Half-wavelength dipole

This is a classical and widely used thin wire antenna: $l = \lambda/2$.

The far field of this dipole is obtained as

$$E_\theta = j\eta I_0 \cdot \frac{e^{-j\beta r}}{2\pi r} \cdot \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

In a similar manner, or by using the established relationship between the E_θ and H_φ in the far field, the total H_φ component can be written as

$$H_\varphi = jI_0 \cdot \frac{e^{-j\beta r}}{2\pi r} \cdot \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

Radiated power density:

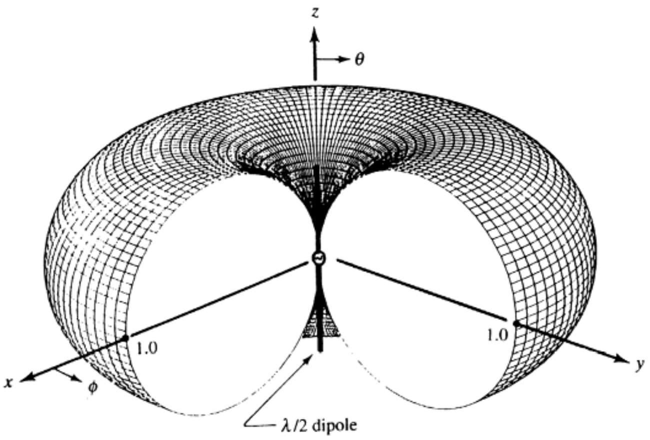
$$P = \frac{1}{2\eta}|E_\theta|^2 = \eta \cdot \frac{|I_0|^2}{8\pi^2 r^2} \cdot \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2 \approx \eta \cdot \frac{|I_0|^2}{8\pi^2 r^2} \cdot \sin^3\theta$$

Radiation intensity:

$$U(\theta,\varphi) = \frac{r^2}{2\eta}|\mathbf{E}|^2$$

$$U(\theta,\varphi) = r^2 P = \frac{r^2}{2\eta}|E_\theta|^2 \approx \eta \cdot \frac{|I_0|^2}{8\pi^2} \cdot \sin^3\theta$$

3-D power pattern (not in dB) of the half-wavelength dipole:



Radiated power:

The radiated power of the half-wavelength dipole is a special case of the integral mentioned in previous lecture:

$$\Pi = \oiint \mathbf{P} \cdot ds = \int_0^{2\pi}\int_0^{\pi} \mathbf{P} \cdot r^2 \sin\theta \, d\theta d\varphi.$$

$$\Pi = \int_0^{2\pi}\int_0^{\pi} \eta \cdot \frac{|I_0|^2}{8\pi^2 r^2} \cdot \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2 \cdot r^2 \sin\theta \, d\theta d\varphi.$$

$$\Pi = \eta \cdot \frac{|I_0|^2}{4\pi} \int_0^{\pi} \frac{\left[\cos\left(\frac{\pi}{2}\cos\theta\right)\right]^2}{\sin\theta} \cdot d\theta = \eta \cdot \frac{|I_0|^2}{8\pi} \underbrace{\int_0^{2\pi} \frac{1-\cos y}{y} \cdot dy}_{\kappa}$$

$$\Pi = \eta \cdot \frac{|I_0|^2}{8\pi} \kappa$$

$$\kappa = 0.5772 + \ln(2\pi) - c_i(2\pi) \approx 2.435$$

$$\Pi = \eta \cdot \frac{|I_0|^2}{8\pi} \cdot 2.435 = 36.525 \cdot |I_0|^2$$

Radiation resistance:

$$R_r = \frac{2\Pi}{|I_0|^2} \approx 73\Omega$$

Directivity:

$$D_{max} = 4\pi \frac{U_{max}}{\Pi} = 4\pi \frac{\eta \cdot \frac{|I_0|^2}{8\pi^2}}{36.525 \cdot |I_0|^2} = \frac{60}{36.525} = 1.643$$

Maximum effective area:

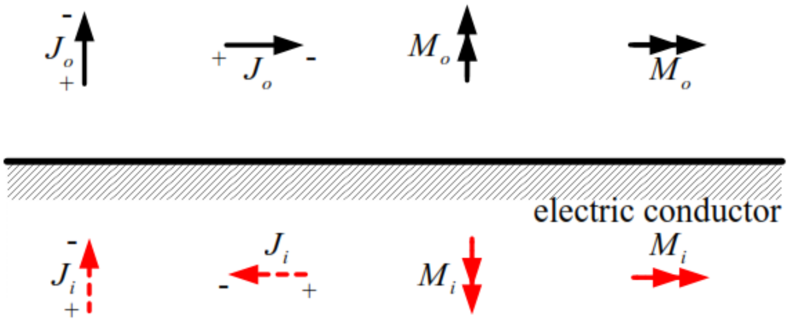
$$A_e = \frac{\lambda^2}{4\pi} D_{max} = 0.13\lambda^2$$

