By Assist Lecturer: Besma Nazar

Digital Systems

References:-

1- Digital fundamentals by Floyd.
2- Digital Design by Morris Mano.
3- Digital logic cct. analysis & design by Victor p. Nelson.
4- Digital Design principles & practice by John F. Wakerly.
5- Digital Electronics principles, Devices and Applications by Anil.

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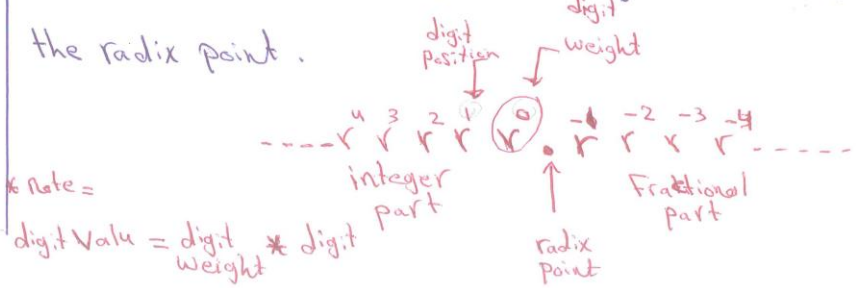
1- -: Number Systems :-

The study of number systems is important from the viewpoint of understanding how data are represented befor they can be processed by any digital system including a digital computer. A number system consists of an ordered set of symbols, called (digits) and the total number of digits allowed in the number system is called the (radix (r)) or (base) of the number system.

Number systems commonly used in digital system design and computer programming include decimal (r=10), binary (r=2), octal (r=8), and hexadecimal (r=16). Any number in a given system may have both an integer part and a fractional part, which are separated by a radix point (.). The place values or weights of different digits in the integer part of the number are given by r^0, r^1, r^2, r^3 and so on, starting with the digit adjacent to the radix point. For the fractional part, these are r^{-1}, r^{-2}, r^{-3} and so on, again starting with the digit next to the radix point.



* Note =
digit Value = digit weight * digit

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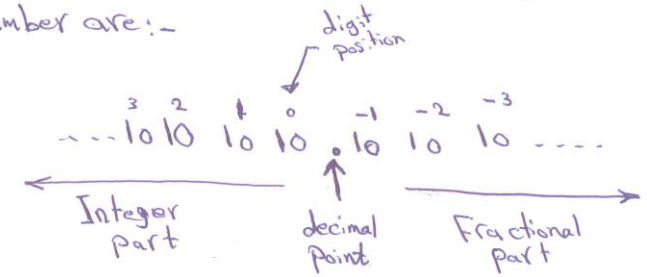
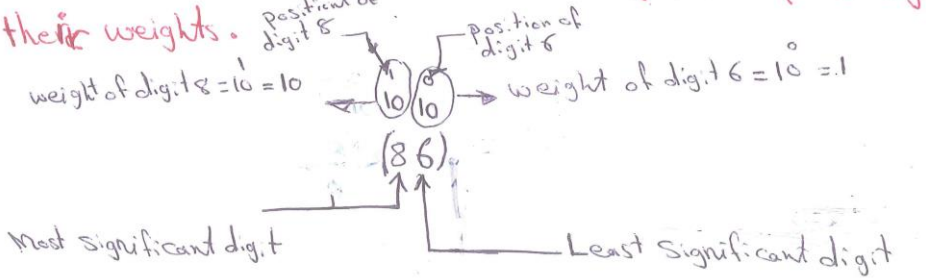
1- Decimal Number System: —

The decimal number system is a radix (10) number system and therfor has 10 different digits or symbols. There are (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). The weights of different digits in a mixed decimal number are:-



The value or magnitude of a given decimal number can be expressed as the Sum of the various digits multiplied by their weights.



Value of digit 8 = $8 * 10^1 = 8 * 10 = 80$
Value of digit 6 = $6 * 10^0 = 6 * 1 = 6$
$\longrightarrow 80 + 6 = 86$

Ex1:- Express each of the following decimal numbers as a sum of the values of each digit.

a) $(47)_{10}$ b) $(568.23)_{10}$

Solution:-

a) 47

$$\begin{matrix} 1 & 0 \\ 10 & 10 \\ 4 & 7 \end{matrix} \quad 47 =$$

$$47 = (4*10^{1}) + (7*10^{0})$$
$$= (4*10) + (7*1)$$
$$= 40 + 7$$

b) 568.23

$$\begin{matrix} 2 & 1 & 0 & \text{decimal point} & -1 & -2 \\ 10 & 10 & 10 & \downarrow & 10 & 10 \\ 5 & 6 & 8 & . & 2 & 3 \end{matrix}$$

$$568.23 = (5*10^{2}) + (6*10^{1}) + (8*10^{0}) + (2*10^{-1}) + (3*10^{-2})$$
$$= (5*100) + (6*10) + (8*1) + (2*0.1) + (3*0.01)$$
$$= 500 + 60 + 8 + 0.2 + 0.03$$

Ex2:- What weight does the digit 7 have in each of the following numbers?

