الكورس الاول
السعـر/

محاضرة / -3-
التاريخ /

Engineering Mechanics

الميكانيك الهندسي

لطلبة الدراسات الاولية

المرحلة الاولى

قسم الهندسة الموارد المائية

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النسخة الأصلية
في مكتب الغدير داخل كلية الهندسة / الفرع الاول
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بأدارة / عادل الكناني

2018 - 2019

Al-Mustansiriya University
College of Engineering
Water Resources Dept.
Engineering Mechanics

The Date:
Lecturer: Dr. Khitam Abdulhussain

Chapter -2-

Resultants of Force Systems

- A resultant of a force system is defined as the simplest force system which can replace the original system without changing its external effect on rigid body.

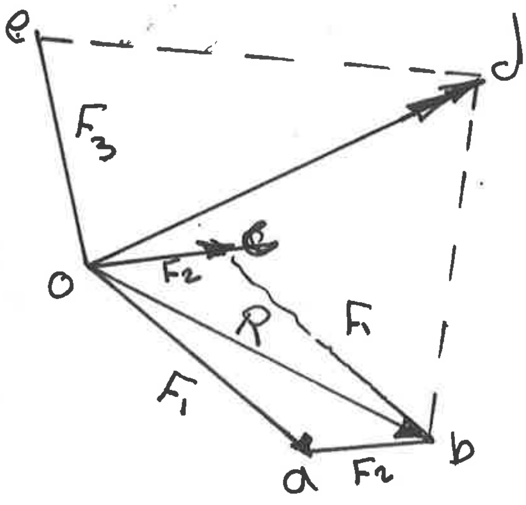
- when the resultant is <u>zero</u>, then the body is in equilibrium and the original force system in this case called <u>a balanced</u>

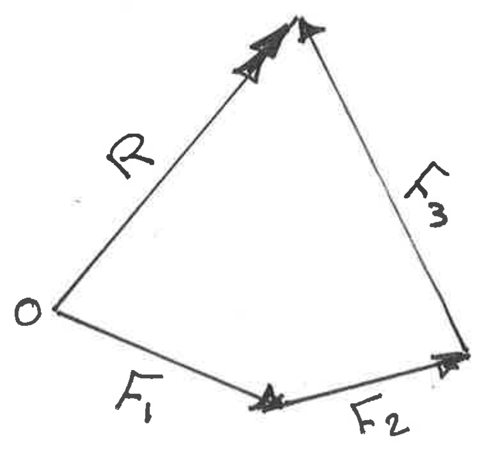
1. Types of force systems

متزامن

1.1 Resultant of a concurrent, coplanar force system

The resultant of a concurrent, coplanar force system is <u>a single</u> force passing through the point of concurrence





(a force polygon)

- The resultant " in a force polygon) is the vector from the tail of the first vector to the tip of the last one, provided the point of concurrence is used as the starting point in the construction.

- the analytical calulation.

if each force is first resolved into a pair of rectangular components.

$$\boxed{R_x = \sum F_x} \quad \text{and} \quad \boxed{R_y = \sum F_y}$$

The magnitude of the resultant is :-

$$\boxed{R = \sqrt{R_x^2 + R_y^2}}$$

The slope of the resultant is :-

$$\boxed{\tan \theta_x = \frac{R_y}{R_x}}$$

where θ_x : is the angle between the resultant and x-axis

and the sense can be determined from the components R_x and R_y

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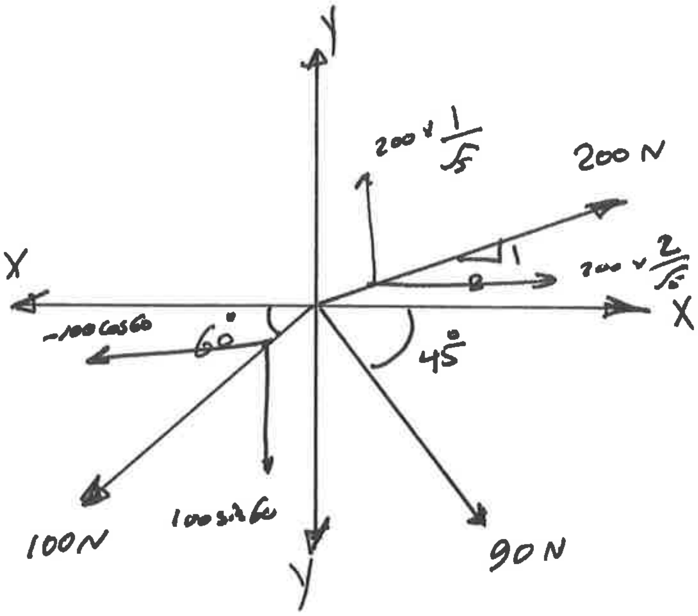
The Resultant.

Ex:- Determine the resultant of the concurrent coplanar force system.



Solution:

$R_x = \Sigma F_x \xrightarrow{+}$

$= -100\cos 60 + 90\cos 45 + 200\left(\frac{2}{\sqrt{5}}\right)$

$= 192.4\,N \longrightarrow$

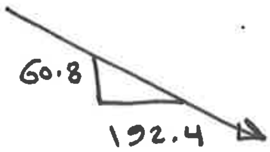
$R_y = \Sigma F_y \uparrow +$

$= -100\sin 60 - 90\sin 45 + 200\left(\frac{1}{\sqrt{5}}\right)$

$= -60.8\,N$

$\therefore R = \sqrt{R_x^2 + R_y^2}$

$= \sqrt{(192.4)^2 + (60.8)^2} = 201.77\,N$



$\theta_x = \tan^{-1}\frac{60.8}{192.4} = 17.5^\circ$

$\therefore R = 201.77\,N$

through o

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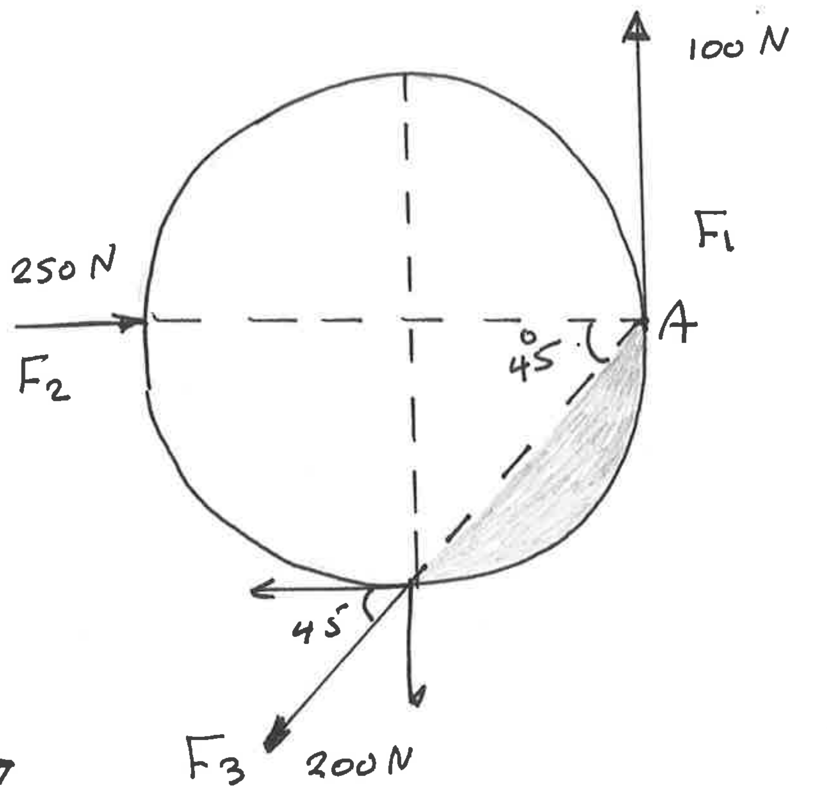
2.2 Determine the resultant of the concurrent, coplanar force system