Input resistance:

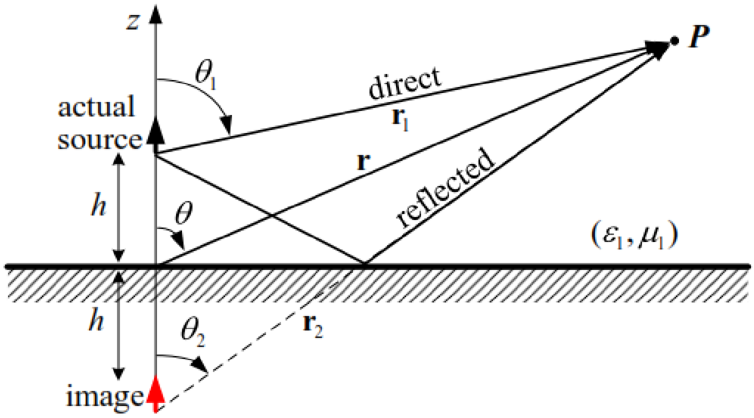
Since $l = \lambda/2$. $\quad R_r = R_{in} \approx 73\Omega$

The imaginary part of the input impedance is approximately $\approx +j42.5\Omega$

Method of images



Vertical electric current element above perfect conductor



The field at the observation point P is a superposition of the fields of the actual source and the image source, both radiating in a homogeneous medium of constitutive parameters (ε_1, μ_1) The actual source is a current element ($I_0\Delta l$) (infinitesimal dipole).

$$E_\theta^d = j\eta \frac{\beta(I_0\Delta l)e^{-j\beta r_1}}{4\pi r_1}\sin\theta_1$$

$$E_\theta^r = j\eta \frac{\beta(I_0\Delta l)e^{-j\beta r_2}}{4\pi r_2}\sin\theta_2$$

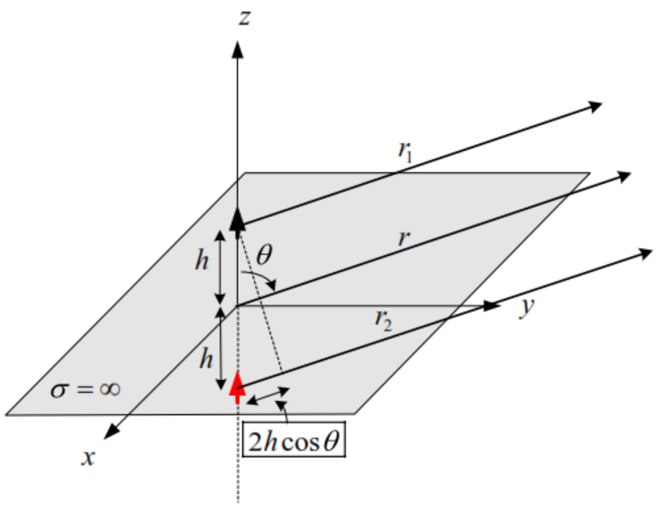
Expressing the distances r1 and r2 in term of r and h (using the cosine theorem) gives

$$r_1 = \sqrt{r^2 + h^2 - 2rh\cos\theta}$$

$$r_2 = \sqrt{r^2 + h^2 - 2rh\cos(\pi - \theta)}$$

We make use of the binomial expansions of r1 and r2 to approximate the amplitude and the phase terms, which simplify the evaluation of the total far field and the VP integral. For the amplitude term,

$$\frac{1}{r_1} \simeq \frac{1}{r_2} \simeq \frac{1}{r}$$



For the phase term, we use the second-order approximation (see also the geometrical interpretation above),

$$r_1 \simeq r - h\cos\theta$$
$$r_2 \simeq r + h\cos\theta$$

The total electrical far field is

$$E_\theta = E_\theta^d + E_\theta^r$$

$$E_\theta = j\eta \frac{\beta(I_0 \Delta l)}{4\pi r} \sin\theta \left[e^{-j\beta(r-h\cos\theta)} + e^{-j\beta(r+h\cos\theta)}\right]$$

$$E_\theta = \underbrace{j\eta \frac{\beta(I_0 \Delta l)}{4\pi r} \sin\theta}_{g(\theta)} \underbrace{[2\cos(\beta h\cos\theta)]}_{f(\theta)}, \qquad z \geq 0$$
$$E_\theta = 0 \qquad , \qquad z < 0$$

We note that the far-field expression can be again decomposed into two factors: the field of the elementary source $g(\theta)$ and the pattern factor $f(\theta)$ (also array factor).

