maximum theoretical value for the work is determined using the entropy balance as follows.

**ENTROPY BALANCE.** The entropy balance for the combined system reduces to give

$$\Delta S_c = \sigma_c$$

where the entropy transfer term is omitted because no heat transfer takes place across the boundary of the combined system, and $\sigma_c$ accounts for entropy production due to irreversibilities as the closed system comes into equilibrium with the environment. $\Delta S_c$ is the entropy change of the combined system, equal to the sum of the entropy changes for the closed system and environment, respectively,

$$\Delta S_c = (S_0 - S) + \Delta S_e$$

where $S$ and $S_0$ denote the entropy of the closed system at the given state and the dead state, respectively. Combining the last two equations

$$(S_0 - S) + \Delta S_e = \sigma_c \tag{7.6}$$

Eliminating $\Delta S_e$ between Eqs. 7.5 and 7.6 results in

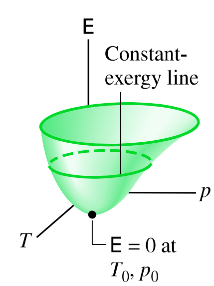
$$W_c = \underline{(E - U_0) + p_0(V - V_0) - T_0(S - S_0)} - T_0\sigma_c \tag{7.7}$$

The value of the underlined term in Eq. 7.7 is determined by the two end states of the closed system—the given state and the dead state—and is independent of the details of the process linking these states. However, the value of the term $T_0\sigma_c$ depends on the nature of the process as the closed system passes to the dead state. In accordance with the second law, $T_0\sigma_c$ is positive when irreversibilities are present and vanishes in the limiting case where there are no irreversibilities. The value of $T_0\sigma_c$ cannot be negative. Hence, the *maximum* theoretical value for the work of the combined system is obtained by setting $T_0\sigma_c$ to zero in Eq. 7.7. By definition, the extensive property exergy, E, is this maximum value. Accordingly, Eq. 7.2 is seen to be the appropriate expression for evaluating exergy.

## ▶ 7.2.4 Exergy Aspects

In this section, we consider several important aspects of the exergy concept, beginning with the following:

▶ Exergy is a measure of the departure of the state of a system from that of the environment. It is therefore an attribute of the system and environment together. However, once the environment is specified, a value can be assigned to exergy in terms of property values for the system only, so exergy can be regarded as a property of the system.




▶ The value of exergy cannot be negative. If a system were at any state other than the dead state, the system would be able to change its condition *spontaneously* toward the dead state; this tendency would cease when the dead state was reached. No work must be done to effect such a spontaneous change. Accordingly, any change in state of the system to the dead state can be accomplished with *at least zero* work being developed, and thus the *maximum* work (exergy) cannot be negative.

▶ Exergy is not conserved but is destroyed by irreversibilities. A limiting case is when exergy is completely destroyed, as would occur if a system were permitted to undergo a spontaneous change to the dead state with no provision to obtain work. The potential to develop work that existed originally would be completely wasted in such a spontaneous process.

▶ Exergy has been viewed thus far as the *maximum* theoretical work obtainable from the combined system of system plus environment as a system passes *from* a given state *to* the dead state while interacting with the environment only. Alternatively, exergy can be regarded as the magnitude of the *minimum* theoretical work *input* required to bring the system *from* the dead state *to* the given state. Using energy and entropy balances as above, we can readily develop Eq. 7.2 from this viewpoint. This is left as an exercise.

Although exergy is an extensive property, it is often convenient to work with it on a unit mass or molar basis. The specific exergy on a unit mass basis, $\mathsf{e}$, is given by

$$\mathsf{e} = (e - u_0) + p_0(v - v_0) - T_0(s - s_0) \tag{7.8}$$

where $e$, $v$, and $s$ are the specific energy, volume, and entropy, respectively, at a given state; $u_0$, $v_0$, and $s_0$ are the same specific properties evaluated at the dead state. With $e = u + V^2/2 + gz$,

$$\mathsf{e} = [(u + \mathrm{V}^2/2 + gz) - u_0] + p_0(v - v_0) - T_0(s - s_0)$$

and the expression for the ***specific exergy*** becomes

*specific exergy*

$$\mathsf{e} = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + \mathrm{V}^2/2 + gz \tag{7.9}$$

By inspection, the units of specific exergy are the same as those of specific energy. Also note that the kinetic and potential energies measured relative to the environment contribute their full values to the exergy magnitude, for in principle each could be completely converted to work were the system brought to rest at zero elevation relative to the environment.

Using Eq. 7.2, we can determine the ***change in exergy*** between two states of a closed system as the difference

*exergy change*

$$\mathsf{E}_2 - \mathsf{E}_1 = (E_2 - E_1) + p_0(V_2 - V_1) - T_0(S_2 - S_1) \tag{7.10}$$

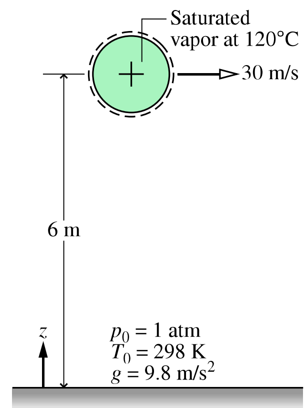
where the values of $p_0$ and $T_0$ are determined by the state of the environment.

When a system is at the dead state, it is in *thermal* and *mechanical* equilibrium with the environment, and the value of exergy is zero. We might say more precisely that the *thermomechanical* contribution to exergy is zero. This modifying term distinguishes the exergy concept of the present chapter from a more general concept introduced in Sec. 13.6, where the contents of a system at the dead state are permitted to enter into chemical reaction with environmental components and in so doing develop additional work. As illustrated by subsequent discussions, the thermomechanical exergy concept suffices for a wide range of thermodynamic evaluations.

## ▶ 7.2.5 Illustrations

We conclude this introduction to the exergy concept with examples showing how to calculate exergy and exergy change. To begin, observe that the exergy of a system at a specified state requires properties at that state and at the dead state. ▶ ***for example...*** let us use Eq. 7.9 to determine the specific exergy of saturated water vapor at 120°C, having a velocity of 30 m/s and an elevation of 6 m, each relative to an exergy reference environment where $T_0 = 298$ K (25°C), $p_0 = 1$ atm, and $g = 9.8$ m/s$^2$. For water as saturated vapor at 120°C, Table A-2 gives $v = 0.8919$ m$^3$/kg, $u = 2529.3$ kJ/kg, $s = 7.1296$ kJ/kg · K. At the dead state, where $T_0 = 298$ K (25°C) and $p_0 = 1$ atm, water is a liquid. Thus, with Eqs. 3.11, 3.12, and

6.7 and values from Table A-2, we get $v_0 = 1.0029 \times 10^{-3}$ m³/kg, $u_0 = 104.88$ kJ/kg, $s_0 = 0.3674$ kJ/kg · K. Substituting values




$$
\begin{aligned}
\mathsf{e} &= (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + \frac{\mathrm{V}^2}{2} + gz \\
&= \left[(2529.3 - 104.88)\frac{\mathrm{kJ}}{\mathrm{kg}}\right] \\
&\quad + \left[\left(1.01325 \times 10^5 \frac{\mathrm{N}}{\mathrm{m}^2}\right)(0.8919 - 1.0029 \times 10^{-3})\frac{\mathrm{m}^3}{\mathrm{kg}}\right]\left|\frac{1\ \mathrm{kJ}}{10^3\ \mathrm{N \cdot m}}\right| \\
&\quad - \left[(298\ \mathrm{K})(7.1296 - 0.3674)\frac{\mathrm{kJ}}{\mathrm{kg \cdot K}}\right] \\
&\quad + \left[\frac{(30\ \mathrm{m/s})^2}{2} + \left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)(6\ \mathrm{m})\right]\left|\frac{1\ \mathrm{N}}{1\ \mathrm{kg \cdot m/s^2}}\right|\left|\frac{1\ \mathrm{kJ}}{10^3\ \mathrm{N \cdot m}}\right| \\
&= (2424.42 + 90.27 - 2015.14 + 0.45 + 0.06)\frac{\mathrm{kJ}}{\mathrm{kg}} = 500\frac{\mathrm{kJ}}{\mathrm{kg}} \blacktriangleleft
\end{aligned}
$$

The following example illustrates the use of Eq. 7.9 together with ideal gas property data.

## EXAMPLE 7.1 Exergy of Exhaust Gas

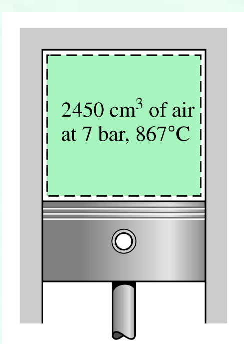
A cylinder of an internal combustion engine contains 2450 cm³ of gaseous combustion products at a pressure of 7 bar and a temperature of 867°C just before the exhaust valve opens. Determine the specific exergy of the gas, in kJ/kg. Ignore the effects of motion and gravity, and model the combustion products as air as an ideal gas. Take $T_0 = 300$ K (27°C) and $p_0 = 1.013$ bar.

### SOLUTION

***Known:*** Gaseous combustion products at a specified state are contained in the cylinder of an internal combustion engine.

***Find:*** Determine the specific exergy.

***Schematic and Given Data:***




***Assumptions:***

1. The gaseous combustion products are a closed system.
2. The combustion products are modeled as air as an ideal gas.
3. The effects of motion and gravity can be ignored.
4. $T_0 = 300$ K (27°C) and $p_0 = 1.013$ bar.

◄ **Figure E7.1**

***Analysis:*** With assumption 3, Eq. 7.9 becomes

$$\mathsf{e} = u - u_0 + p_0(v - v_0) - T_0(s - s_0)$$

The internal energy and entropy terms are evaluated using data from Table A-22, as follows:

$$u - u_0 = 880.35 - 214.07$$
$$= 666.28 \text{ kJ/kg}$$
$$s - s_0 = s^\circ(T) - s^\circ(T_0) - \frac{\overline{R}}{M} \ln \frac{p}{p_0}$$
$$= 3.11883 - 1.70203 - \left(\frac{8.314}{28.97}\right) \ln \left(\frac{7}{1.013}\right)$$
$$= 0.8621 \text{ kJ/kg} \cdot \text{K}$$
$$T_0(s - s_0) = (300 \text{ K})(0.8621 \text{ kJ/kg} \cdot \text{K})$$
$$= 258.62 \text{ kJ/kg}$$

The $p_0(v - v_0)$ term is evaluated using the ideal gas equation of state: $v = (\overline{R}/M)T/p$ and $v_0 = (\overline{R}/M)T_0/p_0$, so

$$p_0(v - v_0) = \frac{\overline{R}}{M}\left(\frac{p_0 T}{p} - T_0\right)$$
$$= \frac{8.314}{28.97}\left[\frac{(1.013)(1140)}{7} - 300\right]$$
$$= -38.75 \text{ kJ/kg}$$

Substituting values into the above expression for the specific exergy

$$\mathsf{e} = 666.28 + (-38.75) - 258.62$$

1 $$= 368.91 \text{ kJ/kg}$$

1 If the gases are discharged directly to the surroundings, the potential for developing work quantified by the exergy value determined in the solution is wasted. However, by venting the gases through a turbine some work could be developed. This principle is utilized by the *turbochargers* added to some internal combustion engines.

The next example emphasizes the fundamentally different characters of exergy and energy, while illustrating the use of Eqs. 7.9 and 7.10.

## EXAMPLE 7.2 Comparing Exergy and Energy

Refrigerant 134a, initially a saturated vapor at −28°C, is contained in a rigid, insulated vessel. The vessel is fitted with a paddle wheel connected to a pulley from which a mass is suspended. As the mass descends a certain distance, the refrigerant is stirred until it attains a state where the pressure is 1.4 bar. The only significant changes of state are experienced by the suspended mass and the refrigerant. The mass of refrigerant is 1.11 kg. Determine

(a) the initial exergy, final exergy, and change in exergy of the refrigerant, each in kJ.

(b) the change in exergy of the suspended mass, in kJ.

(c) the change in exergy of an isolated system of the vessel and pulley–mass assembly, in kJ.

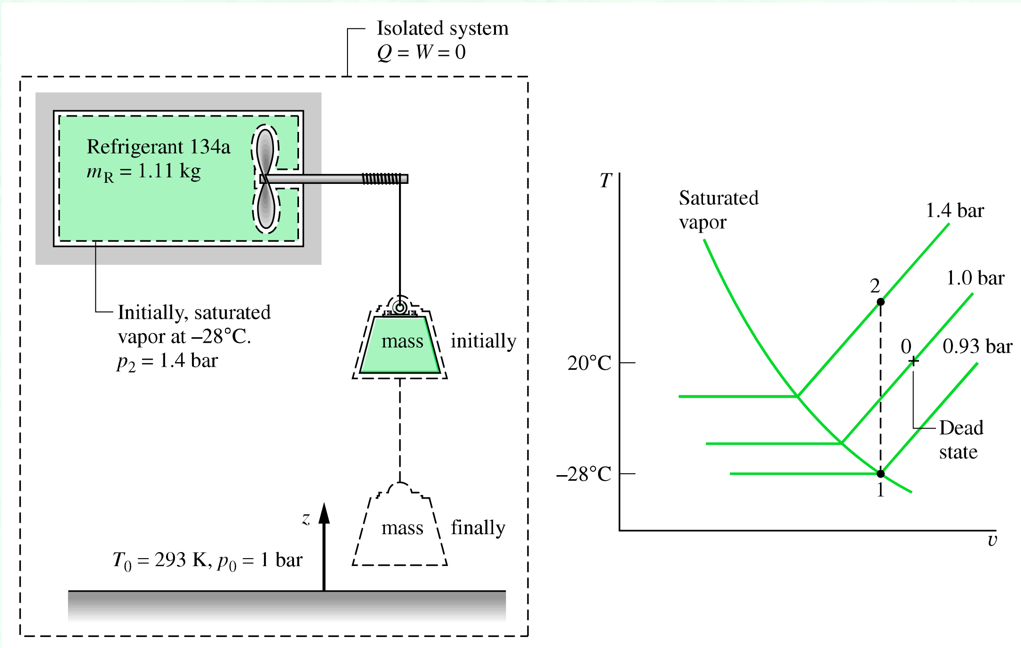
Discuss the results obtained, and compare with the respective energy changes. Let $T_0 = 293$ K (20°C), $p_0 = 1$ bar.

### SOLUTION

***Known:*** Refrigerant 134a in a rigid, insulated vessel is stirred by a paddle wheel connected to a pulley–mass assembly.

***Find:*** Determine the initial and final exergies and the change in exergy of the refrigerant, the change in exergy of the suspended mass, and the change in exergy of the isolated system, all in kJ. Discuss the results obtained.

***Schematic and Given Data:***




◀ **Figure E7.2**

***Assumptions:***

1. As shown in the schematic, three systems are under consideration: the refrigerant, the suspended mass, and an isolated system consisting of the vessel and pulley–mass assembly. For the isolated system $Q = 0$, $W = 0$.
2. The only significant changes of state are experienced by the refrigerant and the suspended mass. For the refrigerant, there is no change in kinetic or potential energy. For the suspended mass, there is no change in kinetic or internal energy. Elevation is the only intensive property of the suspended mass that changes.
3. For the environment, $T_0 = 293$ K (20°C), $p_0 = 1$ bar.

***Analysis:***

(a) The initial and final exergies of the refrigerant can be evaluated using Eq. 7.9. From assumption 2, it follows that for the refrigerant there are no significant effects of motion or gravity, and thus the exergy at the initial state is

$$\mathsf{E}_1 = m_R[(u_1 - u_0) + p_0(v_1 - v_0) - T_0(s_1 - s_0)]$$

The initial and final states of the refrigerant are shown on the accompanying T–$v$ diagram. From Table A-10, $u_1 = u_g(-28°C) = 211.29$ kJ/kg. $v_1 = v_g(-28°C) = 0.2052\ m^3/kg$, $s_1 = s_g(-28°C) = 0.9411$ kJ/kg · K. From Table A-12 at 1 bar, 20°C, $u_0 = 246.67$ kJ/kg, $v_0 = 0.23349\ m^3/kg$, $s_0 = 1.0829$ kJ/kg · K. Then

$$\mathsf{E}_1 = 1.11\ \text{kg}\left[(211.29 - 246.67)\frac{\text{kJ}}{\text{kg}} + \left(10^5\frac{\text{N}}{\text{m}^2}\right)(0.2052 - 0.23349)\frac{\text{m}^3}{\text{kg}}\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| - 293\ \text{K}(0.9411 - 1.0829)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]$$

$$= 1.11\ \text{kg}[(-35.38) + (-2.83) + (41.55)]\frac{\text{kJ}}{\text{kg}} = 3.7\ \text{kJ}$$

The final state of the refrigerant is fixed by $p_2 = 1.4$ bar and $v_2 = v_1$. Interpolation in Table A-12 gives $u_2 = 300.16$ kJ/kg, $s_2 = 1.2369$ kJ/kg · K. Then

❶ $$\mathsf{E}_2 = 1.11\ \text{kg}[(53.49) + (-2.83) + (-45.12)]\frac{\text{kJ}}{\text{kg}} = 6.1\ \text{kJ}$$

For the refrigerant, the change in exergy is

❷ $$(\Delta\mathsf{E})_{\text{refrigerant}} = \mathsf{E}_2 - \mathsf{E}_1 = 6.1\ \text{kJ} - 3.7\ \text{kJ} = 2.4\ \text{kJ}$$

The exergy of the refrigerant increases as it is stirred.

(b) With assumption 2, Eq. 7.10 reduces to give the exergy change for the suspended mass

$$(\Delta \mathsf{E})_{\text{mass}} = (\cancel{\Delta U}^{0} + p_0 \cancel{\Delta V}^{0} - T_0 \cancel{\Delta S}^{0} + \cancel{\Delta \text{KE}}^{0} + \Delta \text{PE})_{\text{mass}}$$
$$= (\Delta \text{PE})_{\text{mass}}$$

Thus, the exergy change for the suspended mass equals its change in potential energy.

3 The change in potential energy of the suspended mass is obtained from an energy balance for the isolated system as follows: The change in energy of the isolated system is the sum of the energy changes of the refrigerant and suspended mass. There is no heat transfer or work, and with assumption 2 we have

$$(\cancel{\Delta \text{KE}}^{0} + \cancel{\Delta \text{PE}}^{0} + \Delta U)_{\text{refrigerant}} + (\cancel{\Delta \text{KE}}^{0} + \Delta \text{PE} + \cancel{\Delta U}^{0})_{\text{mass}} = \cancel{Q}^{0} - \cancel{W}^{0}$$

Solving for $(\Delta \text{PE})_{\text{mass}}$ and using previously determined values for the specific internal energy of the refrigerant

$$(\Delta \text{PE})_{\text{mass}} = -(\Delta U)_{\text{refrigerant}}$$
$$= -(1.11 \text{ kg})(300.16 - 211.29)\left(\frac{\text{kJ}}{\text{kg}}\right)$$
$$= -98.6 \text{ kJ}$$

Collecting results, $(\Delta \mathsf{E})_{\text{mass}} = -98.6$ kJ. The exergy of the mass decreases because its elevation decreases.

(c) The change in exergy of the isolated system is the sum of the exergy changes of the refrigerant and suspended mass. With the results of parts (a) and (b)

$$(\Delta \mathsf{E})_{\text{isol}} = (\Delta \mathsf{E})_{\text{refrigerant}} + (\Delta \mathsf{E})_{\text{mass}}$$
$$= (2.4 \text{ kJ}) + (-98.6 \text{ kJ})$$
$$= -96.2 \text{ kJ}$$

The exergy of the isolated system decreases.

To summarize

| | Energy Change | Exergy Change |
|---|---|---|
| Refrigerant | +98.6 kJ | + 2.4 kJ |
| Suspended mass | −98.6 kJ | −98.6 kJ |
| Isolated system | 0.0 kJ | −96.2 kJ |

4 For the isolated system there is no net change in energy. The increase in the internal energy of the refrigerant equals the decrease in potential energy of the suspended mass. However, the *increase* in exergy of the refrigerant is much less than the *decrease* in exergy of the mass. For the isolated system, exergy decreases because stirring destroys exergy.

1 Exergy is a measure of the departure of the state of the system from that of the environment. At all states, $\mathsf{E} \geq 0$. This applies when $T > T_0$ and $p > p_0$, as at state 2, and when $T < T_0$ and $p < p_0$, as at state 1.

2 The exergy change of the refrigerant can be determined more simply with Eq. 7.10, which requires dead state property values only for $T_0$ and $p_0$. With the approach used in part (a), values for $u_0$, $v_0$, and $s_0$ are also required.

3 The change in potential energy of the suspended mass, $(\Delta \text{PE})_{\text{mass}}$, cannot be determined from Eq. 2.10 (Sec. 2.1) since the mass and change in elevation are unknown. Moreover, for the suspended mass as the system, $(\Delta \text{PE})_{\text{mass}}$ cannot be obtained from an energy balance without first evaluating the work. Thus, we resort here to an energy balance for the isolated system, which does not require such information.

4 As the suspended mass descends, energy is transferred by work through the paddle wheel to the refrigerant, and the refrigerant state changes. Since the exergy of the refrigerant increases, we infer that an exergy *transfer* accompanies the work interaction. The concepts of exergy change, exergy transfer, and exergy destruction are related by the closed system exergy balance introduced in the next section.

## 7.3 Closed System Exergy Balance

A system at a given state can attain a new state through work and heat interactions with its surroundings. Since the exergy value associated with the new state would generally differ from the value at the initial state, transfers of exergy across the system boundary can be inferred to *accompany* heat and work interactions. The change in exergy of a system during a process would not necessarily equal the net exergy transferred, for exergy would be destroyed if irreversibilities were present within the system during the process. The concepts of exergy change, exergy transfer, and exergy destruction are related by the closed system exergy balance introduced in this section. The exergy balance concept is extended to control volumes in Sec. 7.5. These balances are expressions of the second law of thermodynamics and provide the basis for exergy analysis.

