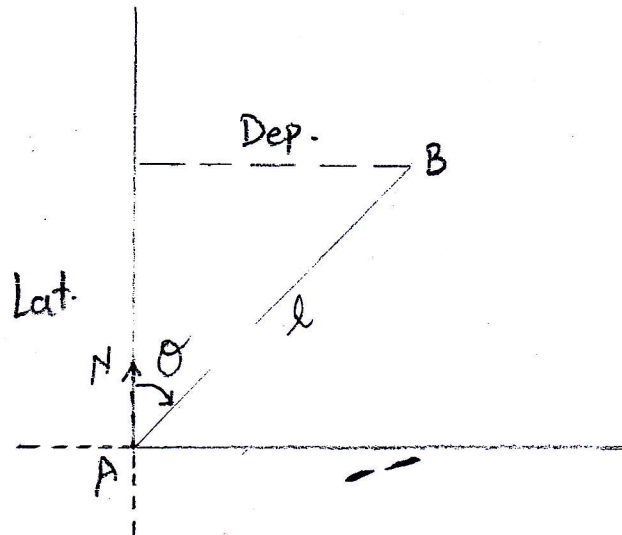


Inverse computation

(1)

To find the length and bearing of line dependent on coordinates or departure and Latitude.



$$\text{Dep.} = l_{AB} \sin \theta$$
$$\text{Lat.} = l_{AB} \cos \theta$$

where: Departure (Dep.)
Latitude (Lat.)

l_{AB} is the length of line AB

$$l_{AB} = \sqrt{(\Delta \text{Dep.})^2 + (\Delta \text{Lat.})^2}$$

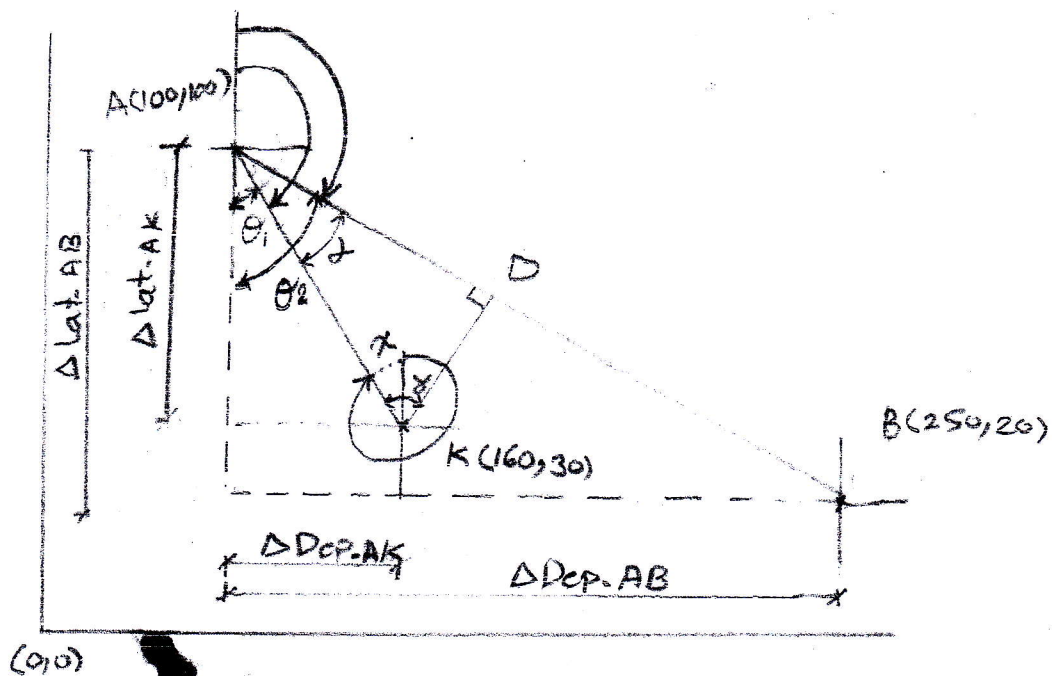
where: $\Delta \text{Dep. } A \rightarrow B$ (the difference between A and B in x-axis)

$\Delta \text{Lat. } A \rightarrow B$ (the difference between A and B in y-axis)

$$\tan \theta = \frac{\Delta \text{Dep. } AB}{\Delta \text{Lat. } AB} \Rightarrow \theta = \tan^{-1} \frac{\Delta \text{Dep. } AB}{\Delta \text{Lat. } AB}$$

(2)

Example 1/ Calculate the length and the α of line KD, which point D located on the line AB and the line KD was perpendicular to the line AB. The coordinate of point A was (100, 100), while the coordinate of point B was (250, 20), and the coordinate of point K was (160, 30).