solution

$R_x = \sum F_x \xrightarrow{+}$

$F_{3x} = -200 \cos 45$

$= -141.4$ N

$F_{3y} = -200 \sin 45$

$= -141.4$ N

$R_x = 250 - 141.4$

$= 108.6$ N $\rightarrow$

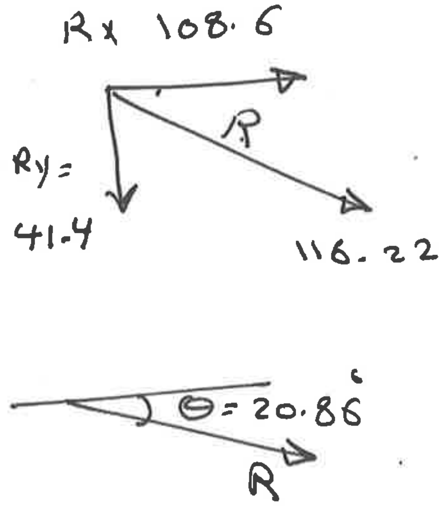
$R_y = \sum F_y \uparrow +$

$= 100 - 141.4 = -41.4$ N $\downarrow$

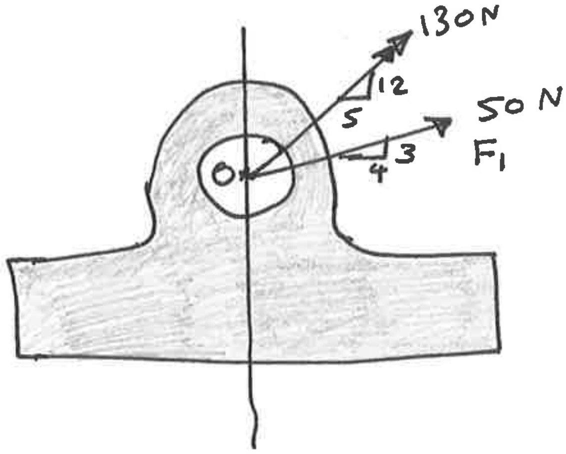
$R = \sqrt{R_x^2 + R_y^2}$

$= 116.2$ N

$\tan \theta_x = \dfrac{41.4}{108.6} = 20.86^\circ$



2.8 The 130 N force is the resultant of two forces, one of which is shown. Determine the other force.



solution

$$R_x = 130 \times \frac{5}{13} = 50\,N \rightarrow$$

$$R_y = 130 \times \frac{12}{13} = 120\,N \uparrow$$

$$R_x = \sum F_x \quad \xrightarrow{+}$$

$$50 = 50 \times \frac{4}{5} + F_2(x) \Longrightarrow F_2x = 10\,N \rightarrow$$

$$R_y = \sum F_y \quad \uparrow +$$

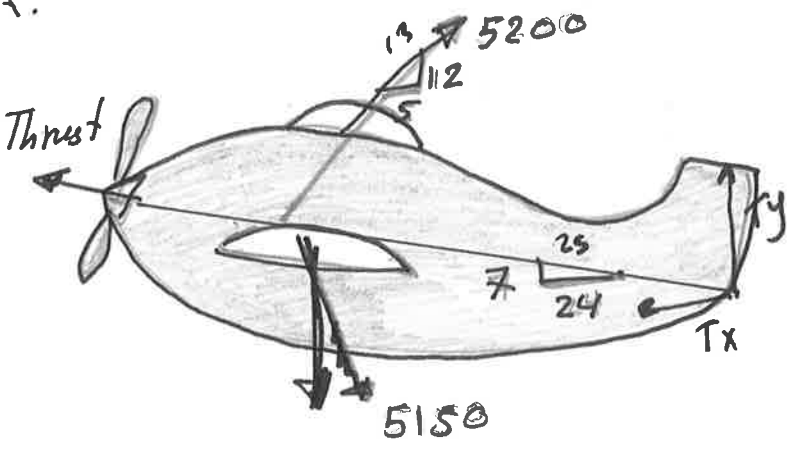
$$120 = 50 \times \frac{3}{5} + F_2y \Longrightarrow F_2y = 90\,N \uparrow$$

$$F_2 = \sqrt{10^2 + 90^2} = 90.55\,N \quad \text{through } O$$

2.9 The resultant of the three forces indicated on the airplane is horizontal. Determine the magnitude and sense of the resultant.

solution

since the resultant is horizontal therfore $R_y = 0$ and $R = R_x$



$$R_y = 0 \;(+\uparrow) = T * \frac{7}{25} - 5150 + 5200 * \frac{12}{13} \quad \text{——— (1)}$$

$$R_x = R \;(\xrightarrow{+}) = -T * \frac{24}{25} + 5200 * \frac{5}{13} \quad \text{——— (2)}$$

From (1) $\longrightarrow T \frac{7}{25} = 5150 - 48800$

$$\therefore T = \frac{25}{7}(350) \Longrightarrow T = 1250 \text{ N}$$

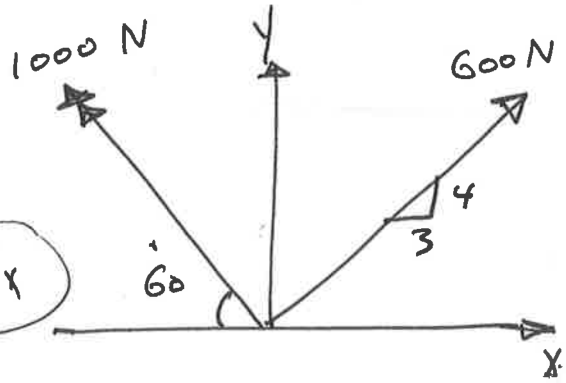
$$\boxed{\therefore \cancel{R = -1250 \text{ N}}}$$

$$R = -1250 * \frac{24}{25} + 5200 \frac{5}{13}$$

$$\therefore R = 800 \text{ N} \longrightarrow$$

2.10 The 1000 N force is the resultant of two forces one of which is 600 N as shown. Determine the other forces.

solution



let the unknown force (F_1)

