محاضرة / -4-
التاريخ /

الكورس الاول
السعـر /

Engineering Mechanics

الميكانيك الهندسي

لطلبة الدراسات الاولية

المرحلة الاولى
قسم الهندسة الموارد المائية

Dr. Khitam Abdulhussein

النسخة الأصلية
في مكتب الغدير داخل كلية الهندسة / الفرع الاول
مكتب الغدير 2 مقابل كلية الهندسة / الفرع الثاني
بأدارة / عادل الكناني

2018 - 2019

Al-Mustansiriya University
College of Engineering
Water Resources Dept.
Engineering Mechanics
Lecturer:
Dr. Khitam

CHAPTER THREE

Centroids and Center of Gravity

Introduction:

- The force of attraction of the earth for a particle is called the weight of the particle

Thus, the resultant weight pass-through one point in the body, for all orientations of the body, and this point is defined as the center of gravity or center of mass of the body

- three coordinates are necessary to indicate the position of the (C.G)
- The center of gravity lies in the plane of symmetry of a homogoneous body.
- The centroid of an area lies on the line of symmetry

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Lecturer: Dr. Whitam
The Date:

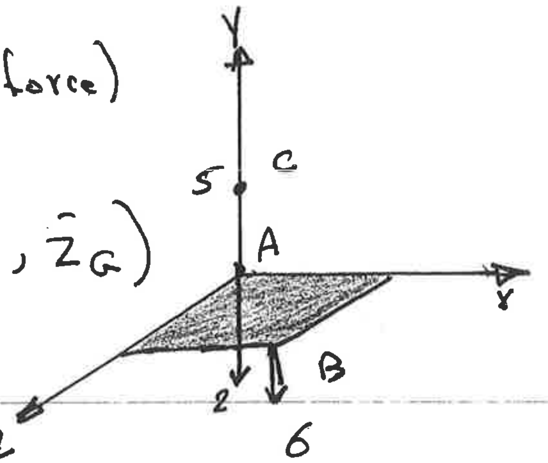
- The center of the C.G of a system of particles may be obtained as follow:

by the below Example.

Ex: Locate the center of gravity of three small bodies (considered as particles) arranged as shown.

particles	weight (N)	x, y, z (cm)
A	2	(0, 0, 0)
B	6	(2, 0, 3)
C	5	(0, 2, 0)

solution



The sum of weight (resultant force)

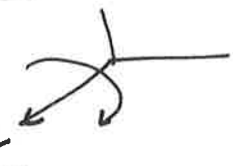
$R = 13N$

let C.G Coordinates $(\acute{x}_G, \acute{y}_G, \bar{z}_G)$

$\Sigma M_x = 6 * 3 = 18 N.cm$

$R . \acute{z} = 18$

$\therefore \acute{z} = \frac{18}{13} = \boxed{1.39 \text{ cm}}$



$\Sigma M_z = 6 * 2 = 12$

$\therefore R . \acute{x} = 12$

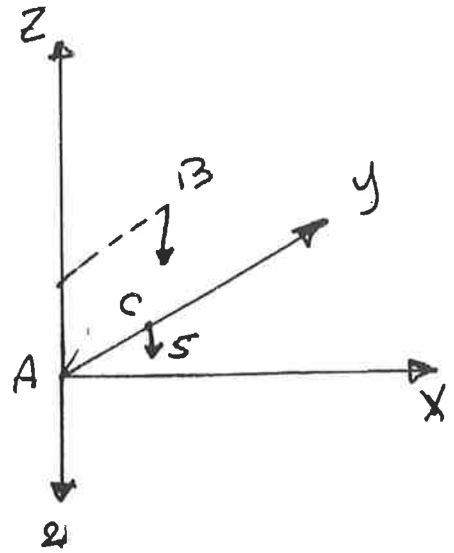
$\acute{x} = \frac{12}{13} = \boxed{0.92 \text{ cm}}$

The system of particles is rotating $90°$ about x-axis inorder to find $(\acute{y}\, G)$ coordinate.

