Laws and Rules of boolean algebra:-

1- $A+B = B+A$

2- $AB = BA$

$\rightarrow$ Commutative Laws

3- $A+(B+C) = (A+B)+C$

4- $A(BC) = (AB)C$

$\rightarrow$ Associative Laws

5- $A(B+C) = AB+AC$ $\rightarrow$ Distributive Law.

6- $A+0 = A$

7- $A+1 = 1$

8- $A+A = A$

9- $A+\bar{A} = 1$

10- $A \cdot 0 = 0$

11- $A \cdot 1 = A$

12- $A \cdot A = A$

13- $A \cdot \bar{A} = 0$

14- $\bar{\bar{A}} = A$

15- $A + AB = A$

Prove:- $A+AB$

$A(\underbrace{1+B}_{1}) = A$

16- $A(A+B) = A$

Prove:- $A \cdot A + AB$

$A + AB$

$A(\underbrace{1+B}_{1}) = A$

17- $A + BC = (A+B)(A+C)$

Prove:- Take the left side:-

$$
\begin{aligned}
(A+B)(A+C) &= AA + AC + AB + BC \\
&= A + AC + AB + BC \\
&= A(1+C) + AB + BC \\
&= A + AB + BC \\
&= A(1+B) + BC \\
&= A + BC
\end{aligned}
$$

18 - $A + A'B = A + B$

Prove:- $A + A'B$

$$\underbrace{(A + A')}_{1}(A + B) = A + B$$

Simplification Using Boolean Algebra:-

Many times in the application of Boolean algebra, you have to reduce a particular expression to its simplest form. A simplified Boolean expression uses the fewest gates possible to implement a given expression.

Ex:- Using Boolean algebra techniques, simplify the following expressions functions.

a) $AB + A(B+C) + B(B+C)$

b) $A\bar{B} + A(\overline{B+C}) + B(\overline{B+C})$

c) $[A\bar{B}(C + BD) + A'B']C$

Solution:- a) $AB + A(B+C) + B(B+C)$

$F = AB + AB + AC + BB + BC$

$F = AB + AC + \underline{B} + \underline{BC}$

$F = AB + AC + B(\underbrace{1 + C}_{1})$

$F = \underline{AB} + AC + \underline{B}$

$F = B(\underbrace{A + 1}_{1}) + AC$

$F = B + AC$

b) $A\bar{B} + A(\overline{B+C}) + B(\overline{B+C})$

$F = A\bar{B} + A(\bar{B} \cdot \bar{C}) + B(\bar{B} \cdot \bar{C})$

$F = A\bar{B} + A\bar{B} + A\bar{C} + \underbrace{B\bar{B}}_{0} + B\bar{C}$

$F = A\bar{B} + A\bar{C} + B\bar{C}$

c) $[A\bar{B}(C + BD) + A'B']C$

$F = [A\bar{B}C + \underbrace{AB'BD}_{0} + A'B']C$

$F = [AB'C + A'B']C$

$F = AB'\underline{CC} + A'B'C$

$F = AB'C + A'B'C$

$F = B'C[\underbrace{A + A'}_{1}]$

$F = B'C$

H.w:- Simplify the following Boolean expression:-

a) $\bar{A}\bar{B}C + \overline{(A+B+\bar{C})} + \bar{A}\bar{B}\bar{C}D$

b) $ABCD + AB(\overline{CD}) + \overline{(AB)}CD$

- Standard form of Boolean expressions:-

All Boolean expressions, regardless of their form, can be converted into either of two standard forms:-

1- The sum of products (SOP)
2- The product of sum (POS)

Standaridization makes the evaluation, simplification, and implementation of Boolean expressions much more systematic and easier.

1- The Sum of Products (SOP):-

SOP is a term consisting of the product of literals (variables) or their complements, when two or more product terms are summed by Boolean addition. A sum of products expression is also known as a minterm expression. It can be obtained from the truth table directly by considering those input combinations that produce a logic 1 at the output.

Ex:- $AB + \bar{A}B\bar{C} + AB \rightarrow SOP$

Ex:- Convert the Boolean expression to SOP form:-

a) $F = (A+B)(B+C+D)$ b) $\overline{\overline{(A+B)}+C}$

Solution:- a) $F = AB + AC + AD + BB + BC + BD$

b) $\overline{\overline{(A+B)}+C} = \overline{\bar{A}\bar{B}+C} = (\bar{\bar{A}}+\bar{\bar{B}})\cdot\bar{C}$

$= (A+B)\bar{C}$

$= A\bar{C} + B\bar{C}$

Note:- (1) Conical or Standard form of Boolean expressions is an Expanded form of Boolean expression, where each term contains all Boolean variables in their true or complemented form and is obtained by including all possible combinations of missing variables.

(2) In Sop to convert it to standard Sop form we ANDed it misses one or more variables with an expression such as $(X+\bar{X})$, where X is one of the missing variables.

Ex:- Convert the following Boolean expression in to standard Sop form:-

$F = A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D$

$F = A\bar{B}C(D+\bar{D}) + \bar{A}\bar{B}(C+\bar{C})(D+\bar{D}) + AB\bar{C}D$

$F = A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}(CD + C\bar{D} + \bar{C}D + \bar{C}\bar{D}) + AB\bar{C}D$

$F = A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$

Ex:- put F in Sop form and simplify it using Boolean algebra and theorem and then implement this function:-

	A	B	F	
m_0	0	0	1	$\bar{A}\bar{B}$
m_1	0	1	1	$\bar{A}B$
m_2	1	0	0	
m_3	1	1	1	AB

$0 \rightarrow \bar{X}$, $1 \rightarrow X$] ← in sop

$F = \sum m_0, m_1, m_3$

$F = \sum 0, 1, 3$

$F = \bar{A}\bar{B} + \bar{A}B + AB$

$F = \bar{A}(\bar{B}+B) + AB$

$F = \bar{A} + AB$

$F = \bar{A} + B$