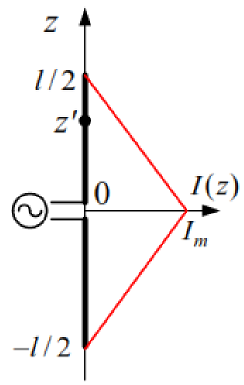
Linear Wire Antennas – Dipoles and Monopoles

The dipole and the monopole are arguably the two most widely used antennas across the UHF, VHF and lower-microwave bands. Arrays of dipoles are commonly used as base-station antennas in land-mobile systems. The monopole and its variations are perhaps the most common antennas for portable equipment, such as cellular telephones, cordless telephones, automobiles, trains, etc. It has attractive features such as simple construction, sufficiently broadband characteristics for voice communication, small dimensions at high frequencies. An alternative to the monopole antenna for hand-held units is the loop antenna, the microstrip patch antenna, the spiral antennas, Inverted-L and Inverted-F antennas, and others.

9.1 Small dipole



the condition for small dipole is $\frac{\lambda}{50} < l \leq \frac{\lambda}{10}$

If we assume that $R \approx r$ and the above condition holds, the maximum phase error in $(R\beta)$ that can occur is

$$e_{max} = \frac{\beta l}{2} = \frac{\pi}{10} = 18^{\circ}$$

At $\theta = 0$, A maximum total phase error of $\pi/8$ is acceptable since it does not affect substantially the integral solution for the vector potential **A**. The assumption $R \approx r$ is made here for both, the amplitude and the phase factors in the kernel of the VP integral.

The current is a triangular function of '*z:*

$$I(z') = \begin{cases} I_m \cdot \left(1 - \dfrac{z'}{l/2}\right), & 0 \le z' \le l/2 \\ I_m \cdot \left(1 + \dfrac{z'}{l/2}\right), & -l/2 \le z' \le 0 \end{cases}$$

The VP integral is obtained as

$$\mathbf{A} = \frac{\mu}{4\pi}\left[\int_{-l/2}^{0} I_m\left(1 + \frac{z'}{l/2}\right)\frac{e^{-j\beta R}}{R}dz' + \int_{0}^{l/2} I_m\left(1 - \frac{z'}{l/2}\right)\frac{e^{-j\beta R}}{R}dz'\right]\hat{a}_z$$

$$\mathbf{A} = \frac{1}{2}\left[\frac{\mu}{4\pi}I_m l\frac{e^{-j\beta R}}{r}\right]\hat{a}_z$$

The further away from the antenna the observation point is, the more accurate the expression in above equation. Note that *the result in it is exactly one-half of the result obtained for* **A** *of an infinitesimal dipole of the same length*, if I_m were the current uniformly distributed along the dipole. This is expected because we made the same approximation for *R*, as in the case of the infinitesimal dipole with a constant current distribution, and we integrated a triangular function along *l*, whose average is obviously $I_0 = I_{av} = 0.5\, I_m$.

Therefore, we need not repeat all the calculations of the field components, power and antenna parameters. We make use of our knowledge of the infinitesimal dipole field. The far-field components of the small dipole are simply half those of the infinitesimal dipole:

$$H_\varphi \approx j\frac{\beta \cdot (I_m l)e^{-j\beta r}}{8\pi r}\sin\theta$$

$$E_\theta \approx j\eta\frac{\beta (I_m l)e^{-j\beta r}}{8\pi r}\sin\theta\,, \beta r \gg 1.$$

$$E_r = H_r = H_\theta = E_\varphi = 0$$

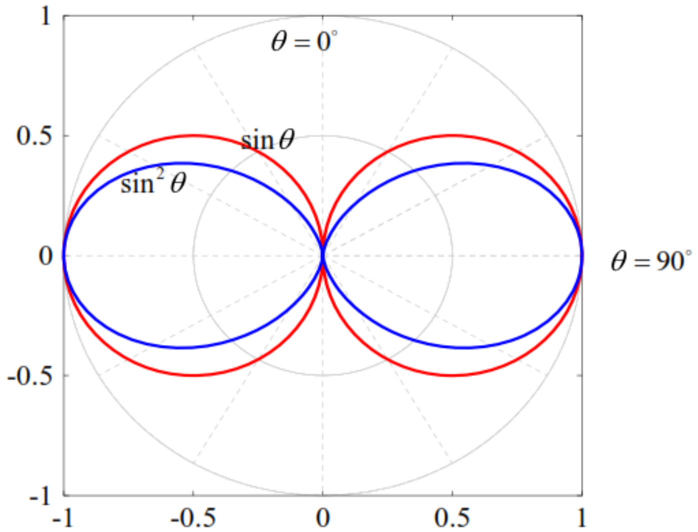
The normalized field pattern is the same as that of the infinitesimal dipole:

$$\bar{E}(\theta,\varphi) = \sin\theta$$

The power pattern:

$$U(\theta,\varphi) = \frac{r^2}{2\eta}|\mathbf{E}|^2$$

$$\bar{U}(\theta,\varphi) = \sin^2\theta$$



The beam solid angle:

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} \bar{U}(\theta,\varphi)\sin\theta\, d\theta d\varphi = \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \cdot \sin\theta\, d\theta d\varphi = \frac{8\pi}{3}$$

The directivity:

$$D_{max} = 4\pi/\,\Omega_A = 3\cdot 4\pi/\,8\pi = 3/\,2 = 1.5$$

As expected, the directivity, the beam solid angle as well as the effective aperture are the same as those of the infinitesimal dipole because the normalized patterns of both dipoles are the same.

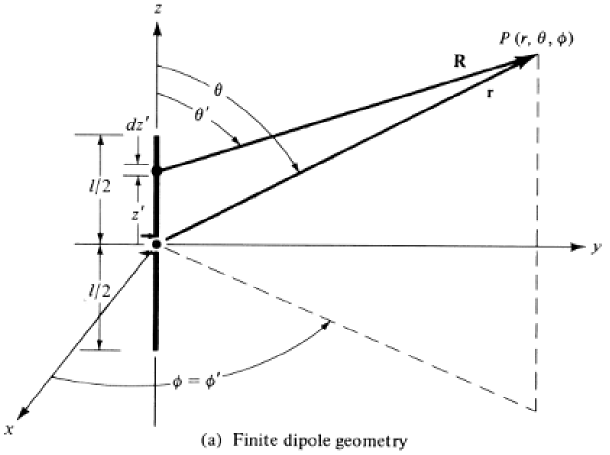
The radiated power is four times less than that of an infinitesimal dipole of the same length and current $I_0=I_m$ because the far fields are twice smaller in magnitude:

$$\Pi = \frac{1}{4}\cdot\frac{\pi}{3}\eta\left(\frac{I_m l}{\lambda}\right)^2 = \frac{\pi}{12}\eta\left(\frac{I_m l}{\lambda}\right)^2$$

As a result, the radiation resistance is also four times smaller than that of the infinitesimal dipole:

$$\Pi = \frac{1}{2}\cdot I_m^2 R_r \Rightarrow R_r = \frac{\pi}{6}\eta\left(\frac{l}{\lambda}\right)^2 = 20\pi^2\left(\frac{l}{\lambda}\right)^2$$

9.2 Finite-length infinitesimally thin dipole



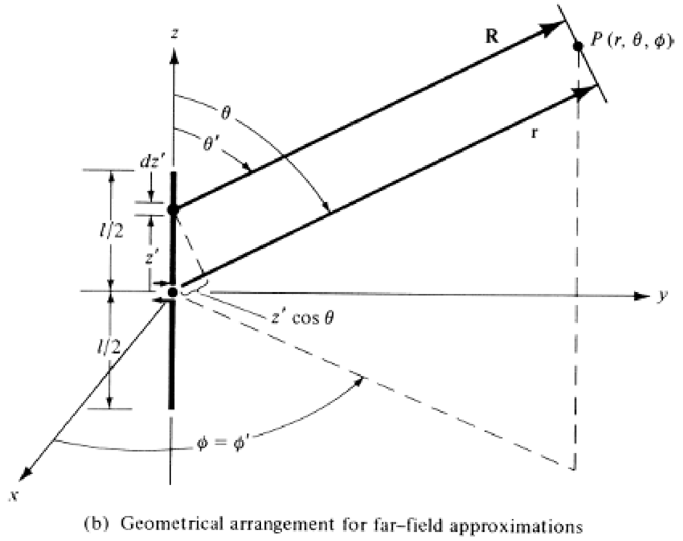


Figure 9.1 Finite dipole geometry and far-field approximations

A good approximation of the current distribution along the dipole's length is the sinusoidal one:

$$I(z') = \begin{cases} I_0 \sin\left[\beta\left(\frac{l}{2} - z'\right)\right], & 0 \le z' \le l/2 \\ I_0 \sin\left[\beta\left(\frac{l}{2} + z'\right)\right], & -l/2 \le z' \le 0. \end{cases}$$

