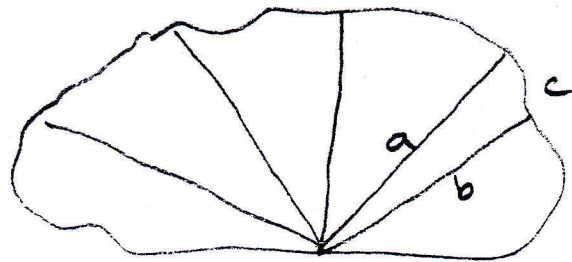


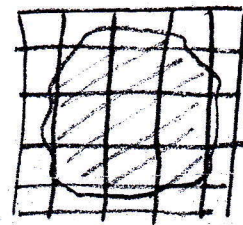
Areas and volumes

1. Calculate the area with Figure method



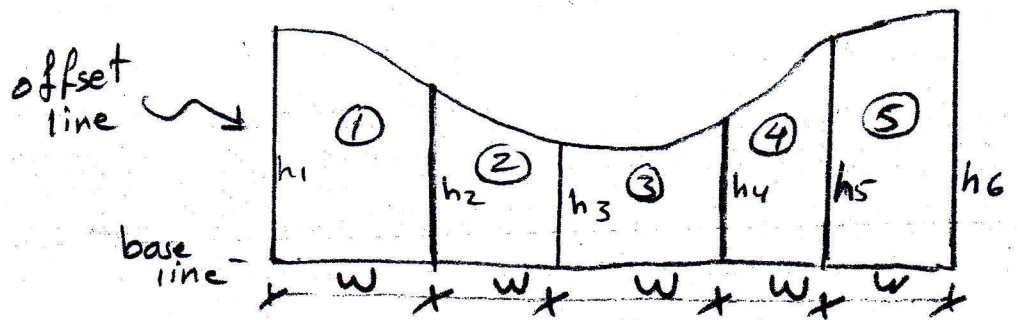
$$S = \frac{a+b+c}{2}$$

$$A = \sqrt{S(S-a)(S-b)(S-c)}$$



Graphical paper

2. Area with arithmetic and engineering method
A. Trapezoidal rule



$$\text{Area of } \textcircled{1} = \frac{h_1+h_2}{2} \times W, \text{ Area of } \textcircled{2} = \frac{h_2+h_3}{2} \times W, \dots$$

$$\text{Total area (A)} = \frac{W}{2} [h_1 + h_n + 2(h_2 + h_3 + \dots + h_{n-1})]$$

W : interval between the offset line.

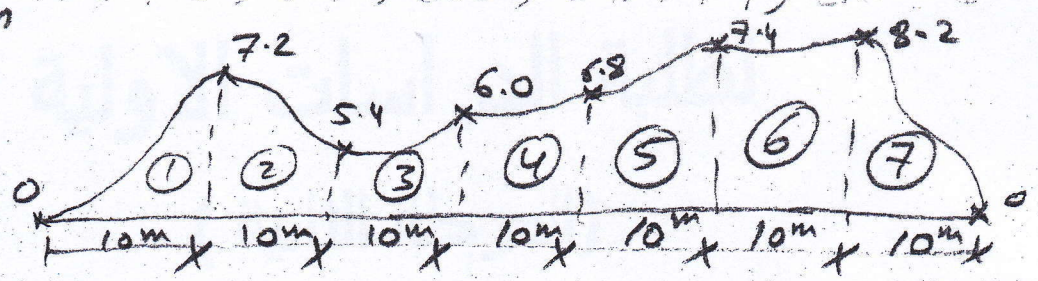
h : offset measurement.

n : number of offset.

B. Simpson's rule [this method is applicable only if the number of offsets is odd]

A = W/3 [h1 + hn + 4 Σ even offset + 2 Σ odd offset]

Example Find the area by 1) Trapezoidal rule 2) Simpson's rule if the offset as follows: 0, 7.2, 5.4, 6.0, 6.8, 7.4, 8.2, 0 the distance between offsets constant and was 10m



1) by the trapezoidal rule

A = W/2 [h1 + hn + 2 (h2 + h3 + ... + hn-1)]
= 10/2 [0 + 0 + 2(7.2 + 5.4 + 6.0 + 6.8 + 7.4 + 8.2)]
= 410 m^2

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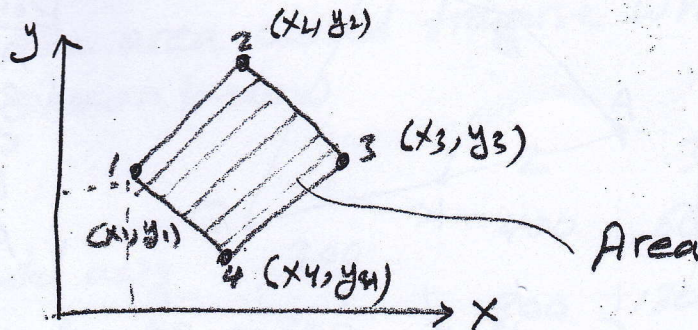
2) by Simpson's rule (division area)