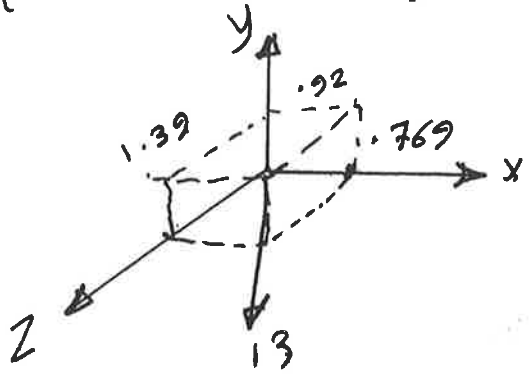
$$\Sigma M_{x'} = 5(2) = 10$$

$$R \cdot \bar{y} = 10$$

$$\therefore \bar{y} = \frac{10}{13} = \boxed{0.769}$$



$\therefore$ C.G.C $(0.92, 0.769, 1.39)$



The Center of Gravity of Abody

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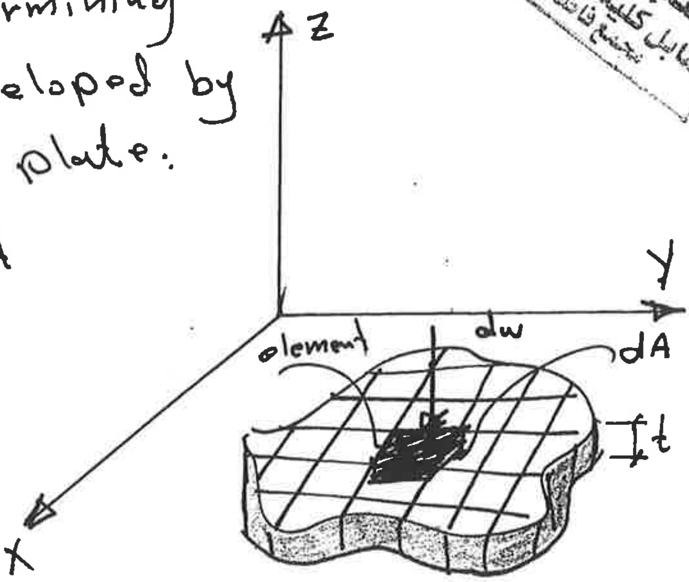
a general method of determining C.G of abody can be developed by considering the following plate:

1. The resultant weight

$$dw = \gamma t \, dA$$

weight of an element



γ = specific weight

$\therefore$ The total weight of the plate is

$$w = \int \gamma t \, dA$$

2- The coordinates of the C.G

$$dM_x = y\,dw = y\gamma t\,dA$$

$$\therefore M_x = \int y\gamma t\,dA$$

from the principle of moments:

$$w\bar{y}_G = M_x \qquad \therefore \boxed{\bar{y}_G = \frac{\int y\gamma t\,dA}{\int \gamma t\,dA}}$$

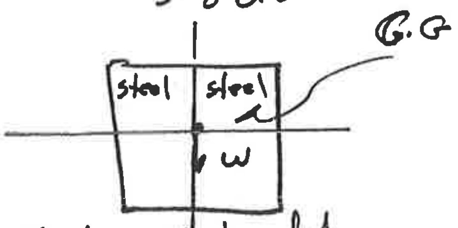
In a similar manner, the x coordinate of a point on the action line of the resultant weight is

$$\boxed{\bar{x}_G = \frac{\int x\gamma t\,dA}{\int \gamma t\,dA}}$$

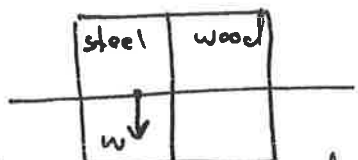
If t and γ are constant, $\bar{z}_G$ may be obtained by symmetry ($\bar{z}_G = t/2$)

If either t or γ is a variable the plate can be rotated, so either x or y axis is vertical and

$$\bar{z}_G = \frac{\int z\gamma\,dv}{\int \gamma\,dv} \qquad \text{where} \quad dv = dA\,dt$$



homegenous body
C.G will coincide



not homegenous body
center and C.G will not coincide

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Centroids.

A general method of determining the centroid for an area is:

1- Find the total Area.

$$A = \int dA$$

2- The coordinates of the centroid are obtained by the principle of moments:

$dM_x = y\,dA$ $dM_y = x\,dA$

$M_x = \int y\,dA$ $M_y = \int x\,dA$

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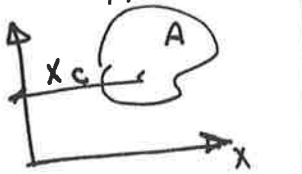
from the principle of moment

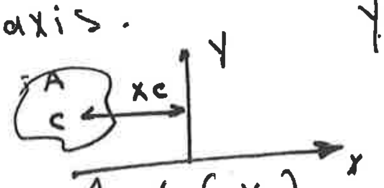
$$\therefore \boxed{\bar{y}_G = \frac{\int y\,dA}{\int dA}} \qquad \therefore \boxed{\bar{X}_G = \frac{\int x\,dA}{\int dA}}$$

The first moment of an area ($\int y\,dA$, $\int x\,dA$) about an axis has units of mm^3, cm^3, m^3

- The Area is a scalar quantity and the sense of moment of an area depends on the moment arm

So it is positive if the area is on one side of the axis and negative if the area on the opposite side of the axis.





A. Xc (-Vc)

Note: The first moment with respect to an axis through the centroid is zero

$\int x\,dA = 0$

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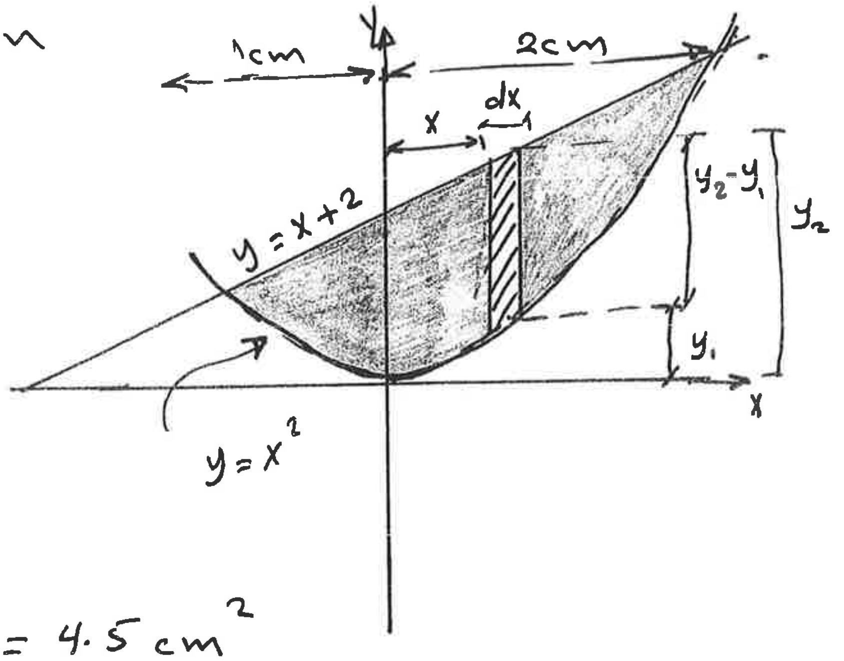
Centroids and Centers of Gravity By Integration