$R_x = \Sigma F_x \quad \xrightarrow{+}$

$$-1000 \cos 60 = 600 \frac{3}{5} + F_{1x}$$

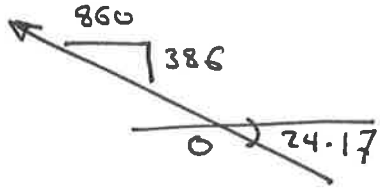
$$F_{1x} = -860 \text{ N} \longrightarrow$$

$$\therefore F_{1x} = 860 \text{ N} \longleftarrow$$

$R_y = \Sigma F_y \quad + \uparrow$

$1000 \sin 60 = 600 \frac{4}{5} + F_{1y}$

$F_{1y} = 386 \text{ N} \uparrow$



$\therefore F_1 = \sqrt{(860)^2 + (386)^2} = 94265 \text{ N}$

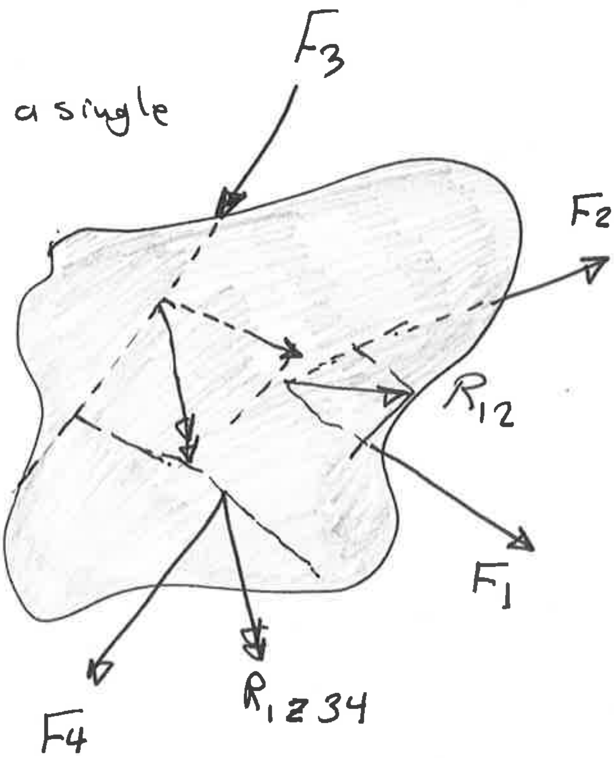
$\tan \theta_x = \frac{-386}{860} \Rightarrow \theta_x = -24.17$

1.2 Resultant of a NonConcurrent coplanar force system

The resultant of this force system is either a single force or a couple.

The resultant in fig is a single force.

The resultant ~~in fig~~ will be a couple whenever the resultant of all but one of the forces (R_{123}) and the remaining force (F_4) form a couple.



To determine the resultant analytically, each force is resolved into rectangular components, then:

① if the algebraic sum of the components in either (x) or (y) direction, or both is different from Zero, the resultant is a force

Then, the magnitude of the force is obtained as following

$$\boxed{R_x = \sum F_x \; ; \quad R_y = \sum F_y \; ; \text{ and } R = \sqrt{R_x^2 + R_y^2}}$$

The angle between the x_axis and the resultant is obtained from the relation.

$$\boxed{\tan \theta_x = \frac{R_y}{R_x}}$$

The location of a point on the action line of the resultant is determined by the principle of moments, thus :-

$$\boxed{R d = \sum M_o}$$

where d: is ⊥ distance from the moment ~~axis~~ through O to the resultant R.

Note : The direction of d is determined from the sense of R and of $\sum M_o$

(2) If the algebraic sum of the components of the forces is zero in two different directions, the resultant cannot be a single force but can be is a couple in the plane of the forces.

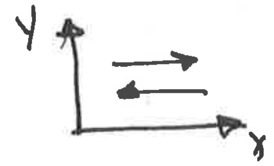
- The magnitude and the sense of rotation of the couple may be obtained as the algebraic sum of the moments of the forces with respect to any point in the plane.

Note:

$Ry = 0$

If $Rx = 0$ the resultant is couple

If $R\cancel{x} \neq 0$ the resultant is a force and it is // to x-axis



Ex:- Determine the resultant of the force systems and shown it on a sketch located with respect to point A.

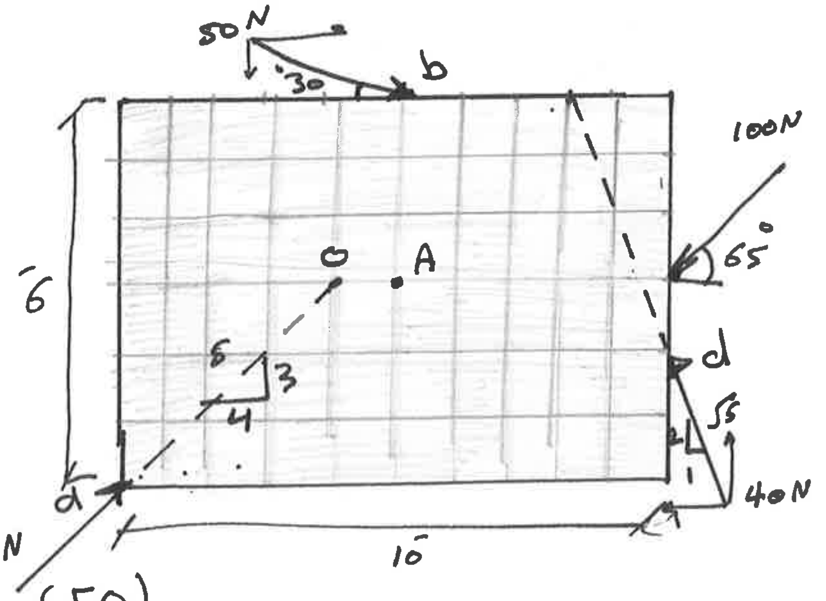
Solution

$\xrightarrow{+}$

$Rx = 75 * \frac{4}{5} + 50\cos 30^{\circ}$

$- 100\cos 65 - 40\left(\frac{1}{\sqrt{5}}\right)$

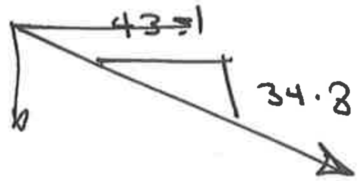
$= 43.1 \text{ N} \longrightarrow$



$$\uparrow^{+} R_y = 75\,\frac{3}{5} - 50\sin 30 - 100\sin 65 + 40\left(\frac{2}{\sqrt{5}}\right)$$

