الكورس الاول
السعـر/

محاضرة / -5-
التاريخ /

Engineering Mechanics

الميكانيك الهندسي

لطلبة الدراسات الاولية

المرحلة الاولى

قسم الـهندسة الموارد المائية

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2018 - 2019

CHAPTER (4)

Equilibrium

- The body is in equilibrium when a system of forces acting on it has no resultant (equal to zero)

- To study the force system acting upon any body or any portion of a body, it is first necessary to recognize both the known and unknown forces acting on the body.

4.1 Free Body Diagrams

A free-body diagram is a sketch of a body or a portion of a body, completely isolated and free from all other bodies.

it has three characteristics :-

1. it is a sketch of the body
2. the body shown is separated (cut free) from all other bodies and from supports.
3. the action on the free body of each body removed is shown as a force or forces on the diagram.

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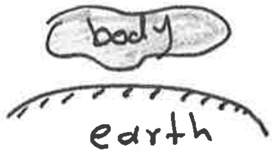
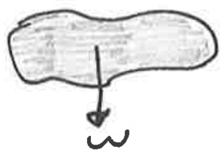
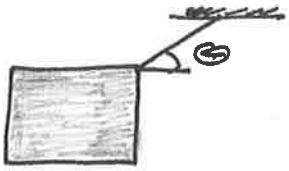
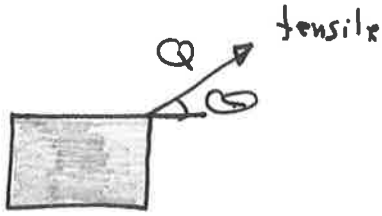
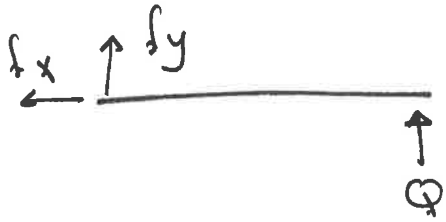
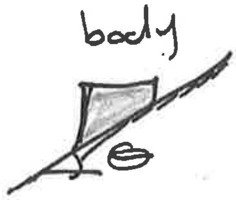
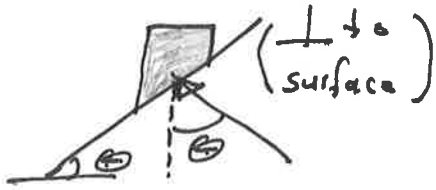
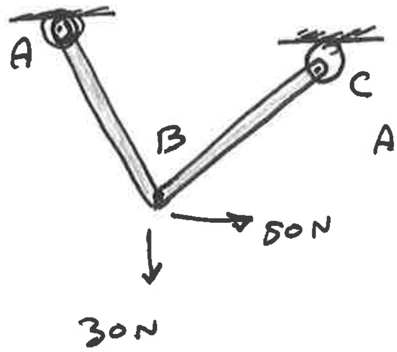
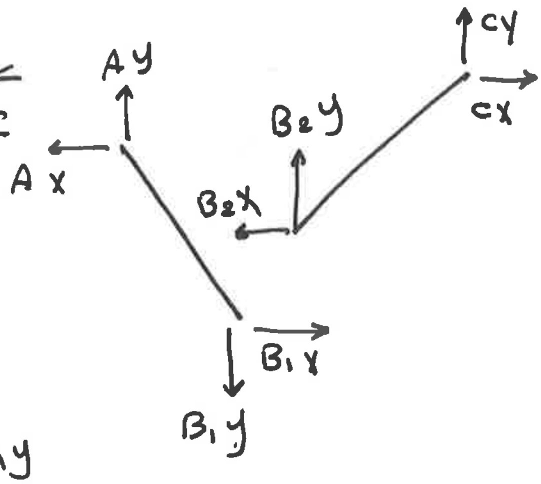
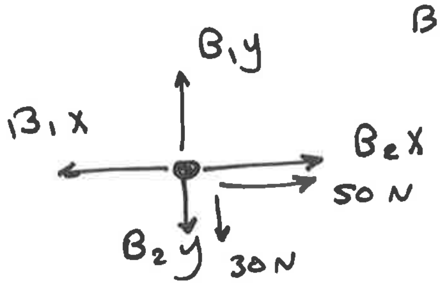
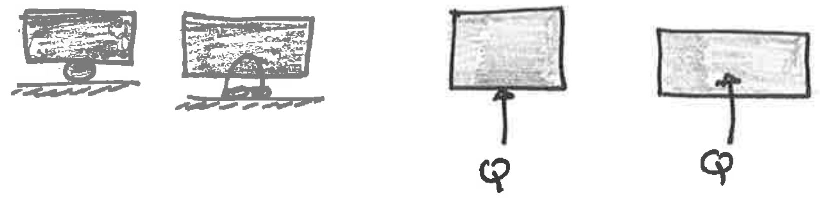
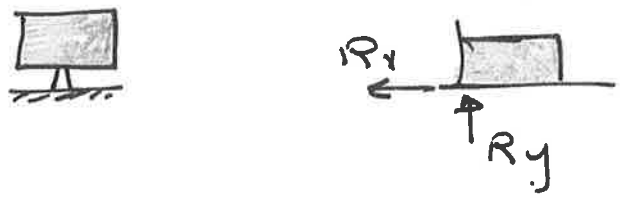
The body to be removed	Sketch of reacting bodies	Action of body removed
Earth.		
rope, cable		
Roller and smoth pin		
smoth surface		
smoth pin with additional forces on pin		



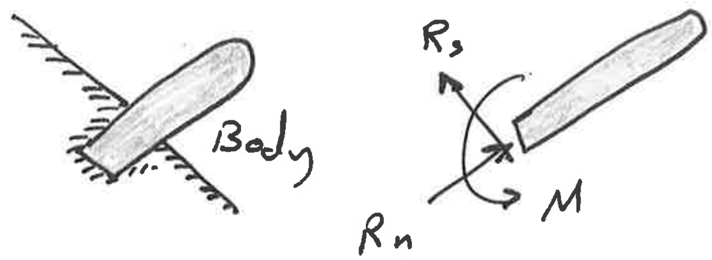
roller or ball



knife edge

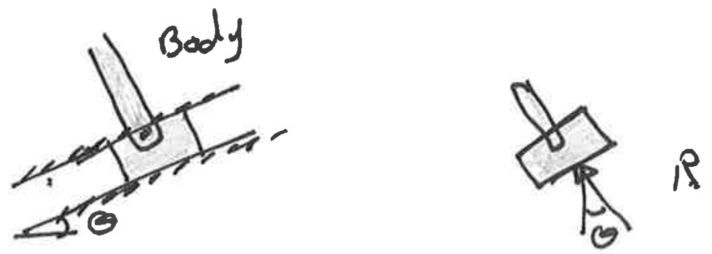


support for a beam or post fixed at the end

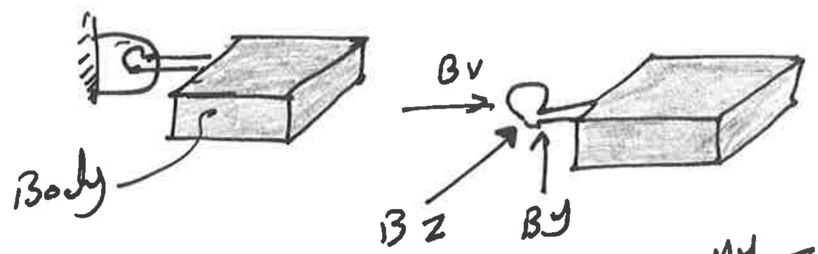


smoth bearing on a shaft

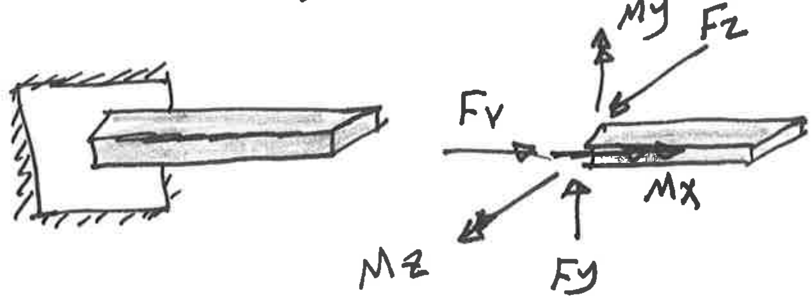
pin or runner in a smoth guide or slot



Ball and socket



fixed end beam (three dimensional)



