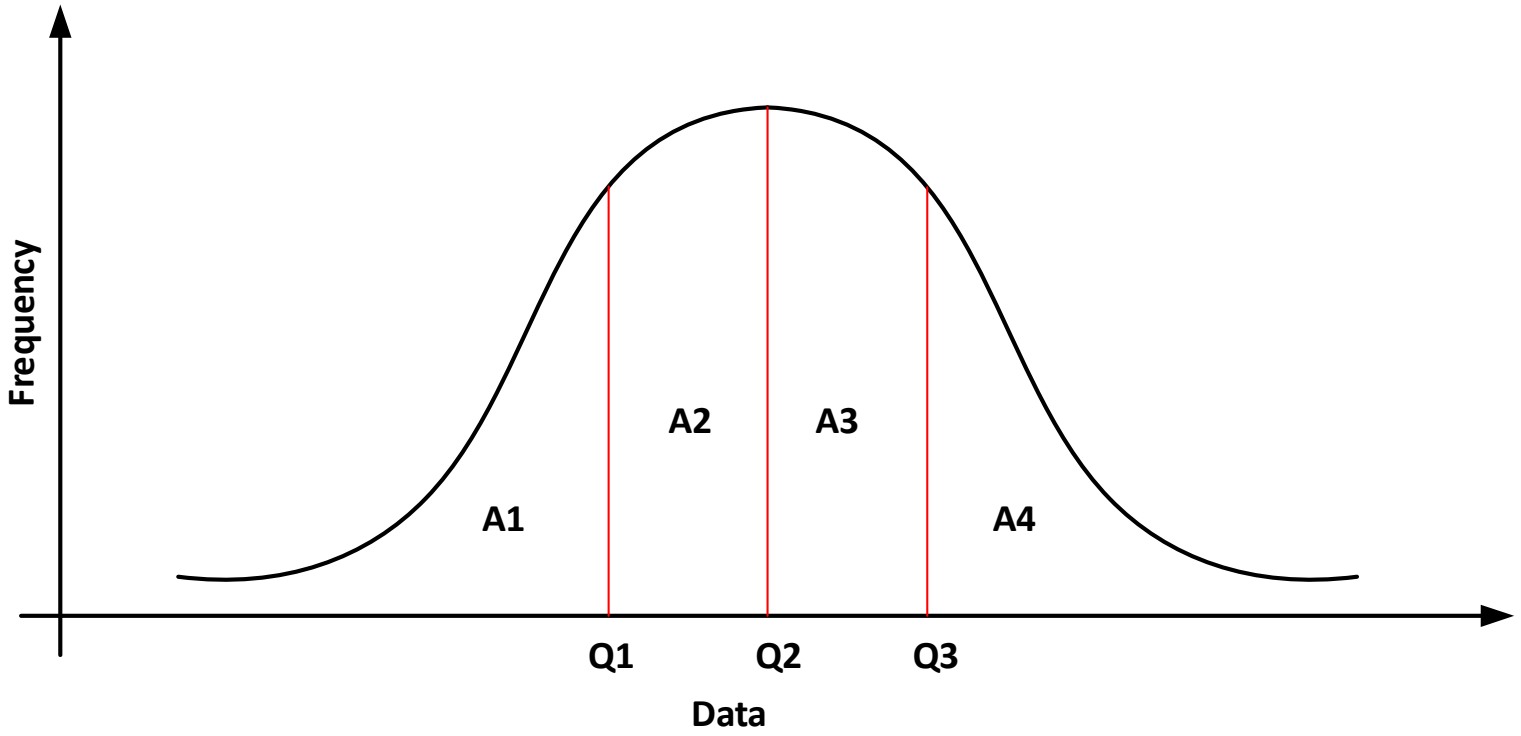