a) $(7051)_{10}$ b) $(58.72)_{10}$

Solution:-

$$\begin{matrix} 3 & 2 & 1 & 0 \\ 10 & 10 & 10 & 10 \\ 7 & 0 & 5 & 1 \end{matrix}$$

a) - the weight of 7 is $=10^{3}$
$=1000$

$$\begin{matrix} 1 & 0 & -1 & -2 \\ 10 & 10 & 10 & 10 \\ 5 & 8 & . & 7 & 2 \end{matrix}$$

b) the weight of 7 is 10^{-1}
$= 0.1$

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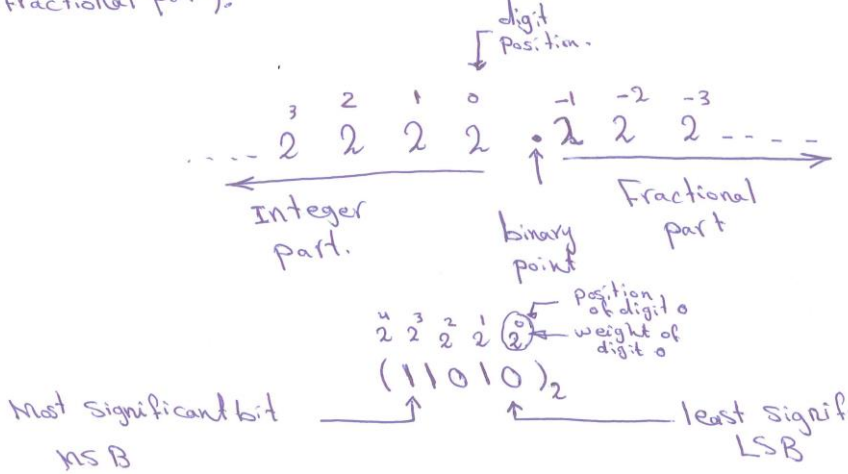
1-2) Binary Number System :-

The binary number system is a radix(2) number system with (0 and 1) as the two independent digits. A binary digit is called a bit. Starting from the binary point, the place values or weights of different bits in mixed binary number are 2^0, 2^1, 2^2 and so on (for the integer part) and 2^{-1}, 2^{-2}, 2^{-3} and so on (for the fractional part).



$$\text{Value} = (1*2^4) + (1*2^3) + (0*2^2) + (1*2^1) + (0*2^0)$$
$$= (1*16) + (1*8) + (0*4) + (1*2) + (0*1)$$

Note:- The largest decimal number that can be represented in binary with n bits equal to $2^n - 1$.

For example :- with five bits (n=5) the largest decimal number

$$2^5 - 1 = 31$$

and with six bits (n=6)

largest decimal number $= 2^6 - 1 = 63$

Ex3:- Determin the weight for the five first bits in binary system:-

Solution:-

b_5	b_4	b_3	b_2	bit1
4	3	2	1	0
2	2	2	2	2
↓	↓	↓	↓	↓
16	8	4	2	1

Ex4:- Determine the weight of the 1 in the binary number $(10000)_2$

Solution:-

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

1 0 0 0 0

The weight of 1 is $2^4 = 16$

Ex5:- Determine the weight of each bit that is a1 in the following fractional binary number $(0.1011)_2$.

Solution:-

$0.1\ 0\ 1\ 1$

$2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4}$

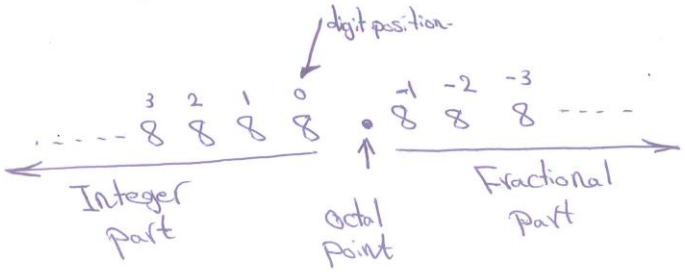
$2^{-4} = 0.0625$

$2^{-3} = 0.125$

$2^{-1} = 0.5$

(1.3) Octal Number System:-

The octel number system has a radix of (8) and therfore has eight distinct digits. The independent digits are (0, 1, 2, 3, 4, 5, 6, 7). The weights of different digits in amixed octal number are:-



8^1 (8) ← position of digit 7
weight of digit 7

$(27)_8$

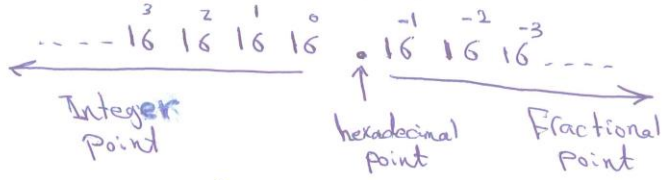
$27 = (2 * 8^1) + (7 * 8^0)$

Value $= (2 * 8) + (7 * 1)$

Note:- Coating in octal is similar to counting in decimal, except that the digits 8 and 9 are not used. To distinguish octal numbers from decimal numbers, we will use the subscript 8 to indicate an octal number For instance. $(15)_8$, $(7)_8$, $(22)_8$. Sometime you may see an "O" or a "Q" following an octal number.

(4) Hexadecimal Number System:-

The hexadecimal number system is radix(16) number system and its 16 basic digits are (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). The first ten digits are borrowed from the decimal system. The letters A, B, C, D, E, F are used for digits to, 11, 12, 13, 14, 15 respectively. The weights of different digits in a mixed hexadecimal number are



$16^2\ 16^1\ 16^0\ 16^{-1}$

$(FD4.E)_{16}$

$= (F \times 16^2) + (D * 16^1) + (4 * 16^0) + (E * 16^{-1})$

$= (15 * 16^2) + (13 * 16^1) + (4 * 16^0) + (14 * 16^{-1})$

$= (15 * 256) + (13 * 16) + (4 * 1) + (14 * 0.0625)$

Note:- To avoid confusion hexa decimal numbers with decimal number or another numbers we will use the subscript 16 to indicide hexadecimal numbers. Sometimes you may see an "h" following a hexadecimal number.

H.w1:- What weight does the digit 0 have in each of the following numbers ?

a) $(51.03)_{10}$

b) $(11011)_2$

c) $(703.12)_8$

d) $(F2E.10A)_{16}$.

H.w2:- How many bits are required to represent the following decimal numbers ?

a) 17 b) 49 c) 114 d) 205