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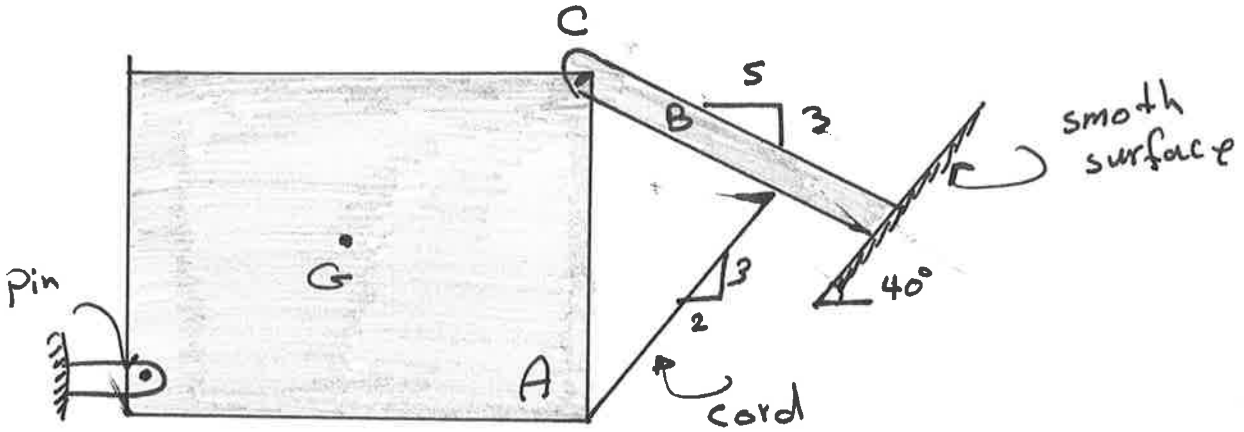
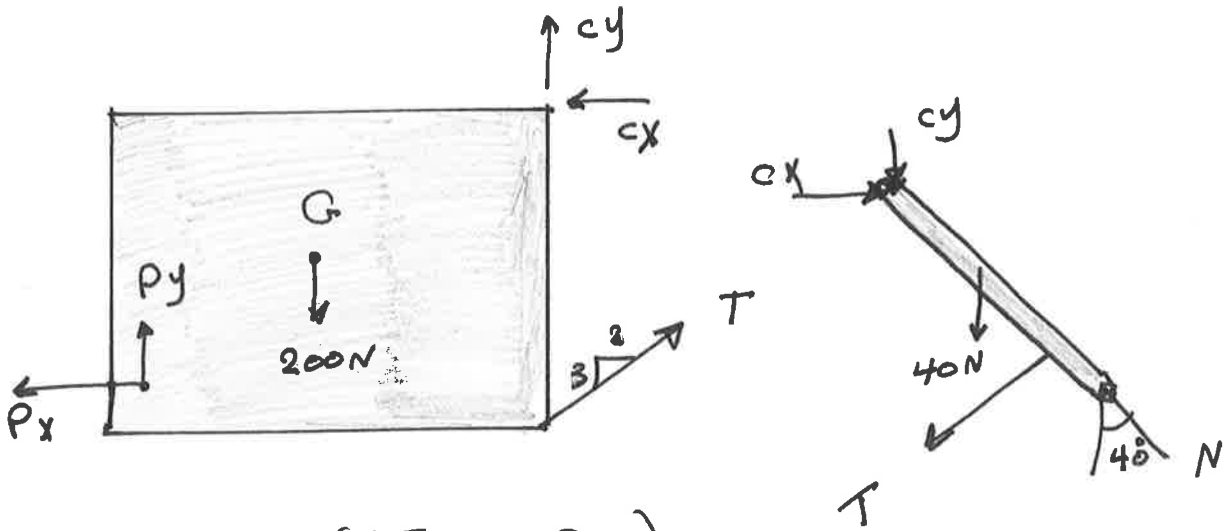
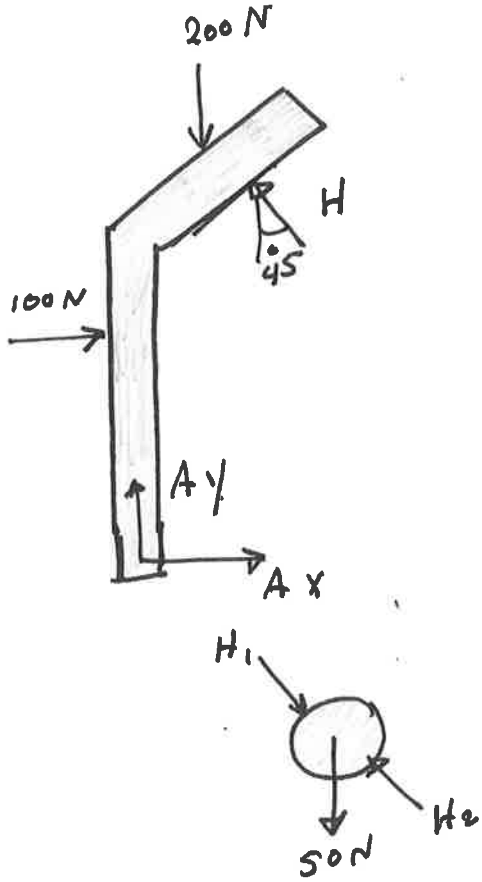
Ex:- Draw a free-body diagram of body A (weights 200N and B weights 40N)

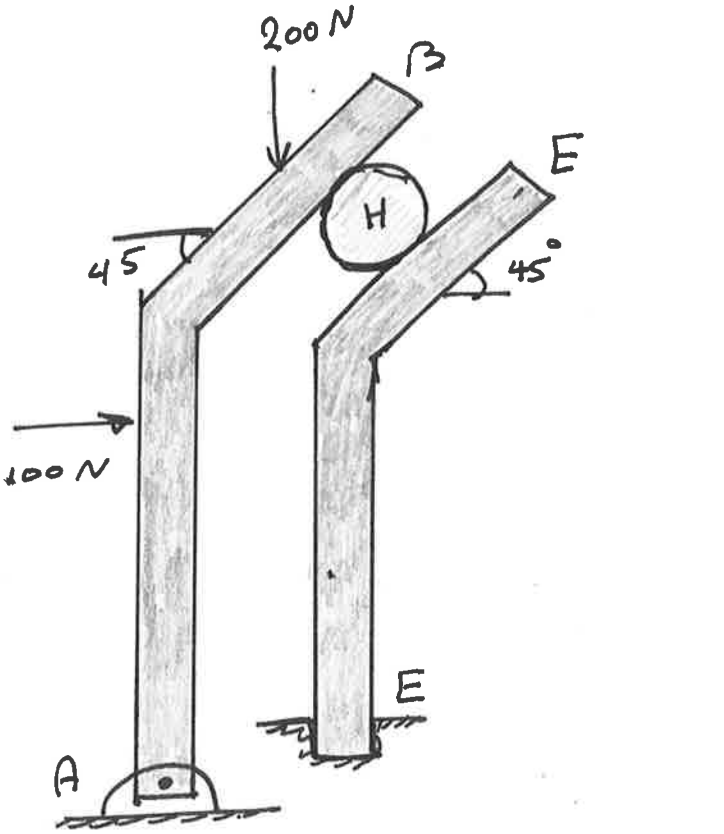




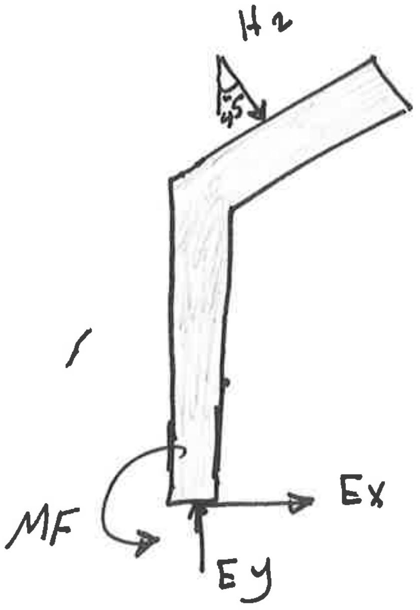
(F.B.D)

4.3 The sphere H weights 50 N, and the weights of both bars may be neglected. Draw a (f.B.D) of all three rigid bodies.





(F.B.D)



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General procedure for the solution of problems in Equilibrium:-

1. Determin the given data and the unknowns
2. Draw F.B.D for the member on which the unknown forces are acting
3. Determine the type of force system acting on the F.B.D and the number of independent equations of Equilibrium
4. compare the no. of unknowns on the F.B.D with the no. of independent equations of equilibrium and
   a) if the no. of equations = the no. of unknowns, then start the solution
   b) if the no. of unknown > the no. of independent equation, then draw F.B.D for another body and repeat step 3 and 4
5. if no. of unknowns in the second F.B.D = the no. of equations then solve the problem. if it is not then repeat step (4-b)

6. If there are still too many unknowns after drawing F.B.D for all bodies, then the problem is statically indeterminate.