The element is chosen so that all parts of it have the same distance from the reference axis or plane (centroidal distance)

Ex: Determin the coordinates of the centroid of the area shown



Solu:

$$dA = (y_2 - y_1)\,dx$$
$$= (x + 2 - x^2)\,dx$$
$$\therefore A = \int_{-1}^{2} (x + 2 - x^2)\,dx$$
$$A = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} = 4.5\ cm^2$$

The mom. of the element with respect to y-axis

$$dM_y = x\,dA = x(x + 2 - x^2)\,dx = (x^2 + 2x - x^3)\,dx$$
$$\therefore M_y = \int_{-1}^{2} (x^2 + 2x - x^3)\,dx = \left.\frac{x^3}{3} + \frac{2x^2}{2} - \frac{x^4}{4}\right]_{-1}^{2} = 2.25\ cm$$
$$\therefore \bar{X}_{c} = \frac{M_y}{A} = 0.5\ cm$$

The location of the centroid of the element with respect to xz plan

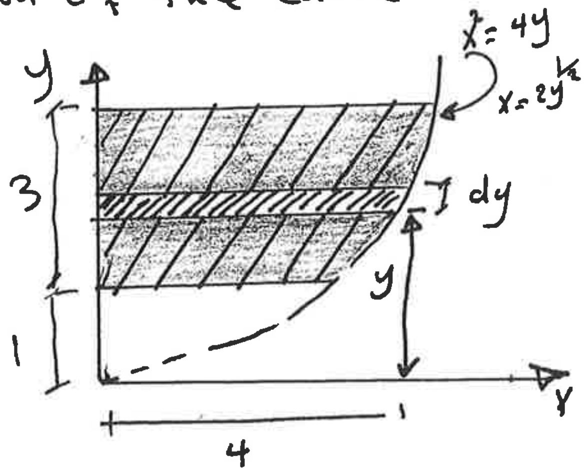
$$dM_x = \frac{y_1 + y_2}{2} dA = \frac{1}{2}(x^2 + 4x + 4 - x^4)\,dx$$

$$\therefore M_x = \frac{1}{2}\int_{-1}^{2}(x^2 + 4x + 4 - x^4)\,dx$$

$$= \frac{1}{2}\left(\frac{x^3}{3} + 2x^2 + 4x - \frac{x^5}{5}\right)_{-1}^{2} = 7.2\ cm^3$$

$$\therefore \bar{y}_{\mathbb{C}} = \frac{M_x}{A} = 1.6\ cm$$

3.8 Determine the coordinates of the centroid of the shaded area. The equation of the curve is $x^2 = 4y$



Solution

$$dA = x\,dy$$
$$dA = 2y^{1/2}\,dy$$

$$A = 2\int_{1}^{4} y^{1/2}\,dy$$

$$= 2\left[\frac{y^{3/2}}{3/2}\right]_1^4 = 9.333$$

$$dM_y = \frac{x}{2}\,dA = \frac{x^2}{2}\,dy = 2y\,dy$$

$$M_y = 2\int_1^4 y\,dy = \left[y^2\right]_1^4 = 15$$

$$\therefore \bar{x} = \frac{15}{9.333} = 1.607\ cm$$

$$dM_x = y\,dA = 2y^{3/2}\,dy$$

$$M_x = 2\int_1^4 y^{3/2}\,dy = 2\left[\frac{y^{5/2}}{5/2}\right]_1^4 = 24.8 \Longrightarrow$$

