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Cyclic Stresses (Fatigue)

The *S-N* Curve

Q1 *A fatigue test was conducted in which the mean stress was 50 MPa (7250 psi) and the stress amplitude was 225 MPa (32,625 psi).*

(a) Compute the maximum and minimum stress levels.

(b) Compute the stress ratio.

(c) Compute the magnitude of the stress range.

Solution

(a) Given the values of σ_m (50 MPa) and σ_a (225 MPa) we are asked to compute σ_{max} and σ_{min}. From Equation 8.14

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 50 \text{ MPa}$$

Or,

$$\sigma_{max} + \sigma_{min} = 100 \text{ MPa}$$

Furthermore, utilization of Equation 8.16 yields

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 225 \text{ MPa}$$

Or,

$$\sigma_{max} - \sigma_{min} = 450 \text{ MPa}$$

Simultaneously solving these two expressions leads to

$$\sigma_{max} = 275 \text{ MPa} \ (40{,}000 \text{ psi})$$

$$\sigma_{min} = -175 \text{ MPa} \ (-25{,}500 \text{ psi})$$

(b) Using Equation 8.17 the stress ratio *R* is determined as follows:

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{-175 \text{ MPa}}{275 \text{ MPa}} = -0.64$$

(c) The magnitude of the stress range σ_r is determined using Equation 8.15 as

$$\sigma_r = \sigma_{max} - \sigma_{min} = 275 \text{ MPa} - (-175 \text{ MPa}) = 450 \text{ MPa} \ (65{,}500 \text{ psi})$$

Q2 *The fatigue data for a brass alloy are given as follows:*

Stress Amplitude (MPa)	*Cycles to Failure*
310	2×10^5
223	1×10^6
191	3×10^6
168	1×10^7
153	3×10^7
143	1×10^8
134	3×10^8
127	1×10^9

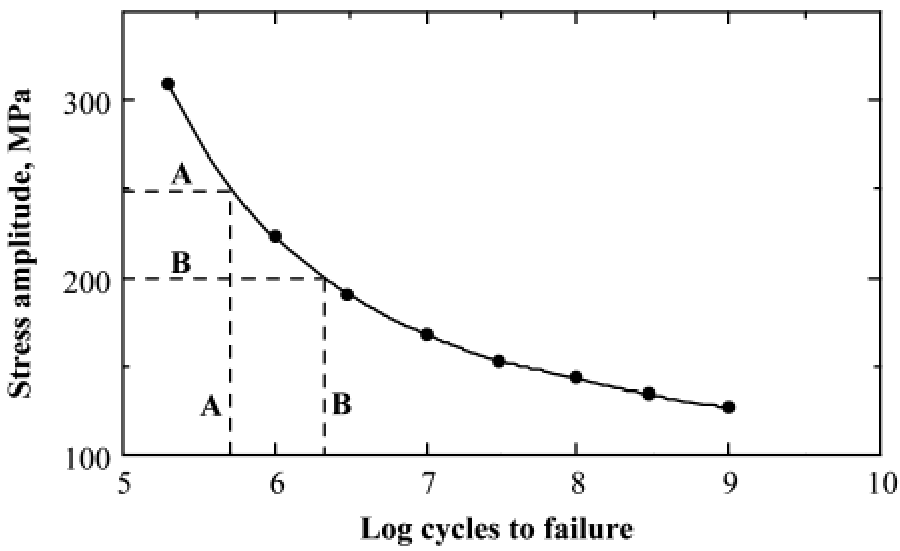
(a) Make an S–N plot (stress amplitude versus logarithm cycles to failure) using these data.

(b) Determine the fatigue strength at 5×10^5 *cycles.*

(c) Determine the fatigue life for 200 MPa.

Solution

(a) The fatigue data for this alloy are plotted below.



(b) As indicated by the "A" set of dashed lines on the plot, the fatigue strength at 5×10^5 cycles [$\log (5 \times 10^5) = 5.7$] is about 250 MPa.