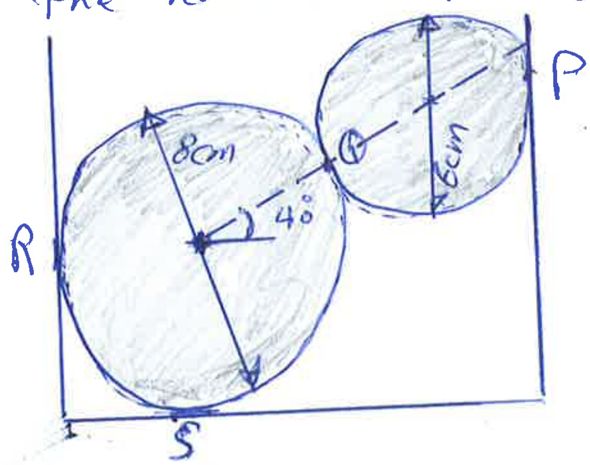
Note: For a collinear force system

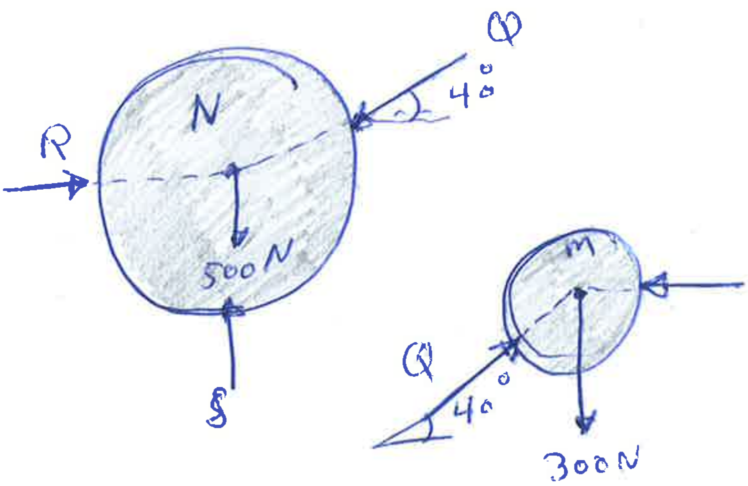
$\Sigma F_x = 0$ [x-axis // to the forces]

or

$\Sigma M_A = 0$ [A is not on the action line of the forces of the collinear system]

Ex:- The 300 N shaft (m) and 500 N shaft (N) are supported as shown. Neglecting friction of the contact surfaces, determine the reactions of R & S on the shaft N





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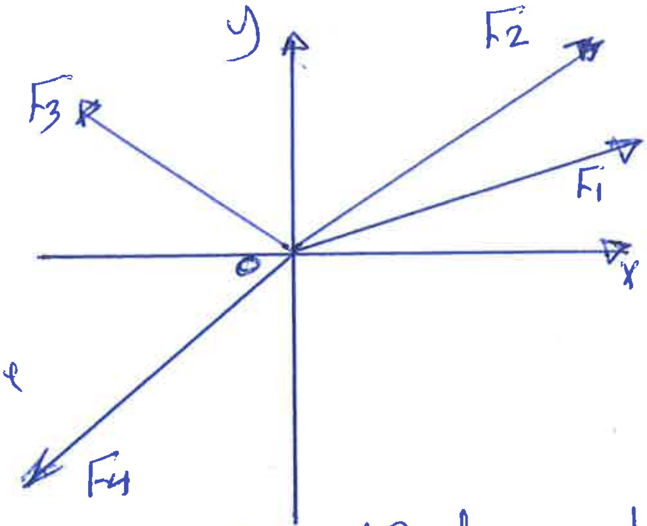
4.2 Equations of Equilibrium for concurrent coplanar force system.

The resultant of a concurrent, coplanar force system is a single force and when this resulted force is zero, the body on which the force system acts is in equilibrium.

The equations necessary to ensure a zero resultant are the equations of equilibrium.

There are three sets of equations of equilibrium for this type of force system.



1. $\sum Fx = 0, \quad \sum Fy = 0$

2. $\sum Fx = 0, \sum MA = 0$
where A is any point in the plane and not on the y-axis

3. $\sum MA = 0 \quad \sum MB = 0$
where line AB dose not pass through the point of concurrence of the forces of the system.

for $\underline{m}$

$\sum F_y = 0 \uparrow$

$Q \sin 40 - 300 = 0$

$\therefore Q = 467\ N \angle 40^\circ$ on m

for $\underline{N}$

$\sum F_y = 0 \uparrow$

$S - 500 - Q \sin 40 = 0$

$S = 500 + 467 \sin 40$

$= 800\ N \uparrow$ on N

from F.B.D of N

$\sum F_x = 0 \xrightarrow{+}$

$R - Q \cos 40 = 0$

$\therefore R = 358\ N \longrightarrow$ on N

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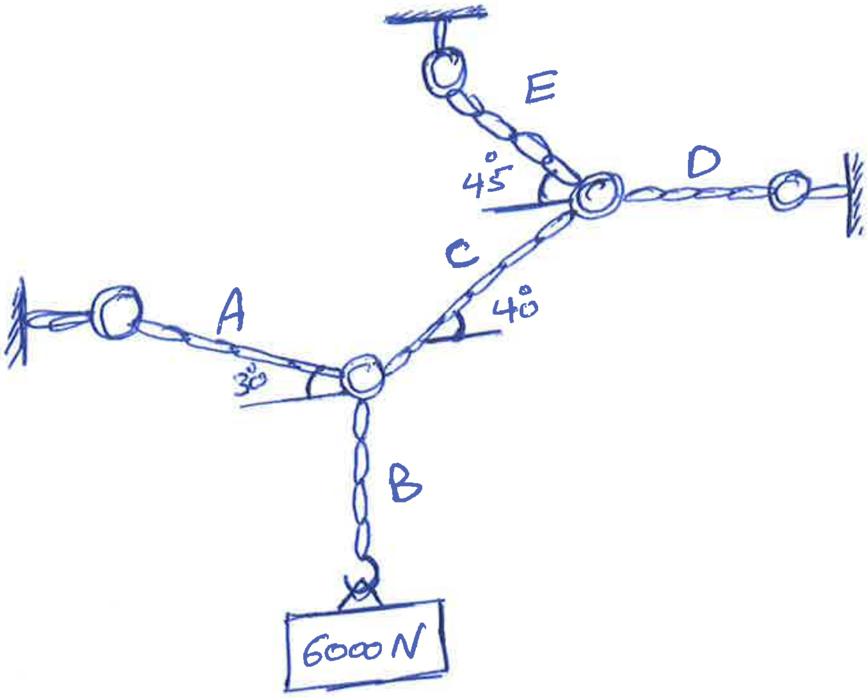
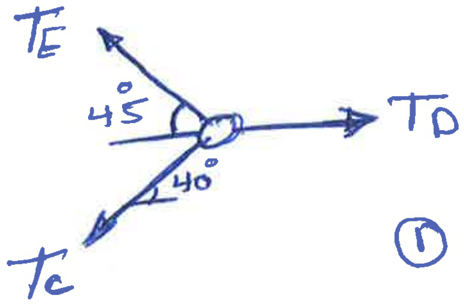
4.12 Determine the tensile force in chain D of the chain system shown



