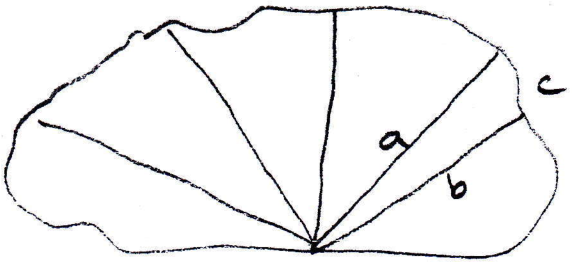
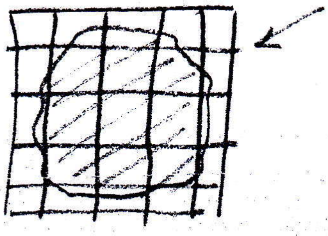
Areas and volumes

1. Calculate the area with figure method



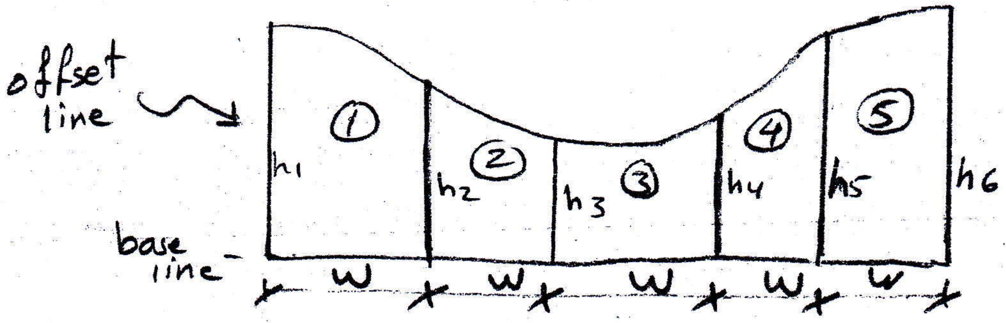
$$S = \frac{a+b+c}{2}$$

$$A = \sqrt{S(S-a)(S-b)(S-c)}$$



2. Area with arethmetic and engineering method

A. Trapezoidal rule



Area of ① $= \frac{h_1+h_2}{2} * w$, Area of ② $= \frac{h_2+h_3}{2} * w$, ...

Total area (A) $= \frac{w}{2}\left[h_1 + h_n + 2(h_2 + h_3 + \dots + h_{n-1})\right]$

W : interval between the offset line.

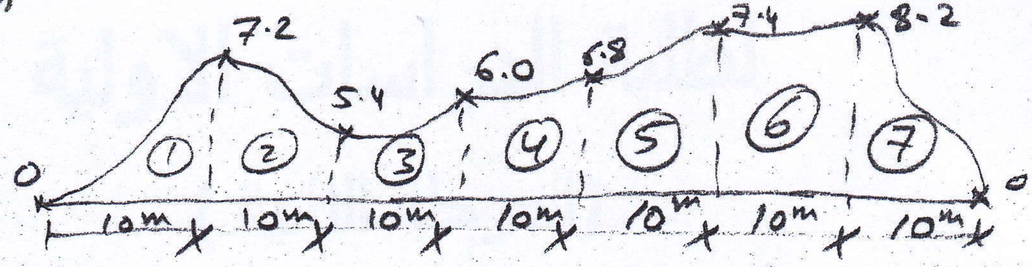
h : offset measurment.

n : number of offset.

B. Simpson's rule [this method is applicable only if the number of offsets is odd]

$$A = \frac{W}{3}\left[h_1 + h_n + 4\sum \text{even offset} + 2\sum \text{odd offset}\right]$$

Example/ Find the area by 1) Trapezoidal rule 2) Simpson's rule if the offset as follows: 0 , 7.2 , 5.4 , 6.0 , 6.8 , 7.4 , 8.2 , 0
the distance between offsets constant and was 10m



1) by the trapezoidal rule

$$A = \frac{W}{2}\left[h_1 + h_n + 2(h_2 + h_3 + \cdots + h_{n-1})\right]$$

$$= \frac{10}{2}\left[0 + 0 + 2(7.2 + 5.4 + 6.0 + 6.8 + 7.4 + 8.2)\right]$$

$$= 410\ m^2$$

2) by simpson's rule (division area)

$$A_T = \underbrace{A_1 \rightarrow 6}_{\text{simpson's rule}} + \underbrace{A_7}_{\text{trapezoidal rule}}$$

strip

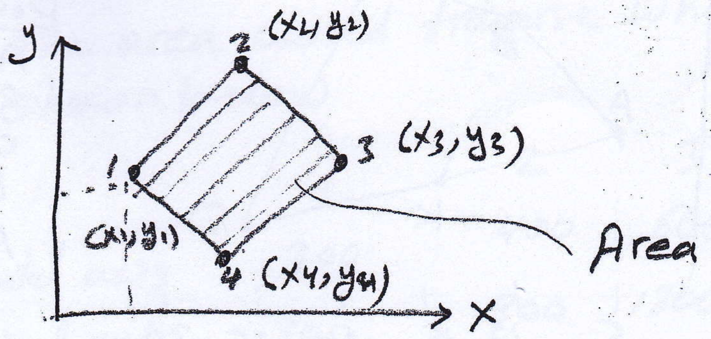
$$A_T = \frac{W}{3}\left[h_1 + h_n + 4\sum \text{even offset} + 2\sum \text{odd offset}\right] + A \text{ by trapozoidal rule}$$

