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Reciprocal levelling



$$\Delta H)_{AB} = a_1 - (b_1 - e) \quad \text{--- (1)}$$



$$\Delta H)_{AB} = (a_2 - e) - b_2 \quad \text{--- (2)}$$

Eq.(1) + Eq.(2) gives:

$$\Delta H)_{AB} = \frac{1}{2} [(a_1 - b_1) + (a_2 - b_2)]$$

H-w Find e from Eq.(1) and Eq.2



Example / In levelling across river, reciprocal levelling observation gave the following result for staff held vertically at X and Y from level stations A and B on each bank.

Instrument position	staff position	staff reading (m)
X	A	1.753
X	B	2.080
Y	A	2.550
Y	B	2.895

if the R.L of A was 90.37, what the R.L of B

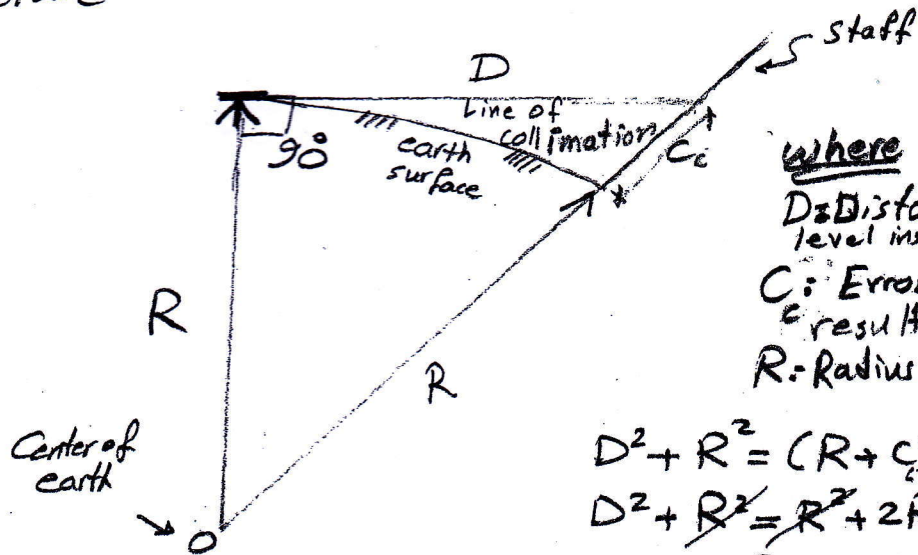
$$\begin{aligned} \Delta H)_{AB} &= \frac{1}{2} [(a_1 - b_1) + (a_2 - b_2)] \\ &= \frac{1}{2} [(1.753 - 2.080) + (2.550 - 2.895)] \\ &= -0.336 \text{ m} \end{aligned}$$

$$\text{R.L)B} = 90.37 - 0.336 = 90.034$$

Effect of Curvature and Refraction

(1)

* Curvature



where
 D : Distance between level instrument, and staff
 C_c : Error in staff reading & result in curvature
 R : Radius of earth

$$D^2 + R^2 = (R + C_c)^2$$

$$D^2 + R^2 = R^2 + 2RC_c + C_c^2$$

$$D^2 = 2RC_c \Rightarrow C_c = \frac{D^2}{2R}$$

$$R = 6370 \text{ km}$$

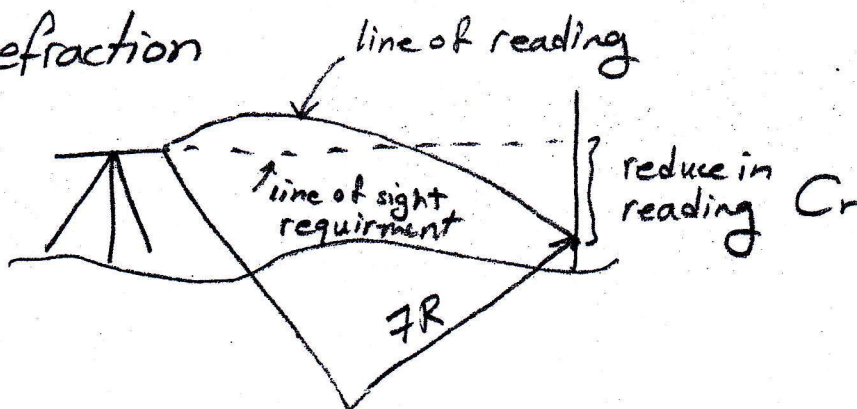
$$= \frac{D^2}{2 \times 6370} \times 1000 \text{ km to m}$$

$$C_c = 0.0785 D^2$$

m km

Cancel because small value

* Refraction



where

C_r : Error in staff Reading result in Refraction

$$\text{Correction of Refraction } (C_r) = \frac{1}{7} \text{ Correction of Curvature } (C_c)$$