### ▶ 7.3.1 Developing the Exergy Balance

The exergy balance for a closed system is developed by combining the closed system energy and entropy balances. The forms of the energy and entropy balances used in the development are, respectively

$$E_2 - E_1 = \int_1^2 \delta Q - W$$

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

where $W$ and $Q$ represent, respectively, work and heat transfers between the system and its surroundings. These interactions do not necessarily involve the environment. In the entropy balance, $T_b$ denotes the temperature on the system boundary where $\delta Q$ is received and the term $\sigma$ accounts for entropy produced by internal irreversibilities.

As the first step in deriving the exergy balance, multiply the entropy balance by the temperature $T_0$ and subtract the resulting expression from the energy balance to obtain

$$(E_2 - E_1) - T_0(S_2 - S_1) = \int_1^2 \delta Q - T_0 \int_1^2 \left(\frac{\delta Q}{T}\right)_b - W - T_0\sigma$$

Collecting the terms involving $\delta Q$ and introducing Eq. 7.10 on the left side, we can rewrite this expression as

$$(\mathsf{E}_2 - \mathsf{E}_1) - p_0(V_2 - V_1) = \int_1^2 \left(1 - \frac{T_0}{T_b}\right)\delta Q - W - T_0\sigma$$

Rearranging, the ***closed system exergy balance*** results

$$\underbrace{\mathsf{E}_2 - \mathsf{E}_1}_{\text{exergy change}} = \underbrace{\int_1^2 \left(1 - \frac{T_0}{T_b}\right)\delta Q - [W - p_0(V_2 - V_1)]}_{\text{exergy transfers}} - \underbrace{T_0\sigma}_{\text{exergy destruction}} \tag{7.11}$$

◀ *closed system exergy balance*

Since Eq. 7.11 is obtained by deduction from the energy and entropy balances, it is not an independent result, but it can be used in place of the entropy balance as an expression of the second law.

## INTERPRETING THE EXERGY BALANCE

For specified end states and given values of $p_0$ and $T_0$, the exergy change $\mathsf{E}_2 - \mathsf{E}_1$ on the left side of Eq. 7.11 can be evaluated from Eq. 7.10. The underlined terms on the right depend explicitly on the nature of the process, however, and cannot be determined by knowing only the end states and the values of $p_0$ and $T_0$. The first underlined term on the right side of Eq. 7.11 is associated with heat transfer to or from the system during the process. It can be interpreted as the ***exergy transfer accompanying heat.*** That is

***exergy transfer accompanying heat***

$$\begin{bmatrix} \textit{exergy} \text{ transfer} \\ \text{accompanying heat} \end{bmatrix} = \int_1^2 \left(1 - \frac{T_0}{T_b}\right) \delta Q \tag{7.12}$$

The second underlined term on the right side of Eq. 7.11 is associated with work. It can be interpreted as the ***exergy transfer accompanying work.*** That is

***exergy transfer accompanying work***

$$\begin{bmatrix} \textit{exergy} \text{ transfer} \\ \text{accompanying work} \end{bmatrix} = [W - p_0(V_2 - V_1)] \tag{7.13}$$

The exergy transfer expressions are discussed further in Sec. 7.3.2. The third underlined term on the right side of Eq. 7.11 accounts for the ***destruction of exergy*** due to irreversibilities within the system. It is symbolized by $\mathsf{E}_d$.

***exergy destruction***

$$\mathsf{E}_d = T_0 \sigma \tag{7.14}$$

To summarize, Eq. 7.11 states that the change in exergy of a closed system can be accounted for in terms of exergy transfers and the destruction of exergy due to irreversibilities within the system.

When applying the exergy balance, it is essential to observe the requirements imposed by the second law on the exergy destruction: In accordance with the second law, the exergy destruction is positive when irreversibilities are present within the system during the process and vanishes in the limiting case where there are no irreversibilities. That is

$$\mathsf{E}_d : \begin{cases} > 0 & \text{irreversibilities present with the system} \\ = 0 & \text{no irreversibilities present within the system} \end{cases} \tag{7.15}$$

The value of the exergy destruction cannot be negative. It is *not* a property. By contrast, exergy is a property, and like other properties, the *change* in exergy of a system can be positive, negative, or zero

$$\mathsf{E}_2 - \mathsf{E}_1 : \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \tag{7.16}$$

To close our introduction to the exergy balance concept, we note that most thermal systems are supplied with exergy inputs derived directly or indirectly from the consumption of fossil fuels. Accordingly, avoidable destructions and losses of exergy represent the waste of these resources. By devising ways to reduce such inefficiencies, better use can be made of fuels. The exergy balance can be used to determine the locations, types, and magnitudes of energy resource waste, and thus can play an important part in developing strategies for more effective fuel use.

## OTHER FORMS OF THE EXERGY BALANCE

As in the case of the mass, energy, and entropy balances, the exergy balance can be expressed in various forms that may be more suitable for particular analyses. A form of the exergy balance that is sometimes convenient is the ***closed system exergy rate balance.***

$$\frac{d\mathsf{E}}{dt} = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \left(\dot{W} - p_0\frac{dV}{dt}\right) - \dot{\mathsf{E}}_d \tag{7.17}$$

***closed system exergy rate balance***

where $d\mathsf{E}/dt$ is the time rate of change of exergy. The term $(1 - T_0/T_j)\dot{Q}_j$ represents the time rate of exergy transfer accompanying heat transfer at the rate $\dot{Q}_j$ occurring at the location on the boundary where the instantaneous temperature is $T_j$. The term $\dot{W}$ represents the time rate of energy transfer by work. The accompanying rate of exergy transfer is given by $(\dot{W} - p_0\, dV/dt)$, where $dV/dt$ is the time rate of change of system volume. The term $\dot{\mathsf{E}}_d$ accounts for the time rate of exergy destruction due to irreversibilities within the system and is related to the rate of entropy production within the system by the expression $\dot{\mathsf{E}}_d = T_0\dot{\sigma}$.

For an *isolated* system, no heat or work interactions with the surroundings occur, and thus there are no transfers of exergy between the system and its surroundings. Accordingly, the exergy balance reduces to give

$$\Delta\mathsf{E}\big]_{isol} = -\mathsf{E}_d\big]_{isol} \tag{7.18}$$

Since the exergy destruction must be positive in any actual process, the only processes of an isolated system that occur are those for which the exergy of the isolated system *decreases*. For exergy, this conclusion is the counterpart of the increase of entropy principle (Sec. 6.5.5) and, like the increase of entropy principle, can be regarded as an alternative statement of the second law.

### ► 7.3.2 Conceptualizing Exergy Transfer

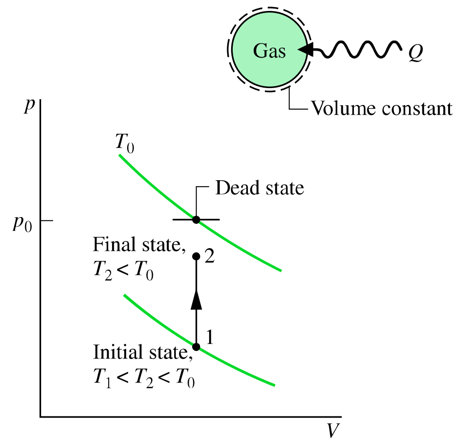
Before taking up examples illustrating the use of the closed system exergy balance, we consider why the exergy transfer expressions take the forms they do. This is accomplished through simple thought experiments. ► ***for example...*** consider a large metal part initially at the dead state. If the part were hoisted from a factory floor into a heat-treating furnace, the exergy of the metal part would increase because its elevation would be increased. As the metal part was heated in the furnace, the exergy of the part would increase further as its temperature increased because of heat transfer from the hot furnace gases. In a subsequent quenching process, the metal part would experience a decrease in exergy as its temperature decreased due to heat transfer to the quenching medium. In each of these processes, the metal part would not actually interact with the environment used to assign exergy values. However, like the exergy values at the states visited by the metal part, the exergy transfers taking place between the part and its surroundings would be evaluated *relative to the environment* used to define exergy. ◄

The following subsections provide means for conceptualizing the exergy transfers that accompany heat transfer and work, respectively.

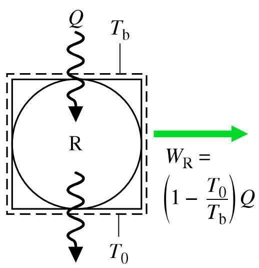
## EXERGY TRANSFER ACCOMPANYING HEAT

Consider a system undergoing a process in which a heat transfer $Q$ takes place across a portion of the system boundary where the temperature $T_b$ is constant at $T_b > T_0$. In accordance with Eq. 7.12, the accompanying exergy transfer is given by

$$\begin{bmatrix}\text{exergy transfer}\\ \text{accompanying heat}\end{bmatrix} = \left(1 - \frac{T_0}{T_b}\right)Q \tag{7.19}$$




◄ **Figure 7.4** Illustration used to discuss an exergy transfer accompanying heat transfer when $T < T_0$.



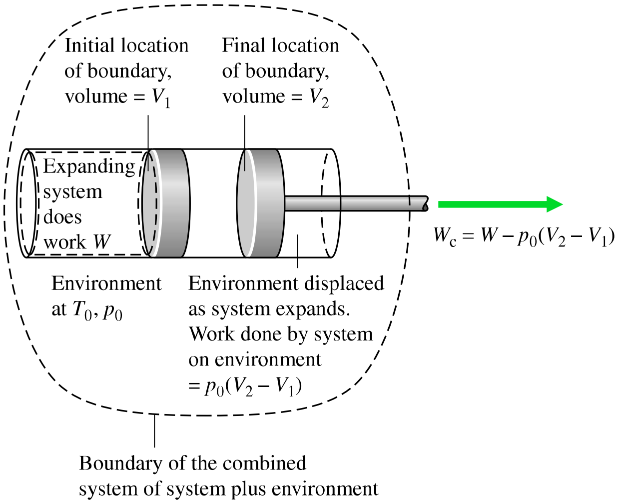

The right side of Eq. 7.19 is recognized from the discussion of Eq. 5.8 as the work, $W_R$, that *could* be developed by a reversible power cycle R receiving $Q$ at temperature $T_b$ and discharging energy by heat transfer to the environment at $T_0$. Accordingly, without regard for the nature of the surroundings with which the system is *actually* interacting, we may interpret the *magnitude* of an exergy transfer accompanying heat transfer as the work that *could* be developed by supplying the heat transfer to a reversible power cycle operating between $T_b$ and $T_0$. This interpretation also applies for heat transfer below $T_0$, but then we think of the *magnitude* of an exergy transfer accompanying heat as the work that *could* be developed by a reversible power cycle receiving a heat transfer from the environment at $T_0$ and discharging $Q$ at temperature $T_b < T_0$.

Thus far, we have considered only the *magnitude* of an exergy transfer accompanying heat. It is necessary to account also for the *direction*. The form of Eq. 7.19 shows that when $T_b$ is greater than $T_0$, the heat transfer and accompanying exergy transfer would be in the *same direction:* Both quantities would be positive, or negative.

However, when $T_b$ is less than $T_0$, the sign of the exergy transfer would be opposite to the sign of the heat transfer, so the heat transfer and accompanying exergy transfer would be *oppositely directed.* ► ***for example...*** refer to Fig. 7.4, which shows a system consisting of a gas heated at constant volume. As indicated by the $p$–$V$ diagram, the initial and final temperatures of the gas are each less than $T_0$. Since the state of the system is brought closer to the dead state in this process, the exergy of the system must decrease as it is heated. Conversely, were the gas cooled from state 2 to state 1, the exergy of the system would increase because the state of the system would be moved farther from the dead state. ◄ In summary, when the temperature at the location where heat transfer occurs is *less* than the temperature of the environment, the heat transfer and accompanying exergy transfer are *oppositely directed*. This becomes significant when studying the performance of refrigerators and heat pumps, where heat transfers can occur at temperatures below that of the environment.

## EXERGY TRANSFER ACCOMPANYING WORK

We conclude the present discussion by taking up a simple example that motivates the form taken by the expression accounting for an exergy transfer accompanying work, Eq. 7.13. ► ***for example...*** consider a closed system that does work $W$ while undergoing a process in which the system volume increases: $V_2 > V_1$. Although the system would not necessarily interact with the environment, the *magnitude* of the exergy transfer is evaluated




◄ **Figure 7.5** Illustration used to discuss the expression for an exergy transfer accompanying work.

as the *maximum* work that *could* be obtained *were* the system and environment interacting. As illustrated by Fig. 7.5, all the work $W$ of the system in the process would not be available for delivery from a combined system of system plus environment because a portion would be spent in pushing aside the environment, whose pressure is $p_0$. Since the system would do work on the surroundings equal to $p_0(V_2 - V_1)$, the maximum amount of work that could be derived from the combined system would thus be

$$W_c = W - p_0(V_2 - V_1)$$

which is in accordance with the form of Eq. 7.13. ◄

As for heat transfer, work and the accompanying exergy transfer can be in the same direction or oppositely directed. If there were no change in the system volume during the process, the transfer of exergy accompanying work would equal the work $W$ of the system.

### ► 7.3.3 Illustrations

Further consideration of the exergy balance and the exergy transfer and destruction concepts is provided by the two examples that follow. In the first example, we reconsider Examples 6.1 and 6.2 to illustrate that exergy is a property, whereas exergy destruction and exergy transfer accompanying heat and work are not properties.

### EXAMPLE 7.3 Exploring Exergy Change, Transfer, and Destruction

Water initially a saturated liquid at 100°C is contained in a piston–cylinder assembly. The water undergoes a process to the corresponding saturated vapor state, during which the piston moves freely in the cylinder. For each of the two processes described below, determine on a unit of mass basis the change in exergy, the exergy transfer accompanying work, the exergy transfer accompanying heat, and the exergy destruction, each in kJ/kg. Let $T_0$ = 20°C, $p_0$ = 1.014 bar.

(a) The change in state is brought about by heating the water as it undergoes an internally reversible process at constant temperature and pressure.

(b) The change in state is brought about adiabatically by the stirring action of a paddle wheel.

## SOLUTION

***Known:*** Saturated liquid at 100°C undergoes a process to the corresponding saturated vapor state.

***Find:*** Determine the change in exergy, the exergy transfers accompanying work and heat, and the exergy destruction for each of two specified processes.

***Schematic and Given Data:*** See Figs. E6.1 and E6.2.

***Assumptions:***

1. For part (a), see the assumptions listed for Example 6.1. For part (b), see the assumptions listed for Example 6.2.
2. $T_0 = 20°\text{C}$, $p_0 = 1.014$ bar.

***Analysis:***

(a) The change in specific exergy is obtained using Eq. 7.9

$$\Delta \text{e} = u_g - u_f + p_0(v_g - v_f) - T_0(s_g - s_f)$$

Using data from Table A-2

$$\begin{aligned}\Delta \text{e} &= 2087.56\frac{\text{kJ}}{\text{kg}} + \left(1.014 \times 10^5 \frac{\text{N}}{\text{m}^2}\right)\left(1.672\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| - (293.15\text{ K})\left(6.048\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\\ &= 484\text{ kJ/kg}\end{aligned}$$

Using the expression for work obtained in the solution to Example 6.1, $W/m = pv_{fg}$, the transfer of exergy accompanying work is

$$\begin{aligned}\begin{bmatrix}\text{exergy transfer}\\ \text{accompanying work}\end{bmatrix} &= \frac{W}{m} - p_0(v_g - v_f)\\ &= (p - p_0)v_{fg} = 0\end{aligned}$$

Although the work has a nonzero value, there is no accompanying exergy transfer in this case because $p = p_0$.

Using the heat transfer value calculated in Example 6.1, the transfer of exergy of accompanying heat transfer in the constant-temperature process is

$$\begin{aligned}\begin{bmatrix}\text{exergy transfer}\\ \text{accompanying heat}\end{bmatrix} &= \left(1 - \frac{T_0}{T}\right)\frac{Q}{m}\\ &= \left(1 - \frac{293.15\text{ K}}{373.15\text{ K}}\right)\left(2257\frac{\text{kJ}}{\text{kg}}\right)\\ &= 484\text{ kJ/kg}\end{aligned}$$

The positive value indicates that exergy transfer occurs in the same direction as the heat transfer.

Since the process is accomplished without irreversibilities, the exergy destruction is necessarily zero in value. This can be verified by inserting the three exergy quantities evaluated above into an exergy balance and evaluating $\text{E}_d/m$.

1 (b) Since the end states are the same as in part (a), the change in exergy is the same. Moreover, because there is no heat transfer, there is no exergy transfer accompanying heat. The exergy transfer accompanying work is

$$\begin{bmatrix}\text{exergy transfer}\\ \text{accompanying work}\end{bmatrix} = \frac{W}{m} - p_0(v_g - v_f)$$

With the net work value determined in Example 6.2 and evaluating the change in specific volume as in part (a)

$$\begin{aligned}\begin{bmatrix}\text{exergy transfer}\\ \text{accompanying work}\end{bmatrix} &= -2087.56\frac{\text{kJ}}{\text{kg}} - \left(1.014 \times 10^5 \frac{\text{N}}{\text{m}^2}\right)\left(1.672\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\\ &= -2257\text{ kJ/kg}\end{aligned}$$

The minus sign indicates that the net transfer of exergy accompanying work is into the system.

Finally, the exergy destruction is determined from an exergy balance. Solving Eq. 7.11 for the exergy destruction per unit mass

❷
$$\frac{\mathsf{E}_d}{m} = -\Delta \mathsf{e} - \left[\frac{W}{m} - p_0(v_g - v_f)\right] = -484 - (-2257) = 1773 \text{ kJ/kg}$$

The numerical values obtained can be interpreted as follows: 2257 kJ/kg of exergy is transferred into the system accompanying work; of this, 1773 kJ/kg is destroyed by irreversibilities, leaving a net increase of only 484 kJ/kg.

❶ Exergy is a property and thus the exergy change during a process is determined solely by the end states. Exergy destruction and exergy transfer accompanying heat and work are not properties. Their values depend on the nature of the process.

❷ Alternatively, the exergy destruction value of part (b) could be determined using $\mathsf{E}_d/m = T_0(\sigma/m)$, where $\sigma/m$ is obtained from the solution to Example 6.2. This is left as an exercise.

In the next example, we reconsider the gearbox of Examples 2.4 and 6.4 from an exergy perspective to introduce *exergy accounting*.

## EXAMPLE 7.4 Exergy Accounting for a Gearbox

For the gearbox of Examples 2.4 and 6.4(a), develop a full exergy accounting of the power input. Let $T_0 = 293$ K.

### SOLUTION

***Known:*** A gearbox operates at steady state with known values for the power input, power output, and heat transfer rate. The temperature on the outer surface of the gearbox is also known.

***Find:*** Develop a full exergy accounting of the input power.

***Schematic and Given Data:*** See Fig. E6.4*a*.

***Assumptions:***

1. See the solution to Example 6.4(a).
2. $T_0 = 293$ K.

***Analysis:*** Since the gearbox volume is constant, the rate of exergy transfer accompanying power, namely $(\dot{W} - p_0\, dV/dt)$, reduces to the power itself. Accordingly, exergy is transferred into the gearbox via the high-speed shaft at a rate equal to the power input, 60 kW, and exergy is transferred out via the low-speed shaft at a rate equal to the power output, 58.8 kW. Additionally, exergy is transferred out accompanying heat transfer and destroyed by irreversibilities within the gearbox.

Let us evaluate the rate of exergy transfer accompanying heat transfer. Since the temperature $T_b$ at the outer surface of the gearbox is uniform with position

$$\left[\begin{array}{c}\text{time rate of exergy} \\ \text{transfer accompanying heat}\end{array}\right] = \left(1 - \frac{T_0}{T_b}\right)\dot{Q}$$

With $\dot{Q} = -1.2$ kW and $T_b = 300$ K from Example 6.4a, and $T_0 = 293$ K

$$\left[\begin{array}{c}\text{time rate of exergy} \\ \text{transfer accompanying heat}\end{array}\right] = \left(1 - \frac{293}{300}\right)(-1.2 \text{ kW})$$
$$= -0.03 \text{ kW}$$

where the minus sign denotes exergy transfer *from* the system.

❶ Next, the rate of exergy destruction is calculated from $\dot{E}_d = T_0\dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production. From the solution to Example 6.4(a), $\dot{\sigma} = 4 \times 10^{-3}$ kW/K. Then

$$\begin{aligned}\dot{E}_d &= T_0\dot{\sigma} \\ &= (293 \text{ K})(4 \times 10^{-3} \text{ kW/K}) \\ &= 1.17 \text{ kW}\end{aligned}$$

The analysis is summarized by the following exergy *balance sheet* in terms of exergy magnitudes on a rate basis:

| | |
|---|---|
| *Rate of exergy in:* | |
| high-speed shaft | 60.00 kW (100%) |
| *Disposition of the exergy:* | |
| ❷ • Rate of exergy out | |
| low-speed shaft | 58.80 kW (98%) |
| heat transfer | 0.03 kW (0.05%) |
| • Rate of exergy destruction | 1.17 kW (1.95%) |
| | 60.00 kW (100%) |

❶ Alternatively, the rate of exergy destruction can be determined from the steady-state form of the exergy rate balance, Eq. 7.17. This is left as an exercise.

❷ The difference between the input and output power is accounted for primarily by the rate of exergy destruction and only secondarily by the exergy transfer accompanying heat transfer, which is small by comparison. The exergy balance sheet provides a sharper picture of performance than the energy balance sheet of Example 2.4, which ignores the effect of irreversibilities within the system and overstates the significance of the heat transfer.

## 7.4 Flow Exergy

The objective of the present section is to develop the *flow exergy* concept. This concept is important for the control volume form of the exergy rate balance introduced in Sec. 7.5.

When mass flows across the boundary of a control volume, there is an *exergy transfer* accompanying mass flow. Additionally, there is an *exergy transfer* accompanying *flow work*. The ***specific flow exergy*** accounts for both of these, and is given by

*specific flow exergy*

$$e_f = h - h_0 - T_0(s - s_0) + \frac{V^2}{2} + gz \tag{7.20}$$

In Eq. 7.20, $h$ and $s$ represent the specific enthalpy and entropy, respectively, at the inlet or exit under consideration; $h_0$ and $s_0$ represent the respective values of these properties when evaluated at the dead state.

