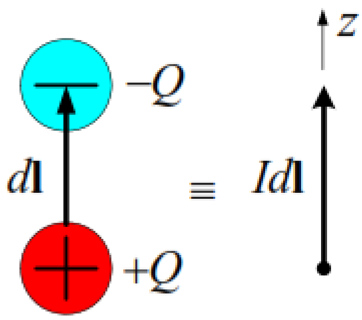
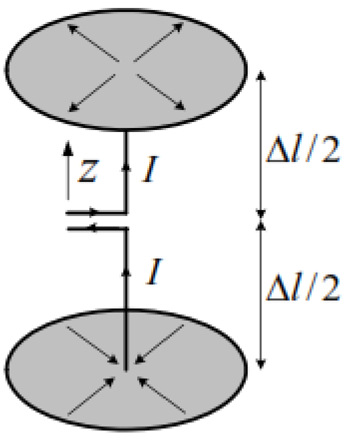
3- Radiation from an infinitesimal dipole (current element)

Definition: The infinitesimal dipole "ideal dipole" is a dipole whose length $\Delta l = \Delta z$ is much smaller than the wavelength λ of the excited wave, i.e. $\Delta l \ll \lambda$ or $(\Delta z \ll \lambda)$ $(\Delta l < \lambda/50)$ or $(\Delta z \le \lambda/50)$. The infinitesimal dipole is equivalent to a current element $I\,\Delta\mathbf{l}$, where

$$I\Delta l = -\frac{dQ}{dt}\Delta l$$



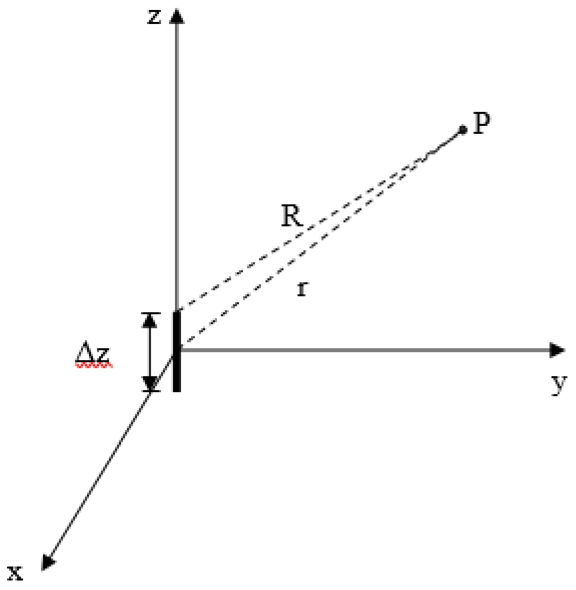
A current element is best illustrated by a very short (compared to λ) piece of infinitesimally thin wire with current I. For simplicity, the current is assumed to have constant magnitude along dl. The ideal current element is practically unrealizable, but a very good approximation of it is the short top-hat antenna. To realize a uniform current distribution along the wire, capacitive plates are used to provide enough charge storage at the end of the wire, so that the current is not zero there.



3.1. Magnetic vector potential (vp) due to a current element

The magnetic vector potential **A** due to a linear source is:

$$A(P) = \int_{\Delta l} \mu I \frac{e^{-j\beta R}}{4\pi R} dl = \hat{z} \frac{\mu_o I}{4\pi} \int_{-\Delta z/2}^{\Delta z/2} \frac{e^{-j\beta R}}{R} dz = A_z$$



Where $\beta$is the wavenumber $\beta = \omega\sqrt{\mu\varepsilon}$, and it is related to the wavelength and as: $\beta = \frac{2\pi}{\lambda}$

Since the dipole is very small, $R \approx r$ both in the exponent term and in the denominator. Therefore,

$$A_z = \mu_o I \Delta z \frac{e^{-j\beta r}}{4\pi r} \hat{z} = \mu_o I \Delta l \frac{e^{-j\beta r}}{4\pi r} \hat{z}$$

In above equation gives the field due to an electric current element (infinitesimal dipole) expressed via **A**. This is an important result because ***the field radiated by any complex antenna in a linear medium can be represented as a superposition of the fields due to the current elements on the antenna surface***.

We represent **A** with its spherical components. In antenna theory, the preferred coordinate system is the spherical one. This is mostly because the ***far field*** radiation is of interest. The field is analyzed so very far from the source, that it is assumed to propagate only radially away from the source. The transformation from rectangular to spherical coordinates is given by