Solution

F.B.D ②



$+\uparrow \Sigma F_y = 0$

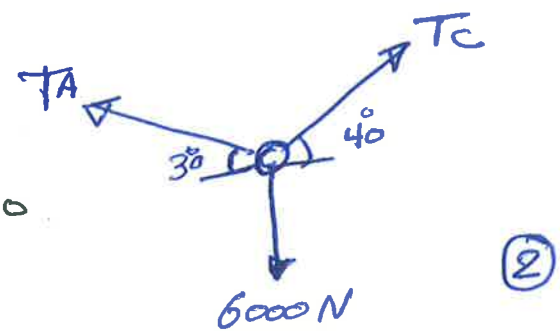
$T_A \sin 30 + T_c \sin 40$

$= 6000$ ---- ①

$\Sigma F_x = 0 \quad \xrightarrow{+}$

$T_c \cos 40 - T_A \cos 30 = 0$

$$\boxed{\therefore T_A = T_c \frac{\cos 40}{\cos 30}}$$



sub in (1)

$$T_C \cdot \frac{\cos 40}{\cos 30} \sin 30 + T_C \sin 40 = 6000$$

$$\boxed{\therefore T_C = 5555.55 \text{ N}}$$

F.B.D (1) :-

$\uparrow^{+} \Sigma F_y = 0$

$$T_E \sin 45 - T_C \sin 40 = 0$$

$$\therefore T_E = 5555.55 \frac{\sin 40}{\sin 45}$$

$$\boxed{T_E = 5050 \text{ N}}$$

$\Sigma F_x = 0 \xrightarrow{+}$

$$T_D = T_E \cos 45 + T_C \cos 40$$

$$= 3570.88 + 4255.798$$

$$\boxed{T_D = 7826.6 \text{ N}}$$

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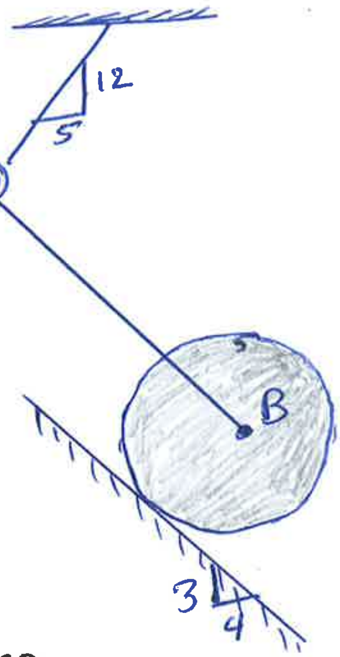
4.15 Determine the force F of which must be applied to ring A in order to keep the $100\,N$ cylinder B in equilibrium.

Solu.:



F.B.D ②

$+\uparrow \Sigma F_y = 0$

$-100 + T_2 \frac{3}{5} + N \frac{4}{5} = 0$ ---- ①

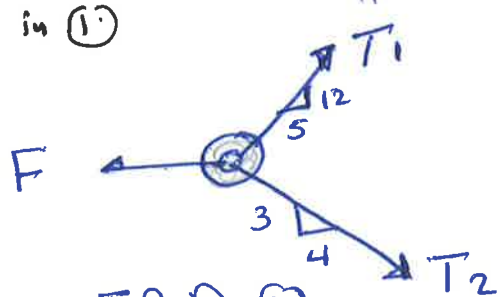
$\Sigma F_x = 0 \quad \xrightarrow{+}$

$N \frac{3}{5} - T_2 \frac{4}{5} = 0$

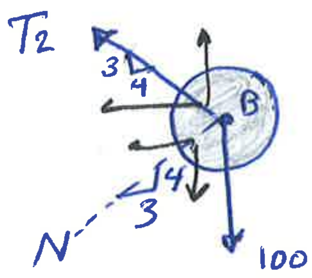
$N = T_2 * \frac{4}{5} * \frac{5}{3} \Rightarrow \boxed{N = T_2 * \frac{4}{3}}$ sub in ①

$100 = T_2 * \frac{3}{5} + T_2 \frac{4}{3} * \frac{4}{5}$

$\boxed{T_2 = 60\,N}$



F.B.D ①



F.B.D ②

F.B.D ①

$\Sigma F_y = 0 \quad \uparrow +$

$T_1 * \frac{12}{13} - T_2 \frac{3}{5} = 0$

$T_1 * \frac{12}{13} - 60 * \frac{3}{5} = 0$

$\boxed{\therefore T_1 = 39\,N}$

$$\sum F_x = 0 \quad \xrightarrow{+}$$

$$-F + T_1 * \frac{5}{13} + T_2 * \frac{4}{5} = 0$$

$$-F_1 + 39 * \frac{5}{13} + 60 * \frac{4}{5} = 0$$

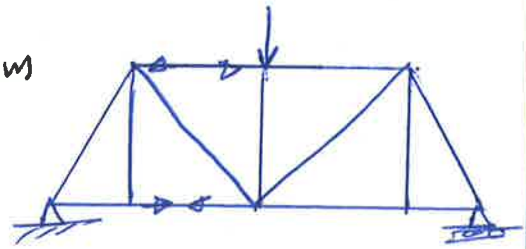
$$\therefore F = 15 + 48$$

$$\boxed{F = 63\,N}$$

4.3 Equilibrium of Bodies Acted on by two Forces or three forces :-

A body acted on by only two forces is called a two-Force body.

If this body is held in equilibrium then the two forces must be Collinear, equal in magnitude, and opposite in sense.

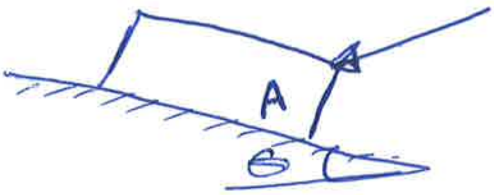


[Ex: each member in truss is a two - force body]

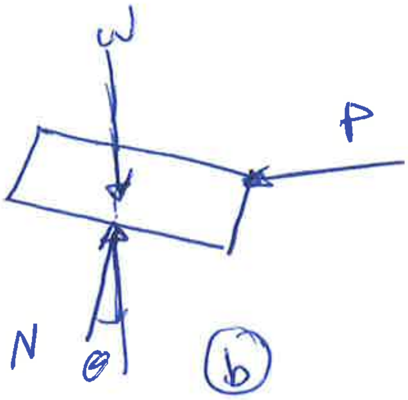
A body acted on by three forces is called a <u>three - force body</u>

when a body is held in equilibrium by three non parallel forces, they must be <u>concurrent</u> and coplanar.

This fact can be used to locate the point of intersection of three forces, and thus provides a simple solution to some problems



a



b

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4.4 Equilibrium of Bodies Acted by Non-Concurrent, Coplanar force systems.

The resultant of this force system is either a single force or a couple.

تحذف

The equations which eliminate all possible resultant are the equations of equil. for this type of force system, there are only three independent equations of equilibrium.

* There are three sets of equations of equilibrium:

① $\sum M_A = 0$ [A is any point in the plane of forces or any axis $\perp$ to the plane]

$\sum F_x = 0$

$\sum F_y = 0$

② $\sum M_A = 0$ [A is any point in the plane]

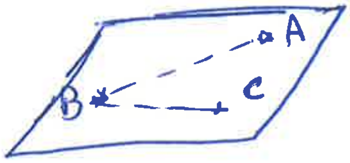
$\sum M_B = 0$ [B is any other point in the plane]

$\sum F_x = 0$ [x- axis is in the plane of forces and not $\perp$ to the line AB

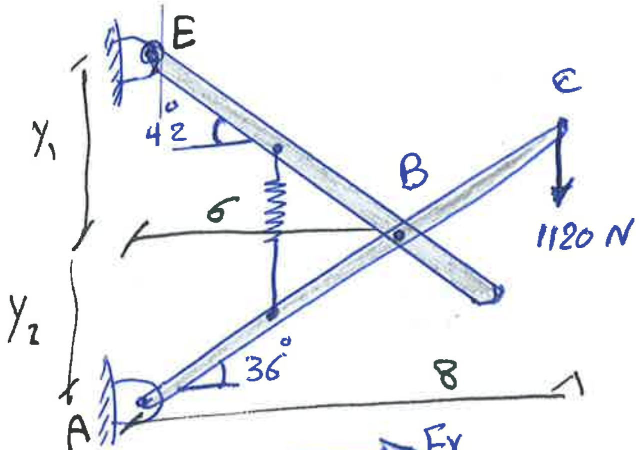
③ $\Sigma M_A = 0$ [point A, B, and c are in the plane and are not collinear]

$\Sigma M_B = 0$

$\Sigma M_C = 0$



Ex:- The tension in the spring is 540 N. The weights of members and friction can be neglected. Determin the horizontal and vertical components of the pin reaction at B on member EB.