$$\therefore \bar{y} = \frac{24.8}{9.333} = 2.66$$

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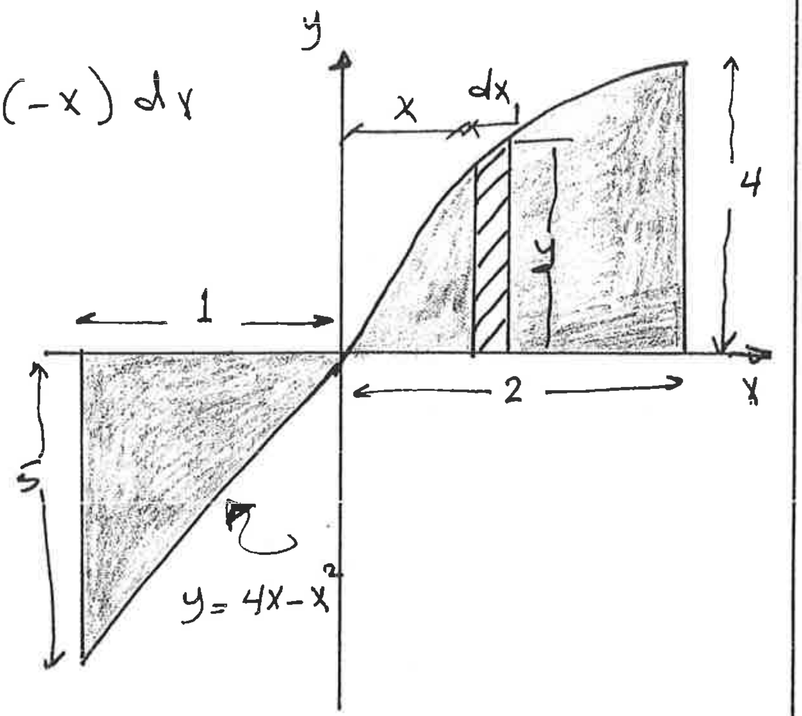
<u>3-12</u> Determin the first moment of the shaded area with respect to y-axis

$$M_y = \int_0^2 y(x)\,dx + \int_{-1}^0 y(-x)\,dx$$

$$= \int_0^2 (4x^2 - x^3)\,dx + \int_{-1}^0 (-4x^2 + x^3)\,dx$$

$$= \left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^2 + \left[-\frac{4x^3}{3} + \frac{x^4}{4}\right]_{-1}^0$$

$$= 6.666 - 1.583 = 5.083\ cm^3$$



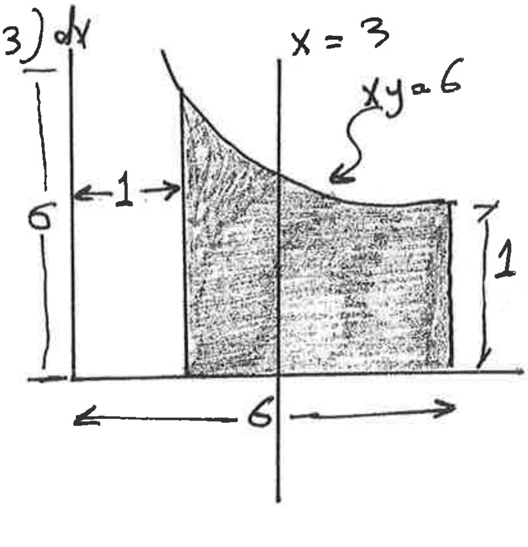
<u>3.15</u> Determine the first moment of the shaded area with respect to the line $x = 3$

$$M)_{x=3} = \int_1^3 y[-(3-x)]\,dx + \int_3^6 y(x-3)\,dx$$

$$= \int_1^3 \frac{6}{x}(x-3)\,dx + \int_3^6 \frac{6}{x}(x-3)\,dx$$

$$= \left[6x - 18\ln x\right]_1^3 + \left[6x - 18\ln x\right]_3^6$$

$$= -2.25\ cm^3$$



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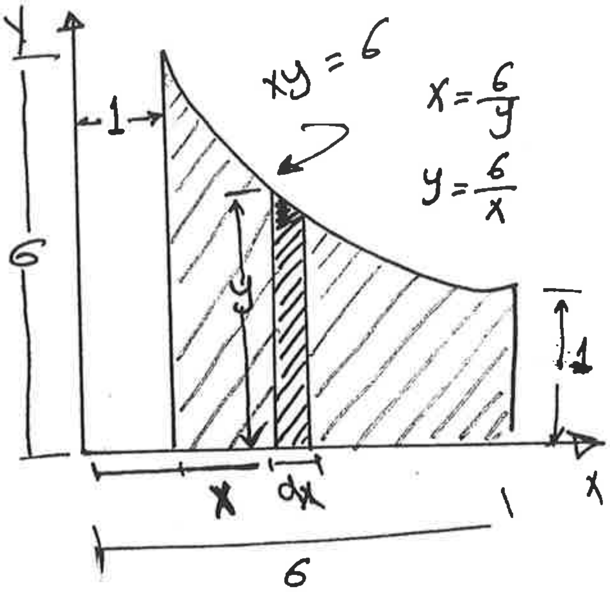
3-14 locate the centroid of the shaded area

Solution



$$dA = y\,dx$$

$$\therefore A = \int_1^6 \frac{6}{x}\,dx = 6\int_1^6 \frac{dx}{x}$$

$$A = 6\Big[\ln x\Big]_1^6$$

$$A = 10.75\ cm^2$$

$$dM_x = y\,\frac{y}{2}\,dx = \frac{y^2}{2}\,dx$$

$$= 18\,\frac{dx}{x^2}$$

$$\therefore M_x = 18\int_1^6 \frac{dx}{x^2} = 18\left[\frac{-1}{x}\right]_1^6 = 15\ cm^3$$

$$\therefore \bar{y} = \frac{15}{10.75} = 1.39\ cm$$

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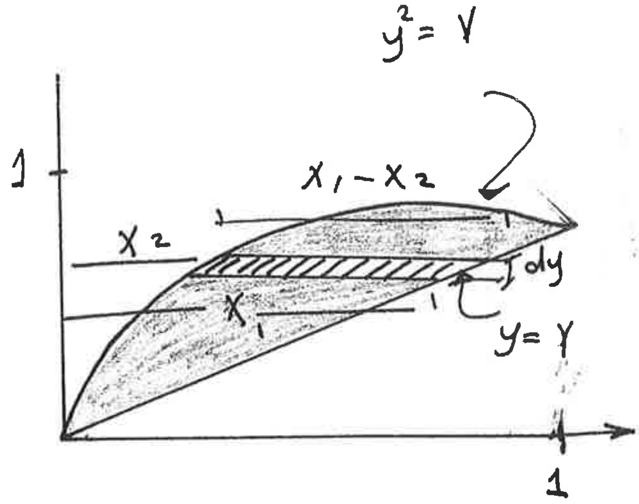
3.16 locate the centroid of the shaded area of the fig.