It can be shown that the VP integral

$$\mathbf{A} = \hat{a}_z \frac{\mu}{4\pi} \int_{-l/2}^{l/2} I(z') \frac{e^{-j\beta R}}{R} d\,z'$$

has an analytical (closed form) solution. Here, however, we follow a standard approach used to calculate the far field for an arbitrary wire antenna. It is based on the solution for the field of the infinitesimal dipole. The finite-length dipole is subdivided into an infinite number of infinitesimal dipoles of length *dz*'. Each such dipole produces the elementary far field as

$$dH_\varphi \approx j\beta \cdot I(z') \frac{e^{-j\beta R}}{4\pi R} \sin\theta \, dz'$$

$$dE_\theta \approx j\eta\beta \cdot I(z') \frac{e^{-j\beta R}}{4\pi R} \sin\theta \, dz'$$

$$dE_r = dH_r = dH_\theta = dE_\varphi = 0$$

where $R= [x^2 + y^2 + (z- z')^2]^{1/2}$ and I (z ') denotes the value of the current element at 'z. Using the far-zone approximations,

$$\begin{cases} \frac{1}{R} \simeq \frac{1}{r}, & \text{for the amplitude factor} \\ R \simeq r - z'\cos\theta, & \text{for the phase factor} \end{cases}$$

the following approximation of the elementary far field is obtained:

$$dE_\theta \approx j\eta\beta \cdot I(z') \frac{e^{-j\beta r}}{4\pi r} \cdot e^{j\beta z' \cos\theta} \cdot \sin\theta \, dz'$$

Using the superposition principle, the total far field is obtained as

$$E_\theta = \int_{-l/2}^{l/2} dE_\theta \approx j\eta\beta \cdot \frac{e^{-j\beta r}}{4\pi r} \cdot \sin\theta \int_{-l/2}^{l/2} I(z') \cdot e^{j\beta z' \cos\theta} \cdot dz'$$

The ***first factor***

$$g(\theta) = j\eta\beta I_0 \cdot \frac{e^{-j\beta r}}{4\pi r} \cdot \sin\theta$$

is called the ***element factor***. The element factor in this case is the far field produced by an infinitesimal dipole of unit current element *I.l*= 1 (A×m). The element factor is the same for any current element, provided the angle θ is always associated with the current axis.

The ***second factor*** is the ***space factor (or pattern factor, array factor)***. The pattern factor is dependent on the amplitude and phase distribution of the current at the antenna.

For the specific current distribution described in the begging of this section, the ***pattern factor*** is

$$f(\theta) = \int_{-l/2}^{0} \sin\left[\beta\left(\frac{l}{2} + z'\right)\right] \cdot e^{j\beta z' \cos\theta} \cdot dz' + \int_{0}^{l/2} \sin\left[\beta\left(\frac{l}{2} - z'\right)\right] \cdot e^{j\beta z' \cos\theta} \cdot dz'$$

The far field of the finite-length dipole is obtained as

$$E_\theta = g(\theta) \cdot f(\theta) = j\eta I_0 \cdot \frac{e^{-j\beta r}}{2\pi r} \cdot \left[\frac{\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{\sin\theta}\right]$$

In a similar manner, or by using the established relationship between the E_θ and H_φ in the far field, the total H_φ component can be written as

$$H_\varphi = jI_0 \cdot \frac{e^{-j\beta r}}{2\pi r} \cdot \left[\frac{\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{\sin\theta}\right]$$

The normalized field pattern:

$$\bar{E}(\theta,\varphi) = \left[\frac{\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{\sin\theta}\right]$$

The power pattern:

$$U(\theta,\varphi) = \frac{r^2}{2\eta}|\mathbf{E}|^2$$

$$\bar{U}(\theta,\varphi) = \left[\frac{\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{\sin\theta}\right]^2 = \frac{\left[\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)\right]^2}{\sin^2\theta}$$

Note: The maximum of $\bar{U}(\theta,\varphi)$ is not necessarily unity, but for $l < 2\lambda$ the major maximum is always at $\theta = 90^o$.