(c) As noted by the "B" set of dashed lines, the fatigue life for 200 MPa is about 2×10^6 cycles (i.e., the log of the lifetime is about 6.3).

Q3 *Suppose that the fatigue data for the brass alloy in Q2 were taken from torsional tests, and that a shaft of this alloy is to be used for a coupling that is attached to an electric motor operating at 1500 rpm. Give the maximum torsional stress amplitude possible for each of the following lifetimes of the coupling: (a) 1 year, (b) 1 month, (c) 1 day, and (d) 2 hours.*

Solution

For each lifetime, first compute the number of cycles, and then read the corresponding fatigue strength from the above plot.

(a) Fatigue lifetime = (1 yr)(365 days/yr)(24 h/day)(60 min/h)(1500 cycles/min) = 7.9×10^8 cycles. The stress amplitude corresponding to this lifetime is about 130 MPa.

(b) Fatigue lifetime = (30 days)(24 h/day)(60 min/h)(1500 cycles/min) = 6.5×10^7 cycles. The stress amplitude corresponding to this lifetime is about 145 MPa.

(c) Fatigue lifetime = (24 h)(60 min/h)(1500 cycles/min) = 2.2×10^6 cycles. The stress amplitude corresponding to this lifetime is about 195 MPa.

(d) Fatigue lifetime = (2 h)(60 min/h)(1500 cycles/min) = 1.8×10^5 cycles. The stress amplitude corresponding to this lifetime is about 315 MPa.

Q4 *The fatigue data for a ductile cast iron are given as follows:*

***Stress Amplitude* [*MPa* (*ksi*)]**	***Cycles to Failure***
248 (36.0)	1×10^5
236 (34.2)	3×10^5
224 (32.5)	1×10^6
213 (30.9)	3×10^6
201 (29.1)	1×10^7
193 (28.0)	3×10^7
193 (28.0)	1×10^8
193 (28.0)	3×10^8

(a) Make an S–N plot (stress amplitude versus logarithm cycles to failure) using these data.

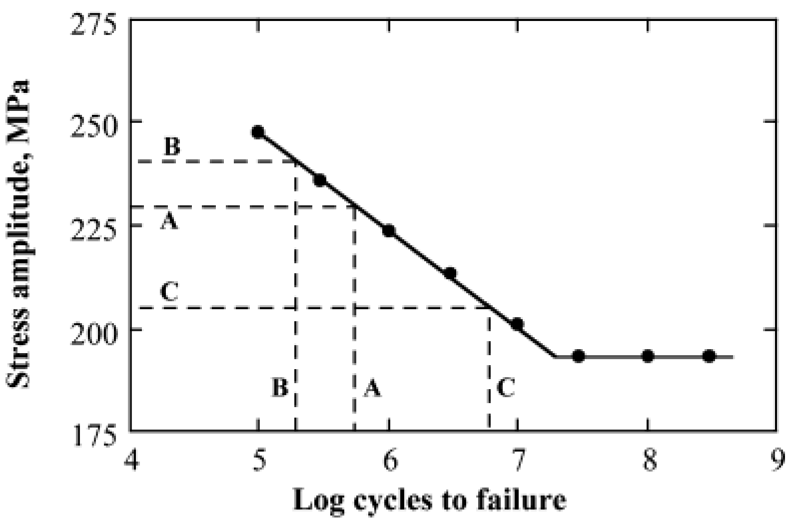
(b) What is the fatigue limit for this alloy?

(c) Determine fatigue lifetimes at stress amplitudes of 230 MPa (33,500 psi) and 175 MPa (25,000 psi).

(d) Estimate fatigue strengths at 2×10^5 and 6×10^6 cycles.

Solution

(a) The fatigue data for this alloy are plotted below.



(b) The fatigue limit is the stress level at which the curve becomes horizontal, which is 193 MPa (28,000 psi).