### EXERGY TRANSFER ACCOMPANYING FLOW WORK

As a preliminary to deriving Eq. 7.20, it is necessary to account for the exergy transfer accompanying flow work.

When one-dimensional flow is assumed, the work at the inlet or exit of a control volume, the *flow work,* is given on a time rate basis by $\dot{m}(pv)$, where $\dot{m}$ is the mass flow rate, $p$ is the pressure, and $v$ is the specific volume at the inlet or exit (Sec. 4.2.1). The following expression accounts for the ***exergy transfer accompanying flow work***

$$\begin{bmatrix}\text{time rate of exergy transfer}\\ \text{accompanying flow work}\end{bmatrix} = \dot{m}(pv - p_0 v) \tag{7.21}$$

***exergy transfer: flow work***

For the development of Eq. 7.21, see box.

## ACCOUNTING FOR EXERGY TRANSFER ACCOMPANYING FLOW WORK

Let us develop Eq. 7.21 for the case pictured in Fig. 7.6. The figure shows a closed system that occupies different regions at time $t$ and a later time $t + \Delta t$. The fixed quantity of matter under consideration is shown in color. During the time interval $\Delta t$, some of the mass initially within the region labeled *control volume* exits to fill the small region $e$ adjacent to the control volume, as shown in Fig. 7.6*b*. We assume that the increase in the volume of the closed system in the time interval $\Delta t$ is equal to the volume of region $e$ and, for further simplicity, that the only work is associated with this volume change. With Eq. 7.13, the exergy transfer accompanying work is

$$\begin{bmatrix}\text{exergy transfer}\\ \text{accompanying work}\end{bmatrix} = W - p_0\,\Delta V \tag{7.22a}$$

where $\Delta V$ is the volume change of the system. The volume change of the system equals the volume of region $e$. Thus, we may write $\Delta V = m_e v_e$, where $m_e$ is the mass within region $e$ and $v_e$ is the specific volume, which is regarded as uniform throughout region $e$. With this expression for $\Delta V$, Eq. 7.22a becomes

$$\begin{bmatrix}\text{exergy transfer}\\ \text{accompanying work}\end{bmatrix} = W - m_e(p_0 v_e) \tag{7.22b}$$

Equation 7.22b can be placed on a time rate basis by dividing each term by the time interval $\Delta t$ and taking the limit as $\Delta t$ approaches zero. That is

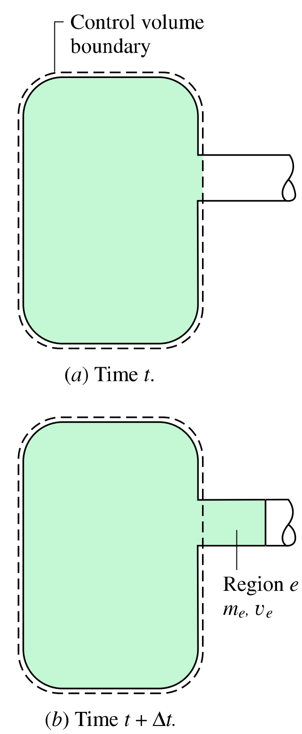
$$\begin{bmatrix}\text{time rate of exergy}\\ \text{transfer accompanying work}\end{bmatrix} = \lim_{\Delta t\to 0}\left(\frac{W}{\Delta t}\right) - \lim_{\Delta t\to 0}\left[\frac{m_e}{\Delta t}(p_0 v_e)\right] \tag{7.23}$$

In the limit as $\Delta t$ approaches zero, the boundaries of the closed system and control volume coincide. Accordingly, in this limit the rate of energy transfer by work from the closed system is also the rate of energy transfer by work from the control volume. For the present case, this is just the flow work. Thus, the first term on the right side of Eq. 7.23 becomes

$$\lim_{\Delta t\to 0}\left(\frac{W}{\Delta t}\right) = \dot{m}_e(p_e v_e) \tag{7.24}$$

where $\dot{m}_e$ is the mass flow rate at the exit of the control volume. In the limit as $\Delta t$ approaches zero, the second term on the right side of Eq. 7.23 becomes

$$\lim_{\Delta t\to 0}\left[\frac{m_e}{\Delta t}(p_0 v_e)\right] = \dot{m}_e(p_0 v_e) \tag{7.25}$$




▲ **Figure 7.6** Illustration used to introduce the flow exergy concept.

In this limit, the assumption of uniform specific volume throughout region $e$ corresponds to the assumption of uniform specific volume across the exit (one-dimensional flow).

Substituting Eqs. 7.24 and 7.25 into Eq. 7.23 gives

$$\begin{bmatrix}\text{time rate of exergy transfer}\\ \text{accompanying flow work}\end{bmatrix} = \dot{m}_e(p_e v_e) - \dot{m}_e(p_0 v_e)$$

$$= \dot{m}_e(p_e v_e - p_0 v_e) \quad (7.26)$$

Extending the reasoning given here, it can be shown that an expression having the same form as Eq. 7.26 accounts for the transfer of exergy accompanying flow work at inlets to control volumes as well. The general result applying at both inlets and exits is given by Eq. 7.21.

## DEVELOPING THE FLOW EXERGY CONCEPT

With the introduction of the expression for the exergy transfer accompanying flow work, attention now turns to the flow exergy. When mass flows across the boundary of a control volume, there is an accompanying energy transfer given by

$$\begin{bmatrix}\text{time rate of energy transfer}\\ \text{accompanying mass flow}\end{bmatrix} = \dot{m}e$$

$$= \dot{m}\left(u + \frac{V^2}{2} + gz\right) \quad (7.27)$$

where $e$ is the specific energy evaluated at the inlet or exit under consideration. Likewise, when mass enters or exits a control volume, there is an accompanying exergy transfer given by

$$\begin{bmatrix}\text{time rate of exergy transfer}\\ \text{accompanying mass flow}\end{bmatrix} = \dot{m}\mathsf{e}$$

$$= \dot{m}[(e - u_0) + p_0(v - v_0) - T_0(s - s_0)] \quad (7.28)$$

where $\mathsf{e}$ is the specific exergy at the inlet or exit under consideration. In writing Eqs. 7.27 and 7.28, one-dimensional flow is assumed. In addition to an exergy transfer accompanying mass flow, an exergy transfer accompanying flow work takes place at locations where mass enters or exits a control volume. Transfers of exergy accompanying flow work are accounted for by Eq. 7.21.

Since transfers of exergy accompanying mass flow and flow work occur at locations where mass enters or exits a control volume, a single expression giving the sum of these effects is convenient. Thus, with Eqs. 7.21 and 7.28,

$$\begin{bmatrix}\text{time rate of exergy transfer}\\ \text{accompanying mass flow } \textit{and} \text{ flow work}\end{bmatrix} = \dot{m}[\mathsf{e} + (pv - p_0 v)]$$

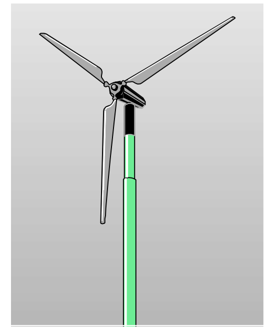
$$= \dot{m}[\underline{(e - u_0) + p_0(v - v_0) - T_0(s - s_0)} \quad (7.29)$$
$$\underline{+ (pv - p_0 v)}]$$

The underlined terms in Eq. 7.29 represent, per unit of mass, the exergy transfer accompanying mass flow and flow work, respectively. The sum identified by underlining is the specific flow exergy $\mathsf{e}_f$. That is

$$\mathsf{e}_f = (e - u_0) + p_0(v - v_0) - T_0(s - s_0) + (pv - p_0 v) \quad (7.30a)$$

## Thermodynamics in the News...

### Wind Power Looming Large



What stands as tall as a 30-story building and produces electricity at a rate that would meet the needs of about 900 typical U.S. homes? One of the world's largest commercial wind turbines. Designed specifically for use in North America, the three-bladed rotor of this wind turbine has a diameter nearly the length of a football field and operates in winds up to 55 miles per hour.

This wind turbine features microprocessor control of all functions and the option of remote monitoring. Other special features include a regulating system ensuring that each blade is always pitched at the correct angle for current wind conditions. Both the rotor and generator can vary their rotational speed during wind gusts, reducing fluctuations in the power provided to the electricity grid and the forces acting on the vital parts of the turbine.

Wind turbines are not without detractors. They are considered unsightly by some and noisy by others; and at some wind turbine sites sufficient winds may not always be available when power is most needed. Cost is another issue. At 4 to 6 cents per kW · h, wind-generated electricity costs up to twice as much as from coal-fired power plants.

Still, wind power is second only to hydroelectric power among the renewable energy resources used by utilities today. Wind energy plants take less time to build than conventional plants and are modular, allowing additional units to be added as warranted. They also produce no carbon dioxide and have minimal environmental impact.

The specific flow exergy can be placed in a more convenient form for calculation by introducing $e = u + V^2/2 + gz$ in Eq. 7.30a and simplifying to obtain

$$\mathsf{e}_f = \left(u + \frac{V^2}{2} + gz - u_0\right) + (pv - p_0 v_0) - T_0(s - s_0)$$

$$= (u + pv) - (u_0 + p_0 v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \qquad (7.30b)$$

Finally, with $h = u + pv$ and $h_0 = u_0 + p_0 v_0$, Eq. 7.30b gives Eq. 7.20, which is the principal result of this section. Equation 7.20 is used in the next section where the exergy rate balance for control volumes is formulated.

A comparison of the current development with that of Sec. 4.2 shows that the flow exergy evolves here in a similar way as does enthalpy in the development of the control volume energy rate balance, and they have similar interpretations: Each quantity is a sum consisting of a term associated with the flowing mass (specific internal energy for enthalpy, specific exergy for flow exergy) and a contribution associated with flow work at the inlet or exit under consideration.

## 7.5 Exergy Rate Balance for Control Volumes

In this section, the exergy balance is extended to a form applicable to control volumes. The control volume form is generally the most useful for engineering analysis.

### GENERAL FORM

The exergy rate balance for a control volume can be derived using an approach like that employed in the box of Sec. 4.1, where the control volume form of the mass rate balance is obtained by transforming the closed system form. However, as in the developments of the energy and entropy rate balances for control volumes, the present derivation is conducted less formally by modifying the closed system rate form, Eq. 7.17, to account for the exergy transfers accompanying mass flow and flow work at the inlets and exits.

The result is the ***control volume exergy rate balance***

*control volume exergy rate balance*

$$\underbrace{\frac{d\mathsf{E}_{\text{cv}}}{dt}}_{\substack{\text{rate of}\\\text{exergy}\\\text{change}}} = \underbrace{\sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \left(\dot{W}_{\text{cv}} - p_0\frac{dV_{\text{cv}}}{dt}\right) + \sum_i \dot{m}_i \mathsf{e}_{\text{f}i} - \sum_e \dot{m}_e \mathsf{e}_{\text{f}e}}_{\substack{\text{rate of}\\\text{exergy}\\\text{transfer}}} - \underbrace{\dot{\mathsf{E}}_{\text{d}}}_{\substack{\text{rate of}\\\text{exergy}\\\text{destruction}}} \tag{7.31}$$

As for control volume rate balances considered previously, $i$ denotes inlets and $e$ denotes exits.

In Eq. 7.31 the term $d\mathsf{E}_{\text{cv}}/dt$ represents the time rate of change of the exergy of the control volume. The term $\dot{Q}_j$ represents the time rate of heat transfer at the location on the boundary where the instantaneous temperature is $T_j$. The accompanying exergy transfer rate is given by $(1 - T_0/T_j)\dot{Q}_j$. The term $\dot{W}_{\text{cv}}$ represents the time rate of energy transfer rate by work *other than flow work*. The accompanying exergy transfer rate is given by $(\dot{W}_{\text{cv}} - p_0\, dV_{\text{cv}}/dt)$, where $dV_{\text{cv}}/dt$ is the time rate of change of volume. The term $\dot{m}_i\mathsf{e}_{\text{f}i}$ accounts for the time rate of exergy transfer accompanying mass flow *and* flow work at inlet $i$. Similarly, $\dot{m}_i\mathsf{e}_{\text{f}e}$ accounts for the time rate of exergy transfer accompanying mass flow *and* flow work at exit $e$. The flow exergies $\mathsf{e}_{\text{f}i}$ and $\mathsf{e}_{\text{f}e}$ appearing in these expressions are evaluated using Eq. 7.20. In writing Eq. 7.31, one-dimensional flow is assumed at locations where mass enters and exits. Finally, the term $\dot{\mathsf{E}}_{\text{d}}$ accounts for the time rate of exergy destruction due to irreversibilities *within* the control volume.

## STEADY-STATE FORMS

Since a great many engineering analyses involve control volumes at steady state, steady-state forms of the exergy rate balance are particularly important. At steady state, $d\mathsf{E}_{\text{cv}}/dt = dV_{\text{cv}}/dt = 0$, so Eq. 7.31 reduces to the ***steady-state exergy rate balance***

*steady-state exergy rate balance: control volumes*

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \dot{W}_{\text{cv}} + \sum_i \dot{m}_i \mathsf{e}_{\text{f}i} - \sum_e \dot{m}_e \mathsf{e}_{\text{f}e} - \dot{\mathsf{E}}_{\text{d}} \tag{7.32a}$$

This equation indicates that the rate at which exergy is transferred into the control volume must *exceed* the rate at which exergy is transferred out, the difference being the rate at which exergy is destroyed within the control volume due to irreversibilities.

Equation 7.32a can be expressed more compactly as

$$0 = \sum_j \dot{\mathsf{E}}_{\text{q}j} - \dot{W}_{\text{cv}} + \sum_i \dot{\mathsf{E}}_{\text{f}i} - \sum_e \dot{\mathsf{E}}_{\text{f}e} - \dot{\mathsf{E}}_{\text{d}} \tag{7.32b}$$

where

$$\dot{\mathsf{E}}_{\text{q}j} = \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j \tag{7.33}$$

$$\dot{\mathsf{E}}_{\text{f}i} = \dot{m}_i \mathsf{e}_{\text{f}i} \tag{7.34a}$$

$$\dot{\mathsf{E}}_{\text{f}e} = \dot{m}_e \mathsf{e}_{\text{f}e} \tag{7.34b}$$

are exergy transfer rates. At steady state, the rate of exergy transfer accompanying the power $\dot{W}_{\text{cv}}$ is the power itself.

If there is a single inlet and a single exit, denoted by 1 and 2, respectively, Eq. 7.32a reduces to

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \dot{W}_{cv} + \dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f2}) - \dot{\mathsf{E}}_d \tag{7.35}$$

where $\dot{m}$ is the mass flow rate. The term $(\mathsf{e}_{f1} - \mathsf{e}_{f2})$ is evaluated using Eq. 7.20 as

$$\mathsf{e}_{f1} - \mathsf{e}_{f2} = (h_1 - h_2) - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \tag{7.36}$$

**METHODOLOGY UPDATE**

When the rate of exergy destruction $\dot{\mathsf{E}}_d$ is the objective, it can be determined either from an exergy rate balance or from $\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production evaluated from an entropy rate balance. The second of these procedures normally requires fewer property evaluations and less computation.

## ILLUSTRATIONS

The following examples illustrate the use of the mass, energy, and exergy rate balances for the analysis of control volumes at steady state. Property data also play an important role in arriving at solutions. The first example involves the expansion of a gas through a valve (a throttling process, Sec. 4.3). From an energy perspective, the expansion of the gas occurs without loss. Yet, as shown in Example 7.5, such a valve is a site of thermodynamic inefficiency quantified by exergy destruction.

## EXAMPLE 7.5 Exergy Destruction in a Throttling Valve

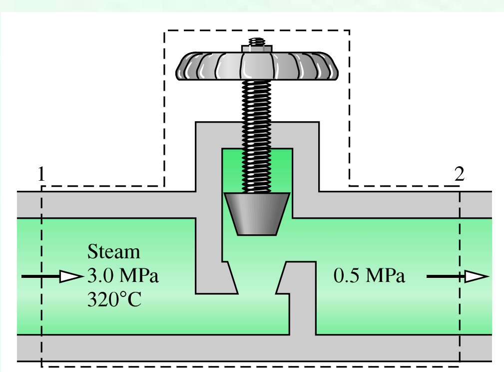
Superheated water vapor enters a valve at 3.0 MPa, 320°C and exits at a pressure of 0.5 MPa. The expansion is a throttling process. Determine the specific flow exergy at the inlet and exit and the exergy destruction per unit of mass flowing, each in kJ/kg. Let $T_0 = 25°C$, $p_0 = 1$ atm.

### SOLUTION

***Known:*** Water vapor expands in a throttling process through a valve from a specified inlet state to a specified exit pressure.

***Find:*** Determine the specific flow exergy at the inlet and exit of the valve and the exergy destruction per unit of mass flowing.

***Schematic and Given Data:***




***Assumptions:***

1. The control volume shown in the accompanying figure is at steady state.
2. For the throttling process, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and kinetic and potential energy effects can be ignored.
3. $T_0 = 25°C$, $p_0 = 1$ atm.

◀ **Figure E7.5**

***Analysis:*** The state at the inlet is specified. The state at the exit can be fixed by reducing the steady-state mass and energy rate balances to obtain

$$h_2 = h_1$$

Thus, the exit state is fixed by $p_2$ and $h_2$. From Table A-4, $h_1 = 3043.4$ kJ/s, $s_1 = 6.6245$ kJ/kg · k. Interpolating at a pressure of 0.5 MPa with $h_2 = h_1$, the specific entropy at the exit is $s_2 = 7.4223$ kJ/kg · k. Evaluating $h_0$ and $s_0$ at the saturated liquid state corresponding to $T_0$, Table A-2 gives $h_0 = 104.89$ kJ/kg, $s_0 = 0.3674$ kJ/kg · k.

Dropping $V^2/2$ and $gz$, we obtain the specific flow exergy from Eq. 7.20 as

$$\mathsf{e}_f = h - h_0 - T_0(s - s_0)$$

Substituting values into the expression for $\mathsf{e}_f$, the flow exergy at the inlet is

$$\mathsf{e}_{f1} = (3043.4 - 104.89) - 298(6.6245 - 0.3674) = 1073.89 \text{ kJ/kg}$$

At the exit

$$\mathsf{e}_{f2} = (3043.4 - 104.89) - 298(7.4223 - 0.3674) = 836.15 \text{ kJ/kg}$$

With assumptions listed, the steady-state form of the exergy rate balance, Eq. 7.35, reduces to

$$0 = \sum_j \cancelto{0}{\left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f2}) - \dot{\mathsf{E}}_d$$

Dividing by the mass flow rate $\dot{m}$ and solving, the exergy destruction per unit of mass flowing is

❶ $$\frac{\dot{\mathsf{E}}_d}{\dot{m}} = (\mathsf{e}_{f1} - \mathsf{e}_{f2})$$

Inserting values

❷ $$\frac{\dot{\mathsf{E}}_d}{\dot{m}} = 1073.89 - 836.15 = 237.7 \text{ kJ/kg}$$

---

❶ Since $h_1 = h_2$, this expression for the exergy destruction reduces to

$$\frac{\dot{\mathsf{E}}_d}{\dot{m}} = T_0(s_2 - s_1)$$

Thus, the exergy destruction can be determined knowing only $T_0$ and the specific entropies $s_1$ and $s_2$. The foregoing equation can be obtained alternatively beginning with the relationship $\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv}$ and then evaluating the rate of entropy production $\dot{\sigma}_{cv}$ from an entropy balance.

❷ Energy is conserved in the throttling process, but exergy is destroyed. The source of the exergy destruction is the uncontrolled expansion that occurs.

Although heat exchangers appear from an energy perspective to operate without loss when stray heat transfer is ignored, they are a site of thermodynamic inefficiency quantified by exergy destruction. This is illustrated in the next example.

## EXAMPLE 7.6 Exergy Destruction in a Heat Exchanger

Compressed air enters a counterflow heat exchanger operating at steady state at 610 K, 10 bar and exits at 860 K, 9.7 bar. Hot combustion gas enters as a separate stream at 1020 K, 1.1 bar and exits at 1 bar. Each stream has a mass flow rate of 90 kg/s.
❶ Heat transfer between the outer surface of the heat exchanger and the surroundings can be ignored. Kinetic and potential energy effects are negligible. Assuming the combustion gas stream has the properties of air, and using the ideal gas model for both streams, determine for the heat exchanger

(a) the exit temperature of the combustion gas, in K.

(b) the net change in the flow exergy rate from inlet to exit of each stream, in MW.

(c) the rate exergy is destroyed, in MW.

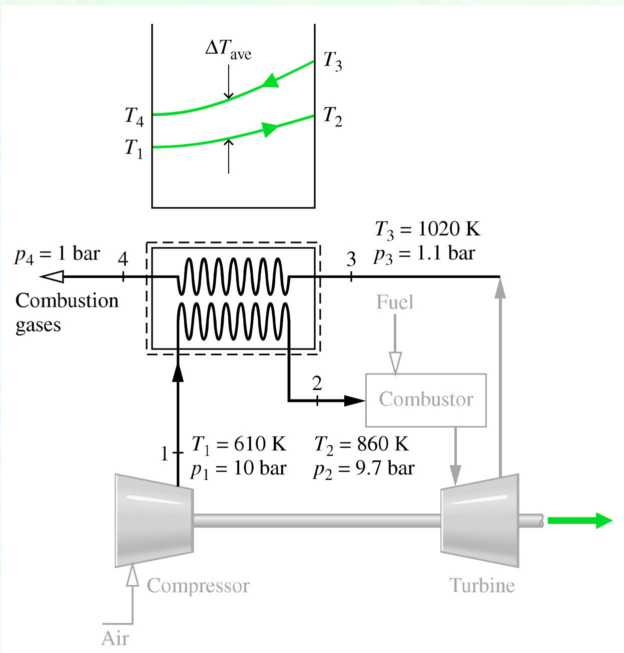
Let $T_0 = 300$ K, $p_0 = 1$ bar.