Solution

F.B.D :

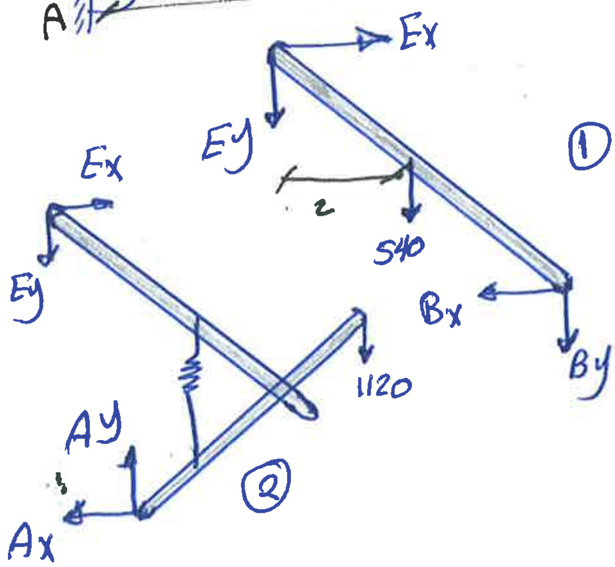
$\Sigma M_A = 0$ ↻+

$E_x(\overset{y_1}{6\tan 42} + \overset{y_2}{6\tan 36}) + 1120(8) = 0$

$9.76\,E_x + 8960 = 0$

$E_x = -918\,N$

$\boxed{\therefore E_x = 918\,N}$ ← on EB



F.B.D ①

$\Sigma F_x = 0 \xrightarrow{+}$

$-918 - B_x = 0$

$B_x = -918\ N$

$\boxed{\therefore B_x = 918\ N \longrightarrow}$ on EB

$\Sigma M_E = 0$

$B_x(6 \tan 42) + B_y(6) + 540(2) = 0$

$\boxed{\therefore B_y = 647}\ \downarrow$ on EB

Ex:- Body G weights 1500 N. Determine the horizontal and vertical components of force at A on AB.

Solution

F.B.D ③

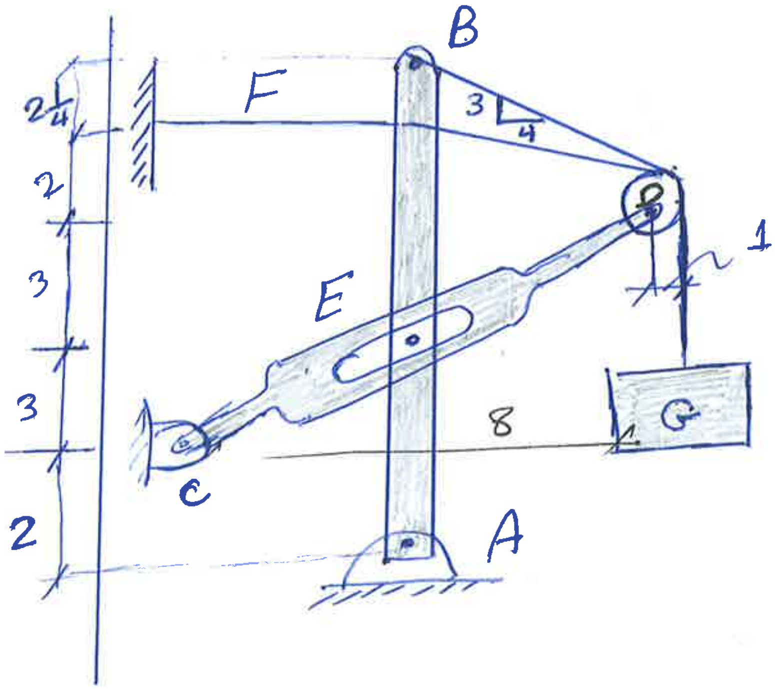
$\Sigma M_D = 0 \Rightarrow \underset{1500 * 1}{T}$

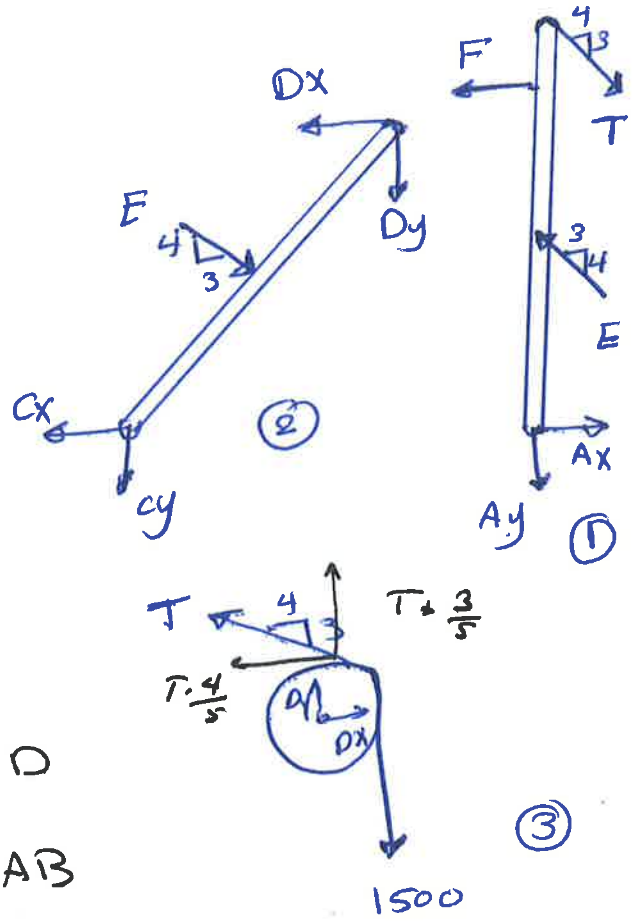
$T = 1500\ N$ as shown

$\Sigma F_x = 0$

$D_x = T \frac{4}{5} = 1200\ N \longrightarrow$ on pully

$D_x = 1200\ N \longleftarrow$ on CD





$\Sigma F_y = 0$

$Dy + \frac{3}{5} \cdot T - 1500 = 0$

$Dy = 600\ N \uparrow$ on pully

$= 600\ N \downarrow$ on CD

F.B.D (2)

$\Sigma M_c = 0 \ \curvearrowright +$

$5E + 8Dy - 6Dx = 0$

$E = 480\ N$ (4, 3) on CD

$E = 480\ N$ (4, 3) on AB

F.B.D $\perp$

$\Sigma F_y = 0 \uparrow +$

معلوم

$-Ay - T * \frac{3}{5} + E * \frac{4}{5} = 0$

$Ay = -516$

$\therefore Ay = 516\ N \uparrow$ on AB

$\Sigma M_F = 0 \ \curvearrowright +$

$T * \frac{4}{5}(2\frac{1}{4}) - Ax * 10 + E * \frac{3}{5} * 5 = 0$

$Ax = 414\ N \longrightarrow$ on AB

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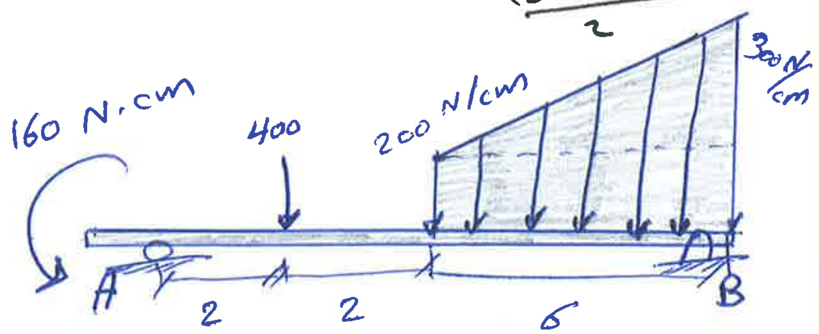
4.27 Determin. the reactions on the beam at A & B.