(c) As noted by the "A" set of dashed lines, the fatigue lifetime at a stress amplitude of 230 MPa is about 5×10^5 cycles (log N = 5.7). From the plot, the fatigue lifetime at a stress amplitude of 230 MPa (33,500 psi) is about 50,000 cycles (log N = 4.7). At 175 MPa (25,000 psi) the fatigue lifetime is essentially an infinite number of cycles since this stress amplitude is below the fatigue limit.

(d) As noted by the "B" set of dashed lines, the fatigue strength at 2×10^5 cycles (log N = 5.3) is about 240 MPa (35,000 psi); and according to the "C" set of dashed lines, the fatigue strength at 6×10^6 cycles (log N = 6.78) is about 205 MPa (30,000 psi).

Q5 *Suppose that the fatigue data for the cast iron in Problem Q4 were taken for bending-rotating tests, and that a rod of this alloy is to be used for an automobile axle that rotates at an average rotational velocity of 750 revolutions per minute. Give maximum lifetimes of continuous driving that are allowable for the following stress levels: (a) 250 MPa (36,250 psi), (b) 215 MPa (31,000 psi), (c) 200 MPa (29,000 psi), and (d) 150 MPa (21,750 psi).*

Solution

For each stress level, first read the corresponding lifetime from the above plot, then convert it into the number of cycles.

(a) For a stress level of 250 MPa (36,250 psi), the fatigue lifetime is approximately 90,000 cycles. This translates into (9×10^4 cycles)(1 min/750 cycles) = 120 min.

(b) For a stress level of 215 MPa (31,000 psi), the fatigue lifetime is approximately 2×10^6 cycles. This translates into (2×10^6 cycles)(1 min/750 cycles) = 2670 min = 44.4 h.

(c) For a stress level of 200 MPa (29,000 psi), the fatigue lifetime is approximately 1×10^7 cycles. This translates into (1×10^7 cycles)(1 min/750 cycles) = 1.33×10^4 min = 222 h.

(d) For a stress level of 150 MPa (21,750 psi), the fatigue lifetime is essentially infinite since we are below the fatigue limit [193 MPa (28,000 psi)].

Q6 *Three identical fatigue specimens (denoted A, B, and C) are fabricated from a nonferrous alloy. Each is subjected to one of the maximum-minimum stress cycles listed below; the frequency is the same for all three tests.*

Specimen	σ_{max} ***(MPa)***	σ_{min} ***(MPa)***
A	*+450*	*–350*
B	*+400*	*–300*
C	*+340*	*–340*

(a) Rank the fatigue lifetimes of these three specimens from the longest to the shortest.

(b) Now justify this ranking using a schematic S–N plot.

Solution

In order to solve this problem, it is necessary to compute both the mean stress and stress amplitude for each specimen. Since from Equation 8.14, mean stresses are the specimens are determined as follows:

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_m(\text{A}) = \frac{450 \text{ MPa} + (-350 \text{ MPa})}{2} = 50 \text{ MPa}$$

$$\sigma_m(\mathrm{B}) = \frac{400 \text{ MPa} + (-300 \text{ MPa})}{2} = 50 \text{ MPa}$$

$$\sigma_m(\mathrm{C}) = \frac{340 \text{ MPa} + (-340 \text{ MPa})}{2} = 0 \text{ MPa}$$

Furthermore, using Equation 8.16, stress amplitudes are computed as

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_a(\mathrm{A}) = \frac{450 \text{ MPa} - (-350 \text{ MPa})}{2} = 400 \text{ MPa}$$

$$\sigma_a(\mathrm{B}) = \frac{400 \text{ MPa} - (-300 \text{ MPa})}{2} = 350 \text{ MPa}$$

$$\sigma_a(\mathrm{C}) = \frac{340 \text{ MPa} - (-340 \text{ MPa})}{2} = 340 \text{ MPa}$$

On the basis of these results, the fatigue lifetime for specimen C will be greater than specimen B, which in turn will be greater than specimen A. This conclusion is based upon the following *S-N* plot on which curves are plotted for two σ_m values.