Asst. Lect. Besma Nazar

Karnaugh Maps (K-map):—

The K-map is simply another form of a truth table and is easiest method for simplifying logical expression and, if properly used, will produce the simplest SOP or POS expression possible. A logic function containing n variables requires 2^n rectangles.

Note:-

1- The main thing to remember is that only one variable can be change between adjacent rectangles when moving in a horizontal or a vertical direction. This is because that the minterms are not arranged in binary sequence but in sequence similar to Gray code. and the characteristic of this sequence is that only one bit changes from 1 → 0 or from 0 → 1 in the listing sequence.

2- The logic expression obtained from K-map must be in the easiest form and cannot be simplified more.

Type of K-map:-

1- Two Variables:-

$F = (A, B)$

msB ↑ ↑— LSB

No. of celles $= 2^n$

$= 2^2$

$= 4$

0 ⟶ 3

$B \backslash A$	$\bar{A}$ 0	A 1
$\bar{B}$ 0	$\bar{A}\bar{B}$ 0	$A\bar{B}$ 2
B 1	$\bar{A}B$ 1	AB 3

2- Three Variables:-

F = (A, B, C)

MSB ↑ A, LSB ↑ C

No. of Celles = 2^3

= 8

0 → 7

$C \backslash AB$	$\bar A\bar B$ 00	$\bar A B$ 01	AB 11	$A\bar B$ 10
$\bar C$ 0	$\bar A\bar B\bar C$ 0	$\bar A B\bar C$ 2	$AB\bar C$ 6	$A\bar B\bar C$ 4
C 1	$\bar A\bar B C$ 1	$\bar A B C$ 3	ABC 7	$A\bar B C$ 5

3- Four Variables:-

F = (A, B, C, D)

MSB ↑ A, LSB ↑ D

No. of Celles = 2^4

= 16

0 → 15

$CD \backslash AB$	$\bar A\bar B$ 00	$\bar A B$ 01	AB 11	$A\bar B$ 10
$\bar C\bar D$ 00	$\bar A\bar B\bar C\bar D$ 0	$\bar A B\bar C\bar D$ 4	$AB\bar C\bar D$ 12	$A\bar B\bar C\bar D$ 8
$\bar C D$ 01	$\bar A\bar B\bar C D$ 1	$\bar A B\bar C D$ 5	$AB\bar C D$ 13	$A\bar B\bar C D$ 9
CD 11	$\bar A\bar B CD$ 3	$\bar A BCD$ 7	$ABCD$ 15	$A\bar B CD$ 11
$C\bar D$ 10	$\bar A\bar B C\bar D$ 2	$\bar A BC\bar D$ 6	$ABC\bar D$ 14	$A\bar B C\bar D$ 10

4- Five Variables:-

F = (A, B, C, D, E)

MSB ↑ A, LSB ↑ E

No. of celles = 2^5

= 32

0 → 31

$DE \backslash ABC$	$\bar A\bar B\bar C$ 000	$\bar A\bar B C$ 001	$\bar A BC$ 011	$\bar A B\bar C$ 010	$AB\bar C$ 110	ABC 111	$A\bar B C$ 101	$A\bar B\bar C$ 100
$\bar D\bar E$ 00	0	4	12	8	24	28	20	16
$\bar D E$ 01	1	5	13	9	25	29	21	17
DE 11	3	7	15	11	27	31	23	19
$D\bar E$ 10	2	6	14	10	26	30	22	18

5- Six Variables:-

F = (A, B, C, D, E, F)