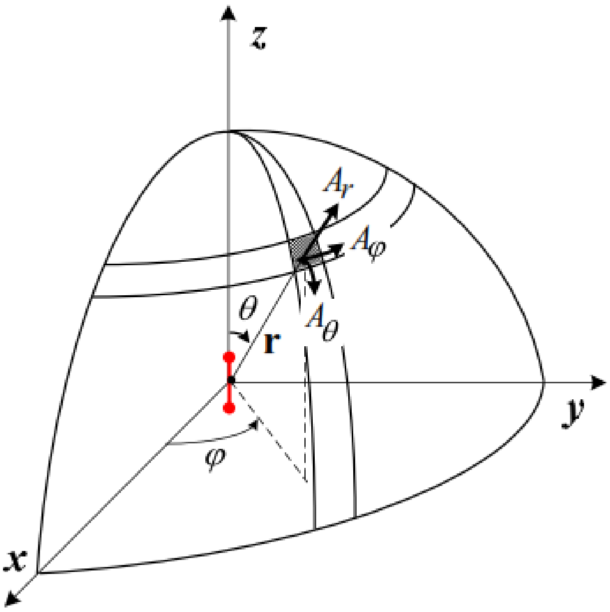
$$\begin{bmatrix} A_r \\ A_\theta \\ A_\varphi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}.$$

Applying transformation to $\mathbf{A_z}$ produces

$$A_r = A_z\cos\theta = \mu_0 I\Delta l\frac{e^{-j\beta r}}{4\pi r}\cos\theta$$

$$A_\theta = -A_z\sin\theta = -\mu_0 I\Delta l\frac{e^{-j\beta r}}{4\pi r}\sin\theta$$

$$A_\varphi = 0$$



Note that:
1) $\mathbf{A_z}$ does not depend on φ (due to the cylindrical symmetry of the dipole);
2) the dependence on r, which is $e^{-j\beta r}/r$, is separable from the dependence on θ.

3.2. Field vectors due to current element radiation

Let us now find the field vectors **H** and **E**.

a) $H = \frac{1}{\mu} \nabla \times A$

The curl operator is expressed in spherical coordinates to obtain

$$H = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(r \cdot A_\theta) - \frac{\partial A_r}{\partial \theta}\right]\hat{\varphi}$$

The magnetic field H has only a φ component. Finally,

$$H_\varphi = j\beta \cdot (I\Delta l)\sin(\theta)\left[1 + \frac{1}{j\beta r}\right]\frac{e^{-j\beta r}}{4\pi r},\ H_\theta = H_r = 0$$

b) $E = \frac{1}{j\omega\varepsilon}\nabla \times H = -j\omega A - \frac{j}{\omega\mu\varepsilon}\nabla\nabla \cdot A$

Explicitly, in spherical coordinates we find:

$$E_r = 2\eta(I\Delta l)\cos\theta\left(\frac{1}{r} + \frac{1}{j\beta r^2}\right)\frac{e^{-j\beta r}}{4\pi r}$$

$$E_\theta = j\eta\beta(I\Delta l)\sin\theta\left(1 + \frac{1}{j\beta r} + \frac{1}{(\beta r)^2}\right)\frac{e^{-j\beta r}}{4\pi r}\ and\ E_\varphi = 0$$

Notes:

1- Equations above show that the EM field generated by the current element is rather complicated unlike the VP A. The use of the VP instead of the field vectors is usually advantageous in antenna studies.

2- The field vectors contain terms, which depend on the distance from the source as $(1/r)$, $(1/r^2)$ and $(1/r^3)$; the higher-order terms can be neglected at large distances from the dipole.

3- The longitudinal $\hat{\boldsymbol{r}}$-components of the field vectors decrease fast as the field propagates away from the source (as $1/r^2$ and $1/r^3$ only): they are neglected in the far zone.

4- The non-zero transverse field components, E_θ and H_φ, are orthogonal to each other, and they have terms, which depend on the distance as 1/r. These terms differ by a factor of η . They represent the so-called far field.

3.3. Power density and overall radiated power of the infinitesimal (ideal) dipole

The complex vector of Poynting $\boldsymbol{P}$ describes the complex power density flux. It is calculated as

$$\boldsymbol{P} = \frac{1}{2}[E \times H^*] = \frac{1}{2}\left[(E_r\widehat{a_r} + E_\theta\widehat{a_\theta}) \times (H_\varphi^*\widehat{a_\varphi})\right] = \frac{1}{2}(E_\theta H_\varphi^*\widehat{a_r} - E_r H_\varphi^*\widehat{a_\theta})$$

Substituting the values of $E_\theta. E_r$ and $H_\varphi$in above equations yields

$$P_r = \frac{\eta}{8}\left|\frac{I\Delta l}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2}\left(1 - j\frac{1}{(\beta r)^3}\right)$$

$$P_\theta = j\eta\beta\frac{|I\Delta l|^2\cos\theta\sin\theta}{16\pi^2 r^3}\left(1 + \frac{1}{(\beta r)^2}\right)$$

The overall power Π is calculated over a sphere, and, therefore, only the radial component of the vector of Poynting P_r contributes:

$$\Pi = \oiint \mathbf{P}\cdot ds = \oiint_S (P_r\widehat{a_r} + P_\theta\widehat{a_\theta})\cdot r^2\sin\theta\, d\theta d\varphi\widehat{a_r}.$$

$$\Pi = \frac{\pi}{3}\eta\left|\frac{I\Delta l}{\lambda}\right|^2\left(1 - \frac{j}{(\beta r)^3}\right).\mathrm{W}$$

The radiated power is equal to the real part of the complex power. Therefore, the radiated power of an infinitesimal dipole is

$$\Pi_{rad} = \frac{\pi}{3}\eta\left|\frac{I\Delta l}{\lambda}\right|^2.\mathrm{W}$$