$$= -34.8\,N \uparrow$$

$$\therefore R_y = +34.8\,N \downarrow$$



$$R = \sqrt{R_x^2 + R_y^2} = 55.4\,N$$

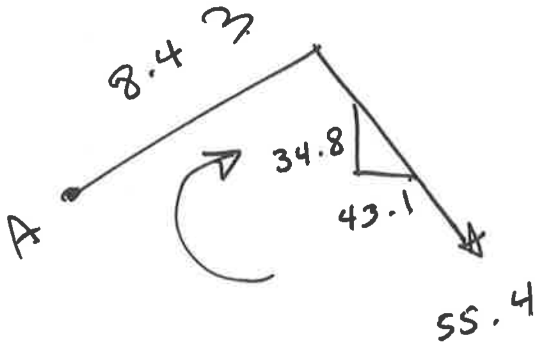
$$R \cdot d = \Sigma M_A$$

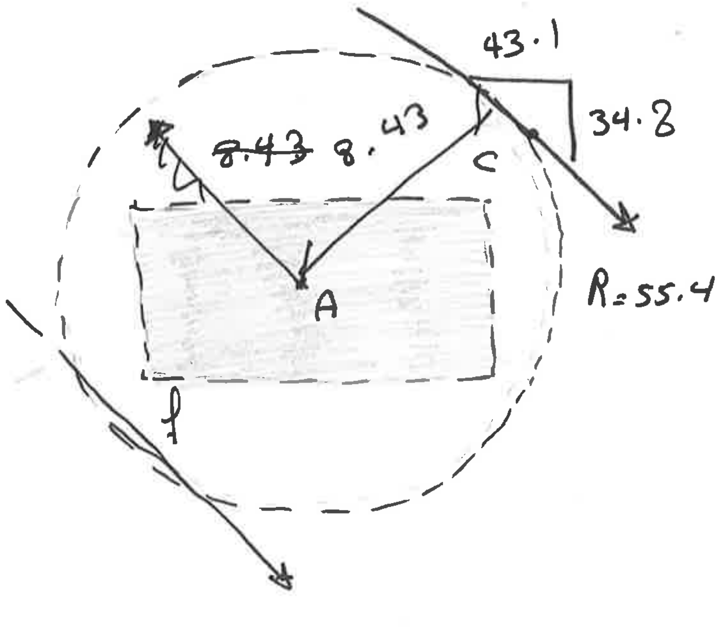
$$\Sigma M_A = 75\left(\frac{3}{5}\right)(1) + (50\cos 30)(3) + (100\sin 65)(5) + 40\left(\frac{1}{\sqrt{5}}\right)(1) - 40\left(\frac{2}{\sqrt{3}}\right)(5)$$

$$= 467\ N.cm$$

$$\therefore d = \frac{\Sigma M_A}{R} = \frac{467}{55.4} = 8.43\,cm$$

$$R = 55.4\,N$$





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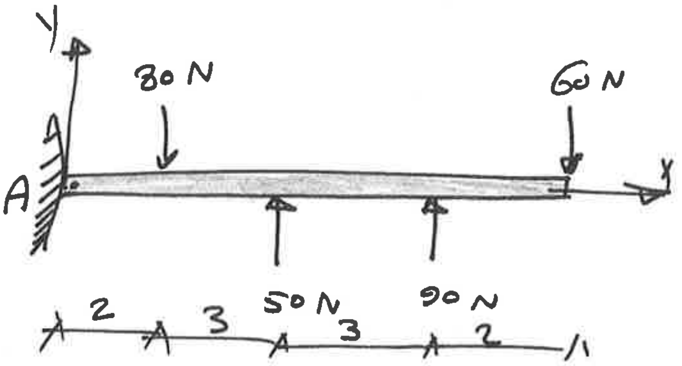
Problems

2.12 Determine the resultant of the parallel forces shown.



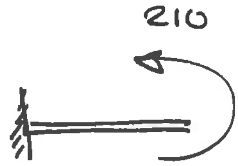
$R_x = 0$

$+\uparrow R_y = 50 + 90 - 80 - 60$
$= 0$

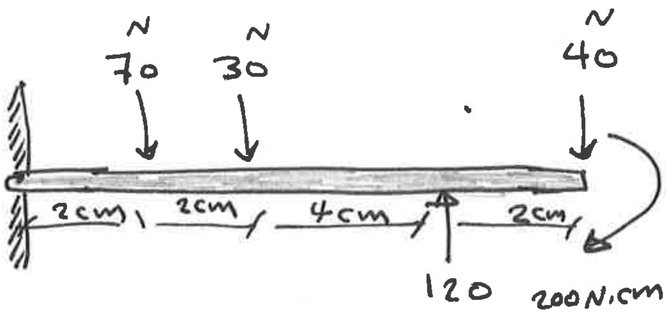
∴ The resultant is a couple

$\circlearrowright + \Sigma M_A = 80 * 2 + 60 * 10 - 50 * 5 - 90 * 8$
$= -210$ N.cm

∴ The couple is 210 N.cm



2.13 Determine the resultant of the forces system shown.



solution

$R_x = 0$

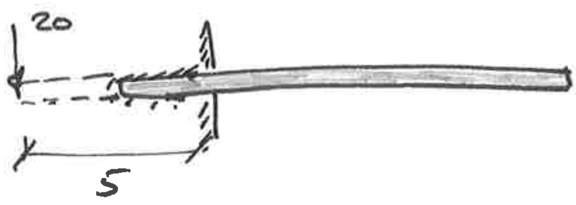
$+\uparrow R_y = 120 - 40 - 70 - 30$
$= -20$ N $\uparrow$

∴ $R_y = 20$ N $\downarrow$

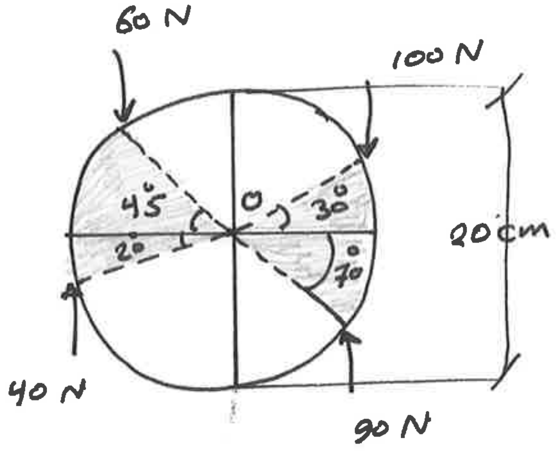
$R q = \Sigma M_o \Longrightarrow 70 * 2 + 30 * 4 + 40 * 10 - 120 * 8 + 200 = -100$ N.cm