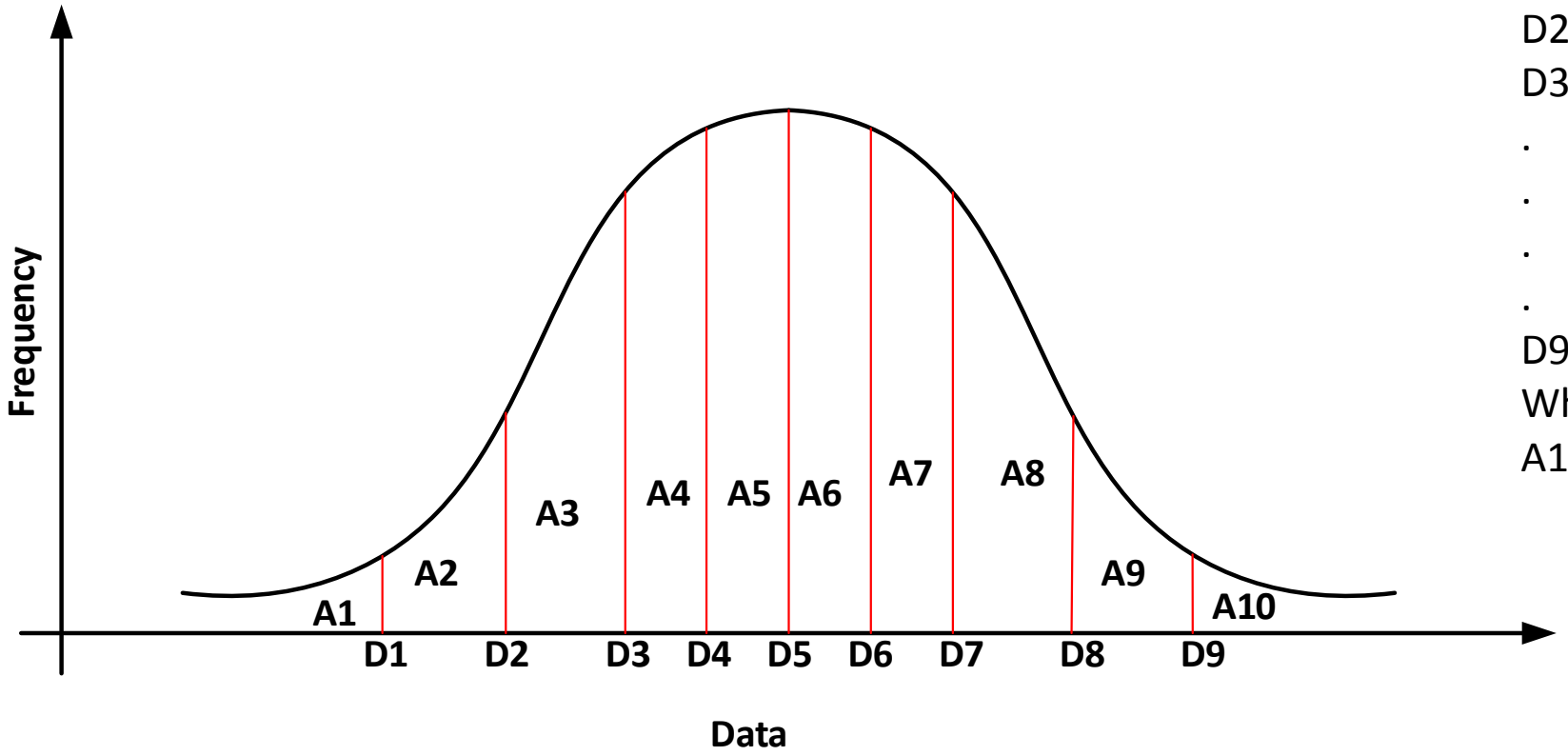
Quartiles, Deciles, and Percentiles