## SOLUTION

***Known:*** Steady-state operating data are provided for a counterflow heat exchanger.

***Find:*** For the heat exchanger, determine the exit temperature of the combustion gas, the change in the flow exergy rate from inlet to exit of each stream, and the rate exergy is destroyed.

***Schematic and Given Data:***




***Assumptions:***

1. The control volume shown in the accompanying figure is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and changes in kinetic and potential energy from inlet to exit are negligible.
3. Each stream has the properties of air modeled as an ideal gas.
4. $T_0 = 300$ K, $p_0 = 1$ bar.

◀ **Figure E7.6**

***Analysis:***

(a) The temperature $T_4$ of the exiting combustion gases can be found by reducing the mass and energy rate balances for the control volume at steady state to obtain

$$0 = \underline{\dot{Q}_{cv}} - \underline{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \underline{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \underline{g(z_1 - z_2)}\right] + \dot{m}\left[(h_3 - h_4) + \underline{\left(\frac{V_3^2 - V_4^2}{2}\right)} + \underline{g(z_3 - z_4)}\right]$$

where $\dot{m}$ is the common mass flow rate of the two streams. The underlined terms drop out by listed assumptions, leaving

$$0 = \dot{m}(h_1 - h_2) + \dot{m}(h_3 - h_4)$$

Dividing by $\dot{m}$ and solving for $h_4$

$$h_4 = h_3 + h_1 - h_2$$

From Table A-22, $h_1 = 617.53$ kJ/kg, $h_2 = 888.27$ kJ/kg, $h_3 = 1068.89$ kJ/kg. Inserting values

$$h_4 = 1068.89 + 617.53 - 888.27 = 798.15 \text{ kJ/kg}$$

❷ Interpolating in Table A-22 gives $T_4 = 778$ K (505°C).

**(b)** The net change in the flow exergy rate from inlet to exit for the compressed air stream can be evaluated using Eq. 7.36, neglecting the effects of motion and gravity. With Eq. 6.21a and data from Table A-22

$$\begin{aligned}\dot{m}(\mathsf{e}_{f2} - \mathsf{e}_{f1}) &= \dot{m}[(h_2 - h_1) - T_0(s_2 - s_1)] \\ &= \dot{m}\left[(h_2 - h_1) - T_0\left(s_2^\circ - s_1^\circ - R\ln\frac{p_2}{p_1}\right)\right] \\ &= 90\frac{\text{kg}}{\text{s}}\left[(888.27 - 617.53)\frac{\text{kJ}}{\text{kg}} - 300\text{ K}\left(2.79783 - 2.42644 - \frac{8.314}{28.97}\ln\frac{9.7}{10}\right)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right] \\ &= 14{,}103\frac{\text{kJ}}{\text{s}}\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right| = 14.1\text{ MW}\end{aligned}$$

Thus, as the air passes from 1 to 2, its flow exergy *increases*.

Similarly, the change in the flow exergy rate from inlet to exit for the combustion gas is

$$\begin{aligned}\dot{m}(\mathsf{e}_{f4} - \mathsf{e}_{f3}) &= \dot{m}\left[(h_4 - h_3) - T_0\left(s_4^\circ - s_3^\circ - R\ln\frac{p_4}{p_3}\right)\right] \\ &= 90\left[(798.15 - 1068.89) - 300\left(2.68769 - 2.99034 - \frac{8.314}{28.97}\ln\frac{1}{1.1}\right)\right] \\ &= -16{,}934\frac{\text{kJ}}{\text{s}}\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right| = -16.93\text{ MW}\end{aligned}$$

Thus, as the combustion gas passes from 3 to 4, its flow exergy *decreases*.

❸ **(c)** The rate of exergy destruction within the control volume can be determined from an exergy rate balance

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j^{\,0} - \dot{W}_{cv}^{\,0} + \dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f2}) + \dot{m}(\mathsf{e}_{f3} - \mathsf{e}_{f4}) - \dot{\mathsf{E}}_d$$

Solving for $\dot{\mathsf{E}}_d$ and inserting known values

$$\begin{aligned}\dot{\mathsf{E}}_d &= \dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f2}) + \dot{m}(\mathsf{e}_{f3} - \mathsf{e}_{f4}) \\ &= (-14.1\text{ MW}) + (16.93\text{ MW}) = 2.83\text{ MW}\end{aligned}$$

❹ Comparing results, we see that the exergy increase of the compressed air stream is less than the exergy decrease of the combustion gas stream, even though each has the same energy change. The difference is accounted for by exergy destruction. Energy is conserved but exergy is not.

---

❶ Heat exchangers of this type are known as *regenerators* (see Sec. 9.7).

❷ The variation in temperature of each stream passing through the heat exchanger is sketched in the schematic.

❸ Alternatively, the rate of exergy destruction can be determined using $\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production evaluated from an entropy rate balance. This is left as an exercise.

❹ Exergy is destroyed by irreversibilities associated with fluid friction and stream-to-stream heat transfer. The pressure drops for the streams are indicators of frictional irreversibility. The temperature difference between the streams is an indicator of heat transfer irreversibility.

The next two examples provide further illustrations of *exergy accounting*. The first involves the steam turbine with stray heat transfer considered previously in Ex. 6.6.

## EXAMPLE 7.7 Exergy Accounting of a Steam Turbine

Steam enters a turbine with a pressure of 30 bar, a temperature of 400°C, a velocity of 160 m/s. Steam exits as saturated vapor at 100°C with a velocity of 100 m/s. At steady state, the turbine develops work at a rate of 540 kJ per kg of steam flowing through the turbine. Heat transfer between the turbine and its surroundings occurs at an average outer surface temperature of 350 K. Develop a full accounting of the *net* exergy carried in by the steam, per unit mass of steam flowing. Neglect the change in potential energy between inlet and exit. Let $T_0 = 25°C$, $p_0 = 1$ atm.

### SOLUTION

***Known:*** Steam expands through a turbine for which steady-state data are provided.

***Find:*** Develop a full exergy accounting of the net exergy carried in by the steam, per unit mass of steam flowing.

***Schematic and Given Data:*** See Fig. E6.6.

***Assumptions:***

1. The turbine is at steady state.
2. Heat transfer between the turbine and the surroundings occurs at a known temperature.
3. The change in potential energy between inlet and exit can be neglected.
4. $T_0 = 25°C$, $p_0 = 1$ atm.

***Analysis:*** The *net* exergy carried in per unit mass of steam flowing is obtained using Eq. 7.36

$$\mathsf{e}_{f1} - \mathsf{e}_{f2} = (h_1 - h_2) - T_0(s_1 - s_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

where the potential energy term is dropped by assumption 3. From Table A-4, $h_1 = 3230.9$ kJ/kg, $s_1 = 6.9212$ kJ/kg · K. From Table A-2, $h_2 = 2676.1$ kJ/kg, $s_2 = 7.3549$ kJ/kg · K. Hence, the net rate exergy is carried in is

$$\begin{aligned}\mathsf{e}_{f1} - \mathsf{e}_{f2} &= \left[(3230.9 - 2676.1) - 298(6.9212 - 7.3549) + \frac{(160)^2 - (100)^2}{2|10^3|}\right] \\ &= 691.84 \text{ kJ/kg}\end{aligned}$$

The net exergy carried in can be accounted for in terms of exergy transfers accompanying work and heat from the control volume and exergy destruction within the control volume. At steady state, the exergy transfer accompanying work is the work itself, or $\dot{W}_{cv}/\dot{m} = 540$ kJ/kg. The quantity $\dot{Q}_{cv}/\dot{m}$ is evaluated in the solution to Example 6.6 using the steady-state forms of the mass and energy rate balances: $\dot{Q}_{cv}/\dot{m} = -22.6$ kJ/kg. The accompanying exergy transfer is

$$\begin{aligned}\frac{\dot{\mathsf{E}}_q}{\dot{m}} &= \left(1 - \frac{T_0}{T_b}\right)\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right) \\ &= \left(1 - \frac{298}{350}\right)\left(-22.6\frac{\text{kJ}}{\text{kg}}\right) \\ &= -3.36\frac{\text{kJ}}{\text{kg}}\end{aligned}$$

where $T_b$ denotes the temperature on the boundary where heat transfer occurs.

The exergy destruction can be determined by rearranging the steady-state form of the exergy rate balance, Eq. 7.35, to give

1 $$\frac{\dot{\mathsf{E}}_d}{\dot{m}} = \left(1 - \frac{T_0}{T_b}\right)\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right) - \frac{\dot{W}_{cv}}{\dot{m}} + (\mathsf{e}_{f1} - \mathsf{e}_{f2})$$

Substituting values

$$\frac{\dot{\mathsf{E}}_d}{\dot{m}} = -3.36 - 540 + 691.84 = 148.48 \text{ kJ/kg}$$

The analysis is summarized by the following exergy *balance sheet* in terms of exergy magnitudes on a rate basis:

| | |
|---|---|
| *Net rate of exergy in*: | 691.84 kJ/kg (100%) |
| *Disposition of the exergy:* | |
| • Rate of exergy out | |
| work | 540.00 kJ/kg (78.05%) |
| heat transfer | 3.36 kJ/kg (0.49%) |
| • Rate of exergy destruction | 148.48 kJ/kg (21.46%) |
| | 691.84 kJ/kg (100%) |

Note that the exergy transfer accompanying heat transfer is small relative to the other terms.

1 The exergy destruction can be determined alternatively using $\dot{E}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy balance. The solution to Example 6.6 provides $\dot{\sigma}_{cv}/\dot{m} = 0.4983$ kJ/kg · K.

The next example illustrates the use of exergy accounting to identify opportunities for improving thermodynamic performance.

## EXAMPLE 7.8 Exergy Accounting of a Waste Heat Recovery System

Suppose the system of Example 4.10 is one option under consideration for utilizing the combustion products discharged from an industrial process.

**(a)** Develop a full accounting of the *net* exergy carried in by the combustion products.

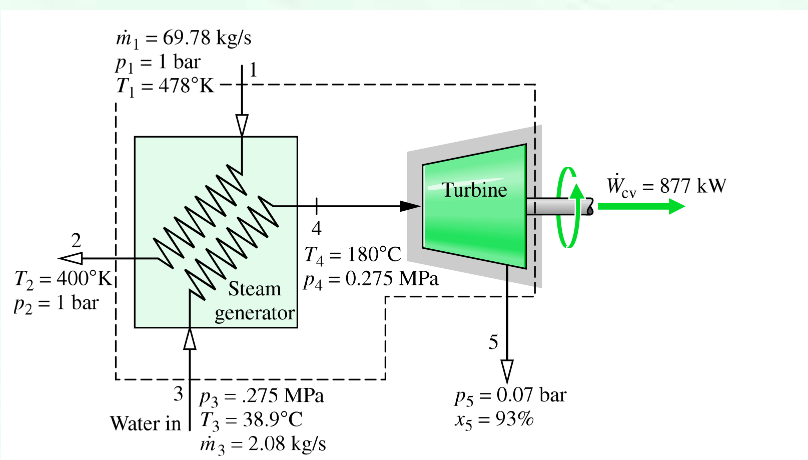
**(b)** Discuss the design implications of the results.

### SOLUTION

***Known:*** Steady-state operating data are provided for a heat-recovery steam generator and a turbine.

***Find:*** Develop a full accounting of the *net* rate exergy is carried in by the combustion products and discuss the implications for design.

***Schematic and Given Data:***




***Assumptions:***

1. See solution to Example 4.10.
2. $T_0 = 298°K$.

◀ **Figure E7.8**

***Analysis:***

(a) We begin by determining the *net* rate exergy is carried *into* the control volume. Modeling the combustion products as an ideal gas, the net rate is determined using Eq. 7.36 together with Eq. 6.21a as

$$\dot{m}_1[\mathsf{e}_{f1} - \mathsf{e}_{f2}] = \dot{m}_1[h_1 - h_2 - T_0(s_1 - s_2)]$$
$$= \dot{m}_1\left[h_1 - h_2 - T_0\left(s_1^\circ - s_2^\circ - R\ln\frac{p_1}{p_2}\right)\right]$$

With data from Table A-22, $h_1 = 480.35$ kJ/kg, $h_2 = 400.97$ kJ/kg, $s_1^\circ = 2.173$ kJ/kg · °K, $s_2^\circ = 1.992$ kJ/kg · °K, and $p_2 = p_1$, we have

$$\dot{m}_1[\mathsf{e}_{f1} - \mathsf{e}_{f2}] = 69.8\ \text{kg/s}\left[(480.35 - 400.97)\frac{\text{kJ}}{\text{kg}} - 298°\text{K}(2.173 - 1.992)\frac{\text{kJ}}{\text{kg}\cdot°\text{C}}\right]\text{kJ/s}$$
$$= 1775.78\ \text{kJ/s}$$

Next, we determine the rate exergy is carried *out* of the control volume. Exergy is carried out of the control volume by work at a rate of 876.8 kJ/s, as shown on the schematic. Additionally, the *net* rate exergy is carried *out* by the water stream is

$$\dot{m}_3[\mathsf{e}_{f5} - \mathsf{e}_{f3}] = \dot{m}_3[h_5 - h_3 - T_0(s_5 - s_3)]$$

From Table A-2E, $h_3 \approx h_f(39°\text{C}) = 162.82$ kg/s, $s_3 \approx s_f(39°\text{C}) = 0.5598$ kJ/kg · °K. Using saturation data at 0.07 bars from Table A-3 with $x_5 = 0.93$ gives $h_5 = 2403.27$ kJ/kg and $s_5 = 7.739$ kJ/kg · °K. Substituting values

$$\dot{m}_3[\mathsf{e}_{f5} - \mathsf{e}_{f3}] = 2.08\frac{\text{kg}}{\text{s}}\left[(2403.27 - 162.82)\frac{\text{kJ}}{\text{kg}} - 298(7.739 - 0.5598)\frac{\text{kJ}}{\text{kg}\cdot°\text{K}}\right]$$
$$= 209.66\ \text{kJ/s}$$

Next, the rate exergy is destroyed in the heat-recovery steam generator can be obtained from an exergy rate balance applied to a control volume enclosing the steam generator. That is

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j^{\,0} - \dot{W}_{cv}^{\,0} + \dot{m}_1(\mathsf{e}_{f1} - \mathsf{e}_{f2}) + \dot{m}_3(\mathsf{e}_{f3} - \mathsf{e}_{f4}) - \dot{\mathsf{E}}_d$$

Evaluating $(\mathsf{e}_{f3} - \mathsf{e}_{f4})$ with Eq. 7.36 and solving for $\dot{\mathsf{E}}_d$

$$\dot{\mathsf{E}}_d = \dot{m}_1(\mathsf{e}_{f1} - \mathsf{e}_{f2}) + \dot{m}_3[h_3 - h_4 - T_0(s_3 - s_4)]$$

The first term on the right is evaluated above. Then, with $h_4 = 2825$ kJ/kg, $s_4 = 7.2196$ kJ/kg · °K at 180°C, .275 MPa from Table A-4, and previously determined values for $h_3$ and $s_3$

$$\dot{\mathsf{E}}_d = 1775.78\frac{\text{kJ}}{\text{s}} + 2.08\frac{\text{kg}}{\text{s}}\left[(162 - 2825)\frac{\text{kJ}}{\text{kg}} - 298(.559 - 7.2196)\frac{\text{kJ}}{\text{kg}\cdot°\text{K}}\right]$$
$$= 366.1\ \text{kJ/s}$$

Finally, the rate exergy is destroyed in the turbine can be obtained from an exergy rate balance applied to a control volume enclosing the turbine. That is

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j^{\,0} - \dot{W}_{cv} + \dot{m}_4(\mathsf{e}_{f4} - \mathsf{e}_{f5}) - \dot{\mathsf{E}}_d$$

Solving for $\dot{\mathsf{E}}_d$, evaluating $(\mathsf{e}_{f4} - \mathsf{e}_{f5})$ with Eq. 7.36, and using previously determined values

$$\dot{\mathsf{E}}_d = -\dot{W}_{cv} + \dot{m}_4[h_4 - h_5 - T_0(s_4 - s_5)]$$

1 $$= -876.8\frac{\text{kJ}}{\text{s}} + 2.08\frac{\text{kg}}{\text{s}}\left[(2825 - 2403)\frac{\text{kJ}}{\text{kg}} - 298°\text{K}(7.2196 - 7.739)\frac{\text{kJ}}{\text{kg}\cdot°\text{C}}\right]$$
$$= 320.2\ \text{kJ/s}$$

The analysis is summarized by the following exergy *balance sheet* in terms of exergy magnitudes on a rate basis:

| | |
|---|---|
| *Net rate of exergy in:* | 1772.8 kJ/s (100%) |
| *Disposition of the exergy:* | |
| • Rate of exergy out | |
| power developed | 876.8 kJ/s (49.5%) |
| water stream | 209.66 kJ/s (11.8%) |
| • Rate of exergy destruction | |
| heat-recovery steam generator | 366.12 kJ/s (20.6%) |
| turbine | 320.2 kJ/s (18%) |

**(b)** The exergy balance sheet suggests an opportunity for improved *thermodynamic* performance because about 50% of the net exergy carried in is either destroyed by irreversibilities or carried out by the water stream. Better thermodynamic performance might be achieved by modifying the design. For example, we might reduce the heat transfer irreversibility by specifying a heat-recovery steam generator with a smaller stream-to-stream temperature difference, and/or reduce friction by specifying a turbine with a higher isentropic efficiency. Thermodynamic performance alone would not determine the *preferred* option, however, for other factors such as cost must be considered, and can be overriding. Further discussion of the use of exergy analysis in design is provided in Sec. 7.7.1.

1 Alternatively, the rates of exergy destruction in control volumes enclosing the heat-recovery steam generator and turbine can be determined using $\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production for the respective control volume evaluated from an entropy rate balance. This is left as an exercise.

In previous discussions we have noted the effect of irreversibilities on *thermodynamic* performance. Some *economic* consequences of irreversibilities are considered in the next example.

## EXAMPLE 7.9 Cost of Exergy Destruction

For the heat pump of Examples 6.8 and 6.14, determine the exergy destruction rates, each in kW, for the compressor, condenser, and throttling valve. If exergy is valued at \$0.08 per kW · h, determine the daily cost of electricity to operate the compressor and the daily cost of exergy destruction in each component. Let $T_0 = 273$ K (0°C), which corresponds to the temperature of the outside air.

### SOLUTION

***Known:*** Refrigerant 22 is compressed adiabatically, condensed by heat transfer to air passing through a heat exchanger, and then expanded through a throttling valve. Data for the refrigerant and air are known.

***Find:*** Determine the daily cost to operate the compressor. Also determine the exergy destruction rates and associated daily costs for the compressor, condenser, and throttling valve.

***Schematic and Given Data:***
See Examples 6.8 and 6.14.

***Assumptions:***

1. See Examples 6.8 and 6.14.
2. $T_0 = 273$ K (0°C).

***Analysis:*** The rates of exergy destruction can be calculated using

$$\dot{\mathsf{E}}_d = T_0\dot{\sigma}$$

together with data for the entropy production rates from Example 6.8. That is

$$(\dot{E}_d)_{comp} = (273\text{ K})(17.5 \times 10^{-4})\left(\frac{\text{kW}}{\text{K}}\right) = 0.478\text{ kW}$$
$$(\dot{E}_d)_{valve} = (273)(9.94 \times 10^{-4}) = 0.271\text{ kW}$$
$$(\dot{E}_d)_{cond} = (273)(7.95 \times 10^{-4}) = 0.217\text{ kW}$$

The costs of exergy destruction are, respectively

$$\left(\begin{array}{c}\text{Daily cost of exergy destruction due}\\ \text{to compressor irreversibilities}\end{array}\right) = (0.478\text{ kW})\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right)\left|\frac{24\text{ h}}{\text{day}}\right| = \$0.92$$

$$\left(\begin{array}{c}\text{Daily cost of exergy destruction due to}\\ \text{irreversibilities in the throttling valve}\end{array}\right) = (0.271)(0.08)|24| = \$0.52$$

$$\left(\begin{array}{c}\text{Daily cost of exergy destruction due to}\\ \text{irreversibilities in the condenser}\end{array}\right) = (0.271)(0.08)|24| = \$0.42$$

From the solution to Example 6.14, the magnitude of the compressor power is 3.11 kW. Thus, the daily cost is

$$\left(\begin{array}{c}\text{Daily cost of electricity}\\ \text{to operate compressor}\end{array}\right) = (3.11\text{ kW})\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right)\left|\frac{24\text{ h}}{\text{day}}\right| = \$5.97$$

1 Note that the total cost of exergy destruction in the three components is about 31% of the cost of electricity to operate the compressor.

---

1 Associating exergy destruction with operating costs provides a rational basis for seeking cost-effective design improvements. Although it may be possible to select components that would destroy less exergy, the trade-off between any resulting reduction in operating cost and the potential increase in equipment cost must be carefully considered.

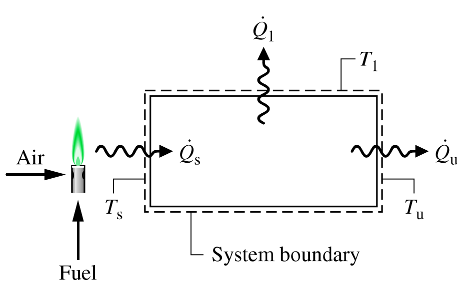
## 7.6 Exergetic (Second Law) Efficiency

The objective of this section is to show the use of the exergy concept in assessing the effectiveness of energy resource utilization. As part of the presentation, the ***exergetic efficiency*** concept is introduced and illustrated. Such efficiencies are also known as *second law* efficiencies.