MSB ↑ A, LSB ↑ F

No. of Celles = 2^6

= 64

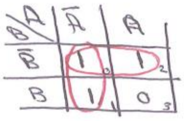
0 → 63

$DEF \backslash ABC$	$\bar A\bar B\bar C$ 000	$\bar A\bar B C$ 001	$\bar A BC$ 011	$\bar A B\bar C$ 010	$AB\bar C$ 110	ABC 111	$A\bar B C$ 101	$A\bar B\bar C$ 100
$\bar D\bar E\bar F$ 000	0	8	24	16	48	56	40	32
$\bar D\bar E F$ 001	1	9	25	17	49	57	41	33
$\bar D EF$ 011	3	11	27	19	51	59	43	35
$\bar D E\bar F$ 010	2	10	26	18	50	58	42	34
$DE\bar F$ 110	6	14	30	22	54	62	46	38
DEF 111	7	15	31	23	55	63	47	39
$D\bar E F$ 101	5	13	29	21	53	61	45	37
$D\bar E\bar F$ 100	4	12	28	20	52	60	44	38

Ex:- Find the Boolean expression for F by using K-map?

AB	F
0 0	1
0 1	1
1 0	1
1 1	0

$F = \sum 0, 1, 2$



B \ A	$\bar{A}$	A
$\bar{B}$	1 $_0$	1 $_2$
B	1 $_1$	0 $_3$

$F = \bar{A} + \bar{B}$

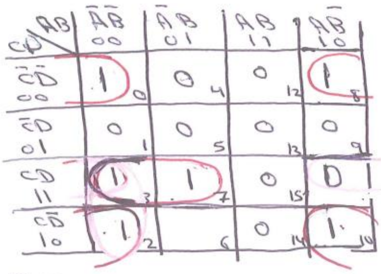
Ex:- Simplify F using K-map?

$F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}CD + A\bar{B}C\bar{D}$

Solution:- $F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}CD(B+\bar{B}) + A\bar{B}C\bar{D}$

$F = \underset{0}{\bar{A}\bar{B}\bar{C}\bar{D}} + \underset{2}{\bar{A}\bar{B}C\bar{D}} + \underset{8}{A\bar{B}\bar{C}\bar{D}} + \underset{7}{\bar{A}BCD} + \underset{3}{\bar{A}\bar{B}CD} + \underset{10}{A\bar{B}C\bar{D}}$

$F = \sum 0, 2, 3, 7, 8, 10$



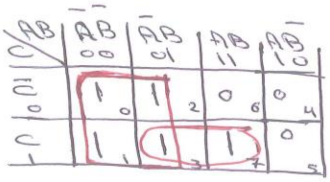
CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	1 $_0$	0 $_4$	0 $_{12}$	1 $_8$
$\bar{C}D$ 01	0 $_1$	0 $_5$	0 $_{13}$	0 $_9$
CD 11	1 $_3$	1 $_7$	0 $_{15}$	D
$C\bar{D}$ 10	1 $_2$	$_6$	0 $_{14}$	1 $_{10}$

$F = \bar{B}\bar{D} + \bar{A}CD$

Ex:- Simplify F using K-map:-

$F = \underset{7}{ABC} + \underset{0}{\bar{A}\bar{B}\bar{C}} + \underset{1}{\bar{A}\bar{B}C} + \underset{2}{\bar{A}B\bar{C}} + \underset{3}{\bar{A}BC}$

$F = \sum 0, 1, 2, 3, 7$



C \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0	1 $_0$	1 $_2$	0 $_6$	0 $_4$
C 1	1 $_1$	1 $_3$	1 $_7$	0 $_5$

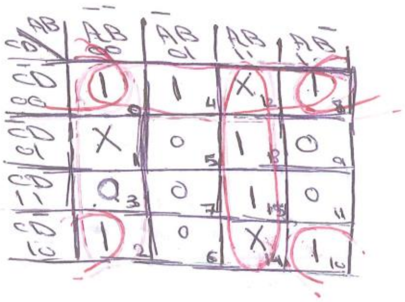
$F = \bar{A} + BC$

Don't care Conditions:-

The presence of don't cares in the design specifications gives us flexibility in selecting the output function that is to be synthesized. When simplifying logic expressions, the Don't care terms are represented by X and may be used as 1's or 0's.