$$= \frac{10}{3}\left[0 + 8.2 + 4(7.2 + 6.0 + 7.4) + 2(5.4 + 6.8)\right] + \frac{10}{2}\left[8.2 + 0 + 2(0)\right]$$

$$= 356 + \quad = 424\ m^2$$

Area from field notes

Area by coordinates



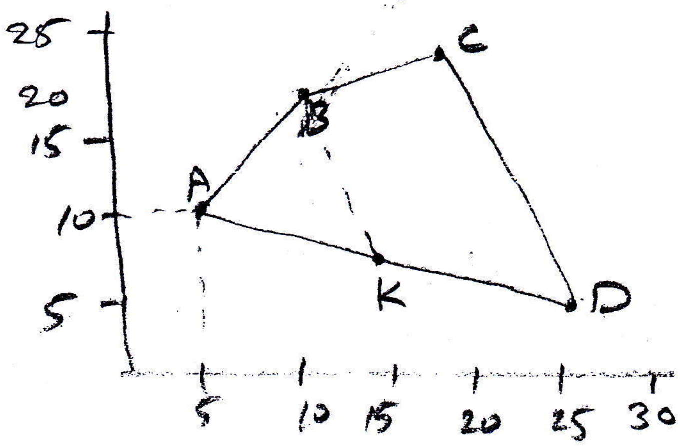
point	x	y
1	x_1	y_1
2	x_2	y_2
3	x_3	y_3
4	x_4	y_4
1	x_1	y_1

$$\text{Area} = \frac{1}{2}\left[(y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1) - (x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1)\right]$$

Example/ The area of closed traverse which coordinates shown below in table was divided into two equal parts by the line BK. Calculate the position of point K where point K placed on the line AD.

point	x	y
A	5	10
B	10	20
C	20	22
D	25	5

Sol.



Point	x	y
A	5	10
B	10	20
C	20	22
D	25	5
A	5	10

$$Area = \frac{1}{2}\left[(10*10 + 20*20 + 22*25 + 5*5) - (5*20 + 10*22 + 20*5 + 25*10)\right]$$

$$Area = 202.5\ m^2$$

Area ABKA = Area BCDKB = $\frac{1}{2}$ total area = $101.25 m^2$

From area ABKB

point	x	y
A	5	10
B	10	20
K	x	y
A	5	10

$$101.25 = \frac{1}{2}\left[(10*10 + 20*X + y*5) - (5*20 + 10*y + \cancel{X*10})\right]$$

$$101.25 = 5X - 2.5y \quad \text{--- (1)}$$

From area BCDKB

point	x	y
B	10	20
C	20	22
D	25	5
K	x	y
B	10	20

$$101.25 = \frac{1}{2}\left[(20*20 + 22*25 + 5*X + y*10) - (10*22 + 20*5 + 25*y + X*20)\right]$$

$$101.25 = 315 - 7.5X - 7.5y \quad \text{--- (2)}$$

From Eq. (1) & Eq. (2)
the coordinate of pk is $x = 23$ m & $y = 5.5$ m

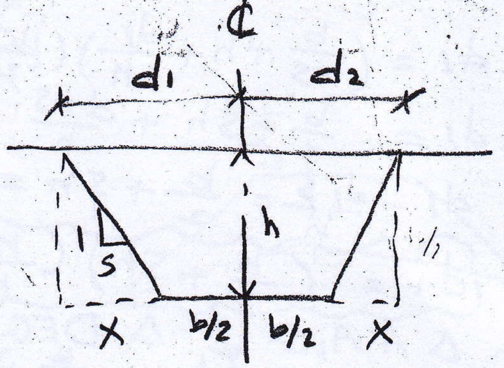
H.W Calculate the area closed traverse which coordinate shown below

Corner	1	2	3	4	5
Distance from horizontal axis	300	400	600	1000	1200
Distance from vertical axis	300	800	1200	1000	400