$\Sigma M_o = 100$ N.cm

∴ $q = \frac{100}{20} = 5$ cm



2.19 Determine the resultant of the parallel force system and locate it with respect to O.



Solution

$R_x = 0$

$R = R_y = \Sigma F_y \quad \uparrow +$

$R = 40 + 90 - 60 - 100$

$= -30 \text{ N} \uparrow$

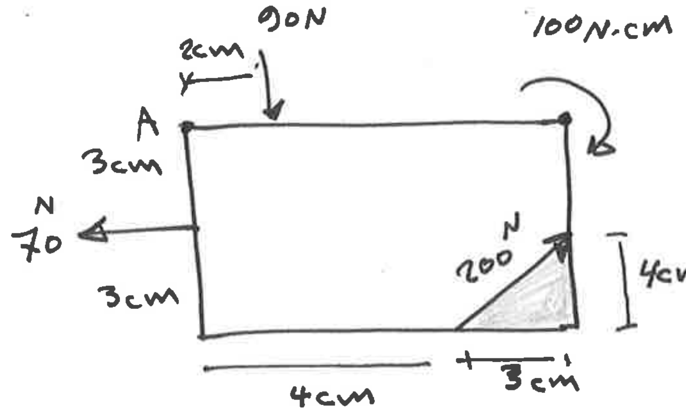
$\therefore R = 30 \text{ N} \downarrow$

$R\,d = \Sigma M_o$

$$d = \frac{100 \times 10 \cos 30 + 40 \times 10 \cos 20 - 90 \times 10 \cos 70 - 60 \times 10 \cos 45}{30}$$

$\therefore d = 17 \text{ cm}$

2.22 Determine the resultant of the force system and locate it with respect to point A.



Solu:

$R_x = \Sigma F_x \quad \xrightarrow{+}$

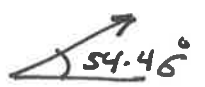
$= 200\left(\frac{3}{5}\right) - 70 = 50 \text{ N} \rightarrow$

$R_y = \Sigma F_y \quad \uparrow +$

$= 200\left(\frac{4}{5}\right) - 90 = 70 \text{ N} \uparrow$

$\therefore R = \sqrt{50^2 + 70^2} = 86 \text{ N}$

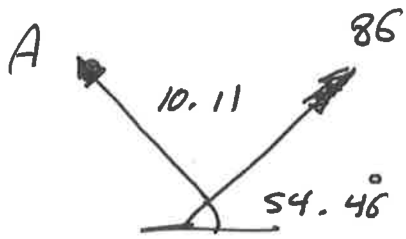
$\tan \theta_x = \frac{70}{50} \Longrightarrow \theta_x = 54.46^\circ$



$$\overset{+}{\curvearrowright} \Sigma M_A = 100 + 90(2) + 70(3) - (200 * \tfrac{4}{5})(4) - (200 * \tfrac{3}{5})(6)$$

$$= -870 \text{ N.cm}$$

$$\therefore d = \frac{\Sigma M_A}{R} = \frac{870}{86} = 10.11$$



2.27

The resultant of the three forces and the couple T in the fig. and an unknown force through point A is vertical 100 N force through point B. Determine the unknown force through A and the magnitude of the couple T.

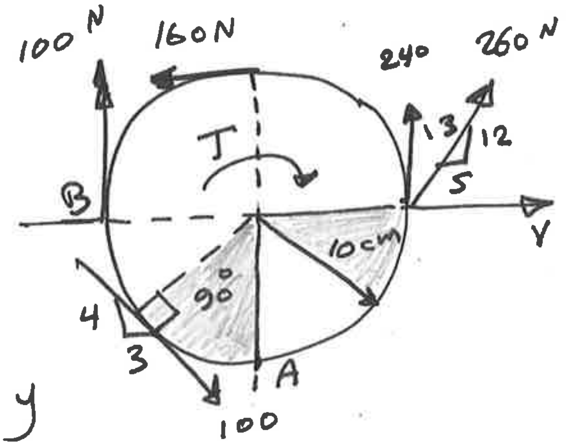
sol:

let the force through A is F_A

$R_y = R \quad : R_x = 0$

$\overset{+}{\rightarrow} R_x = 0 \implies -160 + 260 * \frac{5}{13} + 160 \frac{3}{5} + (F_A)_x$

$\therefore (F_A)_x = 0$



$+\uparrow R_y = R$

$100 = 260 * \frac{12}{13} - 100 \frac{4}{3} + (F_A)_y$

$100 = 240 - 80 + (F_A)_y \qquad \therefore (F_A)_y = -60 \text{ N} \updownarrow$

$\therefore F_A = +60 \text{ N} \downarrow$

The moment of the resultant $= \Sigma$ the moment of this force at O

$\overset{+}{\curvearrowright} 100 * 10 = -160 * 10 - 240 * 10 - 100 * 10 + T$

$1000 = -1600 - 2400 - 1000 + T$

$\therefore T = 6000 \text{ N.cm} \curvearrowright$

1.3 Resultant of a concurrent, Non coplanar force sys.

- The resultant of any set of concurrent forces must be a force passing through the point of concurrence of the forces of the system.

- The components of the resultant in three mutually perpendicular directions are obtained as following

$$R_x = \sum F_x \; ; \; R_y = \sum F_y \; ; \; R_z = \sum F_z$$

the magnitude of the resultant is

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

* To indicate the resultant (R) in space a rectangular parallel ~~lepiped~~ is drawn with one corner at the point of the concurrence, and with edges parallel to the components of the resultant R_x, R_y, R_z

Ex:

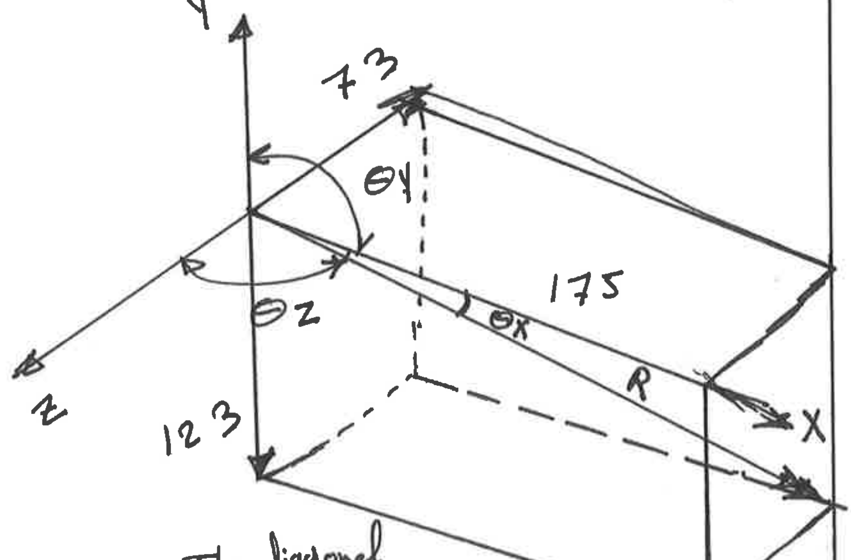
$R_x = 175\ N$ ↘

$R_y = 123\ N$ ↓

$R_z = 73\ N$ ↗

$$R = \sqrt{175^2 + 123^2 + 73^2}$$

$$R = 226\ N$$



The diagonal of the box from the point of concurrence will represent the resultant R.