Here, we introduce the concept of radiation resistance R_r, which can describe the power loss due to radiation in an equivalent circuit of antenna:

$$\Pi_{rad} = \frac{1}{2}R_r I^2 \Rightarrow R_r = \frac{2\Pi}{I^2}$$

$$R_r^{id} = \frac{2\pi}{3}\eta\left(\frac{\Delta l}{\lambda}\right)^2.\Omega$$

3.4. Radiation zones

The space surrounding the antenna is divided into three regions according to the predominant field behavior. The boundaries between the regions are not distinct and the field behavior changes gradually as these boundaries are crossed. In this course, we are mostly concerned with the far-field characteristics of the antennas.

3.4.1.Reactive near-field region

This is the region immediately surrounding the antenna, where the reactive field dominates. For most antennas, it is assumed that this region is a sphere with the antenna at its center, and with a radius of

$$r \approx 0.62\sqrt{D^3 / \lambda}$$

where D is the largest dimension of the antenna, and λ is the wavelength of the radiated field. It must be noted that this limit is most appropriate for wire and waveguide aperture antennas, while it is not valid for electrically large reflector antennas.

At this point, we discuss the general field behavior making use of our knowledge of the infinitesimal electric-dipole field, r is sufficiently small so that $\beta r \ll 1$. Then, the most significant terms in the field expressions are

$$H_\varphi = j\beta \cdot (I\Delta l)\sin(\theta)\left[1 + \frac{1}{j\beta r}\right]\frac{e^{-j\beta r}}{4\pi r} \approx (I\Delta l)\frac{e^{-j\beta r}}{4\pi r^2}\sin(\theta)$$

$$E_r = 2\eta(I\Delta l)\cos\theta\left(\frac{1}{r} + \frac{1}{j\beta r^2}\right)\frac{e^{-j\beta r}}{4\pi r} \approx -j\eta(I\Delta l)\frac{e^{-j\beta r}}{2\pi\beta r^3}\cos\theta$$

$$E_\theta = j\eta\beta(I\Delta l)\sin\theta\left(1 + \frac{1}{j\beta r} - \frac{1}{(\beta r)^2}\right)\frac{e^{-j\beta r}}{4\pi r} \approx -j\eta(I\Delta l)\frac{e^{-j\beta r}}{4\pi\beta r^3}\sin\theta$$

$$H_r = H_\theta = E_\varphi = 0$$

This approximated field is purely reactive. Actually, the term $e^{-j\beta r}$ can be neglected, and then we clearly see that:

(1) H_φ has the distribution of the magnetostatic field of a current filament $I\Delta l$.

(2) E_θ and E_r have the distribution of the electrostatic field of a dipole.

That the field is almost purely reactive in the near zone is obvious from the power equation.

$$\Pi = \frac{\pi}{3}\eta\left|\frac{I\Delta l}{\lambda}\right|^2\left(1-\frac{j}{(\beta r)^3}\right). \mathrm{W}$$

Its imaginary part is

$$\mathrm{Im}\{\Pi\} = -\frac{\pi}{3}\eta\left(\frac{I\Delta l}{\lambda}\right)^2\frac{1}{(\beta r)^3}.$$

$\mathrm{Im}\{\Pi\}$ dominates over the radiated power,

$$\mathrm{Re}\{\Pi\} = \frac{\pi}{3}\eta\left(\frac{I\Delta l}{\lambda}\right)^2 = \Pi_{rad},$$

when $r \to 0$, since Π_{rad} does not depend on r.

The radial reactive power flow density P_r has predominant magnetic character (negative imaginary value) and decreases as $(1/r^5)$, and the P_θ power density component has the same order of dependence on r but has predominant electric nature:

$$P_r^{near} = -j\frac{\eta}{8}\left|\frac{I\Delta l}{\lambda}\right|^2\frac{\sin^2\theta}{\beta^3 r^5}$$

$$P_\theta^{near} = j\eta\frac{|I\Delta l|^2\cos\theta\sin\theta}{16\pi^2\beta r^5} = j\frac{\eta}{8}\left(\frac{I\Delta l}{\lambda}\right)^2\frac{\sin 2\theta}{\beta^3 r^5}$$

3.4.2.Radiating near-field (Fresnel) region

This is an intermediate region between the reactive near-field region and the far-field region, where the radiation field is more significant but the angular field distribution is still dependent on the distance from the antenna. In this region, $\beta r \geq 1$. For most antennas, it is assumed that the Fresnel region is enclosed between two spherical surfaces:

$$0.62\sqrt{\frac{D^3}{\lambda}} \leq r \leq \frac{2D^2}{\lambda}.$$

Here, *D* is the largest dimension of the antenna. This region is called the *Fresnel region* because its field expressions reduce to Fresnel integrals.