*exergetic efficiency*

### ▶ 7.6.1 Matching End Use to Source

Tasks such as space heating, heating in industrial furnaces, and process steam generation commonly involve the combustion of coal, oil, or natural gas. When the products of combustion are at a temperature significantly greater than required by a given task, the end use is not well matched to the source and the result is inefficient use of the fuel burned. To illustrate this simply, refer to Fig. 7.7, which shows a closed system receiving a heat transfer




◀ **Figure 7.7** Schematic used to discuss the efficient use of fuel.

at the rate $\dot{Q}_s$ at a *source* temperature $T_s$ and delivering $\dot{Q}_u$ at a *use* temperature $T_u$. Energy is lost to the surroundings by heat transfer at a rate $\dot{Q}_l$ across a portion of the surface at $T_l$. All energy transfers shown on the figure are in the directions indicated by the arrows.

Assuming that the system of Fig. 7.7 operates at steady state and there is no work, the closed system energy and exergy rate balances reduce, respectively, to

$$\frac{d\cancel{E}^{\,0}}{\cancel{dt}} = (\dot{Q}_s - \dot{Q}_u - \dot{Q}_l) - \cancel{\dot{W}}^{\,0}$$

$$\frac{d\cancel{\mathsf{E}}^{\,0}}{\cancel{dt}} = \left[\left(1 - \frac{T_0}{T_s}\right)\dot{Q}_s - \left(1 - \frac{T_0}{T_u}\right)\dot{Q}_u - \left(1 - \frac{T_0}{T_l}\right)\dot{Q}_l\right] - \left[\cancel{\dot{W}}^{\,0} - p_0 \frac{d\cancel{V}^{\,0}}{\cancel{dt}}\right] - \dot{\mathsf{E}}_d$$

These equations can be rewritten as follows

$$\dot{Q}_s = \dot{Q}_u + \dot{Q}_l \tag{7.37a}$$

$$\left(1 - \frac{T_0}{T_s}\right)\dot{Q}_s = \left(1 - \frac{T_0}{T_u}\right)\dot{Q}_u + \left(1 - \frac{T_0}{T_l}\right)\dot{Q}_l + \dot{\mathsf{E}}_d \tag{7.37b}$$

Equation 7.37a indicates that the energy carried in by heat transfer, $\dot{Q}_s$, is either used, $\dot{Q}_u$, or lost to the surroundings, $\dot{Q}_l$. This can be described by an efficiency in terms of energy rates in the form product/input as

$$\eta = \frac{\dot{Q}_u}{\dot{Q}_s} \tag{7.38}$$

In principle, the value of $\eta$ can be increased by applying insulation to reduce the loss. The limiting value, when $\dot{Q}_l = 0$, is $\eta = 1$ (100%).

Equation 7.37b shows that the exergy carried into the system accompanying the heat transfer $\dot{Q}_s$ is either transferred from the system accompanying the heat transfers $\dot{Q}_u$ and $\dot{Q}_l$ or destroyed by irreversibilities within the system. This can be described by an efficiency in the form product/input as

$$\varepsilon = \frac{(1 - T_0/T_u)\dot{Q}_u}{(1 - T_0/T_s)\dot{Q}_s} \tag{7.39a}$$

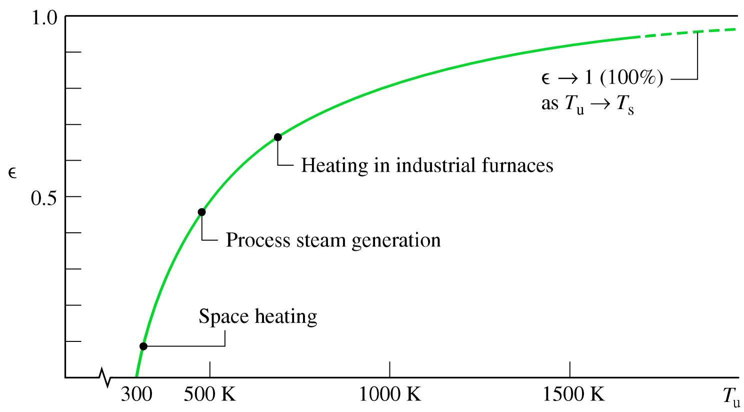
Introducing Eq. 7.38 into Eq. 7.39a results in

$$\varepsilon = \eta\left(\frac{1 - T_0/T_u}{1 - T_0/T_s}\right) \tag{7.39b}$$

The parameter $\varepsilon$, defined with reference to the exergy concept, may be called an *exergetic* efficiency. Note that $\eta$ and $\varepsilon$ each gauge how effectively the input is converted to the product. The parameter $\eta$ does this on an energy basis, whereas $\varepsilon$ does it on an exergy basis. As discussed next, the value of $\varepsilon$ is generally less than unity even when $\eta = 1$.

Equation 7.39b indicates that a value for $\eta$ as close to unity as practical is important for proper utilization of the exergy transferred from the hot combustion gas to the system. However, this alone would not ensure effective utilization. The temperatures $T_s$ and $T_u$ are also important, with exergy utilization improving as the use temperature $T_u$ approaches the source temperature $T_s$. For proper utilization of exergy, therefore, it is desirable to have a value for $\eta$ as close to unity as practical and also a good *match* between the source and use temperatures.

To emphasize the central role of temperature in exergetic efficiency considerations, a graph of Eq. 7.39b is provided in Fig. 7.8. The figure gives the exergetic efficiency $\varepsilon$ versus the use temperature $T_u$ for an assumed source temperature $T_s = 2200$ K. Figure 7.8 shows that $\varepsilon$ tends to unity (100%) as the use temperature approaches $T_s$. In most cases, however, the

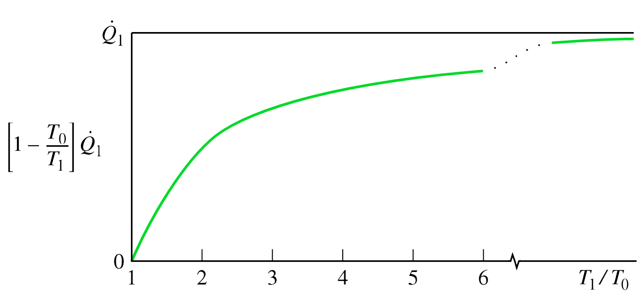



▲ **Figure 7.8** Effect of use temperature $T_u$ on the exergetic efficiency $\varepsilon$ ($T_s$ = 2200 K, $\eta$ = 100%).

use temperature is substantially below $T_s$. Indicated on the graph are efficiencies for three applications: space heating at $T_u$ = 320 K, process steam generation at $T_u$ = 480 K, and heating in industrial furnaces at $T_u$ = 700 K. These efficiency values suggest that fuel is used far more effectively in the higher use-temperature industrial applications than in the lower use-temperature space heating. The especially low exergetic efficiency for space heating reflects the fact that fuel is consumed to produce only slightly warm air, which from an exergy perspective has considerably less utility. The efficiencies given on Fig. 7.8 are actually on the *high* side, for in constructing the figure we have assumed $\eta$ to be unity (100%). Moreover, as additional destruction and loss of exergy would be associated with combustion, the overall efficiency from fuel input to end use would be much less than indicated by the values shown on the figure.

**COSTING HEAT LOSS.** For the system in Fig. 7.7, it is instructive to consider further the rate of exergy loss accompanying the heat loss $\dot{Q}_l$, that is $(1 - T_0/T_l)\dot{Q}_l$. This expression measures the *true* thermodynamic value of the heat loss and is graphed in Fig. 7.9. The figure shows that the thermodynamic value of the heat loss depends *significantly* on the temperature at which the heat loss occurs. Stray heat transfer, such as $\dot{Q}_l$, usually occurs at relatively low temperature, and thus has relatively low thermodynamic value. We might expect that the *economic* value of such a loss varies similarly with temperature, and this is the case. ▶ ***for example...*** since the source of the exergy loss by heat transfer is the fuel input (see Fig. 7.7), the economic value of the loss can be accounted for in terms of the *unit cost* of fuel based on exergy, $c_F$ (in \$/kW · h, for example), as follows

$$\begin{bmatrix}\text{Cost rate of heat loss}\\ \dot{Q}_l \text{ at temperature } T_l\end{bmatrix} = c_F(1 - T_0/T_l)\dot{Q}_l \tag{7.40}$$




◀ **Figure 7.9** Effect of the temperature ratio $T_l/T_0$ on the exergy loss associated with heat transfer.

Equation 7.40 shows that the cost of such a loss is less at lower temperatures than at higher temperatures. ◀

The above example illustrates what we would expect of a rational costing method. It would not be rational to assign the same economic value for a heat transfer occurring near ambient temperature, where the thermodynamic value is negligible, as for an equal heat transfer occurring at a higher temperature, where the thermodynamic value is significant. Indeed, it would be incorrect to assign the *same cost* to heat loss independent of the temperature at which the loss is occurring. For further discussion of exergy costing, see Sec. 7.7.2.

### ▶ 7.6.2 Exergetic Efficiencies of Common Components

Exergetic efficiency expressions can take many different forms. Several examples are given in the current section for thermal system components of practical interest. In every instance, the efficiency is derived by the use of the exergy rate balance. The approach used here serves as a model for the development of exergetic efficiency expressions for other components. Each of the cases considered involves a control volume at steady state, and we assume no heat transfer between the control volume and its surroundings. The current presentation is not exhaustive. Many other exergetic efficiency expressions can be written.

**TURBINES.** For a turbine operating at steady state with no heat transfer with its surroundings, the steady-state form of the exergy rate balance, Eq. 7.35, reduces as follows:

$$0 = \cancelto{0}{\sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j} - \dot{W}_{cv} + \dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f2}) - \dot{\mathsf{E}}_d$$

This equation can be rearranged to read

$$\mathsf{e}_{f1} - \mathsf{e}_{f2} = \frac{\dot{W}_{cv}}{\dot{m}} + \frac{\dot{\mathsf{E}}_d}{\dot{m}} \tag{7.41}$$

The term on the left of Eq. 7.41 is the decrease in flow exergy from turbine inlet to exit. The equation shows that the flow exergy decreases because the turbine develops work, $\dot{W}_{cv}/\dot{m}$, and exergy is destroyed, $\dot{\mathsf{E}}_d/\dot{m}$. A parameter that gauges how effectively the flow exergy decrease is converted to the desired product is the *exergetic turbine efficiency*

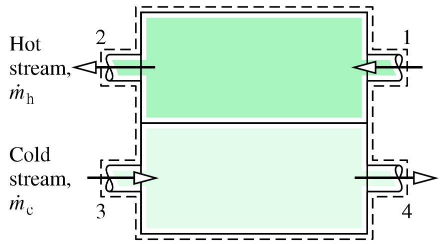
$$\varepsilon = \frac{\dot{W}_{cv}/\dot{m}}{\mathsf{e}_{f1} - \mathsf{e}_{f2}} \tag{7.42}$$

This particular exergetic efficiency is sometimes refered to as the *turbine effectiveness*. Carefully note that the exergetic turbine efficiency is defined differently from the isentropic turbine efficiency introduced in Sec. 6.8.

▶ ***for example...*** the exergetic efficiency of the turbine considered in Example 6.11 is 81.2% when $T_0 = 298$ K. It is left as an exercise to verify this value. ◀

**COMPRESSORS AND PUMPS.** For a compressor or pump operating at steady state with no heat transfer with its surroundings, the exergy rate balance, Eq. 7.35, can be placed in the form

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right) = \mathsf{e}_{f2} - \mathsf{e}_{f1} + \frac{\dot{\mathsf{E}}_d}{\dot{m}}$$




◀ **Figure 7.10** Counterflow heat exchanger.

Thus, the exergy *input* to the device, $-\dot{W}_{cv}/\dot{m}$, is accounted for either as an increase in the flow exergy between inlet and exit or as exergy destroyed. The effectiveness of the conversion from work input to flow exergy increase is gauged by the *exergetic compressor* (or pump) *efficiency*

$$\varepsilon = \frac{e_{f2} - e_{f1}}{(-\dot{W}_{cv}/\dot{m})} \tag{7.43}$$

▶ ***for example...*** the exergetic efficiency of the compressor considered in Example 6.14 is 84.6% when $T_0 = 273$ K. It is left as an exercise to verify this value. ◀

**HEAT EXCHANGER WITHOUT MIXING.** The heat exchanger shown in Fig. 7.10 operates at steady state with no heat transfer with its surroundings and both streams at temperatures above $T_0$. The exergy rate balance, Eq. 7.32a, reduces to

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\overset{0}{\dot{Q}_j} - \overset{0}{\dot{W}_{cv}} + (\dot{m}_h e_{f1} + \dot{m}_c e_{f3}) - (\dot{m}_h e_{f2} + \dot{m}_c e_{f4}) - \dot{E}_d$$

where $\dot{m}_h$ is the mass flow rate of the hot stream and $\dot{m}_c$ is the mass flow rate of the cold stream. This can be rearranged to read

$$\dot{m}_h(e_{f1} - e_{f2}) = \dot{m}_c(e_{f4} - e_{f3}) + \dot{E}_d \tag{7.44}$$

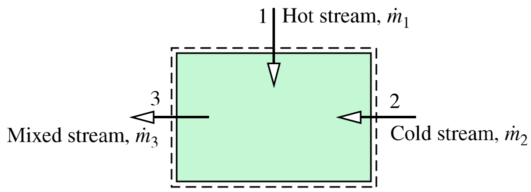
The term on the left of Eq. 7.44 accounts for the decrease in the exergy of the hot stream. The first term on the right accounts for the increase in exergy of the cold stream. Regarding the hot stream as supplying the exergy increase of the cold stream as well as the exergy destroyed, we can write an *exergetic heat exchanger efficiency* as

$$\varepsilon = \frac{\dot{m}_c(e_{f4} - e_{f3})}{\dot{m}_h(e_{f1} - e_{f2})} \tag{7.45}$$

▶ ***for example...*** the exergetic efficiency of the heat exchanger of Example 7.6 is 83.3%. It is left as an exercise to verify this value. ◀

**DIRECT CONTACT HEAT EXCHANGER.** The direct contact heat exchanger shown in Fig. 7.11 operates at steady state with no heat transfer with its surroundings. The exergy rate balance, Eq. 7.32a, reduces to

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\overset{0}{\dot{Q}_j} - \overset{0}{\dot{W}_{cv}} + \dot{m}_1 e_{f1} + \dot{m}_2 e_{f2} - \dot{m}_3 e_{f3} - \dot{E}_d$$




◀ **Figure 7.11** Direct contact heat exchanger.

With $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ from a mass rate balance, this can be written as

$$\dot{m}_1(\mathsf{e}_{f1} - \mathsf{e}_{f3}) = \dot{m}_2(\mathsf{e}_{f3} - \mathsf{e}_{f2}) + \dot{\mathsf{E}}_d \tag{7.46}$$

The term on the left Eq. 7.46 accounts for the decrease in the exergy of the hot stream between inlet and exit. The first term on the right accounts for the increase in the exergy of the cold stream between inlet and exit. Regarding the hot stream as supplying the exergy increase of the cold stream as well as the exergy destroyed by irreversibilities, we can write an *exergetic efficiency* for a direct contact heat exchanger as

$$\varepsilon = \frac{\dot{m}_2(\mathsf{e}_{f3} - \mathsf{e}_{f2})}{\dot{m}_1(\mathsf{e}_{f1} - \mathsf{e}_{f3})} \tag{7.47}$$

### ▶ 7.6.3 Using Exergetic Efficiencies

Exergetic efficiencies are useful for distinguishing means for utilizing energy resources that are thermodynamically effective from those that are less so. Exergetic efficiencies also can be used to evaluate the effectiveness of engineering measures taken to improve the performance of a thermal system. This is done by comparing the efficiency values determined before and after modifications have been made to show how much improvement has been achieved. Moreover, exergetic efficiencies can be used to gauge the potential for improvement in the performance of a given thermal system by comparing the efficiency of the system to the efficiency of like systems. A significant difference between these values would suggest that improved performance is possible.

It is important to recognize that the limit of 100% exergetic efficiency should not be regarded as a practical objective. This theoretical limit could be attained only if there were no exergy destructions or losses. To achieve such idealized processes might require extremely long times to execute processes and/or complex devices, both of which are at odds with the objective of profitable operation. In practice, decisions are usually made on the basis of *total* costs. An increase in efficiency to reduce fuel consumption, or otherwise utilize resources better, normally requires additional expenditures for facilities and operations. Accordingly, an improvement might not be implemented if an increase in total cost would result. The trade-off between fuel savings and additional investment invariably dictates a lower efficiency than might be achieved *theoretically* and may even result in a lower efficiency than could be achieved using the *best available* technology.

*cogeneration*

Various methods are used to improve energy resource utilization. All such methods must achieve their objectives cost-effectively. One method is ***cogeneration,*** which sequentially produces power and a heat transfer (or process steam) for some desired use. An aim of cogeneration is to develop the power and heat transfer using an *integrated* system with a total expenditure that is less than would be required to develop them individually. Further discussions of cogeneration are provided in Secs. 7.7.2 and 8.5. Two other methods employed to improve energy resource utilization are ***power recovery*** and ***waste heat recovery.*** Power recovery can be accomplished by inserting a turbine into a pressurized gas or liquid stream to capture some of the exergy that would otherwise be destroyed in a spontaneous expansion. Waste heat recovery contributes to overall efficiency by using some of the exergy that would otherwise be discarded to the surroundings, as in the exhaust gases of large internal combustion engines. An illustration of waste heat recovery is provided by Example 7.8.

*power recovery*

*waste heat recovery*

# 7.7 Thermoeconomics

*Thermal systems* typically experience significant work and/or heat interactions with their surroundings, and they can exchange mass with their surroundings in the form of hot and cold streams, including chemically reactive mixtures. Thermal systems appear in almost every industry, and numerous examples are found in our everyday lives. Their design involves the application of principles from thermodynamics, fluid mechanics, and heat transfer, as well as such fields as materials, manufacturing, and mechanical design. The design of thermal systems also requires the explicit consideration of engineering economics, for cost is always a consideration. The term ***thermoeconomics*** may be applied to this general area of application, although it is often applied more narrowly to methodologies combining exergy and economics for optimizing the design and operation of thermal systems.

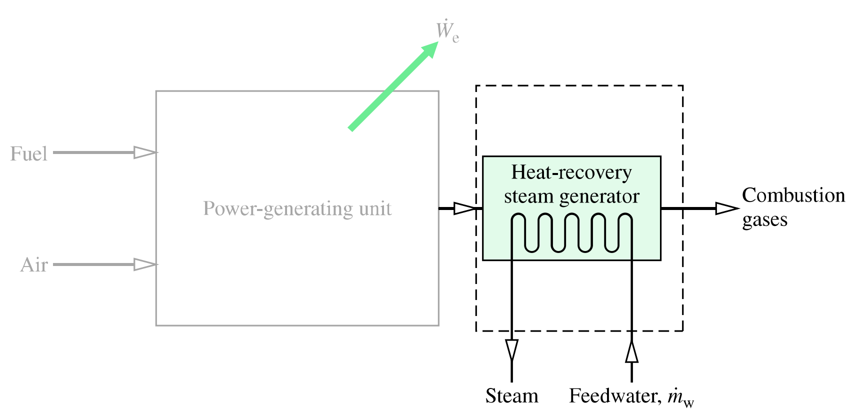
*thermoeconomics*

## ▶ 7.7.1 Using Exergy in Design

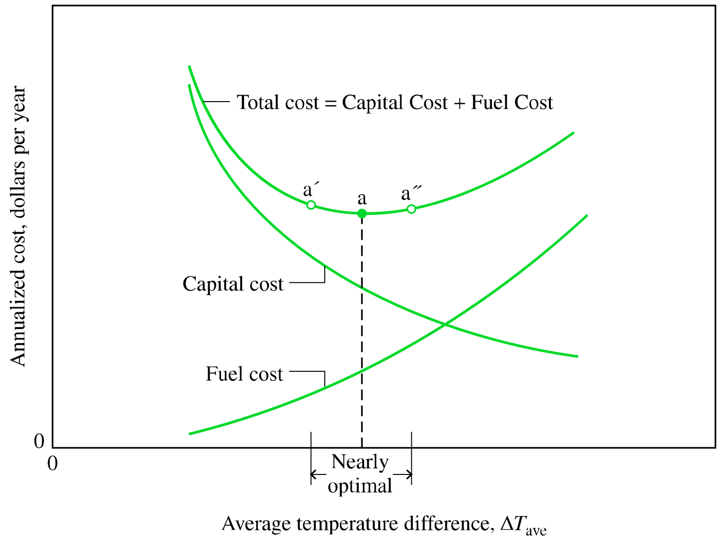
To illustrate the use of exergy in design, consider Fig. 7.12 showing a thermal system consisting of a power-generating unit and a *heat-recovery steam generator*. The power-generating unit develops an electric power output and combustion products that enter the heat recovery unit. Feedwater also enters the heat-recovery steam generator with a mass flow rate of $\dot{m}_w$, receives exergy by heat transfer from the combustion gases, and exits as steam at some desired condition for use in another process. The combustion products entering the heat-recovery steam generator can be regarded as having economic value. Since the source of the exergy of the combustion products is the fuel input (Fig. 7.12), the economic value can be accounted for in terms of the cost of fuel, as we have done in Sec. 7.6.1 when costing heat loss.

From our study of the second law of thermodynamics we know that the average temperature difference, $\Delta T_{ave}$, between two streams passing through a heat exchanger is a measure of irreversibility and that the irreversibility of the heat transfer vanishes as the temperature difference approaches zero. For the heat-recovery steam generator in Fig. 7.12, this source of exergy destruction exacts an economic penalty in terms of fuel cost. Figure 7.13 shows the annual *fuel* cost attributed to the irreversibility of the heat exchanger as a function of $\Delta T_{ave}$. The fuel cost increases with increasing $\Delta T_{ave}$, because the irreversibility is directly related to the temperature difference.

From the study of heat transfer, we know that there is an inverse relation between $\Delta T_{ave}$ and the surface area required for a specified heat transfer rate. More heat transfer area means a larger, more costly heat exchanger—that is, a greater *capital* cost. Figure 7.13 also shows the annualized *capital cost* of the heat exchanger as a function of $\Delta T_{ave}$. The capital cost decreases as $\Delta T_{ave}$ increases.




▲ **Figure 7.12** Figure used to illustrate the use of exergy in design.




▲ **Figure 7.13** Cost curves for a single heat exchanger.