solution

$$dA = (x_1 - x_2)\,dy$$
$$= (y - y^2)\,dy$$
$$A = \int_0^1 (y - y^2)\,dy$$
$$= \left.\frac{y^2}{2} - \frac{y^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}\ cm^2$$

$$dM_y = x\,dA$$
$$= \left(\frac{x_1 - x_2}{2} + x_2\right) dA$$
$$= \left(\frac{x_1}{2} + \frac{x_2}{2}\right) dA \Longrightarrow \frac{y + y^2}{2}(y - y^2)\,dy$$
$$M_y = \frac{1}{2}\int_0^1 (y^2 - y^4)\,dy$$
$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^5}{5}\right]_0^1 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{5}\right] = \frac{1}{2} * \frac{2}{15} = \frac{1}{15}\ cm^3$$

$$\because\ \bar{x} = \frac{1}{15} * \frac{6}{1} = 0.4\ cm$$

$$dM_x = y\,dA \Longrightarrow y(y - y^2)\,dy$$

$$\therefore Mx = \int_0^1 (y^2 - y^3)\,dy = \left[\frac{y^3}{3} - \frac{y^4}{4}\right]_0^1 = \frac{1}{2}$$

$$\therefore \bar{y} = \frac{1}{12} * \frac{6}{1} = 0.5\,cm$$

$\therefore$ The centroid of the area is $(0.4, 0.5)$

2- Centroids and Centers of Gravity of Composite Areas

The centroid or center of gravity of any lines areas, or bodies can be obtained by means of the principle of Moments. The area can be divided into simple shaps (rectangular, traingles, circles), whose area and centroidal coordinates can be obtained. The total area is the sum of the separate areas and the resultant moment about any axis or plane is the algebraic sum of the moments of the component areas.

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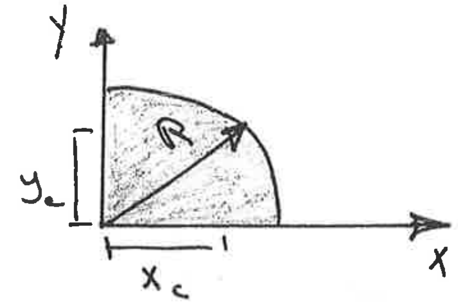
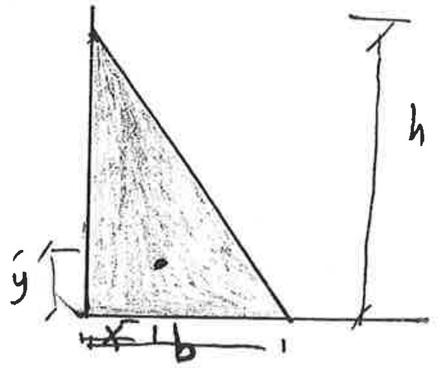
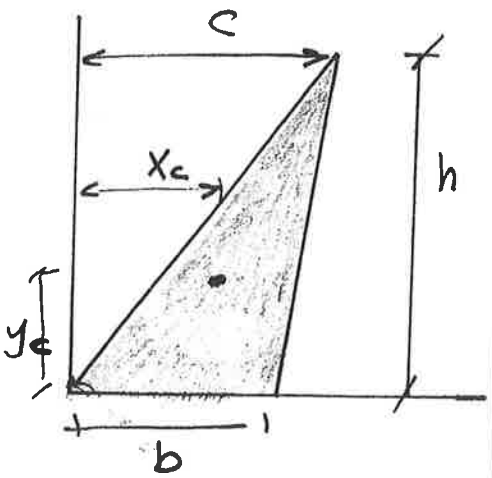
<table>
<tr><th>shape</th><th>area</th><th>$\bar{X}$</th><th>$\bar{Y}$</th></tr>
<tr><td>

Quadrant of a circular arc</td><td>$\frac{\pi R}{2}$</td><td>$\frac{2R}{\pi}$</td><td>$\frac{2R}{\pi}$</td></tr>
<tr><td>

Triangle</td><td>$\frac{bh}{2}$</td><td>$\frac{b}{3}$</td><td>$\frac{h}{3}$</td></tr>
<tr><td>
</td><td>$\frac{bh}{2}$</td><td>$\frac{b+c}{3}$</td><td>$\frac{h}{3}$</td></tr>
</table>