The normalized power pattern is

$$\bar{U} = [\sin\theta \cdot \cos(\beta h \cos\theta)]^2$$

<u>Total radiated power:</u>

$$\Pi = \oiint \mathbf{P} \cdot ds = \int_0^{2\pi}\int_0^{\pi} \mathbf{P} \cdot r^2 \sin\theta \, d\theta d\varphi.$$

$$\Pi = \eta\beta^2 (I_0 \Delta l)^2 \int_0^{\pi/2} [\sin\theta \cdot \cos(\beta h \cos\theta)]^2 \cdot d\theta.$$

$$\boxed{\Pi = \pi\eta \left(\frac{I_0 \Delta l}{\lambda}\right)^2 \left[\frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right]}$$

- As $h\beta \to 0$, the radiated power of the vertical dipole above ground approaches twice the value of the radiated power of a dipole of the same length in free space.
- As $h\beta \to \infty$, the radiated power of both dipoles becomes the same.

That's mean:

$$\lim_{h\to 0}\left[-\frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right] = \frac{1}{3},$$

$$\lim_{h\to \infty}\left[-\frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right] = 0,$$

<u>Radiation resistance:</u>

$$R_r = \frac{2\Pi}{|I_0|^2} = 2\pi\eta \left(\frac{\Delta l}{\lambda}\right)^2 \left[\frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right]$$

- As $h\beta \to 0$, the radiation resistance of the vertical dipole above ground approaches twice the value of the radiation resistance of a dipole of the same length in free space:

$$R_r^{vdp} = 2R_r^{dp} \ when \ \beta h = 0$$

- As $h\beta \to \infty$, the radiation resistance of both dipoles becomes the same.

Radiation intensity:

$$U(\theta,\varphi) = r^2 P = \frac{r^2}{2\eta}|E_\theta|^2 = \frac{\eta}{2}\left(\frac{I_0 \Delta l}{\lambda}\right)^2 [\sin^2\theta \cdot \cos^2(\beta h \cos\theta)]$$

The maximum value of radiation intensity occurs at $\theta = \pi/2$:

$$U_{max} = \frac{\eta}{2}\left(\frac{I_0 \Delta l}{\lambda}\right)^2$$

This value is 4 times greater than U_{max} of a free space dipole of the same length.

Maximum directivity:

$$D_{max} = 4\pi \frac{U_{max}}{\Pi} = \frac{2}{\frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}}$$

If $h\beta$ =0, D_{max} =3, which is twice the maximum directivity of a free-space current element ($D_{\max}^{id} = 1.5$).

The maximum of D_{max} as a function of the height h occurs when $h\beta$ = 2.881 (h=0.4585 λ). Then, D_{max}=$6.566_{/h\beta=2.881}$.

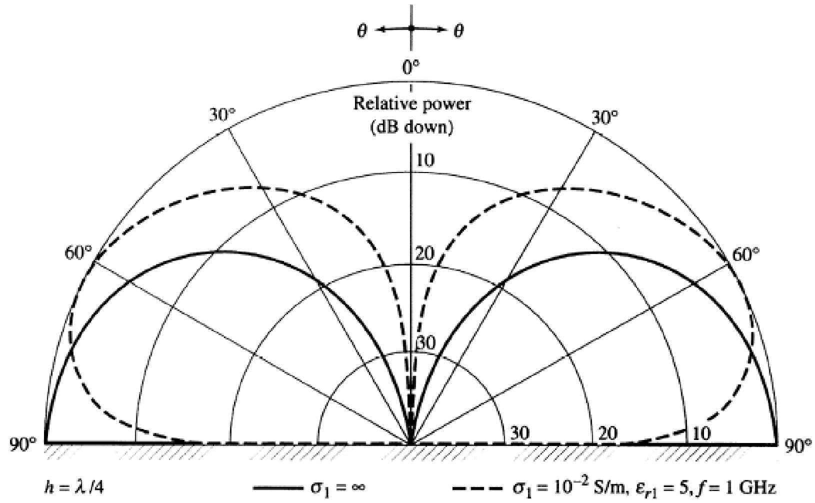
Monopoles

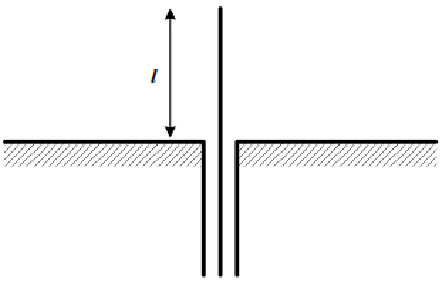
A monopole is a dipole that has been divided into half at its center where it is fed against a ground plane. It is normally λ /4 long (a ***quarter wavelength monopole***), but it might be shorter. Then, the monopole is a ***small monopole*** whose counterpart is the ***small dipole***. Its current has linear distribution with its maximum at the feed point and its null at the end.