* Combined Correction

$$\begin{aligned}C_{\text{combined}} &= C_c - C_r \\&= C_c - \frac{1}{7} C_c \\&= 0.0785 D^2 - \frac{1}{7} (0.0785 D^2) \\C_{\text{combined}} &= 0.0673 D^2\end{aligned}$$

m km

Example / A level instrument was setup between two staff positions A and B were 100 m and 200 m, respectively. The staff readings at A and B were 2.300 and 1.500 m, respectively. Find the correct difference of level between the positions.

$$C_{\text{combined}} = 0.0673 * \left(\frac{200}{1000}\right)^2 = 0.0027$$

(A)

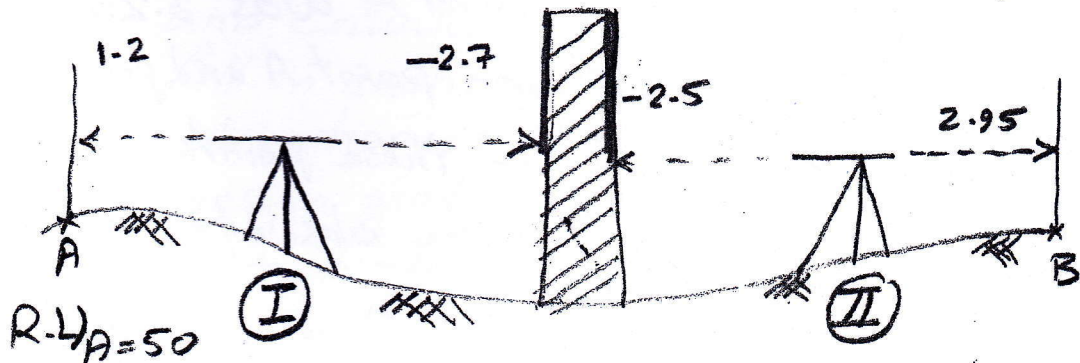
$$\begin{aligned}\text{Correct reading staff at A} &= 2.300 - 0.027 \\&= 2.297 \text{ m}\end{aligned}$$

$$\Delta H)_{AB} = 2.297 - 1.5 = \underline{\underline{0.797 \text{ m}}}$$

Difficulties in levelling

(3)

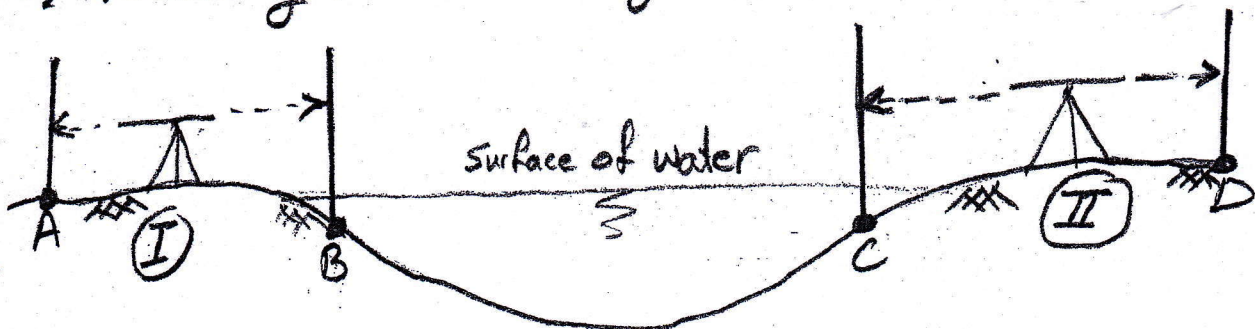
1) levelling across a solid obstruction like high wall



Staff reading			Staff station	H.I	R.L	note
B.S	I.S	F.S				
1.2		(I)	A	51.2	50	
-2.5		-2.7	high wall	51.4	53.9	
		2.95 (II)	B	=	48.45	

H.W/ check the result?

2) levelling across a large lake



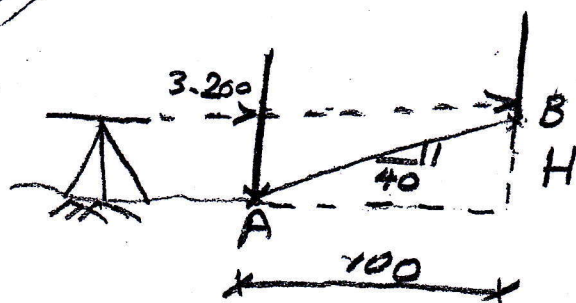
Assume $R.L)_B = R.L)_C$ Then levelling is continued.

Find the Correct Reading through the Slope

(4)

Example / A staff reading at point A was 3.200 m and the distance between point A and point B was 100 m. The slope between these points was 1:40 above. Determine the absolute reading on point B.

$$\frac{H}{100} = \frac{1}{40} \Rightarrow H = 2.5$$



Correct reading at B = $3.200 - 2.5 = 0.7$ m

Sources of Errors in levelling work

1. Instrument error.
2. Error due to displacement of level and staff.
3. Error due to natural causes.