From the definition of median that it's the middle point in the axis frequency distribution curve, and it is divided the area under the curve for two areas have the same area in the left, and in the right. From this may be divided the area under the curve for four equally area and this called quartiles, in the same procedure divided the area for ten equally pieces of area is called deciles, finally where divided the area for hundred equally pieces of area is called percentiles



Q1 = first quartile
Q2 = second quartile
Q3 = third quartile
Where:
A1 = A2 = A3 = A4



D1 = first decile
D2 = second decile
D3 = third decile
.
.
.
.
D9 = ninthe decile
Where:
A1 = A2 = A3 = A4= . . . .=A10

The same procedure for division is done for finding percentiles for any frequency distributed curve

P1 = first percentile
P2 = second percentile
P3 = third percentile
.
.
.
.
.
.
P99 = ninety ninth percentile
Where:
A1 = A2 = A3 = A4= . . . .=A100

To find the quartiles, or desiles, or percentiles we follow the same procedure to fined the median.

Arrangement the data in ascending form only.

If numbering arrangement of quartiles, desiles, and percentiles is fraction then its value is for the number greater than it, if true number the value is the mean of its and the greater numbers

Example:

find the quartiles Q1, Q2, and Q3 of the following data 20, 30, 25, 23, 22, 32, 36

Solution:

Arrange data in ascending form, and n = 7 odd number

Ascending arrangement

20
22
23
25
30
32
36

q1 = (1/4) x n
= (1/4) x 7 = 1.75

q1 = 2

Q1 = 22

q2 = (2/4) x n
= (2/4) x 7 = 3.5

q2 = 4

Q2 = 25

q3 = (3/4) x n
= (3/4) x 7 = 5.25

q3 = 6

Q3 = 32

Example:

find the quartiles Q1, Q2, and Q3 of the following data 20, 30, 25, 23, 22, 32, 36, 18

Solution:

Arrange data in ascending form, and n = 8 even number

Ascending arrangement
18
20
22
23
25
30
32
36

q1 = (1/4) x n
= (1/4) x 8 = 2

q1 = mean of (2), and (3)

Q1 = (20+22)x(1/2) = 21

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q2 = (2/4) x n
= (2/4) x 8 = 4

q2 = mean of (4), and (5)

Q2 = (23+25)x(1/2) = 24

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q3 = (3/4) x n
= (3/4) x 8 = 6

q3 = mean of (6), and (7)

Q3 = (30+32)x(1/2) = 31

Example:

find the the quartiles Q1, Q2, and Q3 of the following data .

Columns Load	Frequency (fi)
50 - 69	3
70 - 89	7
90 - 109	4
110 – 129	4
130 – 149	9

Solution: **1) find the cumulative frequency and the summation of frequencies and real interval limit.**

Columns Load	Frequency (fi)	Cumulative frequency	Real interval	
50 - 69	3	3	49.5 – 69.5	
70 - 89	7	10	69.5 – 89.5	$Q1$
90 - 109	4	14	89.5 – 109.5	
110 – 129	4	18	109.5 – 129.5	
130 – 149	9	27	129.5 – 149.5	

2) find the arrangement number of quartiles to find quartile interval 1.

q1 = (1/4) x n = (1/4) x 27 = 6.75

The interval of quartile number 1 is have the cumulative frequency = 10

$$Q1 = a + \left[\frac{q1 - n_1}{f_q}\right].C$$

a = the real lower limit of quartiles interval = 69.5

n_1 = the cumulative frequency of the previous interval of the quartiles interval = 3

f_q = the frequency of quartiles interval = 7

C = the length of quartiles interval = 20

$$Q1 = 69.5 + \left[\frac{6.75 - 3}{7}\right].20 = 80.2$$

3) find the arrangement number of quartiles to find quartile interval 2.

q2 = (2/4) x n = (2/4) x 27 = 13.5

The interval of quartile number 2 is have the cumulative frequency = 14

$$Q1 = 89.5 + \left[\frac{13.5 - 10}{4}\right].20 = 107$$

4) find the arrangement number of quartiles to find quartile interval 3.