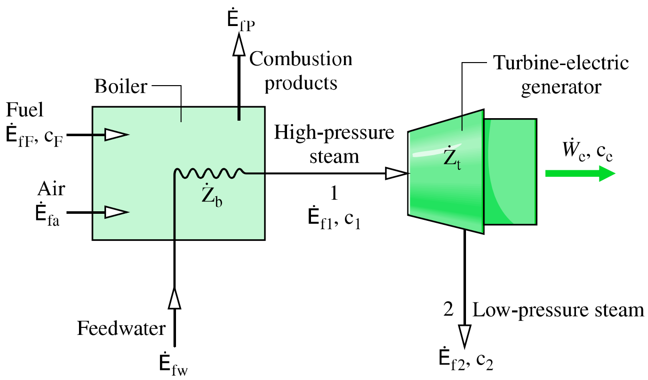
The *total cost* is the sum of the capital cost and the fuel cost. The total cost curve shown in Fig. 7.13 exhibits a minimum at the point labeled a. Notice, however, that the curve is relatively flat in the neighborhood of the minimum, so there is a range of $\Delta T_{ave}$ values that could be considered *nearly optimal* from the standpoint of minimum total cost. If reducing the fuel cost were deemed more important than minimizing the capital cost, we might choose a design that would operate at point a′. Point a″ would be a more desirable operating point if capital cost were of greater concern. Such trade-offs are common in design situations.

The actual design process can differ significantly from the simple case considered here. For one thing, costs cannot be determined as precisely as implied by the curves in Fig. 7.13. Fuel prices may vary widely over time, and equipment costs may be difficult to predict as they often depend on a bidding procedure. Equipment is manufactured in discrete sizes, so the cost also would not vary continuously as shown in the figure. Furthermore, thermal systems usually consist of several components that interact with one another. Optimization of components individually, as considered for the heat exchanger, usually does not guarantee an optimum for the overall system. Finally, the example involves only $\Delta T_{ave}$ as a design variable. Often, several design variables must be considered and optimized simultaneously.

## ▶ 7.7.2 Exergy Costing of a Cogeneration System

Another important aspect of thermoeconomics is the use of exergy for *allocating* costs to the products of a thermal system. This involves assigning to each product the total cost to produce it, namely the cost of fuel and other inputs plus the cost of owning and operating the system (e.g., capital cost, operating and maintenance costs). Such costing is a common problem in plants where utilities such as electrical power, chilled water, compressed air, and steam are generated in one department and used in others. The plant operator needs to know the cost of generating each utility to ensure that the other departments are charged properly according to the type and amount of each utility used. Common to all such considerations are fundamentals from engineering economics, including procedures for annualizing costs, appropriate means for allocating costs, and reliable cost data.

To explore further the costing of thermal systems, consider the simple *cogeneration system* operating at steady state shown in Fig. 7.14. The system consists of a boiler and a turbine, with each having no significant heat transfer to its surroundings. The figure is labeled with exergy




◀ **Figure 7.14** Simple cogeneration system.

transfer rates associated with the flowing streams, where the subscripts F, a, P, and w denote fuel, combustion air, combustion products, and feedwater, respectively. The subscripts 1 and 2 denote high- and low-pressure steam, respectively. Means for evaluating the exergies of the fuel and combustion products are introduced in Chap. 13. The cogeneration system has two principal products: electricity, denoted by $\dot{W}_e$, and low-pressure steam for use in some process. The objective is to determine the cost at which each product is generated.

**BOILER ANALYSIS.** Let us begin by evaluating the cost of the high-pressure steam produced by the boiler. For this, we consider a control volume enclosing the boiler. Fuel and air enter the boiler separately and combustion products exit. Feedwater enters and high-pressure steam exits. The total cost to produce the exiting streams equals the total cost of the entering streams plus the cost of owning and operating the boiler. This is expressed by the following ***cost rate balance*** for the boiler

$$\dot{C}_1 + \dot{C}_P = \dot{C}_F + \dot{C}_a + \dot{C}_w + \dot{Z}_b \tag{7.48}$$

***cost rate balance***

where $\dot{C}$ is the cost rate of the respective stream and $\dot{Z}_b$ accounts for the cost rate associated with owning and operating the boiler (each in \$ per hour, for example). In the present discussion, the cost rate $\dot{Z}_b$ is presumed known from a previous economic analysis.

Although the cost rates denoted by $\dot{C}$ in Eq. 7.48 are evaluated by various means in practice, the present discussion features the use of exergy for this purpose. Since exergy measures the true thermodynamic values of the work, heat, and other interactions between a system and its surroundings as well as the effect of irreversibilities within the system, exergy is a rational basis for assigning costs. With exergy costing, each of the cost rates is evaluated in terms of the associated rate of exergy transfer and a *unit cost*. Thus, for an entering or exiting stream, we write

$$\dot{C} = c\dot{E}_f \tag{7.49}$$

where c denotes the ***cost per unit of exergy*** (in cents per kW · h, for example) and $\dot{E}_f$ is the associated exergy transfer rate.

***exergy unit cost***

For simplicity, we assume the feedwater and combustion air enter the boiler with negligible exergy and cost, and the combustion products are discharged directly to the surroundings with negligible cost. Thus Eq. 7.48 reduces as follows

$$\dot{C}_1 + \cancel{\dot{C}_P}^{\,0} = \dot{C}_F + \cancel{\dot{C}_a}^{\,0} + \cancel{\dot{C}_w}^{\,0} + \dot{Z}_b$$

Then, with Eq. 7.49 we have

$$c_1\dot{E}_{f1} = c_F\dot{E}_{fF} + \dot{Z}_b \tag{7.50a}$$

Solving for $c_1$, the unit cost of the high-pressure steam is

$$c_1 = c_F\left(\frac{\dot{E}_{fF}}{\dot{E}_{f1}}\right) + \frac{\dot{Z}_b}{\dot{E}_{f1}} \tag{7.50b}$$

This equation shows that the unit cost of the high-pressure steam is determined by two contributions related, respectively, to the cost of the fuel and the cost of owning and operating the boiler. Due to exergy destruction and loss, less exergy exits the boiler with the high-pressure steam than enters with the fuel. Thus, $\dot{E}_{fF}/\dot{E}_{f1}$ is invariably greater than one, and the unit cost of the high-pressure steam is invariably greater than the unit cost of the fuel.

**TURBINE ANALYSIS.** Next, consider a control volume enclosing the turbine. The total cost to produce the electricity and low-pressure steam equals the cost of the entering high-pressure steam plus the cost of owing and operating the device. This is expressed by the *cost rate balance* for the turbine

$$\dot{C}_e + \dot{C}_2 = \dot{C}_1 + \dot{Z}_t \tag{7.51}$$

where $\dot{C}_e$ is the cost rate associated with the electricity, $\dot{C}_1$ and $\dot{C}_2$ are the cost rates associated with the entering and exiting steam, respectively, and $\dot{Z}_t$ accounts for the cost rate associated with owning and operating the turbine. With exergy costing, each of the cost rates $\dot{C}_e$, $\dot{C}_1$, and $\dot{C}_2$ is evaluated in terms of the associated rate of exergy transfer and a unit cost. Equation 7.51 then appears as

$$c_e\dot{W}_e + c_2\dot{E}_{f2} = c_1\dot{E}_{f1} + \dot{Z}_t \tag{7.52a}$$

The unit cost $c_1$ in Eq. 7.52a is given by Eq. 7.50b. In the present discussion, the same unit cost is assigned to the low-pressure steam; that is, $c_2 = c_1$. This is done on the basis that the purpose of the turbine is to generate electricity, and thus all costs associated with owning and operating the turbine should be charged to the power generated. We can regard this decision as a part of the *cost accounting* considerations that accompany the thermoeconomic analysis of thermal systems. With $c_2 = c_1$, Eq. 7.52a becomes

$$c_e\dot{W}_e = c_1(\dot{E}_{f1} - \dot{E}_{f2}) + \dot{Z}_t \tag{7.52b}$$

The first term on the right side accounts for the cost of the exergy used and the second term accounts for the cost of the system itself.

Solving Eq. 7.52b for $c_e$, and introducing the exergetic turbine efficiency $\varepsilon$ from Eq. 7.42

$$c_e = \frac{c_1}{\varepsilon} + \frac{\dot{Z}_t}{\dot{W}_e} \tag{7.52c}$$

This equation shows that the unit cost of the electricity is determined by the cost of the high-pressure steam and the cost of owning and operating the turbine. Because of exergy destruction within the turbine, the exergetic efficiency is invariably less than one, and therefore the unit cost of electricity is invariably greater than the unit cost of the high-pressure steam.

**SUMMARY.** By applying cost rate balances to the boiler and the turbine, we are able to determine the cost of each product of the cogeneration system. The unit cost of the electricity is determined by Eq. 7.52c and the unit cost of the low-pressure steam is determined by the expression $c_2 = c_1$ together with Eq. 7.50b. The example to follow provides a detailed illustration. The same general approach is applicable for costing the products of a wide-ranging class of thermal systems.[1]

[1]See A. Bejan, G. Tsatsaronis, and M. J. Moran, *Thermal Design and Optimization,* John Wiley & Sons, New York, 1996.

## EXAMPLE 7.10 Exergy Costing of a Cogeneration System

A cogeneration system consists of a natural gas-fueled boiler and a steam turbine that develops power and provides steam for an industrial process. At steady state, fuel enters the boiler with an exergy rate of 100 MW. Steam exits the boiler at 50 bar, 466°C with an exergy rate of 35 MW. Steam exits the turbine at 5 bar, 205°C and a mass flow rate of 26.15 kg/s. The unit cost of the fuel is 1.44 cents per kW · h of exergy. The costs of owning and operating the boiler and turbine are, respectively, \$1080/h and \$92/h. The feedwater and combustion air enter with negligible exergy and cost. The combustion products are discharged directly to the surroundings with negligible cost. Heat transfer with the surroundings and kinetic and potential energy effects are negligible. Let $T_0 = 298$ K.

**(a)** For the turbine, determine the power and the rate exergy exits with the steam, each in MW.

**(b)** Determine the unit costs of the steam exiting the boiler, the steam exiting the turbine, and the power, each in cents per kW · h of exergy.

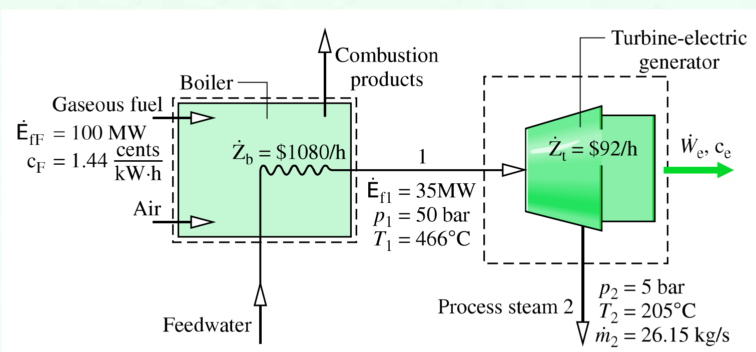
**(c)** Determine the cost rates of the steam exiting the turbine and the power, each in \$/h.

### SOLUTION

***Known:*** Steady-state operating data are known for a cogeneration system that produces both electricity and low-pressure steam for an industrial process.

***Find:*** For the turbine, determine the power and the rate exergy exits with the steam. Determine the unit costs of the steam exiting the boiler, the steam exiting the turbine, and the power developed. Also determine the cost rates of the low-pressure steam and power.

***Schematic and Given Data:***




◄ **Figure E7.10**

***Assumptions:***

1. Each control volume shown in the accompanying figure is at steady state.
2. For each control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.
3. The feedwater and combustion air enter the boiler with negligible exergy and cost.
4. The combustion products are discharged directly to the surroundings with negligible cost.
5. For the environment, $T_0 = 298$ K.

***Analysis:***

**(a)** With assumption 2, the mass and energy rate balances for a control volume enclosing the turbine reduce at steady state to give

$$\dot{W}_e = \dot{m}(h_1 - h_2)$$

From Table A-4, $h_1 = 3353.54$ kJ/kg and $h_2 = 2865.96$ kJ/kg. Thus

$$\dot{W}_e = \left(26.15\frac{\text{kg}}{\text{s}}\right)(3353.54 - 2865.96)\left(\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right|$$
$$= 12.75\text{ MW}$$

Using Eq. 7.36, the difference in the rates exergy enters and exits the turbine with the steam is

$$\dot{\mathsf{E}}_{f2} - \dot{\mathsf{E}}_{f1} = \dot{m}(\mathsf{e}_{f2} - \mathsf{e}_{f1})$$
$$= \dot{m}[h_2 - h_1 - T_0(s_2 - s_1)]$$

Solving for $\dot{\mathsf{E}}_{f2}$

$$\dot{\mathsf{E}}_{f2} = \dot{\mathsf{E}}_{f1} + \dot{m}[h_2 - h_1 - T_0(s_2 - s_1)]$$

With known values for $\dot{\mathsf{E}}_{f1}$ and $\dot{m}$, and data from Table A-4: $s_1 = 6.8773$ kJ/kg · K and $s_2 = 7.0806$ kJ/kg · K, the rate exergy exits with the steam is

$$\dot{\mathsf{E}}_{f2} = 35\text{ MW} + \left(26.15\frac{\text{kg}}{\text{s}}\right)\left[(2865.96 - 3353.54)\frac{\text{kJ}}{\text{kg}} - 298\text{ K}\,(7.0806 - 6.8773)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right|$$
$$= 20.67\text{ MW}$$

**(b)** For a control volume enclosing the boiler, the cost rate balance reduces with assumptions 3 and 4 to give

$$c_1\dot{\mathsf{E}}_{f1} = c_F\dot{\mathsf{E}}_{fF} + \dot{Z}_b$$

where $\dot{\mathsf{E}}_{fF}$ is the exergy rate of the entering fuel, $c_F$ and $c_1$ are the unit costs of the fuel and exiting steam, respectively, and $\dot{Z}_b$ is the cost rate associated with the owning and operating the boiler. Solving for $c_1$ and inserting known values

$$c_1 = c_F\left(\frac{\dot{\mathsf{E}}_{fF}}{\dot{\mathsf{E}}_{f1}}\right) + \frac{\dot{Z}_b}{\dot{\mathsf{E}}_{f1}}$$
$$= \left(1.44\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left(\frac{100\text{ MW}}{35\text{ MW}}\right) + \left(\frac{1080\text{ \$/h}}{35\text{ MW}}\right)\left|\frac{1\text{ MW}}{10^3\text{ kW}}\right|\left|\frac{100\text{ cents}}{1\$}\right|$$
$$= (4.11 + 3.09)\frac{\text{cents}}{\text{kW}\cdot\text{h}} = 7.2\frac{\text{cents}}{\text{kW}\cdot\text{h}}$$

The cost rate balance for the control volume enclosing the turbine is

$$c_e\dot{W}_e + c_2\dot{\mathsf{E}}_{f2} = c_1\dot{\mathsf{E}}_{f1} + \dot{Z}_t$$

where $c_e$ and $c_2$ are the unit costs of the power and the exiting steam, respectively, and $\dot{Z}_t$ is the cost rate associated with owning and operating the turbine. Assigning the same unit cost to the steam entering and exiting the turbine, $c_2 = c_1 =$
❶ 7.2 cents/kW · h, and solving for $c_e$

$$c_e = c_1\left[\frac{\dot{\mathsf{E}}_{f1} - \dot{\mathsf{E}}_{f2}}{\dot{W}_e}\right] + \frac{\dot{Z}_t}{\dot{W}_e}$$

Inserting known values

$$c_e = \left(7.2\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left[\frac{(35 - 20.67)\text{ MW}}{12.75\text{ MW}}\right] + \left(\frac{92\text{\$/h}}{12.75\text{ MW}}\right)\left|\frac{1\text{ MW}}{10^3\text{ kW}}\right|\left|\frac{100\text{ cents}}{1\$}\right|$$

❷
$$= (8.09 + 0.72)\frac{\text{cents}}{\text{kW}\cdot\text{h}} = 8.81\frac{\text{cents}}{\text{kW}\cdot\text{h}}$$

**(c)** For the low-pressure steam and power, the cost rates are, respectively

$$\dot{C}_2 = c_2\dot{E}_{f2}$$
$$= \left(7.2\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(20.67\text{ MW})\left|\frac{10^3\text{ kW}}{1\text{ MW}}\right|\left|\frac{1\$}{100\text{ cents}}\right|$$
❸ $$= \$1488/\text{h}$$
$$\dot{C}_e = c_e\dot{W}_e$$
$$= \left(8.81\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(12.75\text{ MW})\left|\frac{10^3\text{ kW}}{1\text{ MW}}\right|\left|\frac{1\$}{100\text{ cents}}\right|$$
$$= \$1123/\text{h}$$

❶ The purpose of the turbine is to generate power, and thus all costs associated with owning and operating the turbine are charged to the power generated.

❷ Observe that the unit costs $c_1$ and $c_e$ are significantly greater than the unit cost of the fuel.

❸ Although the unit cost of the steam is less than the unit cost of the power, the steam *cost* rate is greater because the associated exergy rate is much greater.

## Chapter Summary and Study Guide

In this chapter, we have introduced the property exergy and illustrated its use for thermodynamic analysis. Like mass, energy, and entropy, exergy is an extensive property that can be transferred across system boundaries. Exergy transfer accompanies heat transfer, work and mass flow. Like entropy, exergy is not conserved. Exergy is destroyed within systems whenever internal irreversibilities are present. Entropy production corresponds to exergy destruction.

The use of exergy balances is featured in this chapter. Exergy balances are expressions of the second law that account for exergy in terms of exergy transfers and exergy destruction. For processes of closed systems, the exergy balance is Eq. 7.11 and a corresponding rate form is Eq. 7.17. For control volumes, rate forms include Eq. 7.31 and the companion steady-state expressions given by Eqs. 7.32. Control volume analyses account for exergy transfer at inlets and exits in terms of flow exergy.

The following checklist provides a study guide for this chapter. When your study of the text and end-of-chapter exercises has been completed you should be able to

- write out meanings of the terms listed in the margins throughout the chapter and understand each of the related concepts. The subset of key concepts listed below is particularly important.
- evaluate exergy at a given state using Eq. 7.2 and exergy change between two states using Eq. 7.10, each relative to a specified reference environment.
- apply exergy balances in each of several alternative forms, appropriately modeling the case at hand, correctly observing sign conventions, and carefully applying SI and English units.
- evaluate the specific flow exergy relative to a specified reference environment using Eq. 7.20.
- define and evaluate exergetic efficiencies for thermal system components of practical interest.
- apply exergy costing to heat loss and simple cogeneration systems.

## Key Engineering Concepts

## Exercises: Things Engineers Think About

**1.** When you hear the term "energy crisis" used by the news media, do the media really mean *exergy* crisis?

**2.** For each case illustrated in Fig. 5.1 (Sec. 5.1), identify the relevant intensive property difference between the system and its surroundings that underlies the *potential for work*. For cases (a) and (b) discuss whether work could be developed if the particular intensive property value for the system were *less* than for the surroundings.

**3.** Is it possible for exergy to be negative? For exergy *change* to be negative?

**4.** Does an airborne, helium-filled balloon at temperature $T_0$ and pressure $p_0$ have exergy?

**5.** Does a system consisting of an evacuated space of volume $V$ have exergy?

**6.** When an automobile brakes to rest, what happens to the exergy associated with its motion?

**7.** Can an energy transfer by heat and the associated exergy transfer be in opposite directions? Repeat for work.

**8.** When evaluating exergy destruction, is it *necessary* to use an exergy balance?

**9.** For a stream of matter, how does the definition of flow exergy parallel the definition of enthalpy?

**10.** Is it possible for the flow exergy to be negative?

**11.** Does the exergetic efficiency given by Eq. 7.45 apply when *both* the hot and cold streams are at temperatures *below* $T_0$?

**12.** A gasoline-fueled generator is claimed by its inventor to produce electricity at a lower unit cost than the unit cost of the fuel used, where each cost is based on exergy. Comment.

**13.** A convenience store sells gasoline and bottled drinking water at nearly the same price per gallon. Comment.

## Problems: Developing Engineering Skills

### Evaluating Exergy

**7.1** A system consists of 5 kg of water at 10°C and 1 bar. Determine the exergy, in kJ, if the system is at rest and zero elevation relative to an exergy reference environment for which $T_0 = 20°C$, $p_0 = 1$ bar.

**7.2** Determine the exergy, in kJ, at 0.7 bar, 90°C for 1 kg of **(a)** water, **(b)** Refrigerant 134a, **(c)** air as an ideal gas with $c_p$ constant. In each case, the mass is at rest and zero evaluation relative to an exergy reference environment for which $T_0 = 20°C$, $p_0 = 1$ bar.

**7.3** Determine the specific exergy, in kJ/kg, at 0.01°C of water as a **(a)** saturated vapor, **(b)** saturated liquid, **(c)** saturated solid. In each case, consider a fixed mass at rest and zero elevation relative to an exergy reference environment for which $T_0 = 20°C$, $p_0 = 1$ bar.

**7.4** A balloon filled with helium at 20°C, 1 bar and a volume of 0.5 $m^3$ is moving with a velocity of 15 m/s at an elevation of 0.5 km relative to an exergy reference environment for which $T_0 = 20°C$, $p_0 = 1$ bar. Using the ideal gas model, determine the specific exergy of the helium, in kJ.

**7.5** Determine the specific exergy, in kJ/kg, at 0.6 bar, −10°C of **(a)** ammonia, **(b)** Refrigerant 22, **(c)** Refrigerant 134a. Let $T_0 = 0°C$, $p_0 = 1$ bar and ignore the effects of motion and gravity.

**7.6** Consider a two-phase solid–vapor mixture of water at −10°C. Each phase present has the same mass. Determine the specific exergy, in kJ/kg, if $T_0 = 20°C$, $p_0 = 1$ atm, and there are no significant effects of motion or gravity.