Sol.

$$\theta_1 = \tan^{-1} \frac{\Delta \text{Dep. AK}}{\Delta \text{Lat. AK}} = \tan^{-1} \frac{160-100}{100-30} = 40^\circ 36' 05''$$

$$AZ \text{ AK} = 180^\circ - 40^\circ 36' 05'' = 139^\circ 23' 55''$$

$$L_{AK} = \sqrt{(\Delta \text{Dep. AK})^2 + (\Delta \text{Lat. AK})^2} = \sqrt{(60)^2 + (70)^2} = 92.2 \text{ m}$$

$$\theta_2 = \tan^{-1} \frac{\Delta \text{Dep. AB}}{\Delta \text{Lat. AB}} = \tan^{-1} \frac{250-100}{100-20} = 61^\circ 55' 39''$$

$$AZ \text{ AB} = 180^\circ - 61^\circ 55' 39'' = 118^\circ 04' 21''$$

$$\alpha = \theta_2 - \theta_1 = 21^\circ 19' 34''$$

$$\begin{aligned}\gamma &= 180^\circ - \alpha - 90^\circ = 180^\circ - 21^\circ 19' 34'' - 90^\circ \\ &= 68^\circ 40' 26''\end{aligned}$$

(3)

$$\frac{AK}{\sin 90^\circ} = \frac{KD}{\sin \alpha} \Rightarrow \frac{92.2}{\sin 90^\circ} = \frac{KD}{\sin 21^\circ 19' 34''}$$

$$\Rightarrow KD = 33.53 \text{ m}$$

$$BA_2 AK = 139^\circ 23' 55'' + 180^\circ = 319^\circ 23' 55''$$

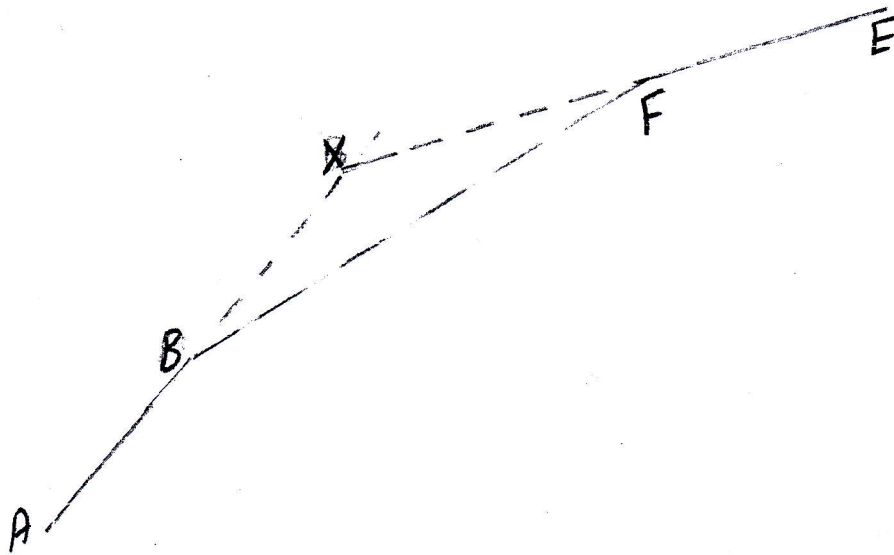
$$\alpha = 360^\circ - 319^\circ 23' 55'' = 40^\circ 36' 05''$$

$$\begin{aligned}A_2 KD &= \gamma - \alpha = 68^\circ 40' 26'' - 40^\circ 36' 05'' \\ &= 28^\circ 04' 21''\end{aligned}$$