AT = A1 -> 6 + A7
Simpson's rule trapezoidal rule

AT = W/3 [h1 + hn + 4 Σ even offset + 2 Σ odd offset] + A by trapezoidal rule
= 10/3 [0 + 8.2 + 4(7.2 + 6.0 + 7.4) + 2(5.4 + 6.8)] + 10/2 [8.2 + 0 + 2(6)]
= 356 + 424 m^2

Area from field notes

Area by coordinates



Point	X	Y
1	x ₁	y ₁
2	x ₂	y ₂
3	x ₃	y ₃
4	x ₄	y ₄
1	x ₁	y ₁



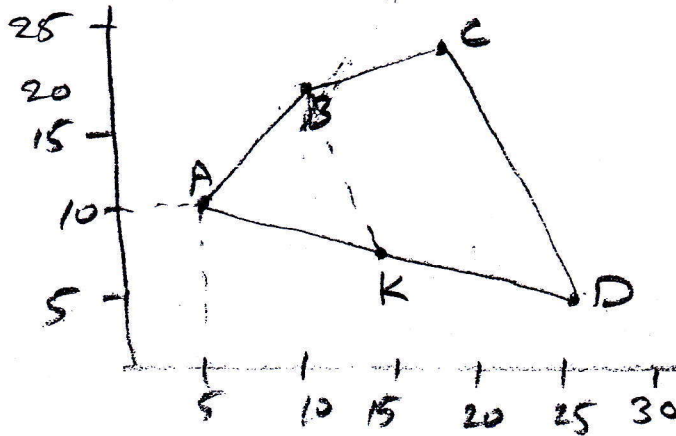
$$\text{Area} = \frac{1}{2} [(y_1 x_2 + y_2 x_3 + y_3 x_4 + y_4 x_1) - (x_1 y_2 + x_2 y_3 + x_3 y_4 + x_4 y_1)]$$

Example/ The area of closed traverse which coordinates shown below in table was divided into two equal parts by the line BK. Calculate the position of point k where point k placed on the line AD.

Point	X	Y
A	5	10
B	10	20
C	20	22
D	25	5

(2)

Sol.



Point	x	y
A	5	10
B	10	20
C	20	22
D	25	5
A	5	10

$$\text{Area} = \frac{1}{2} [(10 \times 10 + 20 \times 20 + 22 \times 25 + 5 \times 5) - (5 \times 20 + 10 \times 22 + 20 \times 5 + 25 \times 10)]$$

$$\text{Area} = 202.5 \text{ m}^2$$

$$\text{Area ABKA} = \text{Area BCDKB} = \frac{1}{2} \text{ total area} = 101.25 \text{ m}^2$$

From area ABKB

point	x	y
A	5	10
B	10	20
K	x	y
A	5	10

$$101.25 = \frac{1}{2} [(10 \times 10 + 20 \times x + y \times 5) - (5 \times 20 + 10 \times y + x \times 10)]$$

$$101.25 = 5x - 2.5y \quad \dots \textcircled{1}$$

From area BCDKB

point	x	y
B	10	20
C	20	22
D	25	5
K	x	y
B	10	20

$$101.25 = \frac{1}{2} [(20 \times 20 + 22 \times 25 + 5 \times x + y \times 10) - (10 \times 22 + 20 \times 5 + 25 \times y + x \times 20)]$$

$$101.25 = 315 - 7.5x - 7.5y \quad \dots \textcircled{2}$$

(3)

From Eq. (1) & Eq. (2)
Area of cross-sections

the coordinate of P is $x = 23 \text{ m}$ & $y = 5.5 \text{ m}$

One-level section

H.W Calculate the area closed traverse which coordinate shown below

Corner	1	2	3	4	5
Distance from horizontal axis	300	400	600	1000	1200
Distance from vertical axis	300	800	1200	1000	400

