Conductors and Dielectrics

In electromagnetic, materials are dividing roughly into two classes: conductors and dielectrics or insulators. The dividing line between the two classes is not sharp and some media (the earth for example) are considered as conductors in one part of the radio frequency range, but as dielectrics (with loss) in another part of the range.

In Maxwell's second equation:

$$\nabla \times H = \sigma E + j\omega\varepsilon E$$

The first term on the right is conduction current density and the second term is displacement current density. The ratio $\sigma/\omega\varepsilon$ is therefore just the ratio of conduction current density to displacement current density in the medium. Hence, $\sigma/\omega\varepsilon = 1$ can be considered to mark the dividing line between conductors and dielectrics .

For good conductors such as metals $\sigma/\omega\varepsilon$ is very much greater than unity over the entire radio frequency spectrum.

For good dielectrics or insulators $\sigma/\omega\varepsilon$ is very much less than unity in the radio frequency range.

Example:

state that the copper is conductor or dielectrics even at the relatively high frequency of 30 GHz, ($\varepsilon = \varepsilon_o, \ \mu = \mu_o \ \ and \ \sigma = 5.8*10^7 \ \ s/m$)

Solution:

$\frac{\sigma}{\omega\varepsilon} = \frac{5.8*10^7}{2*\pi*30*10^9*8.85*10^{-12}} = 3.5*10^7 >> 1$ then the copper is good conductor

Example:

state that the mica at audio or radio frequencies (10KHz) is conductor or dielectrics, ($\varepsilon = \varepsilon_o, \quad \mu = \mu_o \quad and\ \sigma = 10^{-15} \quad s/m$).

Solution:

$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-15}}{2*\pi*10*10^{3}*8.85*10^{-12}} = 0.001798*10^{-6} << 1$ then the mica is good dielectric

Wave propagation in Good Dielectrics

$$\nabla^2 \mathrm{E} = j\omega\mu(\sigma + j\omega\varepsilon)\mathrm{E}$$

$$\nabla^2 \mathrm{H} = j\omega\mu(\sigma + j\omega\varepsilon)\mathrm{H}$$

Let $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

And $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

Then above equations are become

$$\nabla^2 \mathrm{E} = \gamma^2 \mathrm{E}$$

$$\nabla^2 \mathrm{H} = \gamma^2 \mathrm{H}$$

$$\gamma = \alpha + j\beta$$

$$\alpha = real\ part\ of\ \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right)}$$

For this case $\sigma/\omega\varepsilon << 1$ so that it is possible to write to a very good approximation

$$\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} \cong \left(1+\frac{\sigma^2}{2\omega^2\varepsilon^2}\right)$$

where only the first two term of the binomial expansion have been used.

Then α

$$\alpha \cong \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\left(1+\frac{\sigma^2}{2\omega^2\varepsilon^2}\right)-1\right)} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}},$$

$$\boxed{\boldsymbol{\alpha \cong \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}}}$$

The expression for β reduces in a similar manner

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right)}$$

$$\beta \cong \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\left(1+\frac{\sigma^2}{2\omega^2\varepsilon^2}\right)+1\right)} = \omega\sqrt{\mu\varepsilon}\left(1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$\boxed{\beta \cong \omega\sqrt{\mu\varepsilon}\left(1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)}$$

$\omega\sqrt{\mu\varepsilon}$ is the phase shift factor for perfect dielectric. The effect of small amount of loss is to add the second term of above equation as a small correction factor. The velocity of the wave in the dielectric is given by:

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}\left(1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)} \cong v_o\left(1-\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$\boxed{v \cong v_o\left(1-\frac{\sigma^2}{8\omega^2\varepsilon^2}\right)}$$

where $v_o = 1/\sqrt{\mu\varepsilon}$ is the velocity of the wave in the dielectric when the ***conductivity is zero***. The effect of small amount of loss is to reduce slightly the velocity of propagation of the wave.