$\frac{(300-200)\times 6}{2} = 300$

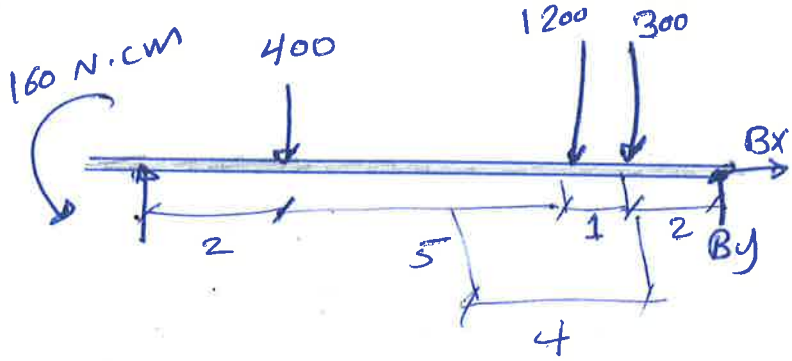


Solution

$\Sigma M_B = 0$ ↻+

$A_y(10) - 160 - 400(8)$
$-1200(3) - 300(2) = 0$

$\boxed{A_y = 756\ N \uparrow}$



$\Sigma F_y = 0 \uparrow +$

$A_y + B_y + 400 - 1200 - 300 = 0$

$\boxed{B_y = 1144\ N \uparrow}$

$\Sigma F_x = 0 \xrightarrow{+}$

$\boxed{\therefore B_x = 0}$

4.30 The weight of pulleys may be neglected

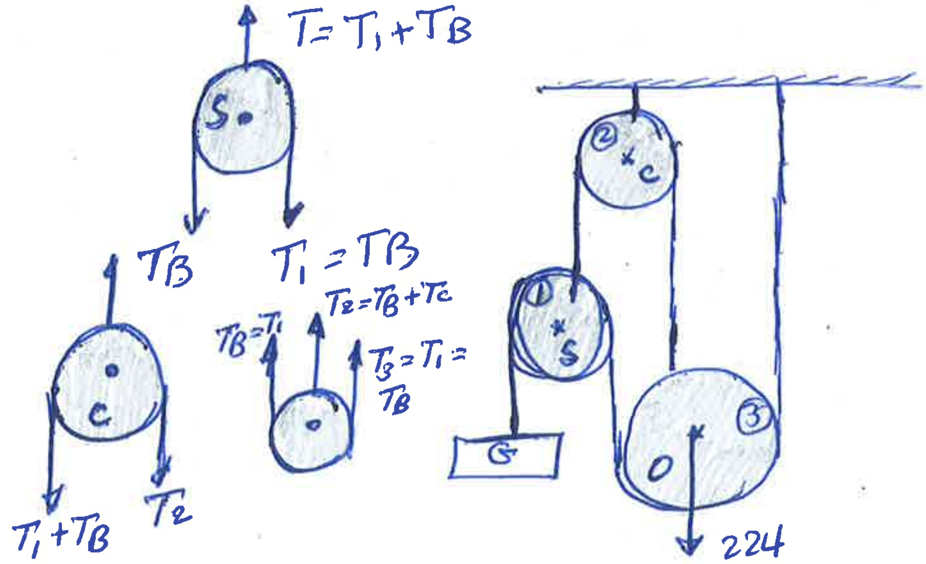
For ①

$\Sigma M_S = 0$

$T_B = T_1$

$\Sigma F_y = 0$

$T = T_B + T_B$



For C

$\Sigma M_c = 0$

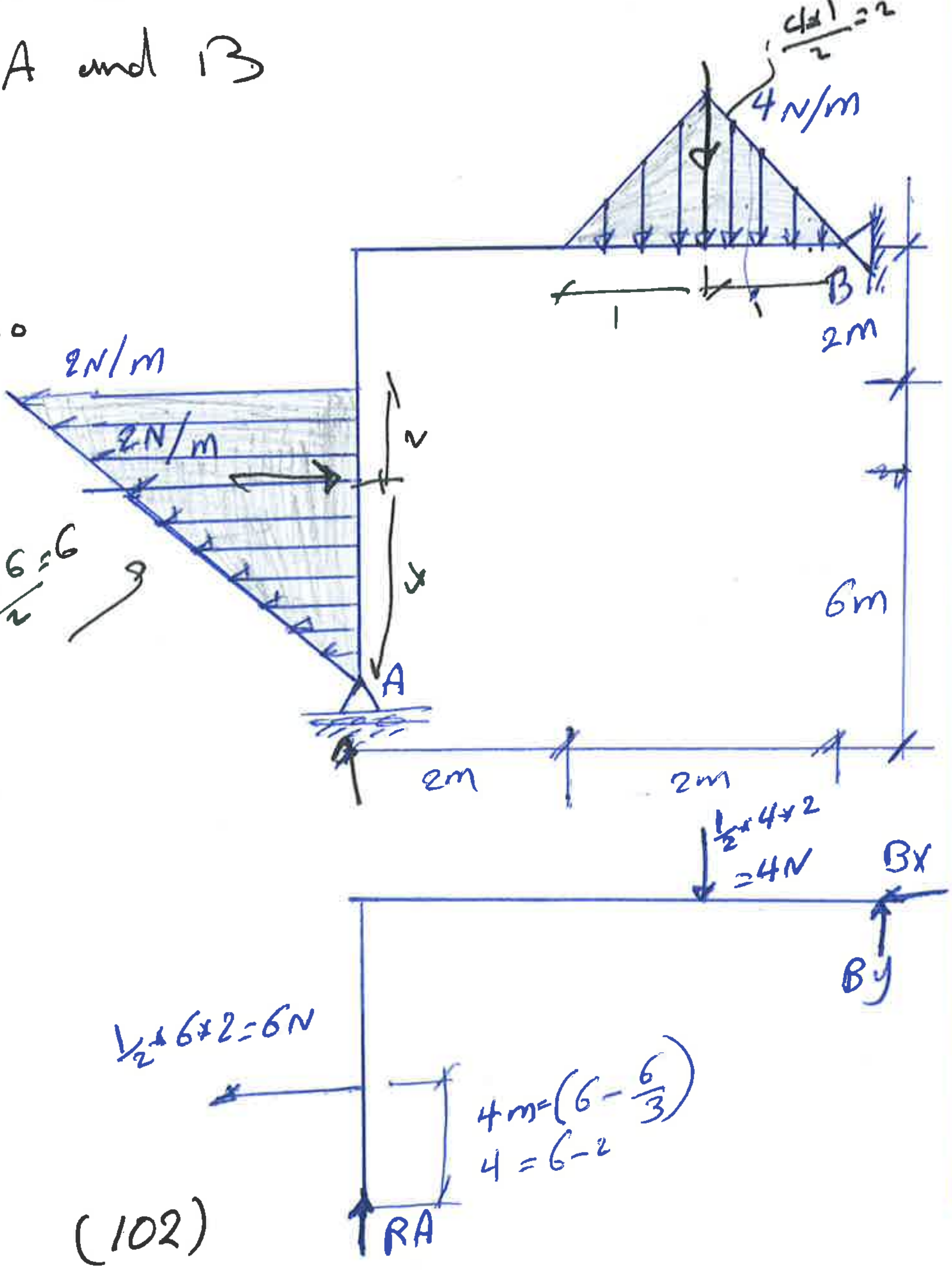
$\therefore T_2 = T_B + T_B$

For O, $\Sigma M_o = 0$

$T_3 = T_1$

$\Sigma F_y = 0 \Longrightarrow -224 + 4T_B = 0$

Ex:- For the rigid structures shown, find reaction force at A and B



$\Sigma M_B = 0$ (+)

$6(4) - 4(1) + RA(4) = 0$

$RA = -5$

$\boxed{RA = 5N \downarrow}$

$\Sigma F_y = 0 \downarrow +$

$5 + 4 - By = 0$

$\boxed{By = 9N \uparrow}$

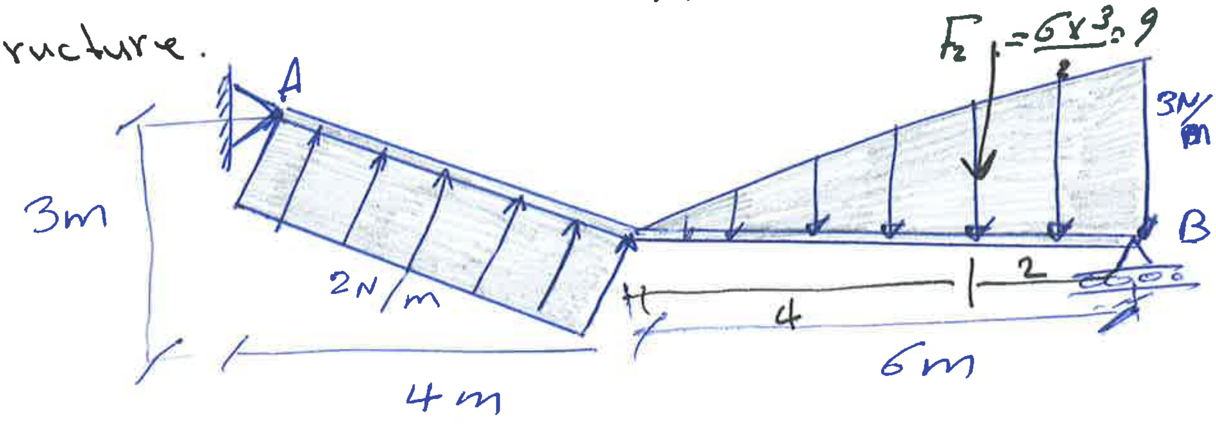
$\Sigma F_x = 0 \xleftarrow{+}$

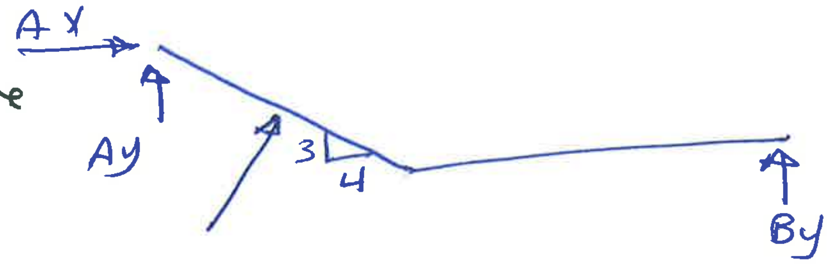
$Bx + 6 = 0$

$Bx = -6N$

$\boxed{Bx = 6N \rightarrow}$

Ex:- Find the forces at support for the rigid structure.