$$A = \frac{1}{2} [b + 2sh] \cdot h$$

$$A = \frac{1}{2} [b + 2sh] \cdot h \rightarrow (1)$$



Area of cross-section

① One-level section

$$\frac{h}{x} = \frac{1}{s} \Rightarrow x = hs$$

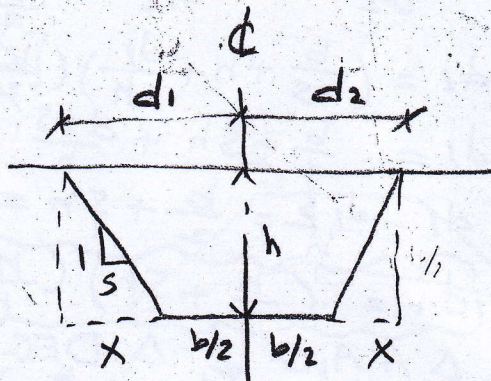
$$d_1 = d_2 = \frac{b}{2} + sh$$

$$A = \frac{1}{2} [b + 2(\frac{b}{2} + sh)] \cdot h$$

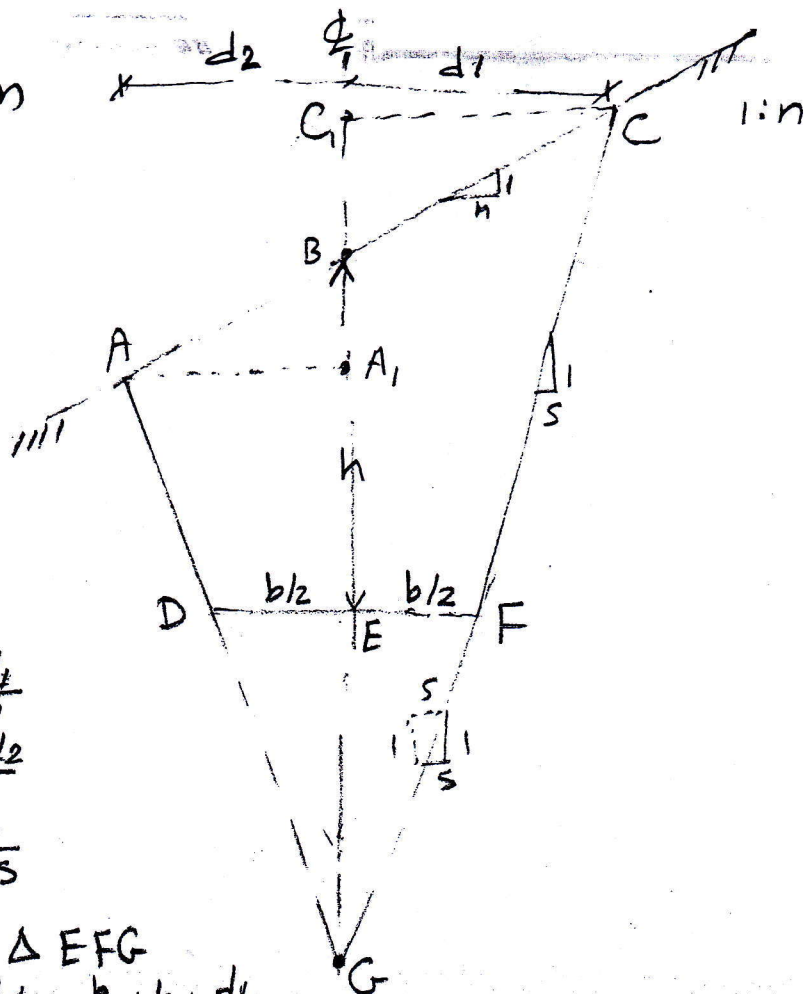
$$= \frac{1}{2} [b + b + 2sh] \cdot h$$

$$= \frac{1}{2} [2b + 2sh] \cdot h$$

$$\therefore A = [b + sh] \cdot h$$



② Two-level section



④

$$\frac{1}{n} = \frac{C_1B}{d_1} \Rightarrow C_1B = \frac{d_1}{n}$$

$$\frac{1}{n} = \frac{A_1B}{d_2} \Rightarrow A_1B = \frac{d_2}{n}$$

$$\frac{b/2}{GE} = \frac{s}{1} \Rightarrow GE = \frac{b}{2s}$$

From ΔC_1CG & ΔEFG

$$\frac{CC_1}{EF} = \frac{GC_1}{GE} \Rightarrow \frac{d_1}{b/2} = \frac{\frac{b}{2s} + h + \frac{d_1}{n}}{\frac{b}{2s}}$$

$$\Rightarrow d_1 = \left(\frac{b}{2s} + h + \frac{d_1}{n}\right) \left(\frac{b}{2}\right) \left(\frac{2s}{b}\right)$$

$$\Rightarrow d_1 = \frac{b}{2} + sh + \frac{d_1 s}{n}$$

$$\Rightarrow d_1 - \frac{d_1 s}{n} = \frac{b}{2} + sh \Rightarrow d_1 \left(1 - \frac{s}{n}\right) = \frac{b}{2} + sh$$

$$\Rightarrow d_1 = \left(\frac{b}{2} + sh\right) \left(\frac{n}{n-s}\right)$$

From ΔAA_1G & ΔDEG

$$\frac{AA_1}{DE} = \frac{GA_1}{GE} \Rightarrow \frac{d_2}{b/2} = \frac{\frac{b}{2s} + h - \frac{d_2}{n}}{\frac{b}{2s}} \Rightarrow d_2 = \left(\frac{b}{2} + sh\right) \left(\frac{n}{n+s}\right)$$

The area of fill or cut of ACFDA is:

Area BCG + Area ABG - Area DFG

$$A = \frac{1}{2} d_1 \left(\frac{b}{2s} + h\right) + \frac{1}{2} d_2 \left(\frac{b}{2s} + h\right) - \frac{1}{2} b * \frac{b}{2s}$$

$$A = \frac{1}{2s} \left[\left(\frac{b}{2} + sh\right) (d_1 + d_2) - \frac{b^2}{2} \right]$$

$$\text{or } A = \frac{1}{2} \left[\left(\frac{b}{2s} + h\right) (d_1 + d_2) - \frac{b^2}{2s} \right]$$