**7.7** Determine the exergy, in kJ, of the contents of a 2-$m^3$ storage tank, if the tank is filled with

**(a)** air as an ideal gas at 400°C and 0.35 bar.
**(b)** water vapor at 400°C and 0.35 bar.

Ignore the effects of motion and gravity and let $T_0 = 17°C$, $p_0 = 1$ atm.

**7.8** Air as an ideal gas is stored in a closed vessel of volume $V$ at temperature $T_0$ and pressure $p$.

**(a)** Ignoring motion and gravity, obtain the following expression for the exergy of the air:

$$\mathsf{E} = p_0 V\left(1 - \frac{p}{p_0} + \frac{p}{p_0}\ln\frac{p}{p_0}\right)$$

**(b)** Using the result of part (a), plot $V$, in $m^3$, versus $p/p_0$ for $\mathsf{E} = 1$ kW · h and $p_0 = 1$ bar.
**(c)** Discuss your plot in the limits as $p/p_0 \to \infty$, $p/p_0 \to 1$, and $p/p_0 \to 0$.

**7.9** An ideal gas is stored in a closed vessel at pressure $p$ and temperature $T$.

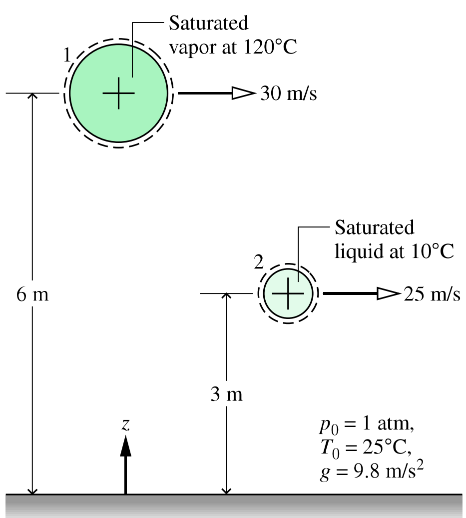
**(a)** If $T = T_0$, derive an expression for the specific exergy in terms of $p$, $p_0$, $T_0$, and the gas constant $R$.
**(b)** If $p = p_0$, derive an expression for the specific exergy in terms of $T$, $T_0$, and the specific heat $c_p$, which can be taken as constant.

Ignore the effects of motion and gravity.

**7.10** Equal molar amounts of carbon dioxide and helium are maintained at the same temperature and pressure. Which has the greater value for exergy relative to the same reference environment? Assume the ideal gas model with constant $c_v$ for each gas. There are no significant effects of motion and gravity.

7.11 Refrigerant 134a vapor initially at 1 bar and 20°C fills a rigid vessel. The vapor is cooled until the temperature becomes −32°C. There is no work during the process. For the refrigerant, determine the heat transfer per unit mass and the change in specific exergy, each in kJ/kg. Comment. Let $T_0 = 20°C$, $p_0 = 0.1$ MPa.

7.12 As shown in Fig. P7.12, two kilograms of water undergo a process from an initial state where the water is saturated vapor at 120°C, the velocity is 30 m/s, and the elevation is 6 m to a final state where the water is saturated liquid at 10°C, the velocity is 25 m/s, and the elevation is 3 m. Determine in kJ, (a) the exergy at the initial state, (b) the exergy at the final state, and (c) the change in exergy. Take $T_0 = 25°C$, $p_0 = 1$ atm and $g = 9.8$ m/s$^2$.




▲ **Figure P7.12**

7.13 Consider 1 kg of steam initially at 20 bar and 240°C as the system. Determine the change in exergy, in kJ, for each of the following processes:

(a) The system is heated at constant pressure until its volume doubles.

(b) The system expands isothermally until its volume doubles.

Let $T_0 = 20°C$, $p_0 = 1$ bar.

7.14 A flywheel with a moment of inertia of 6.74 kg · m$^2$ rotates as 3000 RPM. As the flywheel is braked to rest, its rotational kinetic energy is converted entirely to internal energy of the brake lining. The brake lining has a mass of 2.27 kg and can be regarded as an incompressible solid with a specific heat $c = 4.19$ kJ/kg · K. There is no significant heat transfer with the surroundings. (a) Determine the final temperature of the brake lining, in °C, if its initial temperature is 16°C. (b) Determine the maximum possible rotational speed, in RPM, that could be attained by the flywheel using energy stored in the brake lining after the flywheel has been braked to rest. Let $T_0 = 16°C$.

## Applying the Exergy Balance: Closed Systems

7.15 One kilogram of water initially at 1.5 bar and 200°C cools at constant pressure with no internal irreversibilities to a final state where the water is a saturated liquid. For the water as the system, determine the work, the heat transfer, and the amounts of exergy transfer accompanying work and heat transfer, each in kJ. Let $T_0 = 20°C$, $p_0 = 1$ bar.

7.16 One kilogram of air initially at 1 bar and 25°C is heated at constant pressure with no internal irreversibilities to a final temperature of 177°C. Employing the ideal gas model, determine the work, the heat transfer, and the amounts of exergy transfer accompanying work and heat transfer, each in kJ. Let $T_0 = 298$ K, $p_0 = 1$ bar.

7.17 One kilogram of helium initially at 20°C and 1 bar is contained within a rigid, insulated tank. The helium is stirred by a paddle wheel until its pressure is 1.45 bar. Employing the ideal gas model, determine the work and the exergy destruction for the helium, each in kJ. Neglect kinetic and potential energy and let $T_0 = 20°C$, $p_0 = 1$ bar.

7.18 A rigid, well-insulated tank consists of two compartments, each having the same volume, separated by a valve. Initially, one of the compartments is evacuated and the other contains 0.25 kmol of nitrogen gas at 0.35 MPa and 38°C. The valve is opened and the gas expands to fill the total volume, eventually achieving an equilibrium state. Using the ideal gas model for the nitrogen

(a) determine the final temperature, in °C, and final pressure, in MPa.

(b) evaluate the exergy destruction, in kJ.

(c) What is the cause of exergy destruction in this case?

Let $T_0 = 21°C$, $p_0 = 1$ bar.

7.19 One kilogram of Refrigerant 134a is compressed adiabatically from the saturated vapor state at −10°C to a final state where the pressure is 8 bar and the temperature is 50°C. Determine the work and the exergy destruction, each in kJ/kg. Let $T_0 = 20°C$, $p_0 = 1$ bar.

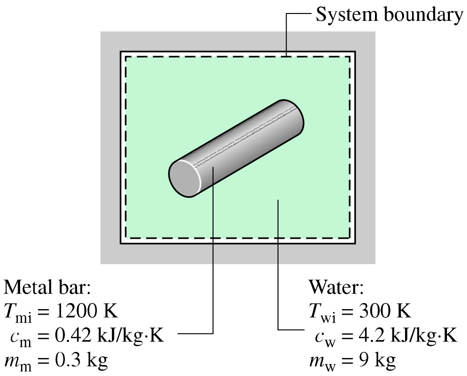
7.20 Two solid blocks, each having mass $m$ and specific heat $c$, and initially at temperatures $T_1$ and $T_2$, respectively, are brought into contact, insulated on their outer surfaces, and allowed to come into thermal equilibrium.

(a) Derive an expression for the exergy destruction in terms of $m$, $c$, $T_1$, $T_2$, and the temperature of the environment, $T_0$.

(b) Demonstrate that the exergy destruction cannot be negative.

(c) What is the cause of exergy destruction in this case?

7.21 As shown in Fig. P7.21, a 0.3 kg metal bar initially at 1200 K is removed from an oven and quenched by immersing it in a closed tank containing 9 kg of water initially at 300 K. Each substance can be modeled as incompressible. An appropriate constant specific heat for the water is $c_w = 4.2$ kJ/kg · K, and an appropriate value for the metal is $c_m = .42$ kJ/kg · K. Heat transfer from the tank contents can be neglected. Determine the exergy destruction, in kJ. Let $T_0 = 25°C$.

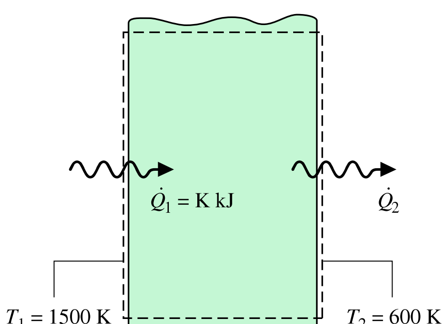



▲ **Figure P7.21**

**7.22** As shown in Fig. P7.22, heat transfer equal to 5 kJ takes place through the inner surface of a wall. Measurements made during steady-state operation reveal temperatures of $T_1 = 1500$ K and $T_2 = 600$ K at the inner and outer surfaces, respectively. Determine, in kJ

(a) the rates of exergy transfer accompanying heat at the inner and outer surfaces of the wall.
(b) the rate of exergy destruction.
(c) What is the cause of exergy destruction in this case?

Let $T_0 = 300$ K.




▲ **Figure P7.22**

**7.23** For the gearbox of Example 6.4(b), develop a full exergy accounting of the power input. Compare with the results of Example 7.4 and discuss. Let $T_0 = 293$ K.

**7.24** The following steady-state data are claimed for two devices:

***Device 1.*** Heat transfer to the device occurs at a place on its surface where the temperature is 52°C. The device delivers electricity to its surroundings at the rate of 10 kW. There are no other energy transfers.

***Device 2.*** Electricity is supplied to the device at the rate of 10 kW. Heat transfer from the device occurs at a place on its surface where the temperature is 52°C. There are no other energy transfers.

For each device, evaluate, in kW, the rates of exergy transfer accompanying heat and work, and the rate of exergy destruction. Can either device operate as claimed? Let $T_0 = 27$°C.

**7.25** For the silicon chip of Example 2.5, determine the rate of exergy destruction, in kW. What causes exergy destruction in this case? Let $T_0 = 293$ K.

**7.26** Two kilograms of a two-phase liquid–vapor mixture of water initially at 300°C and $x_1 = 0.5$ undergo the two different processes described below. In each case, the mixture is brought from the initial state to a saturated vapor state, while the volume remains constant. For each process, determine the change in exergy of the water, the net amounts of exergy transfer by work and heat, and the amount of exergy destruction, each in kJ. Let $T_0 = 300$ K, $p_0 = 1$ bar, and ignore the effects of motion and gravity. Comment on the difference between the exergy destruction values.

(a) The process is brought about adiabatically by stirring the mixture with a paddle wheel.
(b) The process is brought about by heat transfer from a thermal reservoir at 900 K. The temperature of the water at the location where the heat transfer occurs is 900 K.

**7.27** For the water heater of Problem 6.48, determine the exergy transfer and exergy destruction, each in kJ, for

(a) the water as the system.
(b) the overall water heater including the resistor as the system.

Compare the results of parts (a) and (b), and discuss. Let $T_0 = 20$°C, $p_0 = 1$ bar.

**7.28** For the electric motor of Problem 6.50, evaluate the rate of exergy destruction and the rate of exergy transfer accompanying heat, each in kW. Express each quantity as a percentage of the electrical power supplied to the motor. Let $T_0 = 293$ K.

**7.29** A thermal reservoir at 1200 K is separated from another thermal reservoir at 300 K by a cylindrical rod insulated on its lateral surfaces. At steady state, energy transfer by conduction takes place through the rod. The rod diameter is 2 cm, the length is $L$, and the thermal conductivity is 0.4 kW/m · K. Plot the following quantities, each in kW, versus $L$ ranging from 0.01 to 1 m: the rate of conduction through the rod, the rates of exergy transfer accompanying heat transfer into and out of the rod, and the rate of exergy destruction. Let $T_0 = 300$ K.

**7.30** A system undergoes a refrigeration cycle while receiving $Q_C$ by heat transfer at temperature $T_C$ and discharging energy $Q_H$ by heat transfer at a higher temperature $T_H$. There are no other heat transfers.

(a) Using an exergy balance, show that the net work input to the cycle cannot be zero.
(b) Show that the coefficient of performance of the cycle can be expressed as

$$\beta = \left(\frac{T_C}{T_H - T_C}\right)\left(1 - \frac{T_H \mathsf{E}_d}{T_0(Q_H - Q_C)}\right)$$

where $\mathsf{E}_d$ is the exergy destruction and $T_0$ is the temperature of the exergy reference environment.

(c) Using the result of part (b), obtain an expression for the maximum theoretical value for the coefficient of performance.

## Applying the Exergy Balance: Control Volumes

7.31 The following conditions represent the state at the inlet to a control volume. In each case, evaluate the specific exergy and the specific flow exergy, each in kJ/kg. The velocity is relative to an exergy reference environment for which $T_0 = 20°C$, $p_0 = 1$ bar. The effect of gravity can be neglected.

(a) water vapor at 100 bar, 520°C, 100 m/s.
(b) Ammonia at 3 bar, 0°C, 5 m/s.
(c) nitrogen ($N_2$) as an ideal gas at 50 bar, 527°C, 200 m/s.

7.32 For an ideal gas with constant specific heat ratio $k$, show that in the absence of significant effects of motion and gravity the specific flow exergy can be expressed as

$$\frac{\mathsf{e}_f}{c_p T_0} = \frac{T}{T_0} - 1 - \ln\frac{T}{T_0} + \ln\left(\frac{p}{p_0}\right)^{(k-1)/k}$$

(a) For $k = 1.2$ develop plots of $\mathsf{e}_f/c_pT_0$ versus $T/T_0$ for $p/p_0 = 0.25, 0.5, 1, 2, 4$. Repeat for $k = 1.3$ and 1.4.
(b) The specific flow exergy can take on negative values when $p/p_0 < 1$. What does a negative value mean physically?

7.33 A geothermal source provides a stream of liquid water at temperature $T$ ($\geq T_0$) and pressure $p$. Using the incompressible liquid model, develop a plot of $\mathsf{e}_f/cT_0$, where $\mathsf{e}_f$ is the specific flow exergy and $c$ is the specific heat, versus $T/T_0$ for $p/p_0 = 1.0, 1.5$, and 2.0. Neglect the effects of motion and gravity. Let $T_0 = 60°F$, $p_0 = 1$ atm.

7.34 The state of a flowing gas is defined by $h$, $s$, V, and $z$, where velocity and elevation are relative to an exergy reference environment for which the temperature is $T_0$ and the pressure is $p_0$. Determine the *maximum* theoretical work, per unit mass of gas flowing, that could be developed by any one-inlet, one-exit control volume at steady state that would reduce the stream to the dead state at the exit while allowing heat transfer only at $T_0$. Using your final expression, interpret the specific flow exergy.

7.35 Steam exits a turbine with a mass flow rate of $2 \times 10^5$ kg/h at a pressure of 0.008 MPa, a quality of 94%, and a velocity of 70 m/s. Determine the *maximum* theoretical power that could be developed, in MW, by any one-inlet, one-exit control volume at steady state that would reduce the steam to the dead state at the exit while allowing heat transfer only at temperature $T_0$. The velocity is relative to an exergy reference environment for which $T_0 = 15°C$, $p_0 = 0.1$ MPa. Neglect the effect of gravity.

7.36 Water at 25°C, 1 bar is drawn from a mountain lake 1 km above a valley and allowed to flow through a hydraulic turbine-generator to a pond on the valley floor. For operation at steady state, determine the minimum theoretical mass flow rate, in kg/s, required to generate electricity at a rate of 1 MW. Let $T_0 = 25°C$, $p_0 = 1$ bar.

7.37 Water vapor enters a valve with a mass flow rate of 2.7 kg/s at a temperature of 280°C and a pressure of 30 bar and undergoes a throttling process to 20 bar.

(a) Determine the flow exergy rates at the valve inlet and exit and the rate of exergy destruction, each in kW.
(b) Evaluating exergy at 8 cents per kW · h, determine the annual cost associated with the exergy destruction, assuming 7500 hours of operation annually.

Let $T_0 = 25°C$, $p_0 = 1$ atm.

7.38 Steam enters a turbine operating at steady state at 6 MPa, 500°C with a mass flow rate of 400 kg/s. Saturated vapor exits at 8 kPa. Heat transfer from the turbine to its surroundings takes place at a rate of 8 MW at an average surface temperature of 180°C. Kinetic and potential energy effects are negligible.

(a) For a control volume enclosing the turbine, determine the power developed and the rate of exergy destruction, each in MW.
(b) If the turbine is located in a facility where the ambient temperature is 27°C, determine the rate of exergy destruction for an enlarged control volume that includes the turbine and its immediate surroundings so the heat transfer takes place from the control volume at the ambient temperature. Explain why the exergy destruction values of parts (a) and (b) differ.

Let $T_0 = 300$ K, $p_0 = 100$ kPa.

7.39 Air at 1 bar, 17°C, and a mass flow rate of 0.3 kg/s enters an insulated compressor operating at steady state and exits at 3 bar, 147°C. Determine, the power required by the compressor and the rate of exergy destruction, each in kW. Express the rate of exergy destruction as a percentage of the power required by the compressor. Kinetic and potential energy effects are negligible. Let $T_0 = 17°C$, $p_0 = 1$ bar.

7.40 Refrigerant 134a at −10°C, 1.4 bar, and a mass flow rate of 280 kg/h enters an insulated compressor operating at steady state and exits at 9 bar. The isentropic compressor efficiency is 82%. Determine

(a) the temperature of the refrigerant exiting the compressor, in °C.
(b) the power input to the compressor, in kW.
(c) the rate of exergy destruction expressed as a percentage of the power required by the compressor.

Neglect kinetic and potential energy effects and let $T_0 = 20°C$, $p_0 = 1$ bar.

7.41 Water vapor at 4.0 MPa and 400°C enters an insulated turbine operating at steady state and expands to saturated vapor at 0.1 MPa. Kinetic and potential energy effects can be neglected.

(a) Determine the work developed and the exergy destruction, each in kJ per kg of water vapor passing through the turbine.
(b) Determine the *maximum* theoretical work per unit of mass flowing, in kJ/kg, that could be developed by any one-inlet, one-exit control volume at steady state that has water vapor entering and exiting at the specified states, while allowing heat transfer only at temperature $T_0$.

Compare the results of parts (a) and (b) and comment. Let $T_0 = 27°C$, $p_0 = 0.1$ MPa.

**7.42** Air enters an insulated turbine operating at steady state at 8 bar, 500 K, and 150 m/s. At the exit the conditions are 1 bar, 320 K, and 10 m/s. There is no significant change in elevation. Determine

**(a)** the work developed and the exergy destruction, each in kJ per kg of air flowing.

**(b)** the *maximum* theoretical work, in kJ per kg of air flowing, that could be developed by any one-inlet, one-exit control volume at steady state that has air entering and exiting at the specified states, while allowing heat transfer only at temperature $T_0$.

Compare the results of parts (a) and (b) and comment. Let $T_0 = 300$ K, $p_0 = 1$ bar.

**7.43** For the compressor of Problem 6.84, determine the rate of exergy destruction and the rate of exergy transfer accompanying heat, each in kJ per kg of air flow. Express each as a percentage of the work input to the compressor. Let $T_0 = 20°C$, $p_0 = 1$ atm.

**7.44** A compressor fitted with a water jacket and operating at steady state takes in air with a volumetric flow rate of 900 $m^3$/h at 22°C, 0.95 bar and discharges air at 317°C, 8 bar. Cooling water enters the water jacket at 20°C, 100 kPa with a mass flow rate of 1400 kg/h and exits at 30°C and essentially the same pressure. There is no significant heat transfer from the outer surface of the water jacket to its surroundings, and kinetic and potential energy effects can be ignored. For the water-jacketed compressor, perform a full exergy accounting of the power input. Let $T_0 = 20°C$, $p_0 = 1$ atm.

**7.45** Steam at 1.4 MPa and 350°C with a mass flow rate of 0.125 kg/s enters an insulated turbine operating at steady state and exhausts at 100 kPa. Plot the temperature of the exhaust steam, in °C, the power developed by the turbine, in kW, and the rate of exergy destruction within the turbine, in kW, each versus the isentropic turbine efficiency ranging from 0 to 100%. Neglect kinetic and potential energy effects. Let $T_0 = 20°C$, $p_0 = 0.1$ MPa.

**7.46** If the power-recovery device of Problem 6.102 develops a net power of 6 kW, determine, in kW

**(a)** the rate exergy enters accompanying heat transfer.

**(b)** the *net* rate exergy is carried in by the steam.

**(c)** the rate of exergy destruction within the device.

Let $T_0 = 293$ K, $p_0 = 1$ bar.

**7.47** A counterflow heat exchanger operating at steady state has ammonia entering at 60°C, 14 bar with a mass flow rate of 0.5 kg/s and exiting as saturated liquid at 14 bar. Air enters in a separate stream at 300 K, 1 bar and exits at 335 K with a negligible change in pressure. Heat transfer between the heat exchanger and its surroundings is negligible as are changes in kinetic and potential energy. Determine

**(a)** the change in the flow exergy rate of each stream, in kW.

**(b)** the rate of exergy destruction in the heat exchanger, in kW.

Let $T_0 = 300$ K, $p_0 = 1$ bar.

**7.48** Saturated water vapor at 0.008 MPa and a mass flow rate of $2.6 \times 10^5$ kg/h enters the condenser of a 100-MW power plant and exits as a saturated liquid at 0.008 MPa. The cooling water stream enters at 15°C and exits at 35°C with a negligible change in pressure. At steady state, determine

**(a)** the *net* rate energy exits the plant with the cooling water stream, in MW.

**(b)** the *net* rate exergy exits the plant with the cooling water stream, in MW.

Compare these values. Is the *loss* with the cooling water significant? What are some possible uses for the exiting cooling water? Let $T_0 = 20°C$, $p_0 = 0.1$ MPa.

**7.49** Air enters a counterflow heat exchanger operating at steady state at 22°C, 0.1 MPa and exits at 7°C. Refrigerant 134a enters at 0.2 MPa, a quality of 0.2, and a mass flow rate of 30 kg/h. Refrigerant exits at 0°C. There is no significant change in pressure for either stream.

**(a)** For the Refrigerant 134a stream, determine the rate of heat transfer, in kJ/h.

**(b)** For each of the streams, evaluate the change in flow exergy rate, in kJ/h. Compare the values.

Let $T_0 = 22°C$, $p_0 = 0.1$ MPa, and ignore the effects of motion and gravity.