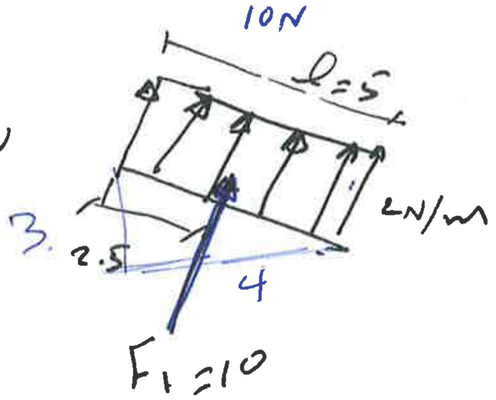


For rectangular pressure

$\sqrt{3^2+4^2} = l$

$l = 5m$

$F_1 = 5(2) = 10 N$



For triangular

$F_2 = \frac{1}{2} * 6 * 3$

$\Sigma MA = 0$

$B\cancel{x}(10) - 9(8) + 10(2.5) = 0$

$\boxed{B\cancel{x} = 4.7 \ N \uparrow}$

$\Sigma Fy = 0 \uparrow +$

$4.7 + Ay - 9 + 10 * \frac{4}{5} = 0 \implies Ay = -3.7$

$\boxed{Ay = 3.7 \downarrow}$

$\Sigma Fx = 0 \longrightarrow$

$Ax = +10 \times \frac{3}{5} = 0$

$Ax = -6$

$\boxed{Ax = 6\ N \longleftarrow}$

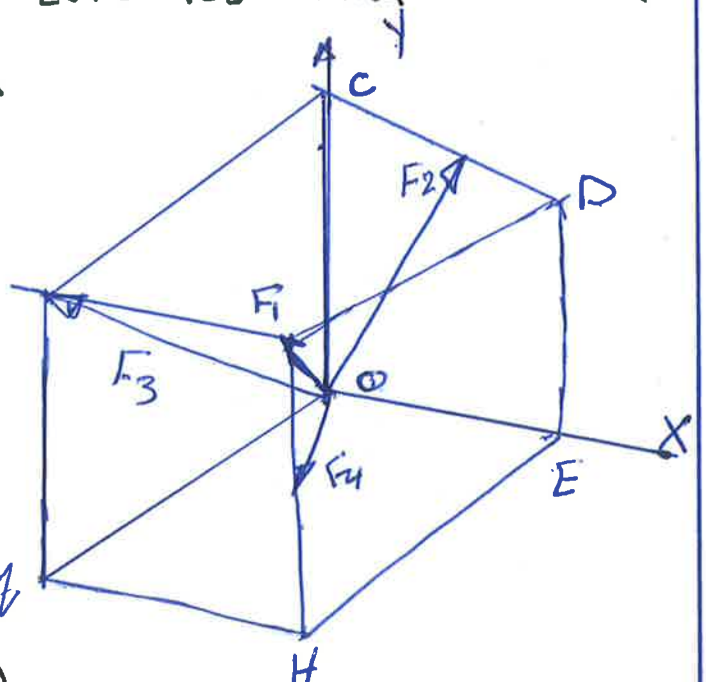
Types of Equilibrium

1- Equilibrium of Bodies Acted on by concurrent, Non coplanar force system.

The resultant of a concurrent force system in space is a single force through the point of concurrenc. The equations necessary for a zero resultant are the equations of equilibrium

A complete set of equations of equilibrium for this force system is

1. $\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma Fz = 0$



one or more of the above force equations can be replaced by the same number of moment equations, for example:

2. $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_{AB} = 0$ [AB & CD is not // to z-axis and it does not intersed the z axis]
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ CD

Not: line CD or DE might be more convenient.

3. $\Sigma F_x = 0$, $\Sigma M_{HE} = 0$, $\Sigma M_{CD} = 0$

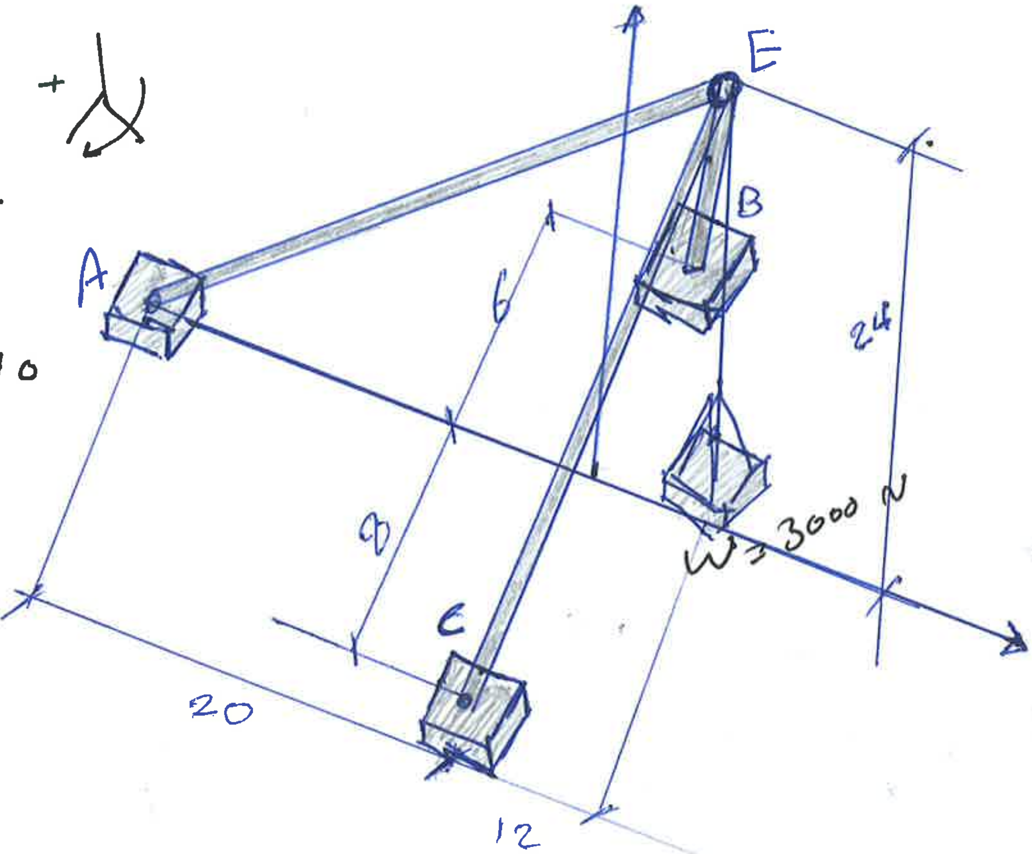
4. $\Sigma M_{HG} = 0$, $\Sigma M_{HE} = 0$, $\Sigma M_{CD} = 0$

4.97 The shear-leg shown in the fig supports a load w of 3000 N. Determin the forces in legs BE and CE and in the . AE . Member AE and the vertical load are in the x-y Plane . all connections are ball-and-sockd joints.