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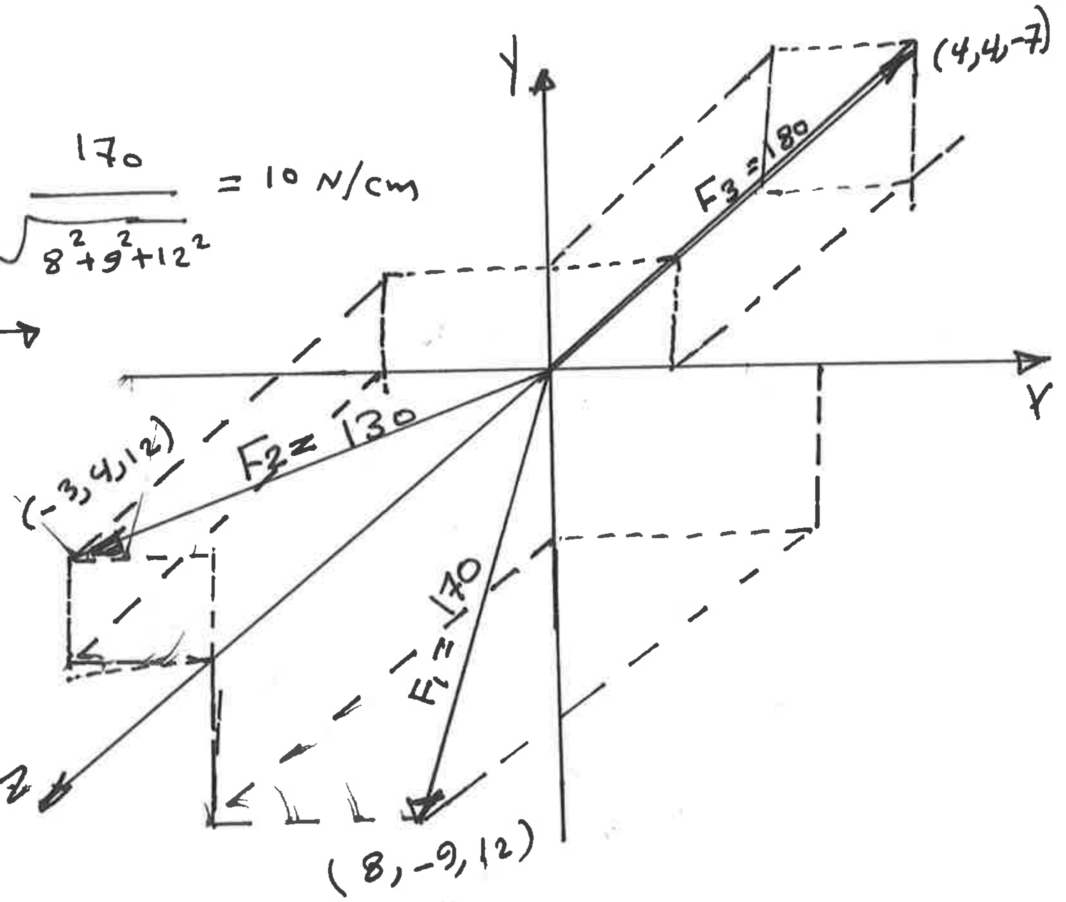
PROBLEMS

2.36 The forces in this problem are concurrent at the origin and directed away from it toward the point designated by its x, y, z coordinate. Determine the resultant of the system
170 N (8, −9, 12), 130 N (−3, 4, 12) and 180 N (4, 4, −7)

solution



scale for $f_1 = \dfrac{170}{\sqrt{8^2+9^2+12^2}} = 10\ N/cm$

$\therefore f_1)_x = 80\ N \rightarrow$

$f_1)_y = -9 * 10$

$= -90\ N \uparrow$

$f_1)_y = 90\ N \downarrow$

$f_1)_z = 12 * 10$

$= 120\ N \swarrow$

scale for $f_2 = \dfrac{130}{\sqrt{3^2+4^2+12^2}} = 10\ N/cm$

$F_2)x = -3 \times 10 = -30\ N \rightarrow\ = 30\ N \leftarrow$

$F_2)y = 40\ N \uparrow$

$F_2)z = 120\ N \swarrow$

scale for $f_3 = \dfrac{180}{\sqrt{16+16+49}} = 20\ N/cm$

$F_3)_x = 4 * 20 = 80\ N \rightarrow$

$F_3)_y = 80N \uparrow$

$F_3)_z = -7 * 20 = -140\ N$
$= 140\ N \nearrow$

$R_x = \Sigma F_x$

$= 80 - 30 + 80 = 130N \rightarrow$

$R_y = \Sigma F_y$

$= -90 + 40 + 80 = 30 \uparrow$

$R_z = \Sigma F_z$

$= 120 + 120 - 140 = 100N \swarrow$

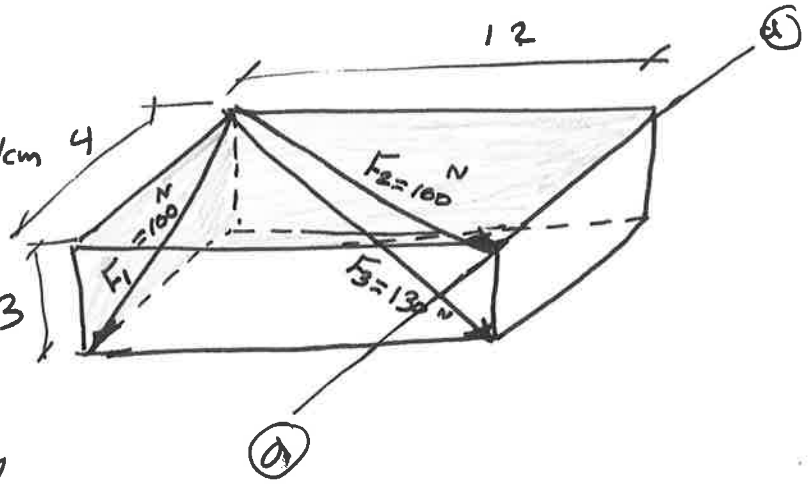
$\therefore R = \sqrt{130^2 + 30^2 + 100^2} = 166.7\ N$ from o through

point (13, 3, 10).

2.39 (a) Determine the resultant of the force system shown in fig.