The fields of an infinitesimal dipole in the Fresnel region are obtained by neglecting the higher-order $(1/r)^n$ -terms in fields equations:

$$H_\varphi = j\beta \cdot (I\Delta l)\sin(\theta)\left[1 + \frac{1}{j\beta r}\right]\frac{e^{-j\beta r}}{4\pi r} \approx \frac{j\beta \cdot (I\Delta l)e^{-j\beta r}}{4\pi r}\sin(\theta)$$

$$E_r = 2\eta(I\Delta l)\cos\theta\left(\frac{1}{r} + \frac{1}{j\beta r^2}\right)\frac{e^{-j\beta r}}{4\pi r} \approx \eta\frac{(I\Delta l)e^{-j\beta r}}{2\pi r^2}\cos\theta$$

$$E_\theta = j\eta\beta(I\Delta l)\sin\theta\left(1 + \frac{1}{j\beta r} + \frac{1}{(\beta r)^2}\right)\frac{e^{-j\beta r}}{4\pi r} \approx j\eta\frac{\beta(I\Delta l)e^{-j\beta r}}{4\pi r}\sin\theta$$

$$H_r = H_\theta = E_\varphi = 0$$

The radial component E_r is still not negligible, but the transverse components (E_θ and H_φ) are dominant.

3.4.3.Far-field (Fraunhofer) region

Only terms ~$1/r$ are considered when $\beta r >> 1$. The angular field distribution does not depend on the distance from the source any more, i.e., the *far-field pattern* is already well established. The field is a transverse EM wave. For most antennas, the far-field region is defined as

$$r \geq 2D^2 / \lambda$$

The far-field of the infinitesimal dipole is obtained as

$$H_\varphi = j\beta \cdot (I\Delta l)\sin(\theta)\left[1 + \frac{1}{j\beta r}\right]\frac{e^{-j\beta r}}{4\pi r} \approx j\frac{\beta \cdot (I\Delta l)e^{-j\beta r}}{4\pi r}\sin\theta$$

$$E_\theta = j\eta\beta(I\Delta l)\sin\theta\left(1 + \frac{1}{j\beta r} + \frac{1}{(\beta r)^2}\right)\frac{e^{-j\beta r}}{4\pi r} \approx j\eta\frac{\beta(I\Delta l)e^{-j\beta r}}{4\pi r}\sin\theta$$

$$E_r = 2\eta(I\Delta l)\cos\theta\left(\frac{1}{r} + \frac{1}{j\beta r^2}\right)\frac{e^{-j\beta r}}{4\pi r} \approx 0$$