Ex:- Simplify:- $F = \sum 0,2,4,8,10,13,15 + \sum_d 1,12,14$, using K-map

solution:-

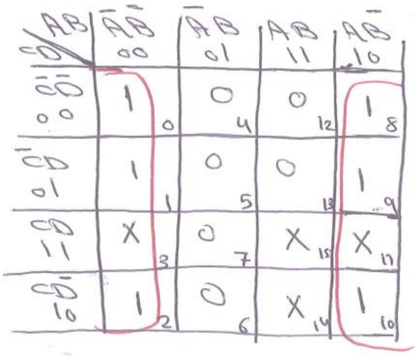


CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	1_{0}	1_{4}	X_{12}	1_{8}
$\bar{C}D$ 01	X_{1}	0_{5}	1_{13}	0_{9}
CD 11	0_{3}	0_{7}	1_{15}	0_{11}
$C\bar{D}$ 10	1_{2}	0_{6}	X_{14}	1_{10}

$F = \bar{C}\bar{D} + AB + \bar{B}\bar{D}$.

Ex:- Simplify: $F = \sum 0,1,2,8,9,10 + \sum_d 3,11,14,15$ using K-map

solution:-



CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	1_{0}	0_{4}	0_{12}	1_{8}
$\bar{C}D$ 01	1_{1}	0_{5}	0_{13}	1_{9}
CD 11	X_{3}	0_{7}	X_{15}	X_{11}
$C\bar{D}$ 10	1_{2}	0_{6}	X_{14}	1_{10}

$F = \bar{B}$

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Ex:- Design a logic cct using NAND gates only to convert ㊾ BCD to. EX-3 code: -

Solution:-

ABCD	WXYZ
0000	0011
0001	0100
0010	0101
0011	0110
0100	0111
0101	1000
0110	1001
0111	1010
1000	1011
1001	1100
1010	xxxx
1011	xxxx
1100	xxxx
1101	xxxy
1110	xxxx
1111	xxxx