Area of cross_section

① One-level section

$$\frac{h}{x} = \frac{1}{s} \Rightarrow x = hs$$



$$d_1 = d_2 = \frac{b}{2} + Sh$$

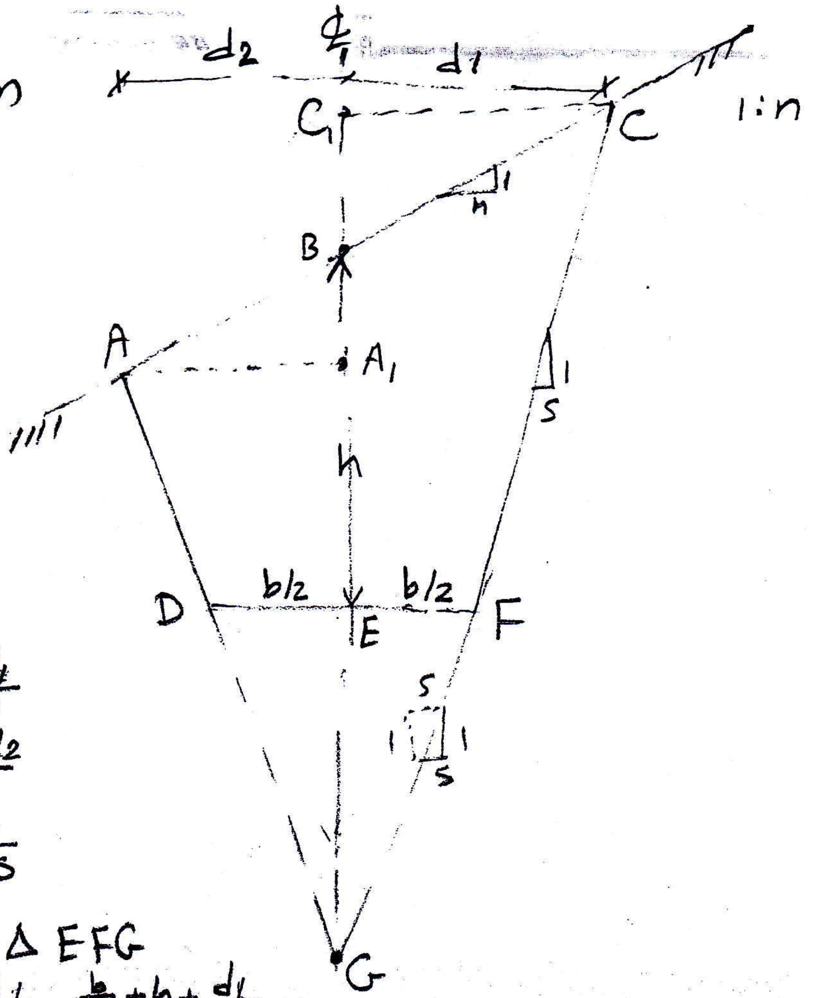
$$A = \frac{1}{2}\left[b + 2\left(\frac{b}{2} + Sh\right)\right] \cdot h$$

$$= \frac{1}{2}\left[b + b + 2Sh\right] \cdot h$$

$$= \frac{1}{2}\left[2b + 2Sh\right] \cdot h$$

$$\therefore A = [b + Sh] \cdot h$$

2 Two-level section



$$\frac{1}{n}=\frac{C_1B}{d_1} \Rightarrow C_1B=\frac{d_1}{n}$$

$$\frac{1}{n}=\frac{A_1B}{d_2} \Rightarrow A_1B=\frac{d_2}{n}$$

$$\frac{b/2}{GE}=\frac{s}{1} \Rightarrow GE=\frac{b}{2s}$$

From ΔC_1CG & ΔEFG

$$\frac{CC_1}{EF}=\frac{GC_1}{GE} \Rightarrow \frac{d_1}{b/2}=\frac{\frac{b}{2s}+h+\frac{d_1}{n}}{\frac{b}{2s}}$$

$$\Rightarrow d_1=\left(\frac{b}{2s}+h+\frac{d_1}{n}\right)\left(\frac{\cancel{b}}{2}\right)\left(\frac{2s}{\cancel{b}}\right)$$

$$\Rightarrow d_1=\frac{b}{2}+sh+\frac{d_1 s}{n}$$

$$\Rightarrow d_1-d_1\frac{s}{n}=\frac{b}{2}+sh \Rightarrow d_1\left(1-\frac{s}{n}\right)=\frac{b}{2}+sh$$

$$\Rightarrow d_1=\left(\frac{b}{2}+sh\right)\left(\frac{n}{n-s}\right)$$

From ΔAA_1G & ΔDEG

$$\frac{AA_1}{DE}=\frac{GA_1}{GE} \Rightarrow \frac{d_2}{b/2}=\frac{\frac{b}{2s}+h-\frac{d_2}{n}}{\frac{b}{2s}} \Rightarrow d_2=\left(\frac{b}{2}+sh\right)\left(\frac{n}{n+s}\right)$$

The area of fill or Cut of ACFDA is:

Area BCG + Area ABG − Area DFG

$$A=\frac{1}{2}d_1\left(\frac{b}{2s}+h\right)+\frac{1}{2}d_2\left(\frac{b}{2s}+h\right)-\frac{1}{2}\,b*\frac{b}{2s}$$

$$A=\frac{1}{2s}\left[\left(\frac{b}{2}+sh\right)(d_1+d_2)-\frac{b^2}{2}\right]$$

or

$$A=\frac{1}{2}\left[\left(\frac{b}{2s}+h\right)(d_1+d_2)-\frac{b^2}{2s}\right]$$