The general expression for the intrinsic or characteristics impedance of the medium which has a finite conductivity is

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

Using the same approximations as above, this becomes for a good dielectric

$$\eta = \sqrt{\frac{\mu}{\varepsilon}\left(\frac{1}{1+\frac{\sigma}{j\omega\varepsilon}}\right)} \cong \sqrt{\frac{\mu}{\varepsilon}}\left(1+j\frac{\sigma}{2\omega\varepsilon}\right)$$

$$\boxed{\eta \cong \sqrt{\frac{\mu}{\varepsilon}}\left(1+j\frac{\sigma}{2\omega\varepsilon}\right)}$$

Since $\sqrt{\mu/\varepsilon}$ is the intrinsic impedance of the dielectric when $\sigma = 0$

Wave propagation in Good Conductor

for this case $\sigma/\omega\varepsilon >> 1$ so that the expression for γ may be written

$$\gamma = \sqrt{(j\omega\mu\sigma)\left(1 + j\frac{\omega\varepsilon}{\sigma}\right)} \cong \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma}e^{j45}$$

Therefore

$$\boldsymbol{\beta = \alpha = \sqrt{\frac{\omega\mu\varepsilon}{2}}}$$

The velocity of the wave in the conductor is given by:

$$\boldsymbol{v = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\varepsilon}}}$$

The intrinsic impedance of the conductor is given by:

$$\boldsymbol{\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}e^{j45}}$$

Example:

Consider the possibility of propagating a plane wave through seawater ($\sigma = 4,\ \varepsilon_r = 81$). Find $\alpha, \beta, v, and\ \lambda$ at 1MHz.

Solution:

$$\sigma/\omega\varepsilon = 888$$

$$\beta = \alpha = \sqrt{\frac{\omega\mu\varepsilon}{2}} = \sqrt{\frac{2\pi \times 10^6 \times 4\pi \times 10^{-7} \times 81 \times 8.85 \times 10^{-12}}{2}}$$

$$= 5.32 \times 10^{-5}\ rad/sec$$

$$v = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{5.32 \times 10^{-5}} = 118 \times 10^9 m/s$$

$$\lambda = \frac{2\pi}{\beta} = 118km$$

Example:

A uniform plane wave at a frequency of 1GHz is traveling in a large block of Teflon ($\mu_r = 1, \varepsilon_r = 2.1, \sigma = 0$) . Determine *v*, intrinsic impedance (η), phase constant (β), and wavelength (λ).

Solution:

$$\sigma/\omega\varepsilon = 0$$

At σ=0 , $\alpha = 0$

$$\beta = \omega\sqrt{\mu\varepsilon} = 2\pi f\sqrt{\mu_o\varepsilon_o}\sqrt{\mu_r\varepsilon_r} = \frac{2\pi f}{3*10^8}\sqrt{\mu_r\varepsilon_r} = \frac{2\pi*10^9}{3*10^8}\sqrt{2.1} = 30.4 \quad rad/\sec$$

$$v_o = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi*10^9}{30.4} = 2.04*10^8 \quad m/sec$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r}}\sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi\sqrt{\frac{1}{2.1}} = 260\Omega$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{30.4} = 0.204 \quad m \approx 20.4 \quad cm$$

Depth of penetration

The depth of penetration (δ) is defined as that depth in which the wave has been attenuated to $1/e$ or approximately 37 percent of its original value.

$$\alpha\delta = 1 \quad \text{or} \quad \delta = \frac{1}{\alpha}$$

The general expression for depth of penetration is

$$\delta = \frac{1}{\alpha} = \frac{1}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right)}}$$

For good conductors the depth of penetration is

$$\delta = \frac{1}{\alpha} \cong \sqrt{\frac{2}{\omega\mu\sigma}}$$

Example:

Find the depth of penetration of 1 MHz wave in copper which has conductivity $\sigma = 5.8 * 10^7$ mhos/meter and permeability approximately equal to that of free space.

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{2\pi * 10^6 * 4\pi * 10^{-7} * 5.8 * 10^7}} = 0.0667mm$$

At $f = 100MHz$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{2\pi * 100 * 10^6 * 4\pi * 10^{-7} * 5.8 * 10^7}} = 0.00667mm$$

At $f = 60MHz$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{2\pi * 60 * 10^6 * 4\pi * 10^{-7} * 5.8 * 10^7}} = 0.06608mm$$

Example:

find the depth of penetration of 1 MHz wave in sea water which has conductivity $\sigma = 4$ mhos/meter and permeability approximately equal to that of free space.

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{2\pi * 10^6 * 4\pi * 10^{-7} * 4}} = 25cm$$