q3 = (3/4) x n = (3/4) x 27 = 20.25

The interval of quartile number 3 is have the cumulative frequency = 27

$$Q1 = 129.5 + \left[\frac{20.25 - 18}{9}\right].20 = 134.5$$

Columns Load	Frequency (fi)	Cumulative frequency	Real interval	
50 - 69	3	3	49.5 – 69.5	
70 - 89	7	10	69.5 – 89.5	$Q1$
90 - 109	4	14	89.5 – 109.5	$Q2$
110 – 129	4	18	109.5 – 129.5	
130 – 149	9	27	129.5 – 149.5	$Q2$

Example:

find the desiles D1, D5, and D8 of the following data 20, 30, 25, 23, 22, 32, 36

Solution:

Arrange data in ascending form, and n = 7 odd number

Ascending arrangement

20
22
23
25
30
32
36

d1 = (1/10) x n
= (1/10) x 7 = 0.7

d1 = 1

D1 = 20

d5 = (5/10) x n
= (5/10) x 7 = 3.5

d5 = 4

D5 = 25

d8 = (8/10) x n
= (8/10) x 7 = 5.6

d8 = 6

D8 = 32

Example:

find the desiles D1, D5, and D8 of the following data 20, 30, 25, 23, 22, 32, 36, 18

Solution:

Arrange data in ascending form, and n = 8 even number

Ascending arrangement

18
20
22
23
25
30
32
36

d1 = (1/10) x n
= (1/10) x 8 = 0.8

d1 = 1

D1 = 18

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d5 = (5/10) x n
= (5/10) x 8 = 4

d5 = mean of (4), and (5)

D5 = (23+25)x(1/2) = 24

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d8 = (8/10) x n
= (8/10) x 8 = 6.4

d8 = 7

D8 = 32

Example:

find the desiles D1, D5, and D9 of the following data .

Columns Load	Frequency (fi)
50 - 69	3
70 - 89	7
90 - 109	4
110 – 129	4
130 – 149	9

Solution: **1) find the cumulative frequency and the summation of frequencies and real interval limit.**

Columns Load	Frequency (fi)	Cumulative frequency	Real interval	
50 - 69	3	3	49.5 – 69.5	$D1$
70 - 89	7	10	69.5 – 89.5	
90 - 109	4	14	89.5 – 109.5	
110 – 129	4	18	109.5 – 129.5	
130 – 149	9	27	129.5 – 149.5	

2) find the arrangement number of desiles to find desiles interval D1.

d1 = (1/10) x n = (1/10) x 27 = 2.7

The interval of desiles number 1 is have the cumulative frequency = 3

$$D = a + \left[\frac{d - n_1}{f_d}\right].C$$

a = the real lower limit of desiles interval = 49.5

n_1 = the cumulative frequency of the previous interval of the desiles interval = 0

f_d = the frequency of desiles interval = 3

C = the length of desiles interval = 20

$$D1 = 49.5 + \left[\frac{3-0}{3}\right].20 = 69.5$$

3) find the arrangement number of desiles to find desiles interval D5.

d5 = (5/10) x n = (5/10) x 27 = 13.5

The interval of desiles number D5 is have the cumulative frequency = 14

$$D5 = 89.5 + \left[\frac{13.5-10}{4}\right].20 = 107$$

4) find the arrangement number of desiles to find desiles interval D9.

d9 = (9/10) x n = (9/10) x 27 = 24.3

The interval of desiles number D5 is have the cumulative frequency = 27

$$D9 = 129.5 + \left[\frac{24.3-18}{9}\right].20 = 143.278$$

Columns Load	Frequency (fi)	Cumulative frequency	Real interval	
50 - 69	3	3	49.5 – 69.5	$D1$
70 - 89	7	10	69.5 – 89.5	
90 - 109	4	14	89.5 – 109.5	$D2$
110 – 129	4	18	109.5 – 129.5	
130 – 149	9	27	129.5 – 149.5	$D9$