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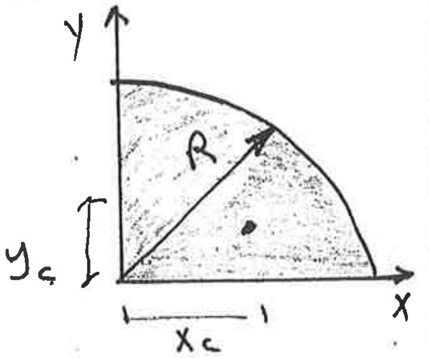
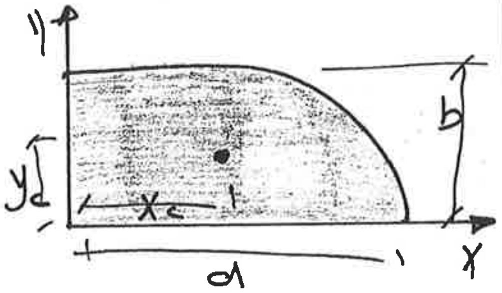
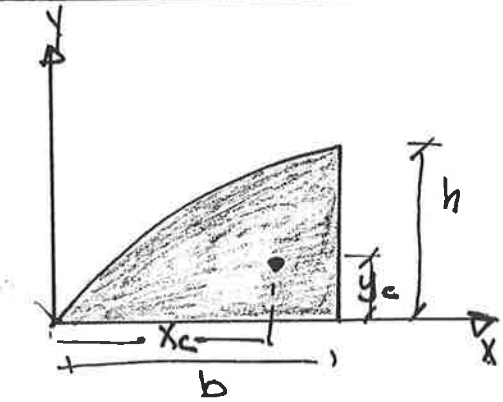
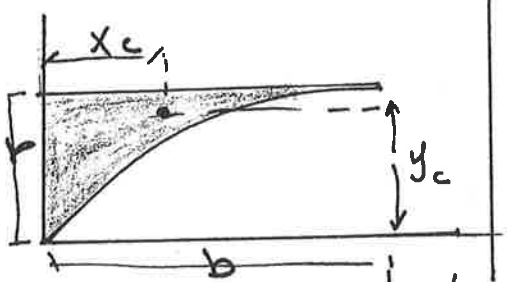
 Quadrant of a Circle.	$\frac{\pi R^2}{4}$	$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$
 Quadrant of ellipse	$\frac{\pi ab}{4}$	$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$
 Quadrant of a parabola	$\frac{2bh}{3}$	$\frac{3b}{5}$	$\frac{3h}{8}$
 Parabola spandrel	$\frac{bh}{3}$	$\frac{3b}{10}$	$\frac{3}{4}h$

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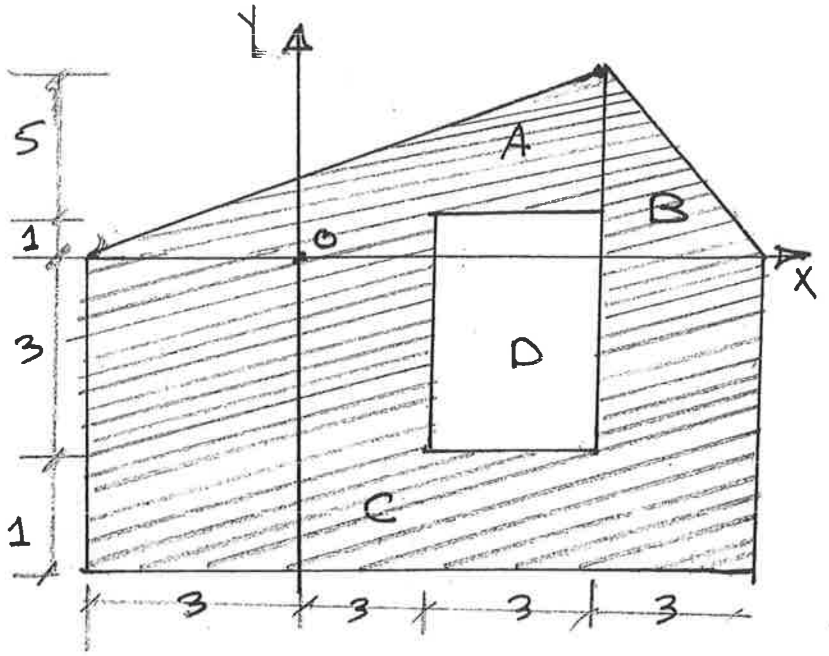
Ex: Determine the Coordinates of the centroid of the shaded area shown.

$$\acute{x} = \frac{M_y}{A} = \frac{234}{72} = 2.25\,cm$$

$$\acute{y} = \frac{M_x}{A} = \frac{-12}{72} = -0.1667\,cm$$

or 0.1667 cm below the x-axis



symbol	Area (cm^2)	X cm Coordinate	M_y cm^2	y cm Coordinate	M_x cm^2
A	27	3	81	2	54
B	9	7	63	2	18
C	48	3	144	−2	−96
D	−12	4.5	−54	−1	12
total	Σ = 72		Σ = 234		