**7.50** Determine the rate of exergy destruction, in kW, for

**(a)** the computer of Example 4.8, when air exits at 32°C.

**(b)** the computer of Problem 4.70, ignoring the change in pressure between the inlet and exit.

**(c)** the water-jacketed electronics housing of Problem 4.71, when water exits at 24°C.

Let $T_0 = 293$ K, $p_0 = 1$ bar.

**7.51** Determine the rate of exergy destruction, in kW, for the electronics-laden cylinder of Problems 4.73 and 6.108. Let $T_0 = 293$ K, $p_0 = 1$ bar.

**7.52** Helium gas enters an insulated nozzle operating at steady state at 1300 K, 4 bar, and 10 m/s. At the exit, the temperature and pressure of the helium are 900 K and 1.45 bar, respectively. Determine

**(a)** the exit velocity, in m/s.

**(b)** the isentropic nozzle efficiency.

**(c)** the rate of exergy destruction, in kJ per kg of gas flowing through the nozzle.

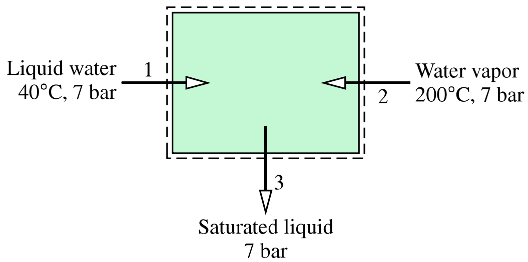
Assume the ideal gas model for helium and ignore the effects of gravity. Let $T_0 = 20°C$, $p_0 = 1$ atm.

**7.53** As shown schematically in Fig. P7.53, an open feedwater heater in a vapor power plant operates at steady state with liquid entering at inlet 1 with $T_1 = 40°C$ and $p_1 = 7.0$ bar. Water vapor at $T_2 = 200°C$ and $p_2 = 7.0$ bar enters at inlet 2. Saturated liquid water exits with a pressure of $p_3 = 7.0$ bar. Ignoring heat transfer with the surroundings and all kinetic and potential energy effects, determine

**(a)** the ratio of mass flow rates, $\dot{m}_1/\dot{m}_2$.

**(b)** the rate of exergy destruction, in kJ per kg of liquid exiting.

Let $T_0 = 25°C$, $p_0 = 1$ atm.




▲ **Figure P7.53**

**7.54** Reconsider the open feedwater heater of Problem 6.87a. For an exiting mass flow rate of 1 kg/s, determine the cost of the exergy destroyed for 8000 hours of operation annually. Evaluate exergy at 8 cents per kW · h. Let $T_0 = 20°C$, $p_0 =$ 1 atm.

**7.55** Steam at 3 MPa and 700°C is available at one location in an industrial plant. At another location, steam at 2 MPa, 400°C, and a mass flow rate of 1 kg/s is required for use in a certain process. An engineer suggests that steam at this condition can be provided by allowing the higher-pressure steam to expand through a valve to 2 MPa and then flow through a heat exchanger where the steam cools at constant pressure to 400°C by heat transfer to the surroundings, which are at 20°C.

**(a)** Determine the total rate of exergy destruction, in kW, that would result from the implementation of this suggestion.

**(b)** Evaluating exergy at 8 cents per kW · h, determine the annual cost of the exergy destruction determined in part (a) for 8000 hours of operation annually.

Would you endorse this suggestion? Let $T_0 = 20°C$, $p_0 =$ 0.1 MPa.

**7.56** For the compressor and turbine of Problem 6.110, determine the rates of exergy destruction, each in kJ per kg of air flowing. Express each as a percentage of the net work developed by the power plant. Let $T_0 = 22°C$, $p_0 = 0.95$ bar.

**7.57** For the turbines and heat exchanger of Problem 4.57, determine the rates of exergy destruction, each in kW. Place in rank order, beginning with the component contributing most to inefficient operation of the overall system. Let $T_0 = 300$ K, $p_0 = 1$ bar.

**7.58** For the turbine, condenser, and pump of Problem 4.59, determine the rates of exergy destruction, each in kW. Place in rank order, beginning with the component contributing most to inefficient operation of the overall system. Let $T_0 = 293$ K, $p_0 = 1$ bar.

**7.59** If the gas turbine power plant of Problem 6.74 develops a net power output of 0.7 MW, determine, in MW,

**(a)** the rate of exergy transfer accompanying heat transfer to the air flowing through the heat exchanger.

**(b)** the *net* rate exergy is carried out by the air stream.

**(c)** the *total* rate of exergy destruction within the power plant.

Let $T_0 = 295$ K (22°C), $p_0 = 0.95$ bar.

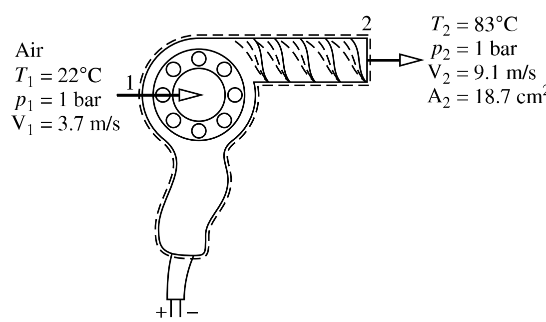
**7.60** For the compressor and heat exchanger of Problem 6.88, develop a full exergy accounting, in kW, of the compressor power input. Let $T_0 = 300$ K, $p_0 = 96$ kPa.

## Using Exergetic Efficiencies

**7.61** Plot the exergetic efficiency given by Eq. 7.39b versus $T_u/T_0$ for $T_s/T_0 = 8.0$ and $\eta = 0.4, 0.6, 0.8, 1.0$. What can be learned from the plot when $T_u/T_0$ is fixed? When $\varepsilon$ is fixed? Discuss.

**7.62** The temperature of water contained in a closed, well-insulated tank is increased from 15 to 50°C by passing an electric current through a resistor within the tank. Devise and evaluate an exergetic efficiency for this water heater. Assume that the water is incompressible and the states of the resistor and the enclosing tank do not change. Let $T_0 = 15°C$.

**7.63** Measurements during steady-state operation indicate that warm air exits a hand-held hair dryer at a temperature of 83°C with a velocity of 9.1 m/s through an area of 18.7 cm$^2$. As shown in Fig. P7.63, air enters the dryer at a temperature of 22°C and a pressure of 1 bar with a velocity of 3.7 m/s. No significant change in pressure between inlet and exit is observed. Also, no significant heat transfer between the dryer and its surroundings occurs, and potential energy effects can be ignored. Let $T_0 = 22°C$. For the hair dryer **(a)** evaluate the power $\dot{W}_{cv}$, in kW, and **(b)** devise and evaluate an exergetic efficiency.




▲ **Figure P7.63**

**7.64** From an input of electricity, an electric resistance furnace operating at steady state delivers energy by heat transfer to a process at the rate $\dot{Q}_u$ at a *use* temperature $T_u$. There are no other significant energy transfers.

**(a)** Devise an exergetic efficiency for the furnace.

**(b)** Plot the efficiency obtained in part (a) versus the use temperature ranging from 300 to 900 K. Let $T_0 = 20°C$.

**7.65** Hydrogen at 25 bar, 450°C enters a turbine and expands to 2 bar, 160°C with a mass flow rate of 0.2 kg/s. The turbine operates at steady state with negligible heat transfer with its surroundings. Assuming the ideal gas model with $k = 1.37$ and neglecting kinetic and potential energies, determine

**(a)** the isentropic turbine efficiency.

(b) the exergetic turbine efficiency.

Let $T_0 = 25°C$, $p_0 = 1$ atm.

**7.66** An ideal gas with constant specific heat ratio $k$ enters a turbine operating at steady state at $T_1$ and $p_1$ and expands adiabatically to $T_2$ and $p_2$. When would the value of the exergetic turbine efficiency exceed the value of the isentropic turbine efficiency? Discuss. Ignore the effects of motion and gravity.

**7.67** Air enters an insulated turbine operating at steady state with a pressure of 4 bar, a temperature of 450 K, and a volumetric flow rate of 5 $m^3/s$. At the exit, the pressure is 1 bar. The isentropic turbine efficiency is 84%. Ignoring the effects of motion and gravity, determine

(a) the power developed and the exergy destruction rate, each in kW.
(b) the exergetic turbine efficiency.

Let $T_0 = 20°C$, $p_0 = 1$ bar.

**7.68** Argon enters an insulated turbine operating at steady state at 1000°C and 2 MPa and exhausts at 350 kPa. The mass flow rate is 0.5 kg/s. Plot each of the following versus the turbine exit temperature, in °C

(a) the power developed, in kW.
(b) the rate of exergy destruction in the turbine, in kW.
(c) the exergetic turbine efficiency.

Neglect kinetic and potential energy effects. Let $T_0 = 20°C$, $p_0 = 1$ bar.

**7.69** A compressor operating at steady state takes in 1980 kg/h of air at 1 bar and 25°C and compresses it to 10 bar and 200°C. The power input to the compressor is 160 kW, and heat transfer occurs from the compressor to the surroundings at an average surface temperature of 60°C.

(a) Perform a full exergy accounting of the power input to the compressor.
(b) Devise and evaluate an exergetic efficiency for the compressor.
(c) Evaluating exergy at 8 cents per kW · h, determine the hourly costs of the power input, exergy loss associated with heat transfer, and exergy destruction.

Neglect kinetic and potential energy changes. Let $T_0 = 25°C$, $p_0 = 1$ bar.

**7.70** In the boiler of a power plant are tubes through which water flows as it is brought from 0.8 MPa, 150°C to 240°C at essentially constant pressure. The total mass flow rate of the water is 100 kg/s. Combustion gases passing over the tubes cool from 1067 to 547°C at essentially constant pressure. The combustion gases can be modeled as air as an ideal gas. There is no significant heat transfer from the boiler to its surroundings. Assuming steady state and neglecting kinetic and potential energy effects, determine

(a) the mass flow rate of the combustion gases, in kg/s.
(b) the rate of exergy destruction, in kJ/s.
(c) the exergetic efficiency given by Eq. 7.45.

Let $T_0 = 25°C$, $p_0 = 1$ atm.

**7.71** Liquid water at 95°C, 1 bar enters a direct-contact heat exchanger operating at steady state and mixes with a stream of liquid water entering at 15°C, 1 bar. A single liquid stream exits at 1 bar. The entering streams have equal mass flow rates. Neglecting heat transfer with the surroundings and kinetic and potential energy effects, determine for the heat exchanger

(a) the rate of exergy destruction, in kJ per kg of liquid exiting.
(b) the exergetic efficiency given by Eq. 7.47.

Let $T_0 = 15°C$, $p_0 = 1$ bar.

**7.72** Refrigerant 134a enters a counterflow heat exchanger operating at steady state at −20°C and a quality of 35% and exits as saturated vapor at −20°C. Air enters as a separate stream with a mass flow rate of 4 kg/s and is cooled at a constant pressure of 1 bar from 300 to 260 K. Heat transfer between the heat exchanger and its surroundings can be ignored, as can all changes in kinetic and potential energy.

(a) As in Fig. E7.6, sketch the variation with position of the temperature of each stream. Locate $T_0$ on the sketch.
(b) Determine the rate of exergy destruction within the heat exchanger, in kW.
(c) Devise and evaluate an exergetic efficiency for the heat exchanger.

Let $T_0 = 300$ K, $p_0 = 1$ bar.

**7.73** Determine the exergetic efficiencies of the turbines and heat exchanger of Problem 4.57. Let $T_0 = 300$ K, $p_0 = 1$ bar.

**7.74** Determine the exergetic efficiencies of the compressor and condenser of the heat pump system of Examples 6.8 and 6.14. Let $T_0 = 273$ K, $p_0 = 1$ bar.

**7.75** Determine the exergetic efficiencies of the compressor and heat exchanger of Problem 6.88. Let $T_0 = 300$ K, $p_0 =$ 96 kPa.

**7.76** Determine the exergetic efficiencies of the steam generator and turbine of Examples 4.10 and 7.8. Let $T_0 = 298°C$, $p_0 = 1$ atm.

## Considering Thermoeconomics

**7.77** The total cost rate for a device varies with the pressure drop for flow through the device, $(p_1 - p_2)$, as follows:

$$\dot{C} = c_1(p_1 - p_2)^{-1/3} + c_2(p_1 - p_2)$$

where the c's are constants incorporating economic factors. The first term on the right side of this equation accounts for the capital cost and the second term on the right accounts for the operating cost (pumping power).

(a) Sketch a plot of $\dot{C}$ versus $(p_1 - p_2)$.
(b) At the point of minimum total cost rate, evaluate the contributions of the capital and operating cost rates to the total cost rate, each in percent. Discuss.

**7.78** A system operating at steady state generates electricity at the rate $\dot{W}_e$. The cost rate of the fuel input is $\dot{C}_F = c_F\dot{E}_{fF}$, where $c_F$ is the unit cost of fuel based on exergy. The cost of owning and operating the system is

$$\dot{Z} = c\left(\frac{\varepsilon}{1-\varepsilon}\right)\dot{W}_e$$

where $\varepsilon = \dot{W}_e/\dot{E}_{fF}$, and c is a constant incorporating economic factors. $\dot{C}_F$ and $\dot{Z}$ are the only significant cost rates for the system.

(a) Derive an expression for the unit cost of electricity, $c_e$, based on $\dot{W}_e$ in terms of $\varepsilon$ and the ratios $c_e/c_F$ and $c/c_F$ only.

(b) For fixed $c/c_F$, derive an expression for the value of $\varepsilon$ corresponding to the minimum value of $c_e/c_F$.

(c) Plot the ratio $c_e/c_F$ versus $\varepsilon$ for $c/c_F = 0.25$, 1.0, and 4.0. For each specified $c/c_F$, evaluate the minimum value of $c_e/c_F$ and the corresponding value of $\varepsilon$.

**7.79** At steady state, a turbine with an exergetic efficiency of 85% develops $18 \times 10^7$ kW · h of work annually (8000 operating hours). The annual cost of owning and operating the turbine is $\$5.0 \times 10^5$. The steam entering the turbine has a specific flow exergy of 645 Btu/lb, a mass flow rate of $32 \times 10^4$ lb/h, and is valued at \$0.0182 per kW · h of exergy.

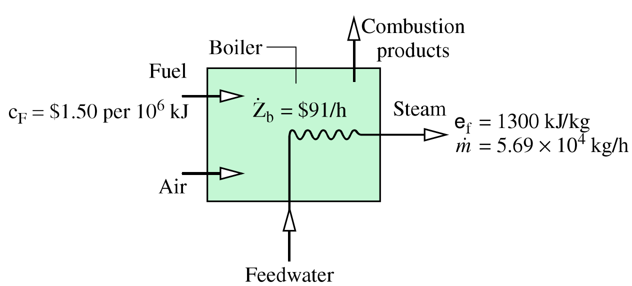
(a) Evaluate the unit cost of the power developed, in \$ per kW · h.

(b) Evaluate the unit cost based on exergy of the steam entering and exiting the turbine, each in cents per lb of steam flowing through the turbine.

**7.80** Figure P7.80 shows a boiler at steady state. Steam having a specific flow exergy of 1300 kJ/kg exits the boiler at a mass flow rate of $5.69 \times 10^4$ kg/h. The cost of owning and operating the boiler is \$91/h. The ratio of the exiting steam exergy to the entering fuel exergy is 0.45. The unit cost of the fuel based on exergy is \$1.50 per $10^6$ kJ. If the cost rates of the combustion air, feedwater, heat transfer with the surroundings, and exiting combustion products are ignored, develop

(a) an expression for the unit cost based on exergy of the steam exiting the boiler.

(b) Using the result of part (a), determine the unit cost of the steam, in cents per kg of steam flowing.



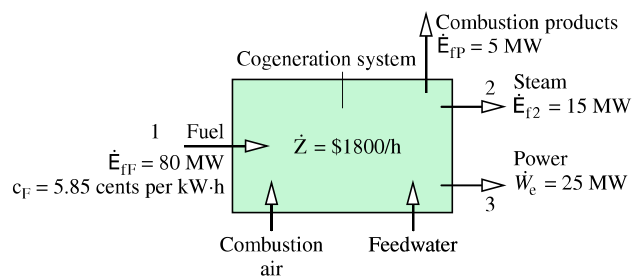

▲ **Figure P7.80**

**7.81** A cogeneration system operating at steady state is shown schematically in Fig. P7.81. The exergy transfer rates of the entering and exiting streams are shown on the figure, in MW. The fuel, produced by reacting coal with steam, has a unit cost of 5.85 cents per kW · h of exergy. The cost of owning and operating the system is \$1800/h. The feedwater and combustion air enter with negligible exergy and cost. The combustion products are discharged directly to the surroundings with negligible cost. Heat transfer with the surroundings can be ignored.

(a) Determine the rate of exergy destruction within the cogeneration system, in MW.

(b) Devise and evaluate an exergetic efficiency for the system.

(c) Assuming the power and steam each have the same unit cost based on exergy, evaluate the unit cost, in cents per kW · h. Also evaluate the cost rates of the power and steam, each in \$/h.




▲ **Figure P7.81**

**7.82** Consider an *overall* control volume comprising the boiler and steam turbine of the cogeneration system of Example 7.10. Assuming the power and process steam each have the same unit cost based on exergy: $c_e = c_2$, evaluate the unit cost, in cents per kW · h. Compare with the respective values obtained in Example 7.10 and comment.

**7.83** The table below gives alternative specifications for the state of the process steam exiting the turbine of Example 7.10. The cost of owning and operating the turbine, in \$/h, varies with the power $\dot{W}_e$, in MW, according to $\dot{Z}_t = 7.2\dot{W}_e$. All other data remain unchanged.

| $p_2$ (bar) | 40 | 30 | 20 | 9 | 5 | 2 | 1 |
|---|---|---|---|---|---|---|---|
| $T_2$(°C) | 436 | 398 | 349 | 262 | 205 | 128 | sat |

Plot versus $p_2$, in bar

(a) the power $\dot{W}_e$, in MW.

(b) the unit costs of the power and process steam, each in cents per kW · h of exergy.

(c) the unit cost of the process steam, in cents per kg of steam flowing.

## Design & Open Ended Problems: Exploring Engineering Practice

**7.1D** A utility charges households the same per kW · h for space heating via steam radiators as it does for electricity. Critically evaluate this costing practice and prepare a memorandum summarizing your principal conclusions.

**7.2D** For what range of steam mass flow rates, in kg/s, would it be economically feasible to replace the throttling valve of Example 7.5 with a power recovery device? Provide supporting calculations. What type of device might you specify? Discuss.

**7.3D** A *vortex tube* is a device having no moving mechanical parts that converts an inlet stream of compressed air at an intermediate temperature into two exiting streams, one cold and one hot.

(a) A product catalogue indicates that 20% of the air entering a vortex tube at 21°C and 5 bar exits at −37°C and 1 bar while the rest exits at 34.2°C and 1 bar. An inventor proposes operating a power cycle between the hot and cold streams. Critically evaluate the feasibility of this proposal.

(b) Obtain a product catalogue from a vortex tube vendor located with the help of the *Thomas Register of American Manufacturers*. What are some of the applications of vortex tubes?

**7.4D** A government agency has solicited proposals for technology in the area of *exergy* harvesting. The aim is to develop small-scale devices to generate power for rugged-duty applications with power requirements ranging from hundreds of milliwatts to several watts. The power must be developed from only *ambient sources,* such as thermal and chemical gradients, naturally occurring fuels (tree sap, plants, waste matter, etc.), wind, solar, sound and vibration, and mechanical motion including human motion. The devices must also operate with little or no human intervention. Devise a system that would meet these requirements. Clearly identify its intended application and explain its operating principles. Estimate its size, weight, and expected power output.

**7.5D** In one common arrangement, the exergy input to a power cycle is obtained by heat transfer from hot products of combustion cooling at approximately constant pressure, while exergy is discharged by heat transfer to water or air at ambient conditions. Devise a theoretical power cycle that at steady state develops the *maximum theoretical* net work per cycle from the exergy supplied by the cooling products of combustion and discharges exergy by heat transfer to the natural environment. Discuss practical difficulties that require actual power plant operation to depart from your theoretical cycle.

**7.6D** Define and evaluate an exergetic efficiency for an electric heat pump system for a 2500 $ft^2$ dwelling in your locale. Use manufacturer's data for heat pump operation.

**7.7D** Using the key words *exergetic efficiency, second law efficiency,* and *rational efficiency,* develop a bibliography of recent publications discussing the definition and use of such efficiencies for power systems and their components. Write a critical review of one of the publications you locate. Clearly state the principal contribution(s) of the publication.

**7.8D** The initial plans for a new factory space specify 1000 fluorescent light fixtures, each with two 8-ft conventional tubes sharing a single magnetic ballast. The lights will operate from 7 AM to 10 PM, 5 days per week, 350 days per year. More expensive high-efficiency tubes are available that require more costly electronic ballasts but use considerably less electricity to operate. Considering both initial and operating costs, determine which lighting system is best for this application, and prepare a report of your findings. Use manufacturer's data and industrial electric rates in your locale to estimate costs. Assume that comparable lighting levels would be achieved by the conventional and high-efficiency lighting.

**7.9D** A factory has a 120 kW screw compressor that takes in 0.5 $m^3/s$ of ambient air and delivers compressed air at 1 MPa for actuating pneumatic tools. The factory manager read in a plant engineering magazine that using compressed air is more expensive than the direct use of electricity for operating such tools and asks for your opinion. Using exergy costing with electric rates in your locale, what do you say?

**7.10D** *Pinch analysis* (or *pinch technology*) is a popular methodology for optimizing the design of heat exchanger networks in complex thermal systems. Pinch analysis uses a primarily graphical approach to implement second-law reasoning. Write a paper discussing the role of pinch analysis within *thermoeconomics*.

**7.11D** **Wind Power Looming Large** (see box Sec. 7.4) Identify sites in your state where wind turbines for utility-scale electrical generation are feasible, considering both engineering and societal issues. Write a report including at least three references.