BCD count from 0 → 9

10 → 15 novalued

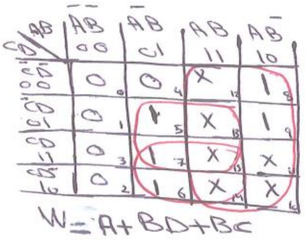
$d = \sum 10, 11, 12, 13, 14, 15$

$W = \sum 5, 6, 7, 8, 9$

$X = \sum 1, 2, 3, 4, 9$

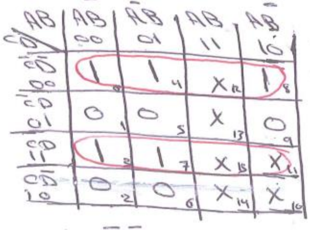
$y = \sum 0, 3, 4, 7, 8$

$Z = \sum 0, 2, 4, 6, 8$

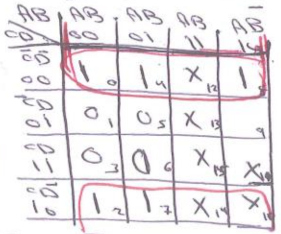


$W = A + BD + BC$

$X = B\bar{C}\bar{D} + \bar{B}D + \bar{B}C$



$y = \bar{C}\bar{D} + CD$



$Z = \bar{D}$

$W = \overline{\overline{A + BD + BC}}$

$W = \overline{\bar{A} \cdot \overline{BD} \cdot \overline{BC}}$ ✓

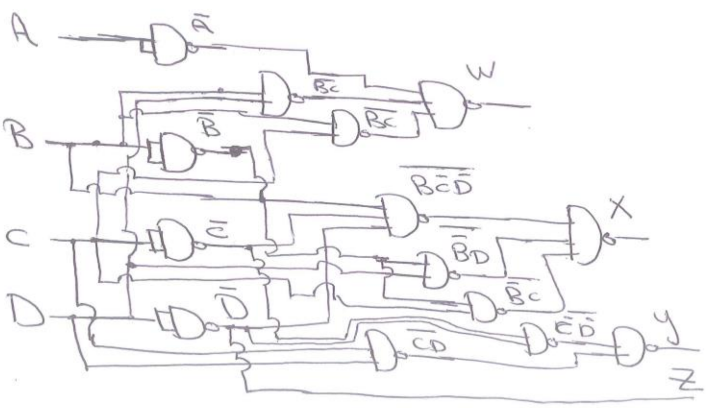
$X = \overline{\overline{B\bar{C}\bar{D} + \bar{B}D + \bar{B}C}}$

$X = \overline{\overline{B\bar{C}\bar{D}} \cdot \overline{\bar{B}D} \cdot \overline{\bar{B}C}}$ ✓

$y = \overline{\overline{\bar{C}\bar{D} + CD}}$

$y = \overline{\overline{\bar{C}\bar{D}} \cdot \overline{CD}}$ ✓

$Z = \bar{D}$ ✓



Ex:- Use K-map to design a logic circuit to convert from 5421 code to 8421 code using NAND gates only.

	ABCD	WXYZ
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
8	1000	0101
9	1001	0110
10	1010	0111
11	1011	1000
12	1100	1001

$d=\sum 5,6,7,13,14,15$

$W=\sum 11,12$

$X=\sum 4,8,9,10$

$y=\sum 2,3,9,10$

$Z=\sum 1,3,8,10,12$

AB CD	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0_{0}	0_{4}	1_{12}	0_{8}
$\bar{C}D$	0_{1}	X_{5}	X_{13}	0_{9}
CD	0_{3}	X_{7}	X_{15}	1_{11}
$C\bar{D}$	0_{2}	X_{6}	X_{14}	0_{10}

$W=AB+ACD$

AB CD	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0_{0}	1_{4}	0_{12}	1_{8}
$\bar{C}D$	0_{1}	X_{5}	X_{13}	1_{9}
CD	0_{3}	X_{7}	X_{15}	0_{11}
$C\bar{D}$	0_{2}	X_{6}	X_{14}	1_{10}

$X=\bar{A}B+A\bar{B}\bar{C}+AC\bar{D}$

AB CD	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0_{0}	0_{4}	1_{12}	1_{8}
$\bar{C}D$	1_{1}	X_{5}	X_{13}	0_{9}
CD	1_{3}	X_{7}	X_{15}	0_{11}
$C\bar{D}$	0_{2}	X_{6}	X_{14}	1_{10}

$Z=\bar{A}D+A\bar{D}$

AB CD	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0_{0}	0_{4}	0_{12}	0_{8}
$\bar{C}D$	0_{1}	X_{5}	X_{13}	1_{9}
CD	1_{3}	X_{7}	X_{15}	0_{11}
$C\bar{D}$	1_{2}	X_{6}	X_{14}	1_{10}

$y=C\bar{D}+\bar{A}C+A\bar{C}D$

$W=\overline{\overline{AB+ACD}}=\overline{\overline{AB}\cdot\overline{ACD}}$ ✓

$X=\overline{\overline{\bar{A}B+A\bar{B}\bar{C}+AC\bar{D}}}$
$=\overline{\overline{\bar{A}B}\cdot\overline{A\bar{B}\bar{C}}\cdot\overline{AC\bar{D}}}$ ✓

$y=\overline{\overline{C\bar{D}+\bar{A}C+A\bar{C}D}}=\overline{\overline{C\bar{D}}\cdot\overline{\bar{A}C}\cdot\overline{A\bar{C}D}}$ ✓

$Z=\overline{\overline{\bar{A}D+A\bar{D}}}=\overline{\overline{\bar{A}D}\cdot\overline{A\bar{D}}}$

Draw the cct as H.w