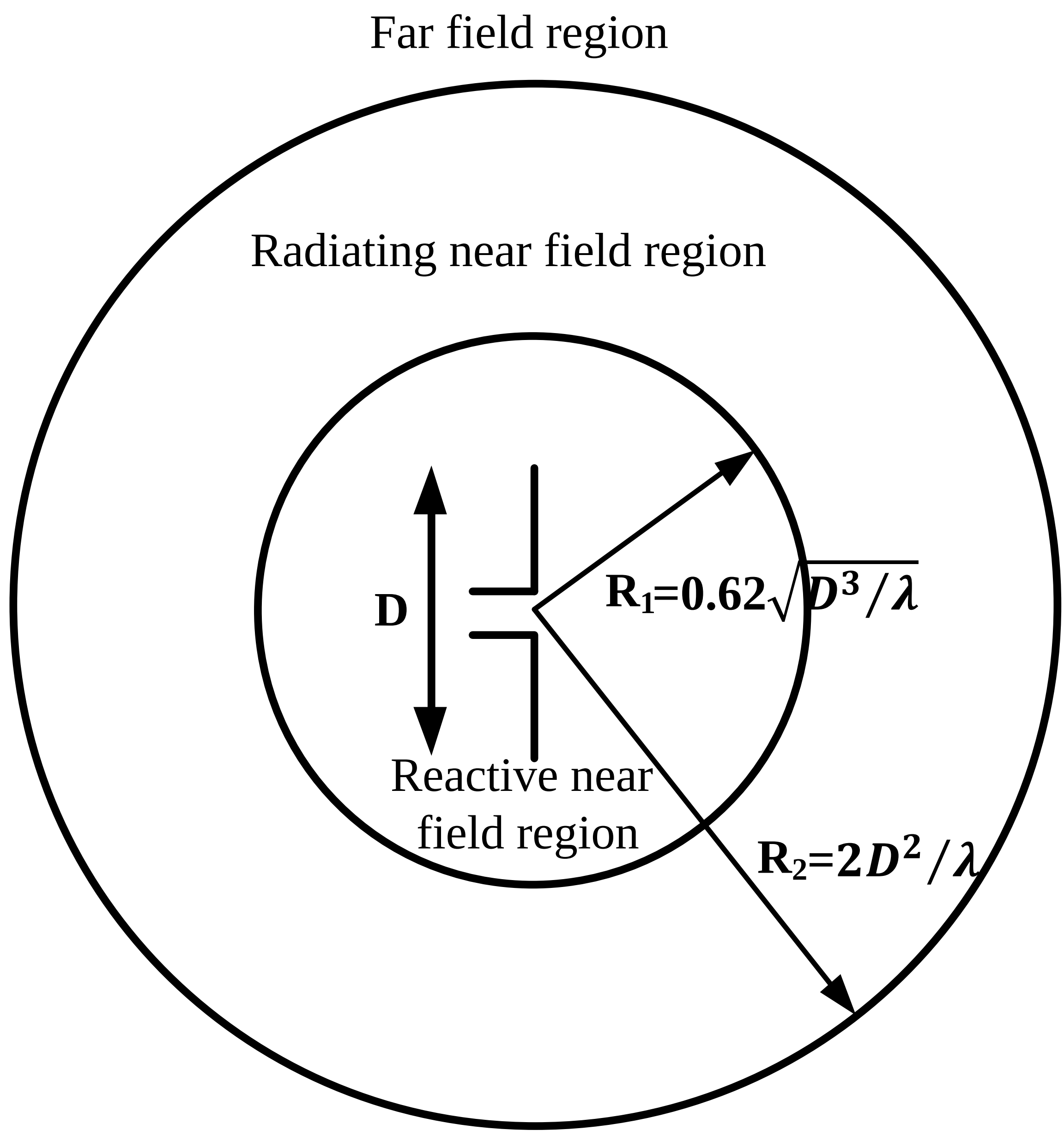
$$H_r = H_\theta = E_\varphi = 0$$

η is the intrinsic impedance of the medium.

$\eta = \frac{E_\theta}{H\varphi} = \frac{\omega\mu}{\beta} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}$, At free space η is given by $\eta_o = 120\pi\Omega \approx 377\Omega$

Important features of the far field:

1. No radial components.
2. The angular field distribution is independent of ***r***.
3. $E \perp H$.
4. $\eta = \frac{E_\theta}{H\varphi}$.
5. $\mathbf{P} = \frac{(\boldsymbol{E}\times\boldsymbol{H}^*)}{2} = \frac{1}{2}|\boldsymbol{E}_\theta|^2/\boldsymbol{\eta}\,\hat{\boldsymbol{a}}_r = \frac{1}{2}\boldsymbol{\eta}|\boldsymbol{H}_\varphi|^2\hat{\boldsymbol{a}}_r$


Far field region
Radiating near field region
D
R1=0.62√D3/λ
Reactive near
field region
R2=2D2/λ

3.5. Fundamental Antenna Parameters

The antenna parameters describe the antenna performance with respect to space distribution of the radiated energy, power efficiency, matching to the feed circuitry, etc. Many of these parameters are interrelated.

3.5.1. Radiation Pattern

The radiation pattern (RP) (or antenna pattern) is a graphical representation of the radiation (far-field) properties of the antenna as a function of the angular coordinates.

The RP is measured in the far-field region, where the spatial (angular) distribution of the radiated power does not depend on the distance. We usually measure and plot the field intensity, e.g. $\sim|E(\theta.\varphi)|$, or the received power $\sim|E(\theta.\varphi)|^2/\eta = \eta|H(\theta.\varphi)|^2$

The power pattern is the trace of the spatial variation of the received/radiated power at a constant radius from the antenna.

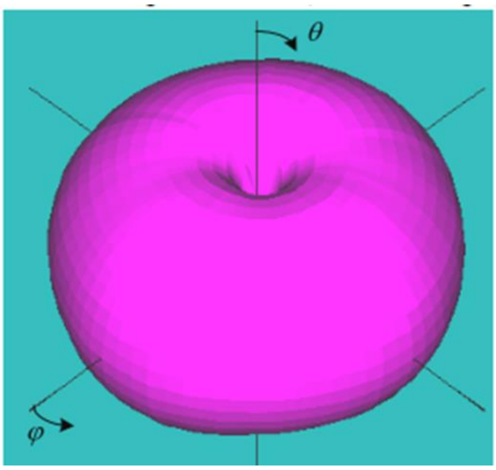
The amplitude field is the trace of the spatial variation of the magnitude of electric (magnetic) field at a constant radius from the antenna.

Usually, the pattern describes the *normalized* field (power) values with respect to the maximum value.

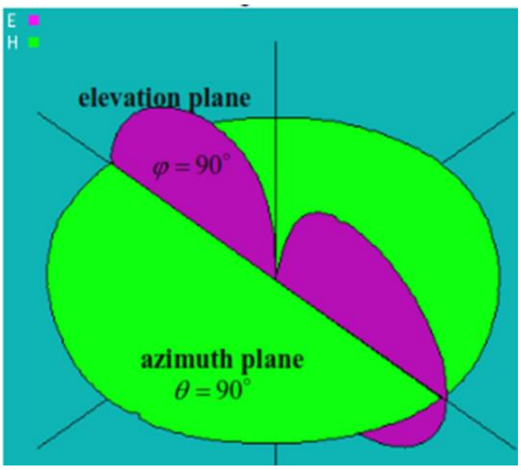
Note: The power pattern and the amplitude field pattern are the same when computed and plotted in dB.

The pattern can be a 3-D plot (both θ *and* φ vary), or a 2-D plot.

A 2-D plot is obtained as an intersection of the 3-D RP with a given plane, usually a θ =*constant* plane or a φ =*constant* plane that must contain the pattern's maximum.