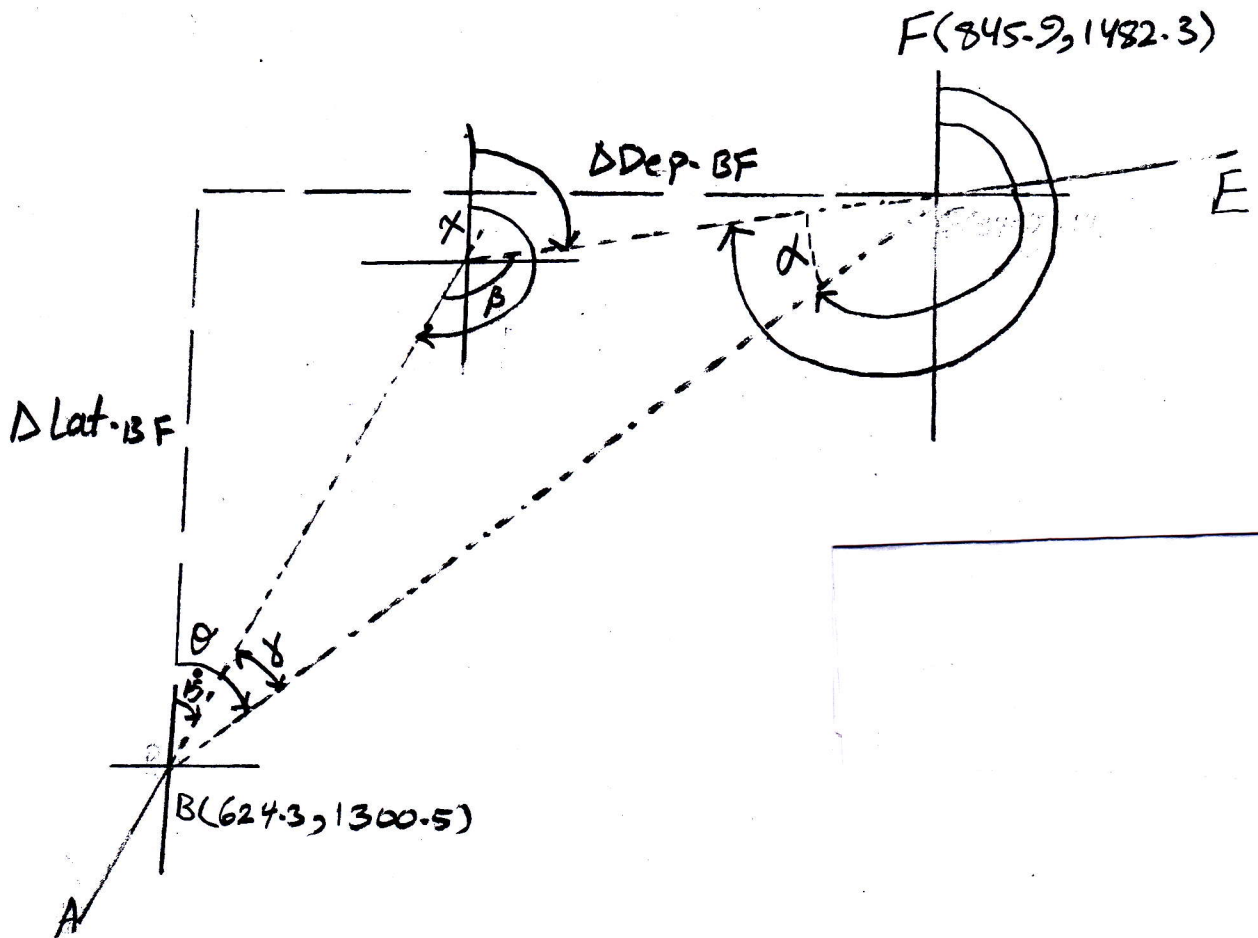
H.W/ Find the coordinate of point D in example 1.

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Example 2/ Through excavation in ground of specified location, the telephone cable from A to B and from E to F were placed. For technical purpose the line of cable AB was extended to point X and it connected from this point to the line of cable EF, as shown in figure below. The bearing of AB was $15^{\circ} 00'$, while the bearing of EF was $265^{\circ} 00'$. Besides to, the coordinates of B and F were $(624.30, 1300.5)$, and $(845.90, 1482.3)$, respectively. Find the length of excavation from X to F and from B to X.



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$$Az BF \Rightarrow \theta = \tan^{-1} \frac{\Delta Dep. BF}{\Delta Lat. BF} = \tan^{-1} \frac{845.9 - 624.3}{1482.3 - 1300.5} = 50^{\circ} 38' 05''$$

$$\gamma = 50^{\circ} 38' 05'' - 15^{\circ} 00' = 35^{\circ} 38' 05''$$

$$l_{BF} = \sqrt{(\Delta Dep. BF)^2 + (\Delta Lat. BF)^2} = \sqrt{(221.6)^2 + (181.8)^2} = 286.63 \text{ m.}$$

$$BAz BF = 50^{\circ} 38' 05'' + 180^{\circ} = 230^{\circ} 38' 05''$$

$$\circ \circ Az EF = 265^{\circ}$$

$$\circ \circ \alpha = Az EF - BAz BF = 265^{\circ} 00' - 230^{\circ} 38' 05'' = 34^{\circ} 21' 55''$$

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$$BA_2 Ax = 15^\circ 00' + 180^\circ 00' = 195^\circ 00'$$

$$A_2 XF = 265^\circ 00' - 180^\circ 00' = 85^\circ 00'$$

$$\beta = 195^\circ 00' - 85^\circ 00' = 110^\circ 00'$$

Must be the sum of triangle is 180° ; therefore

$$\gamma + \beta + \alpha = 180^\circ.$$

For checking

$$35^\circ 38' 05'' + 110^\circ 00' + 34^\circ 21' 55'' = 180^\circ 00' \text{ o.k.}$$

$$\frac{XF}{\sin \gamma} = \frac{BF}{\sin \beta} \Rightarrow XF = \frac{BF \sin \gamma}{\sin \beta}$$

$$= \frac{286.63 \sin 35^\circ 38' 05''}{\sin 110^\circ 00'}$$

$$= 177.712 \text{ m}$$

$$\frac{BX}{\sin \alpha} = \frac{BF}{\sin \beta} \Rightarrow BX = \frac{BF \sin \alpha}{\sin \beta}$$

$$= \frac{286.63 \sin 34^\circ 21' 55''}{\sin 110^\circ 00'}$$

$$= 172.176 \text{ m}$$