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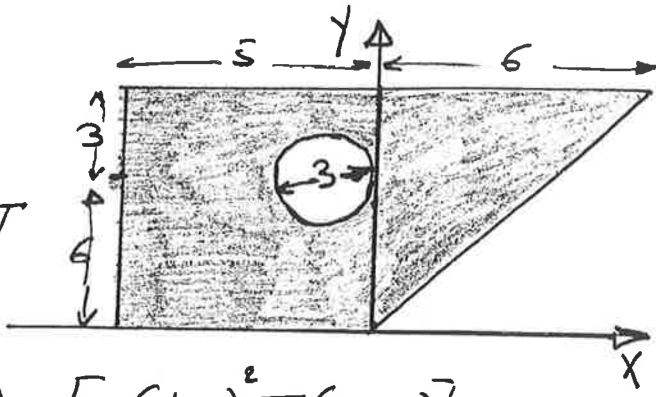
3.27 Locat the centroid of the shaded area shown



Solution

$$A = \frac{6\times 9}{2} + 9\times 5 - (1.5^2)\pi$$

$$= 65\ cm^2$$

$$M_y = \frac{6\times 9}{2}(2) + 5\times 9(-2.5) + [-(1.5)^2\pi(-1.5)]$$

$$= -47.89\ cm^3$$

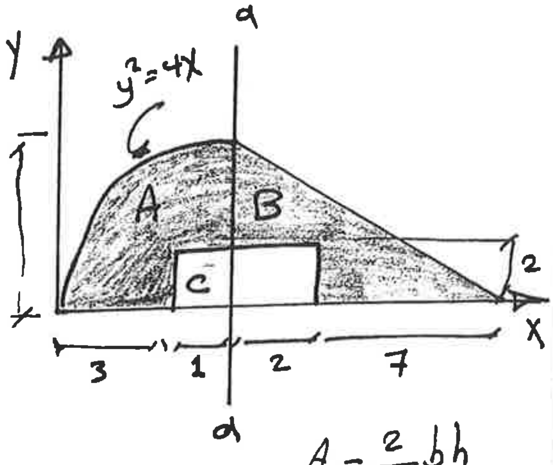
$$\therefore \bar{x} = \frac{M_y}{A} = -0.73$$

$$M_x = \frac{6\times 9}{2}(6) + 5\times 9(4.5) + [-(1.5)^2\pi(6)] = 322.08\ cm^3$$

$$\bar{y} = 4.95\ cm$$

$\therefore$ The coordent $(-0.73, 4.95)$

3.31 Determine the first moment of the shaded area with respect to the (a-axis)



Solution

$$A = \frac{2}{3}*4*4 + \frac{4*9}{2} - 3\times 2$$

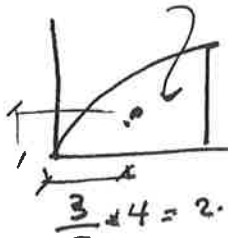
$$= 22.66\ cm^2$$

$$M_{a-a} = \frac{2}{3}*4*4*[-(4-2-4)]$$

$$+ \frac{4*9}{2}(3) + (-3*2)(.5)$$

$$= 34\ cm$$

$$A = \frac{2}{3}bh$$



$$\frac{3}{8}*4 = 1.5$$

$$\frac{3}{5}*4 = 2.4$$

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	Area	y	mx	x	my
A	10.666	1.5	15.999	2.4	25.59
B	18	$\frac{4}{3}$	24	7	126
C	−6	1	−6	4.5	−27
	$\Sigma = 22.66$		$\Sigma = 34$		$\Sigma = 124.59$

$$\therefore \bar{y} = \frac{Mx}{A} = \frac{34}{22.66} = 1.5$$

$$\bar{x} = \frac{My}{A} = \frac{124.59}{22.66} = 5.49$$

3.33 For the cross section of a built-up member shown, locate the centroid of the cross-sectional area. If the section composed of 3 steel plate and 4 steel angles $(5 * 3 * \frac{1}{2})$

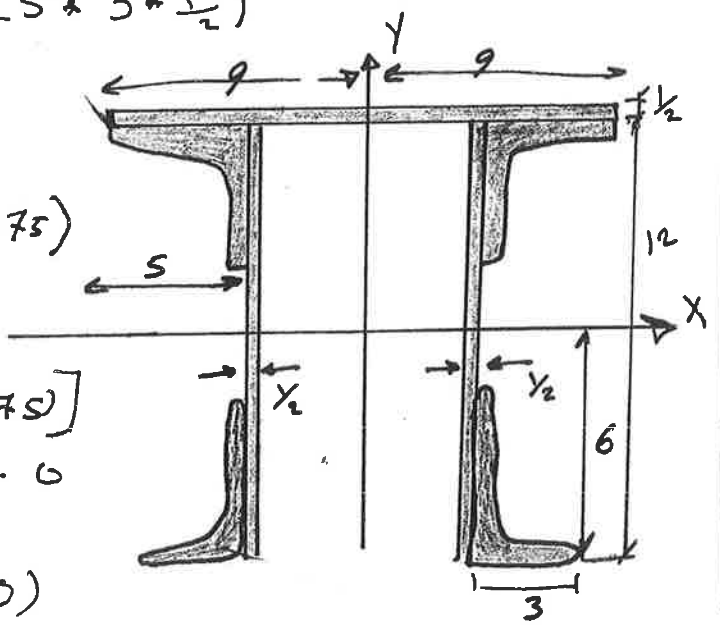
Solu.