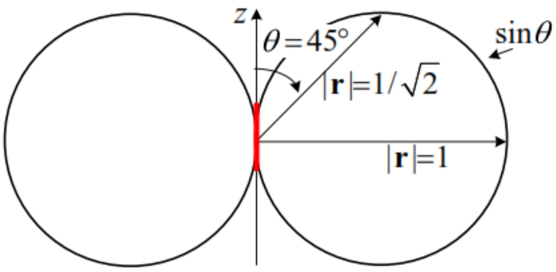
3D radiation pattern



2D radiation pattern in H and E plane

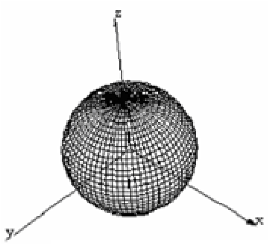
Plotting the pattern: the trace of the pattern is obtained by setting the length of the radius-vector $|r(\theta.\varphi)|$ corresponding to the $(\theta.\varphi)$ point of the RP proportional to the strength of the field $|E(\theta.\varphi)|$ (in the case of an amplitude field pattern) or proportional to the power density $|E(\theta.\varphi)|^2$ (in the case of a power pattern).

Elevation Plane: $\varphi = const$



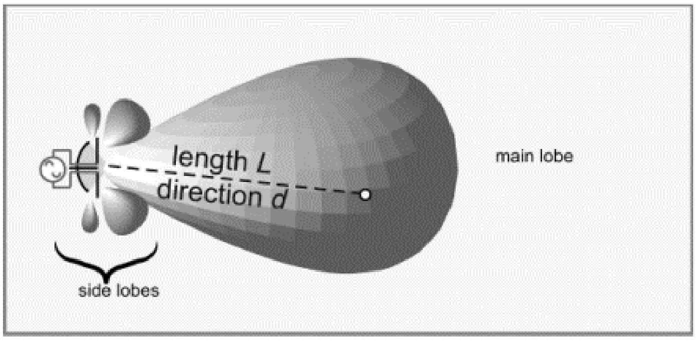
Types of Radiation Pattern:

a) ***Isotropic pattern***: is the pattern of an antenna having equal radiation in all directions. This is an ideal (not physically achievable) concept. However, it is used to define other antenna parameters. It is represented simply by a sphere whose center coincides with the location of the isotropic radiator.

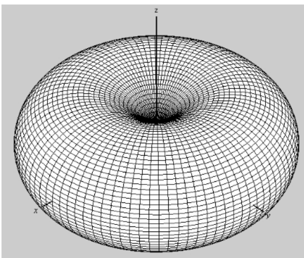


Isotropic Antenna Pattern

b) ***Directional antenna*** is an antenna, which radiates (receives) much more efficiently in some directions than in others. Usually, this term is applied to antennas whose directivity is much higher than that of a half-wavelength dipole.

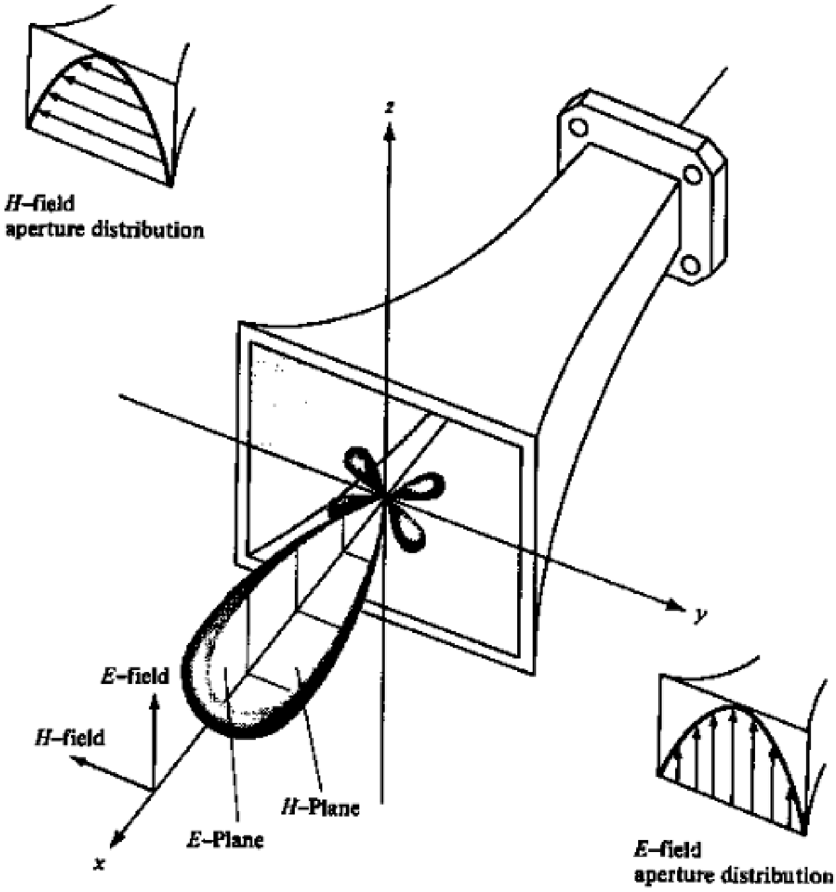


c) ***Omnidirectional antenna*** is an antenna, which has a non-directional pattern in a given plane, and a directional pattern in any orthogonal plane (e.g. single-wire antenna).