(b) Determin the moment of the resultant with respect to the a-axis

solution



scale for $F_1 = \frac{100}{\sqrt{3^2+4^2}} = 20\,N/cm$

$F_1)_z = 20 * 4 = 80\,N$
$= 100 * \frac{4}{5} = 80\,N$

$F_1)_y = 20 * -3 = -60\,N$
$= 60\,N$

F scale for $F_2 = \frac{100}{\sqrt{12^2+4^2}} = 7.9\,N/cm$

$F_2)_x = 94.8\,N \rightarrow$ $7.9 * 12$

$F_2)_z = 31.6\,N$ $7.9 * 4$

scale for $f_3 = \frac{130}{\sqrt{4^2+3^2+12^2}} = 10\,N/cm$

$F_3)_x = 120\,N \rightarrow$ $10 * 12$

$F_3)_y = -3 * 10 = -30\,N = 30$

$F_3)_z = 40\,N$

B

$R_x = \Sigma F_x$
$= 120 + 94.8 = 214.8\ N \rightarrow$

$R_y = \Sigma F_y = -60 - 30 = -90$
$= +90 \downarrow$

$R_z = 80 + 31.6 + 40 = 151.6\ N \swarrow$

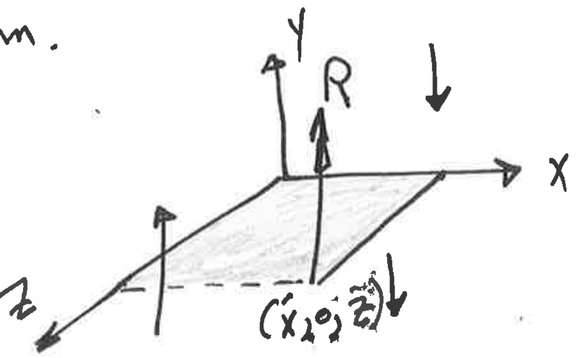
$R = \sqrt{214.8^2 + 90^2 + 151.6^2}$

$= 277.88\ N$ from o through point $(21.48, -9.0, 15.16)$

(b) $M_{a-a\ axis} = -R_y\,(12)$
$= -90\,(12)$
$= -10.80\,N$
$= 1080\ N.cm$

1.4 Resultant of a parallel, Non coplanar force syst.
① If the resultant is a single force

- equal to the algebraic sum of the forces of the system, and its position in space can be determined by the coordinates of the intersection of its action line with a plane perpendicular to
- the force of the system.



- the resultant of forces is parallel to y-axis.
and it is completely determined by the equations,

$R = \Sigma F_y$

$R\bar{x} = \Sigma M_z$

$R\bar{z} = \Sigma M_x$

(2) If the sum of forces is zero and the sum of the moments about one or both of two rectangular axes in a plane perpendicular to the forces is not zero, <u>the resultant is a couple</u>

For forces // to y-axis, the resultant is a couple.

if $\Sigma F_y = 0$ and $\Sigma M_x \neq 0$ or $\Sigma M_z \neq 0$

or $\Sigma M_x \neq 0$ and $\Sigma M_z \neq 0$

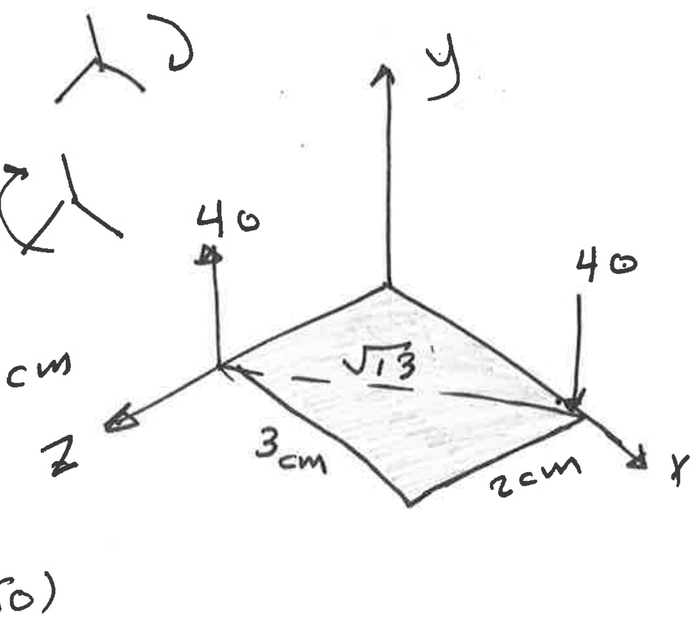
The resultant couple can be shown on a sketch by placing an upward force on one moment axis and an equal downward force on the other moment axis, so spaced as to produce the proper resultant moment.

for ex: $\Sigma F_y = 0$

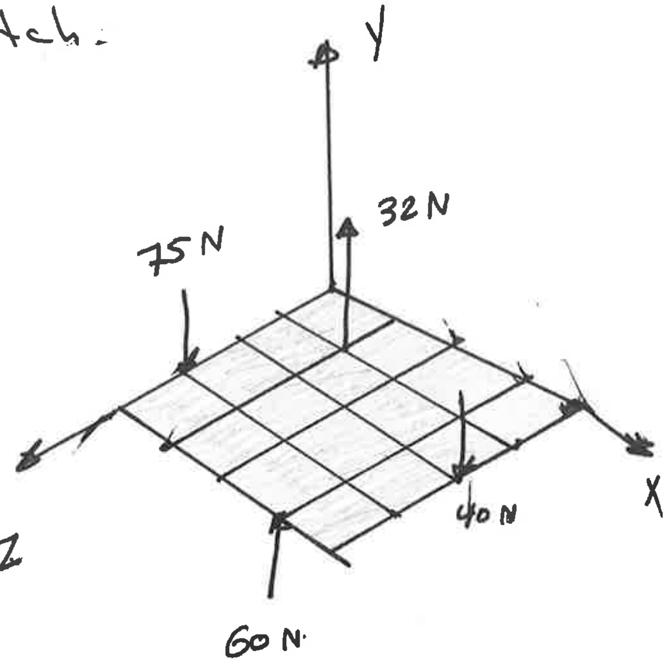
$\Sigma M_z = 120$ N.cm

$\Sigma M_x = 80$ N.cm

$M = 40\sqrt{13} = 144.2$ N.cm



Ex:- Determine the resultant of the four parallel forces and show it on a sketch.



solu:-

$+\uparrow R = 60 + 32 - 75 - 40$

$= -23\,N \uparrow$

$= 23\,N \downarrow$

The moment of the forces at x-axis can be calculated as:

$\Sigma M_x = -60(4) - 75(3) - 32(1) + 40(2)$

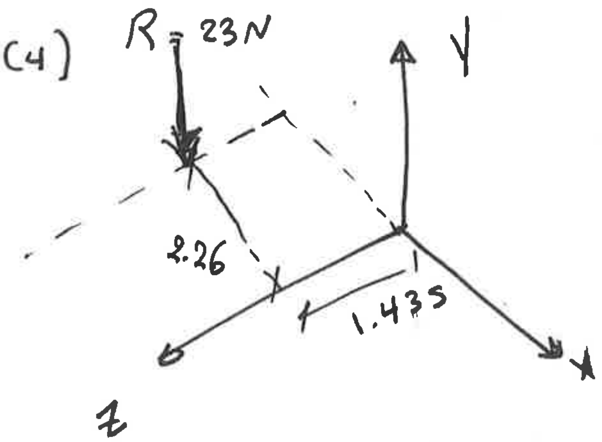
$= 33\ N.cm$

$R\bar{z} = \Sigma M_x$

$R\bar{z} = 33$ $\therefore z = \frac{33}{23} = 1.435\,cm$

$+\Sigma M_z = 60(3) + 32(1) - 40(4)$

$= 52\ N.cm$



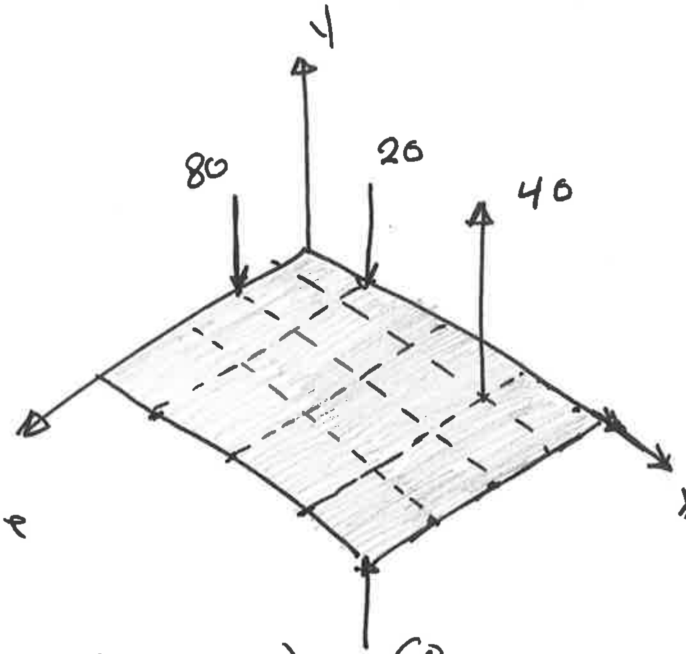
$R x' = 52$

$\therefore x' = \frac{52}{23} = 2.26\,cm$

$\therefore R = 23\,N \downarrow$

$R = 23\,N \downarrow$ through $\rightarrow (-2.26, 0, 1.435)$

Ex: Determin the resultant of the force system



$R = \sum F_y \quad \uparrow +$

$= 60 + 40 - 80 - 20$

$R = 0$

The resultant is a couple

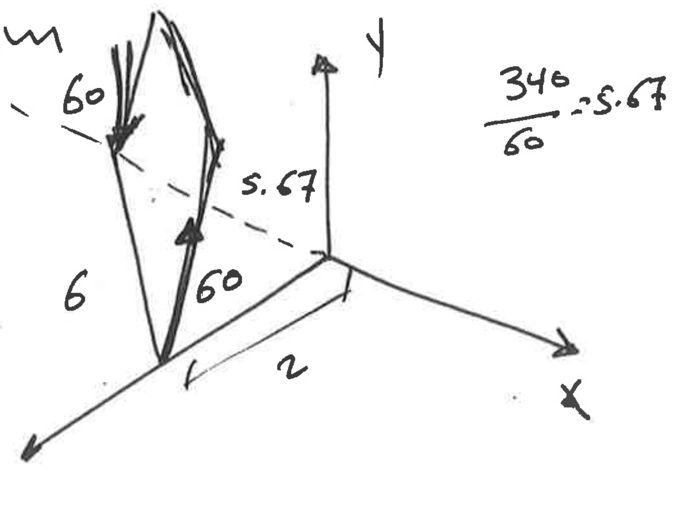
$+ \sum M_x = 80(2) - 40(1) - 60(4)$

$= -120$ N.cm

$= 120$ N.cm

$\sum M_z = 40(3) + 60(4) - 20(1)$

$= 340$ N.cm



$C = M = 60 * 6$

$= 360$ N.cm

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The date:

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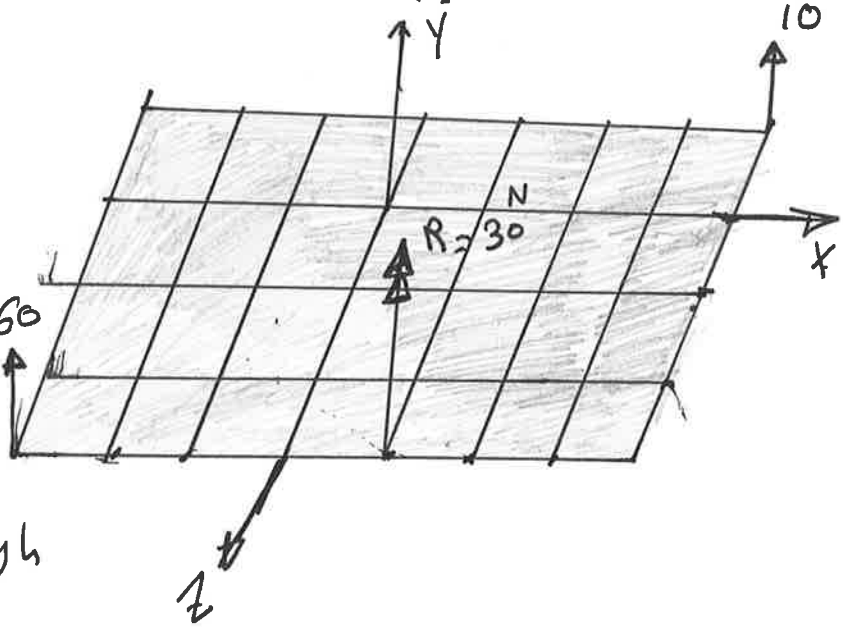
2.47 In the parallel force system, the 30 N force is the resultant of three forces two of which are shown. Determin the third force and locate it on a sketch.



$R = \Sigma F_y \uparrow +$

$30 = 10 + 60 + F$

$\therefore F = -40\ N$

$F = 40\ N \downarrow$

let F pass through $(\bar{x}, 0, \bar{z})$

$+\circlearrowleft \quad R\bar{x} = \Sigma M_z$

$30(1) = 10(4) - 60(3) + (M_F)_z$

$(M_F)_z = 30 - 40 + 180$

$= 170\ N.cm \circlearrowleft$

$x(F) = 170$

$\therefore x = \frac{170}{40} = 4.25\ cm$

$x = -4.25\ cm$ [From the direction of moment $(M_F)_z$]

$$R\bar{z} = \Sigma M_y$$

$$-30(3) = 10(1) - 60(3) + (M_f)_x$$

$$(M_f)_x = -90 - 10 + 180$$
$$= 80 \text{ N.cm}$$

$$(z)_f = 80$$

$$z = \frac{80}{40} = 2 \text{ cm}$$

$\therefore$ $z = 2$ cm (from the direction of moment $(M_f)_x$)

$\therefore$ $F = 40$ N $\downarrow$ through $(-4.25, 0, 2)$