From symetry $\bar{x} = 0$

$$A = 18 * \frac{1}{2} + 2\left(12 * \frac{1}{2}\right) + 4(3.75)$$
$$= 36 \text{ cm}^2$$

$$mx = 9(6.25) + 2\left[3.75(6 - 0.75)\right] + 2\left[3.75(-(6 - 1.75))\right] + 0$$
$$= 63.75 \text{ cm}^3$$

$$\therefore \bar{y} = \frac{63.75}{36} = 1.77 \text{ cm} \quad (80)$$

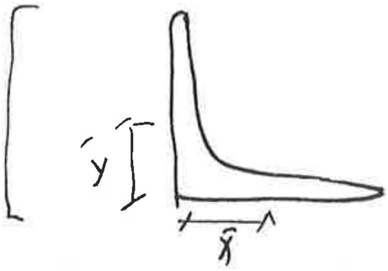


$A = 3.75$

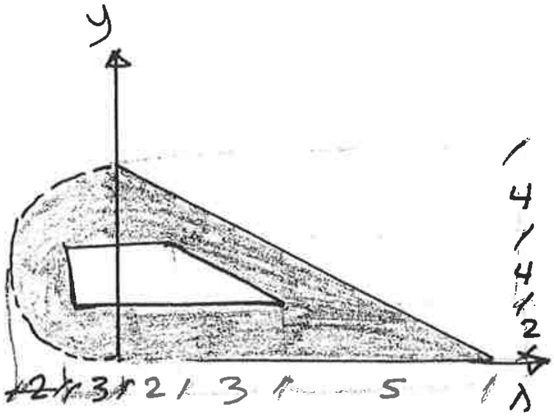
$\bar{x} = 0.75$

$\bar{y} = 1.75$

From table P(100).



3.35 Determine the x coordinate of the centroid of the shaded area.



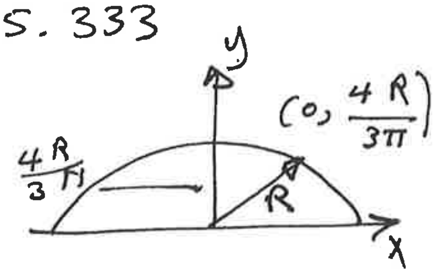
Solution

$$A = \frac{5^2\pi}{2} + \frac{10*10}{2} - (5)(4) - \frac{3\times4}{2}$$

$$= 63.26 \text{ cm}^2$$

$$my = \frac{25\pi}{2} * \left(\frac{-4*-5}{3\pi}\right) + 50\left(\frac{10}{3}\right)$$

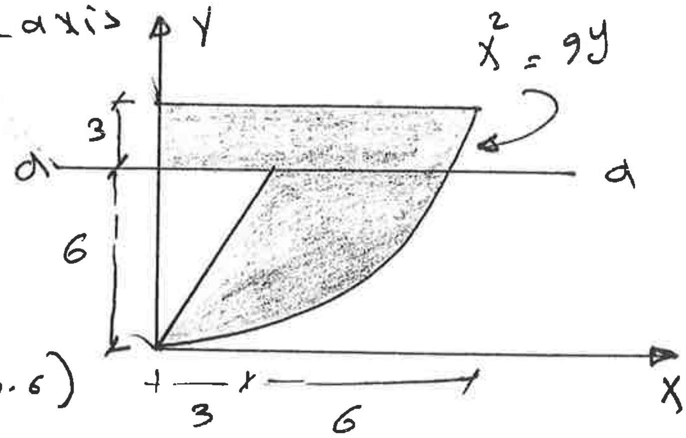
$$+ (-20(-0.5)) + (-6(3)) = 75.333$$



$$\therefore \bar{x} = \frac{75.333}{63.26} = 1.19 \text{ cm}$$

AL Mustansiriyah University
College of Engineering
Water Resources Dept.

3.36 Determin the first moment of the area with respect to a-axis



$$A = \frac{2}{3} 9 * 9 - \frac{3 * 6}{2} = 45$$

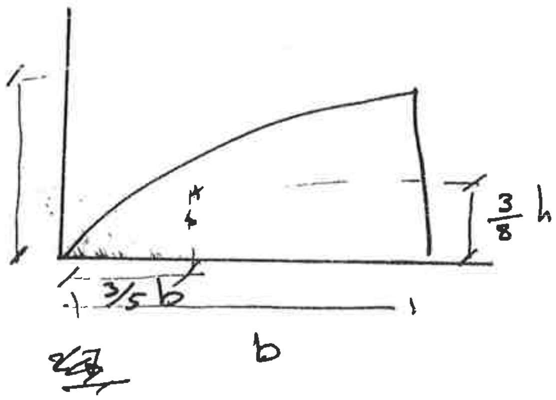
$$M_{a-a} = -9(-2) + \frac{2}{3} 81(-0.6)$$

$$= 18 - 32.4$$

$$= -14.4 \ cm^3$$

$\frac{3}{5} * 9$

$\frac{27}{5} = 5.4$



3.38 Determin the y coordinate of the area

$$A = \frac{4}{4}\pi + 2 - \frac{2}{3} * 1 * 1 + (\frac{1}{2} * 2 * 2)$$

$$= 4.4749 \ cm^2$$

$$mx = \pi\left(-\frac{4 * 2}{3\pi}\right) + 2 * \frac{2}{3} + \left(-\frac{2}{3}\right)\left(\frac{3}{8} * 1\right)$$

$$= -1.5833$$

$$\bar{y} = -\frac{1.5833}{4.4749} = -0.353$$