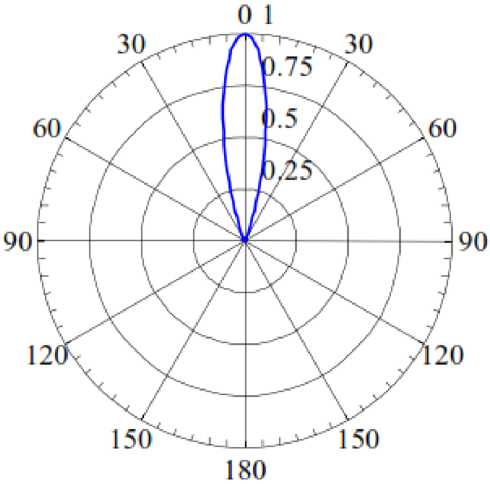


Principal patterns:

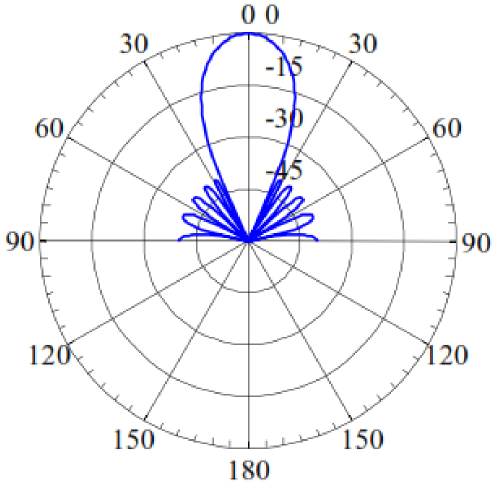
There are two types of principal patterns, these types are the 2-D patterns of linearly polarized antennas, measured in the ***E-plane*** (a plane parallel to the **E** vector and containing the direction of maximum radiation) and in the ***H-plane*** (a plane parallel to the **H** vector, orthogonal to the *E*-plane, and containing the direction of maximum radiation).



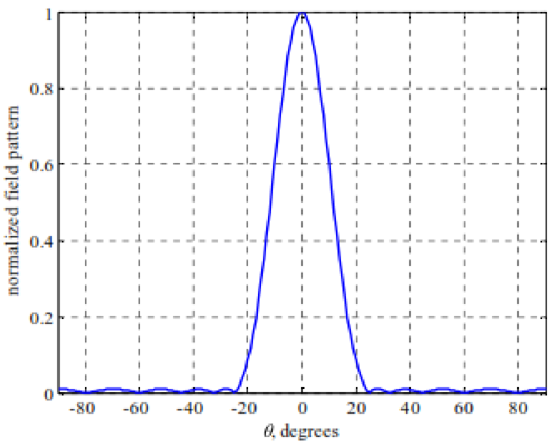
2-D patterns can be polar or linear



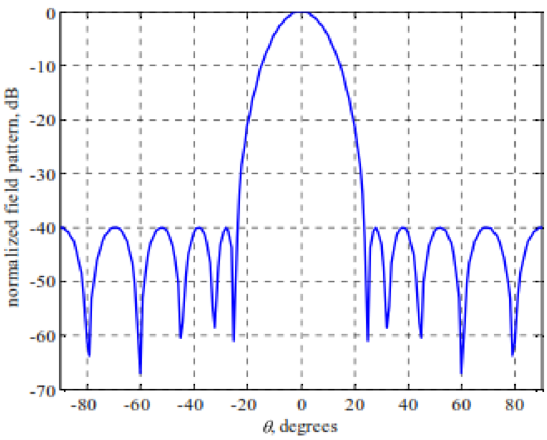
Polar Pattern (linear scale)



Polar Pattern (dB scale)



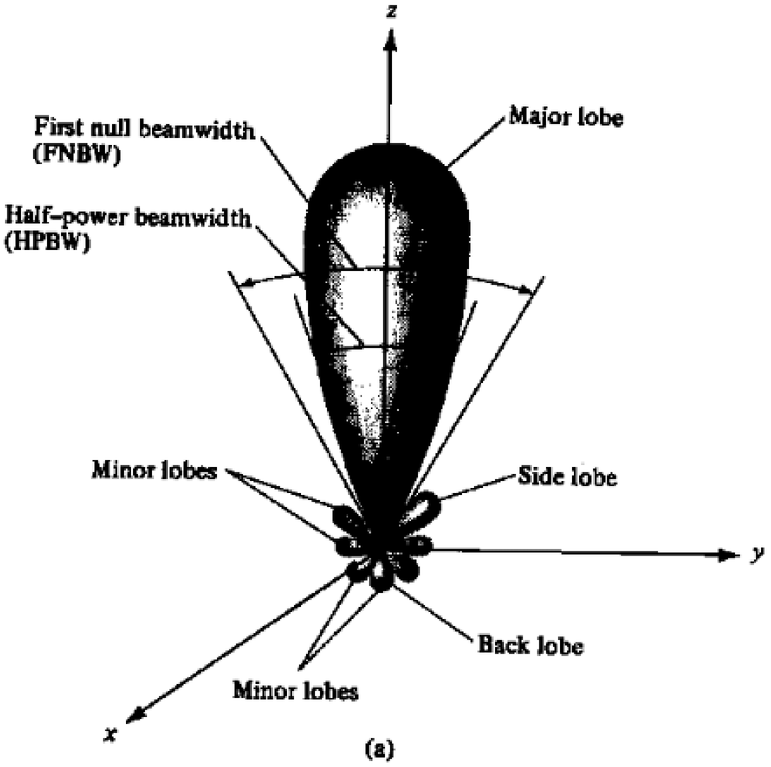
Rectangular Pattern (linear scale)