The vertical monopole is extensively used for AM broadcasting (f = 500 to 1500 kHz, λ = 200 to 600 m), because it is the shortest most efficient antenna at these frequencies as well as because vertically polarized waves suffer less attenuation at close-to-ground propagation. Vertical monopoles are widely used as base-station antennas in mobile communications, too.

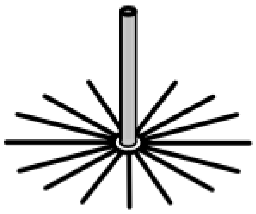
Monopoles at base stations and radiobroadcast stations are supported by towers and guy wires. The guy wires must be separated into short enough (≤ λ /8) pieces insulated from each other to suppress any parasitic currents.

Special care is taken when grounding the monopole. Usually, multiple radial wire rods, each 0.25−0.35 λ long, are buried at the monopole base in the ground to simulate a perfect ground plane, so that the pattern approximates closely the theoretical one, i.e., the pattern of the λ/2 -dipole. Losses in the ground plane cause undesirable deformation of the pattern as shown below (infinitesimal dipole above an imperfect ground plane).





Monopole fed against a large solid ground plane



Practical monopole with radial wires to simulate perfect ground

Several important conclusions follow from the image theory and the above discussion:

- The field distribution in the upper half-space is the same as that of the respective free-space dipole.
- The currents and charges on a monopole are the same as on the upper half of its dipole counterpart but the terminal voltage is only half that of the dipole.
- The input impedance of a monopole is therefore only half that of the respective dipole:

$$Z_{in}^{mp} = \frac{1}{2} Z_{in}^{dp}$$

- The total radiated power of a monopole is half the power radiated by its dipole counterpart since it radiates in half-space (but its field is the same). As a result, the beam solid angle of the monopole is half that of the respective dipole and its directivity is twice the directivity of the dipole:

$$D_{max}^{mp} = \frac{4\pi}{\Omega_A^{mp}} = \frac{4\pi}{\frac{1}{2}\Omega_A^{dp}} = 2D_{max}^{dp}$$

The quarter-wavelength monopole:

This is a straight wire of length $l = \lambda/4$ mounted over a ground plane. From the discussion above, it can be expected that the quarter-wavelength monopole is very similar to the half-wavelength dipole (in the hemisphere above the ground plane).

- Its radiation pattern is the same as that of a free-space $\lambda/2$ -dipole, but it is non-zero only for $0^o < \theta \leq 90^o$ (above ground).
- The field expressions are the same as those of the $\lambda/2$ -dipole.
- The radiated power of the $\lambda/4$ -monopole is half that of the $\lambda/2$ -dipole.
- The radiation resistance of the $\lambda/4$-monopole is half that of the $\lambda/2$ -dipole:

$$Z_{in}^{mp} = \frac{1}{2} Z_{in}^{dp} = 0.5(73 + j42.5) = 36.5 + j21.25, \Omega$$

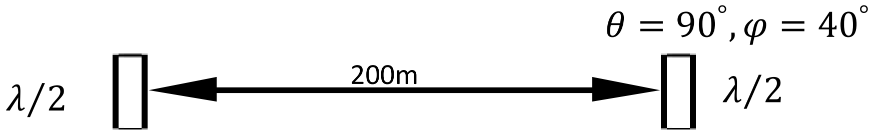
- The directivity of the $\lambda/4$ -monopole is:

$$D_{max}^{mp} = 2D_{max}^{dp} = 2 * 1.643 = 3.286$$

Example:

A $\lambda/2$ dipole situated with its center at the origin radiates a time-averaged power of 600w at a frequency of 300 MHz. A second $\lambda/2$ dipole is placed with its center at a point $P(r, \theta, \varphi)$, where r = 200m, $\theta = 90^\circ, \varphi = 40^\circ$. It is oriented so that its axis is parallel to that of the transmitting antenna. what is the available power at the terminals of the second (receiving) dipole?

Solution:



$$P_r = P_t e_t e_r \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{300 \times 10^6} = 1m$$

$$r_{far\ field} = \frac{2D^2}{\lambda} = \frac{2(\frac{\lambda}{2})^2}{\lambda} = \frac{\lambda}{2} = 0.5m$$

$$r_{far\ field} \ll 200m$$

For lossless antenna D_t=G_t and D_r=G_r

For half wave dipole D_{max}=1.643

$$P_r = P_t \left(\frac{\lambda}{4\pi R}\right)^2 D_t D_r = 600 * \left(\frac{1}{4\pi \times 200}\right)^2 \times 1.643 \times 1.643 = 0.256mw$$