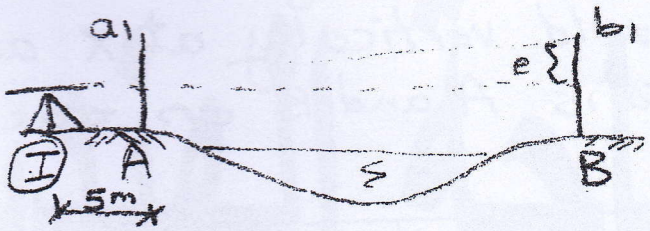
Reciperiocal levelling



$$(\Delta H)_{AB} = a_1 - (b_1 - e) \quad \text{---------- (1)}$$

$$(\Delta H)_{AB} = (a_2 - e) - b_2 \quad \text{---------- (2)}$$

Eq.(1) + Eq.(2) gives:

$$(\Delta H)_{AB} = \frac{1}{2}[(a_1 - b_1) + (a_2 - b_2)]$$

H.W/ Find e from Eq.(1) and Eq.(2)

Example / In levelling across river, reciperiocal Levelling observation gave the following result for staff held vertically at X and Y from level stations A and B on each bank.

Instrument position	staff position	staff readi (m)
X	A	1.753
X	B	2.080
Y	A	2.550
Y	B	2.895

if the R.L of A was 90.37, what the R.L of

$$\Delta H)_{AB} = \frac{1}{2}[(a_1 - b_1) + (a_2 - b_2)]$$

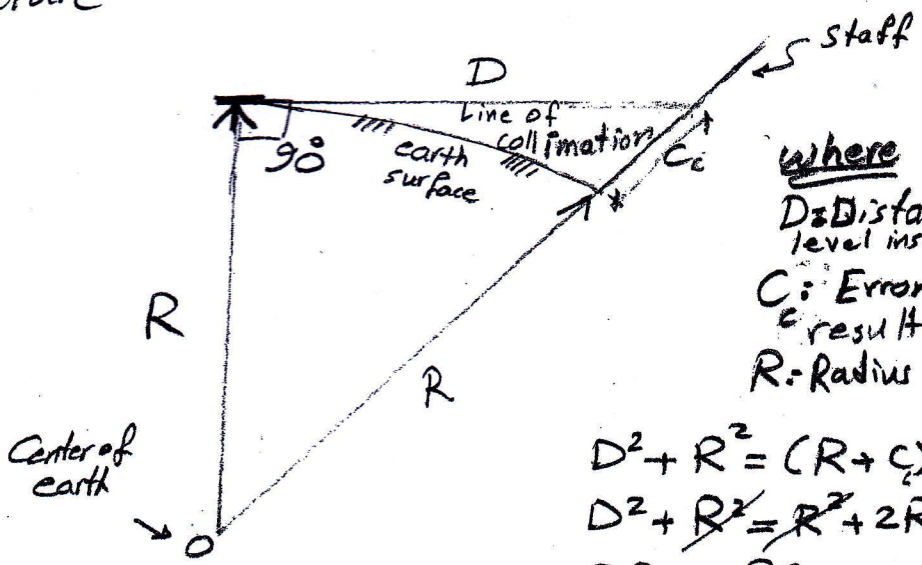
$$= \frac{1}{2}[(1.753 - 2.080) + (2.550 - 2.895)]$$

$$= -0.336 \text{ m}$$

$$R.L)_B = 90.37 - 0.336 = 90.034$$

Effect of Curvature and Refraction

* Curvature



Where

D: Distance between level instrument and staff

C_c: Error in staff reading result in curvature

R: Radius of earth

$$D^2 + R^2 = (R + C_c)^2$$

Cancel because small value

$$D^2 + R^2 = R^2 + 2RC_c + C_c^2$$

$$D^2 = 2RC_c \Rightarrow C_c = \frac{D^2}{2R}$$

R = 6370 km

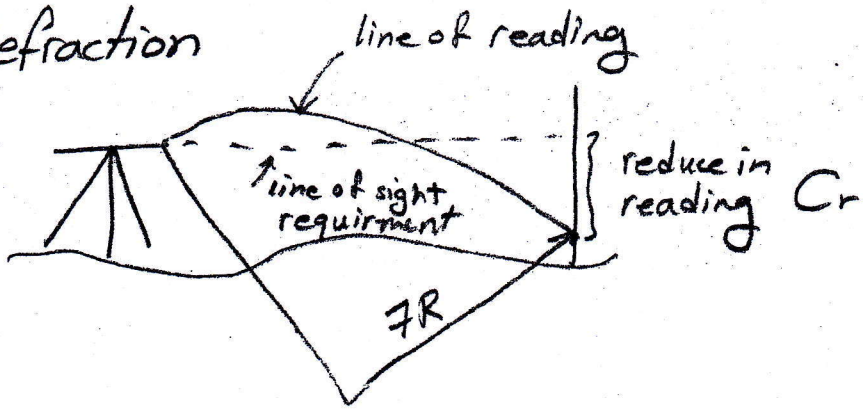
km to m

$$C_c = \frac{D^2}{2 * 6370} * 1000$$

$$C_c = 0.0785\, D^2$$

C_c (m), D (km)

* Refraction



Where

Cr: Error in staff Reading result in Refraction

$$\text{Correction of Refraction } (C_r) = \frac{1}{7} \text{ Correction of Curvature } (C_c)$$

* Combined Correction

$$C_{combined} = C_c - C_r$$
$$= C_c - \frac{1}{7} C_c$$
$$= 0.0785 D^2 - \frac{1}{7}(0.0785 D^2)$$

$C_{combined} = 0.0673 D^2$

m, km

Example / A level instrument was setup between two staff positions A and B were 100 m and 200 m, respectively. The staff readings at A and B were 2.300 and 1.500 m, respectively. Find the correct difference of level between the positions.