The radiated power:

First, the far-zone power flux density is calculated as

$$\mathbf{P} = \frac{1}{2\eta}|E_\theta|^2\hat{a}_r = \eta \cdot \frac{I_0^2}{8\pi^2 r^2} \cdot \left[\frac{\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{\sin\theta}\right]^2 \hat{a}_r$$

The total radiated power is given by the integral

$$\Pi = \oiint \mathbf{P}\cdot ds = \int_0^{2\pi}\int_0^{\pi} \mathbf{P}\cdot r^2\sin\theta\, d\theta d\varphi.$$

$$\Pi = \int_0^{2\pi}\int_0^{\pi} \eta \cdot \frac{I_0^2}{8\pi^2 r^2} \cdot \left[\frac{\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{\sin\theta}\right]^2 \cdot r^2\sin\theta\, d\theta d\varphi.$$

$$\Pi = \eta \cdot \frac{I_0^2}{4\pi} \underbrace{\int_0^{\pi} \frac{\left[\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)\right]^2}{\sin\theta} \cdot d\theta}_{\mathcal{L}}$$

Thus, the radiated power can be written as

$$\Pi = \eta \cdot \frac{I_0^2}{4\pi} \cdot \mathcal{L}$$

The radiation resistance is defined as

$$\Pi = \frac{1}{2} \cdot I_m^2 R_r \Rightarrow R_r = \frac{2\Pi}{I_m^2}$$

$$R_r = \frac{2\eta \cdot \frac{I_0^2}{4\pi} \cdot \mathcal{L}}{I_m^2} = \frac{\eta}{2\pi} \cdot \frac{I_0^2}{I_m^2} \cdot \mathcal{L}$$

where I_m is the maximum current magnitude along the dipole. If the dipole is half-wavelength long or longer ($l \geq \lambda/2$), $I_m = I_0$. However, if $l < \lambda/2$, then $I_m < I_0$. This can be easily understood from the current distribution equation. If $l < \lambda/2$ holds, the maximum current is at the center (the feed point $z'=0$) and its value is

$$I_m = I_{(z'=0)} = I_0 \sin(\beta l/2)$$

where $\beta l/2 < \pi/2$, and, therefore $\sin(\beta l/2) < 1$. Therefore,

$$R_r = \frac{\eta}{2\pi} \cdot \frac{\mathcal{L}}{\sin^2(\beta l/2)}, \quad if\ l < \lambda/2$$

$$R_r = \frac{\eta}{2\pi} \cdot \mathcal{L}, \quad if\ l \geq \lambda/2$$

<u>The directivity is obtained as</u>

$$D_{max} = 4\pi \frac{U_{max}}{\Pi}$$

$$D_{max} = 4\pi \Big/ \int_0^{2\pi}\int_0^{\pi} \bar{U}(\theta, \varphi) \sin\theta \, d\theta d\varphi$$

$$D_{max} = \frac{2U_{max}}{\mathcal{L}}$$

Input resistance:

The radiation resistance given above is not necessarily equal to the input resistance because the current at the dipole center I_{in} (if its center is the feed point) is not necessarily equal to I_m. In particular, $I_{in} \neq I_m$ if $l > \lambda/2$ and $l \neq (2n+1)\lambda/2$. Note that in this case $I_m = I_0$. If the dipole is lossless, the input power is equal to the radiated power. Therefore,

$$P_{in} = \frac{1}{2} \cdot I_{in}^2 R_{in} = \Pi = \frac{1}{2} \cdot I_0^2 R_r \quad l > \lambda/2$$

Since the current at the center of the dipole ($z' = 0$) is

$$I_{in} = I_{(z'=0)} = I_0 \sin(\beta l/2)$$

then

$$R_{in} = \frac{R_r}{\sin^2(\beta l/2)} = \frac{\eta}{2\pi} \cdot \frac{\mathcal{L}}{\sin^2(\beta l/2)}, \quad l > \lambda/2.$$

For a short dipole ($l \leq \lambda/2$), $I_{in} = I_m$ and therefore;

$$R_{in} = R_r = \frac{\eta}{2\pi} \cdot \frac{\mathcal{L}}{\sin^2(\beta l/2)}, \quad l \leq \lambda/2.$$

In summary, the dipole input resistance, regardless of its length, depends on the integral $\mathcal{L}$ as in above equations, as long as it is fed at its center.