Example:

find the percentiles P8, P50, and P85 of the following data 20, 30, 25, 23, 22, 32, 36

Solution:

Arrange data in ascending form, and n = 7 odd number

Ascending arrangement

20
22
23
25
30
32
36

p8 = (8/100) x n
= (8/100) x 7 = 0.56

p8 = 1

P8 = 20

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p50 = (50/100) x n
= (50/100) x 7 = 3.5

p50 = 4

P50 = 25

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p85 = (85/100) x n
= (85/100) x 7 = 5.95

p85 = 6

P85 = 32

Example:

find the percentiles P8, P50, and P85 of the following data 20, 30, 25, 23, 22, 32, 36, 18

Solution:

Arrange data in ascending form, and n = 8 even number

Ascending arrangement
18
20
22
23
25
30
32
36

p8 = (8/100) x n

= (8/100) x 8 = 0.64

p8 = 1

P8 = 18

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p50 = (50/100) x n

= (50/100) x 8 = 4

p50 = mean of (4), and (5)

P50 = (23+25)x(1/2) = 24

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p85 = (85/100) x n

= (85/100) x 8 = 6.8

p85 = 7

P85 = 32

Example:

find the percentiles P8, P50, and P85 of the following data .

Columns Load	Frequency (fi)
50 - 69	3
70 - 89	7
90 - 109	4
110 – 129	4
130 – 149	9

Solution: **1) find the cumulative frequency and the summation of frequencies and real interval limit.**

Columns Load	Frequency (fi)	Cumulative frequency	Real interval	
50 - 69	3	3	49.5 – 69.5	$P8$
70 - 89	7	10	69.5 – 89.5	
90 - 109	4	14	89.5 – 109.5	
110 – 129	4	18	109.5 – 129.5	
130 – 149	9	27	129.5 – 149.5	

2) find the arrangement number of percentiles to find percentiles interval P8.

p8 = (8/100) x n = (8/100) x 27 = 2.16

The interval of percentiles number 8 is have the cumulative frequency = 3

$$P = a + \left[\frac{p - n_1}{f_p}\right].C$$

a = the real lower limit of percentiles interval = 49.5

n_1 = the cumulative frequency of the previous interval of the percentiles interval = 0

f_p = the frequency of percentiles interval = 3

C = the length of percentiles interval = 20

$$P8 = 49.5 + \left[\frac{3-0}{3}\right].20 = 69.5$$

3) find the arrangement number of percentiles to find percentiles interval P50.

p50 = (50/100) x n = (50/100) x 27 = 13.5

The interval of percentiles number P50 is have the cumulative frequency = 14

$$P50 = 89.5 + \left[\frac{13.5-10}{4}\right].20 = 107$$

4) find the arrangement number of percentiles to find percentiles interval P85.

p85 = (85/100) x n = (85/100) x 27 = 22.95

The interval of percentiles number P85 is have the cumulative frequency = 27

$$P85 = 129.5 + \left[\frac{22.95-18}{9}\right].20 = 140.5$$

Columns Load	Frequency (fi)	Cumulative frequency	Real interval	
50 - 69	3	3	49.5 – 69.5	$P8$
70 - 89	7	10	69.5 – 89.5	
90 - 109	4	14	89.5 – 109.5	$P50$
110 – 129	4	18	109.5 – 129.5	
130 – 149	9	27	129.5 – 149.5	$P85$

Example:

find the percentiles arrangement of 115 in the above data .

$$P = a + \left[\frac{p - n_1}{f_p}\right].C$$

a = the real lower limit of percentiles interval = 109.5

n_1 = the cumulative frequency of the previous interval of the percentiles interval = 14

f_p = the frequency of percentiles interval = X

C = the length of percentiles interval = 20

$$115 = 109.5 + \left[\frac{X - 14}{4}\right].20 \qquad then\ , X = 15.1$$

$$then\ , percentiles\ arrangment = \frac{15.1}{27} x100\% = 56\%$$