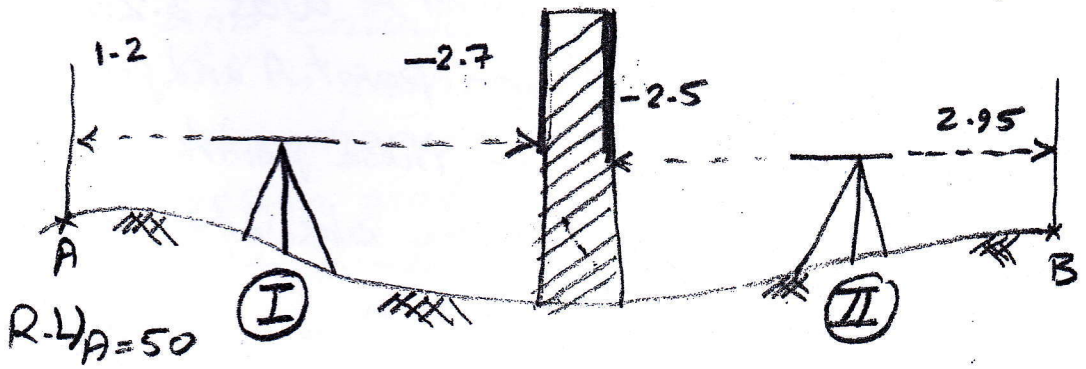
$$C_{combined\ (A)} = 0.0673 * \left(\frac{200}{1000}\right)^2 = 0.0027$$

Correct reading staff at A = 2.300 − 0.027
= 2.297 m

$$\Delta H)_{AB} = 2.297 - 1.5 = \underline{0.797\ m}$$

Difficulties in levelling

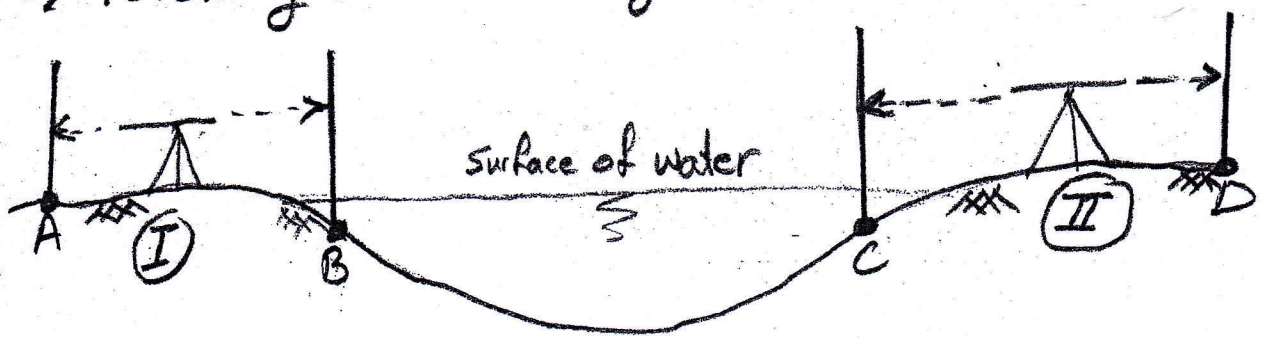
1) levelling acrross a solid obstruction like high wall



Staff reading			Staff station	H.I	R.L	note
B.S	I.S	F.S				
1.2		(I)	A	51.2	50	
-2.5		-2.7	high wall	51.4	53.9	
		2.95 (II)	B	=	48.45	

H.W/ check the result?

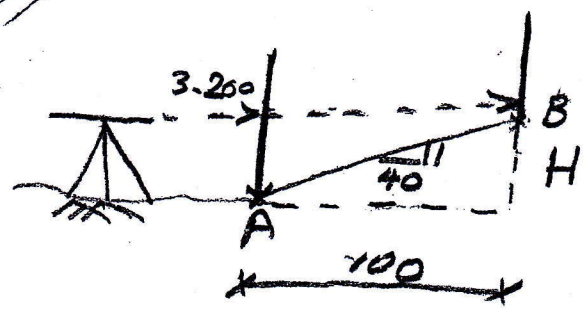
2) levelling across a large Lake



Assume R.L)B = R.L)C Then levelling is continued.

Find the Correct Reading through the Slope

Example/ A staff reading at point A was 3.200 m and the distance between point A and point B was 100 the slope between these points was 1:40 above. Determine the absolute reading on point B.



$$\frac{H}{100} = \frac{1}{40} \Rightarrow H = 2.5$$

Correct reading at B = 3.200 − 2.5 = 0.7 m

Sources of Errors in levelling work

1. Instrument error.
2. Error due to displacement of level and staff
3. Error due to natural causes.