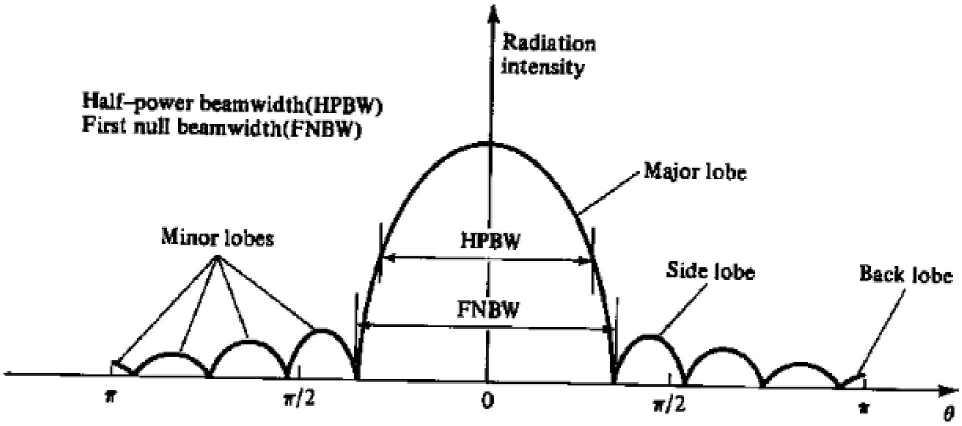
Rectangular Pattern (dB)

Pattern lobe:

Pattern lob is a portion of the RP whose local radiation intensity maximum is relatively weak.
Lobes are classified as: major, minor, side lobes, back lobes.



(a)



(b)

Side Lobe level(SLL): is a measure of how well the power is concentrated into the main lobe.

$$SLL_{dB} = 20 \ \log \frac{|E(SLL)|}{|E(\max)|}$$

Pattern beamwidth:

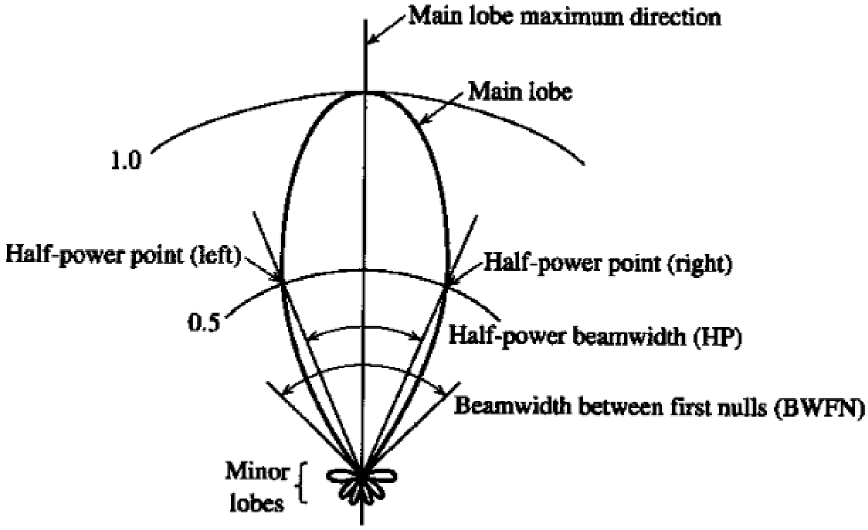
a) ***Half-power beamwidth*** (HPBW):

HPBW is the angle between two vectors, originating at the pattern's origin and passing through these points of the major lobe where the radiation intensity is half its maximum.

b) ***First-null beamwidth*** (FNBW):

FNBW is the angle between two vectors, originating at the pattern's origin and tangent to the main beam at its base.

Often, it is true that FNBW≈2HPBW.



The HPBW is the best parameter to describe the antenna resolution properties. In radar technology as well as in radio-astronomy, the antenna resolution capability is of primary importance.

$$HPBW = \left|\theta_{HPleft} - \theta_{HPright}\right|$$

For example, in the case of the ideal dipole:

$$\theta_{right} = 45^o \text{ and } \theta_{left} = 135^o$$

$$\text{Then } HPBW = \left|135^o - 45^0\right| = 90^o$$