$\Sigma M_{CB} = 0$ +

The length AE

$AE = \sqrt{24^2 + 32^2} = 40$



$\therefore \Sigma M_{CB} = 3000(12) + \left(\frac{TA}{40}\right)(24)(12) - \left(\frac{32}{40}\right)TA(24) = 0$

$TA = 3000$ N Tension

joint E +

$\Sigma F_z = 0$

The length of $Ec = \sqrt{8^2+24^2+12^2} = 28$

" " " $EB = \sqrt{6^2+12^2+24^2} = 27.495$

$$\frac{8Tc}{28} - \frac{6}{27.495} TB = 0$$

$$\boxed{\therefore Tc = 0.7637 \; TB.}$$

$\Sigma Fy = 0 \quad \uparrow +$

$$-3000 - \left(\frac{3000}{40}\right)24 - \left(\frac{Tc}{28}\right)24 - \left(\frac{TB}{27.495}\right)24 = 0$$

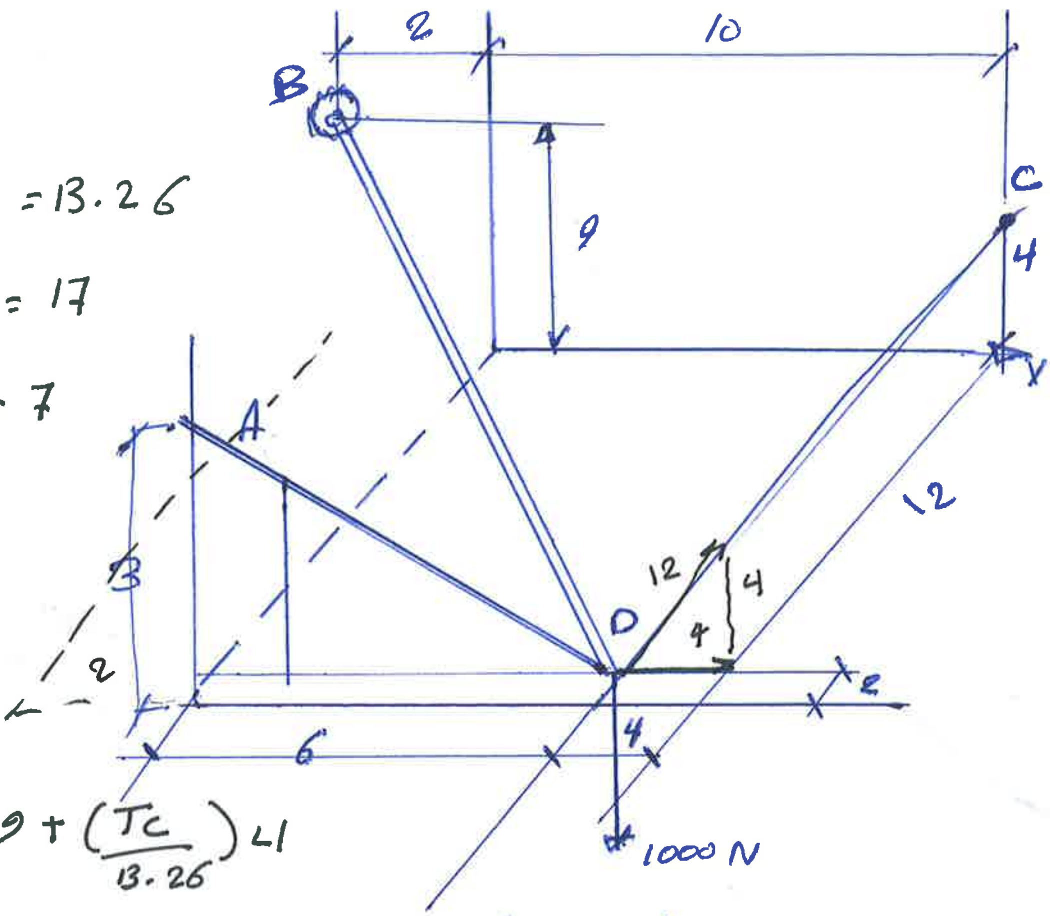
$$-4800 - 0.6545 \; TB - 0.8728 \; TB = 0$$

$$\therefore TB = -3142.43 \Longrightarrow \boxed{TB = 3142.43 \; Tc}$$

$$\therefore Tc = .7637(-3142.43)$$

$$\therefore Tc = 2399.87 \; N$$

4.99 The 1000 N force is held in equilibrium by two wires AD and CD and a compression member BC. Determine the tension in wire AD.



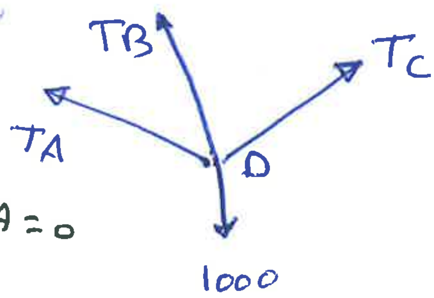
$$D_C = \sqrt{4^2+4^2+12^2} = 13.26$$

$$DB = \sqrt{8^2+9^2+12^2} = 17$$

$$D_A = \sqrt{6^2+3^2+2^2} = 7$$

$\Sigma F_y = 0 \uparrow$

$$-1000 - \left(\frac{TB}{17}\right)9 + \left(\frac{Tc}{13.26}\right)4 + \left(\frac{TA}{7}\right)3 = 0$$



8. $-1000 - 0.53\,TB + 0.3\,Tc + .42\,TA = 0$ —— (1)

$\Sigma F_y = 0 \xrightarrow{+}$

$$\left(\frac{TB}{17}\right)8 + \left(\frac{Tc}{13.26}\right)(4) - \left(\frac{TA}{7}\right)6 = 0$$

$0.47\,TB + 0.3\,Tc - 0.85\,TA = 0$ —— (2)

$\Sigma F_z = 0 \swarrow^{+}$

$$\left(\frac{T_B}{17}\right)12 - \left(\frac{T_C}{13.26}\right)12 + \left(\frac{T_A}{7}\right)2 = 0$$

$$0.7T_B - 0.9T_C + 0.28T_A = 0 \quad \text{——— (3)}$$

from (3)

$$T_B = 1.2T_C - 0.4T_A \quad \text{--- (4)}$$

from (2) and (4)

$$0.47(1.28T_C - 0.4T_A) + 0.3T_C - 0.85T_A = 0$$

$$\therefore T_C = 1.15T_A$$

$$\therefore T_B = 1.28(1.15T_A) - 0.4T_A$$

$$\therefore T_B = 1.072T_A$$

By sub. $T_C + T_B$ values in equation (1)

$$-1000 - 0.53(1.072T_A) + 0.3(1.15T_A) + 0.42T_A = 0$$

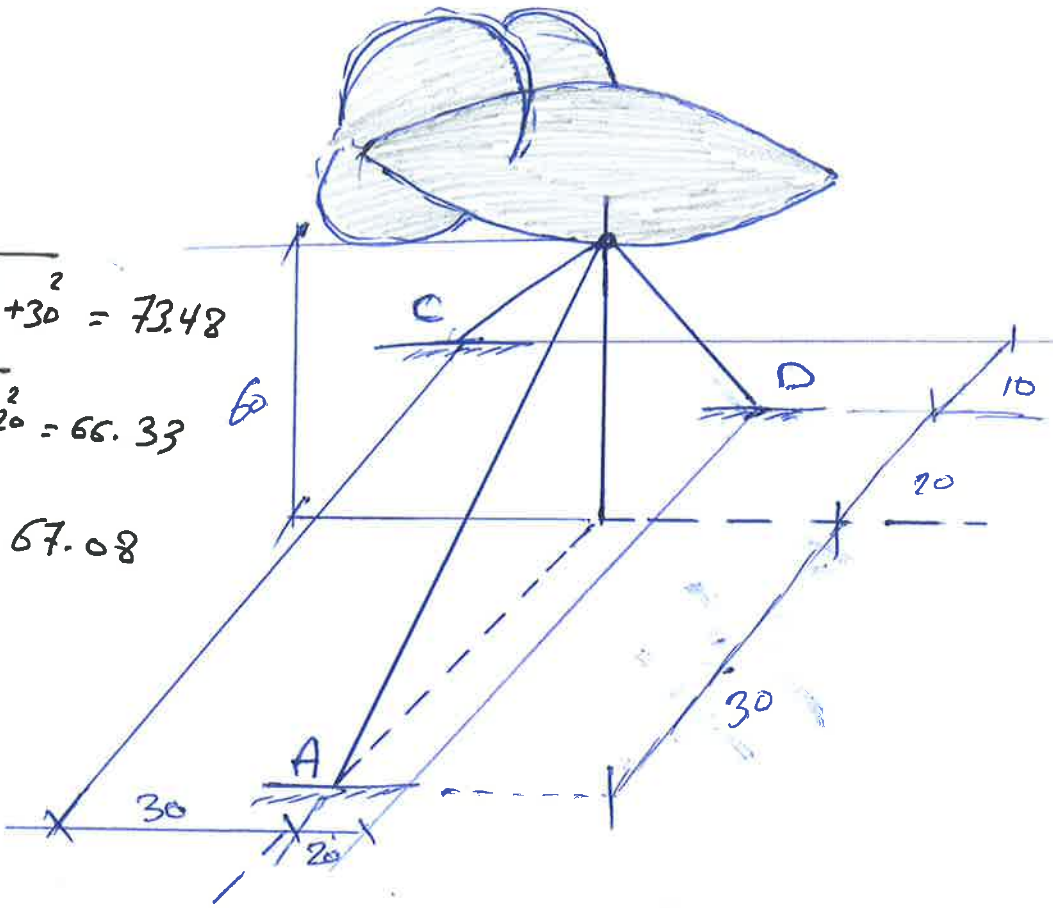
$$\therefore T_A = 5080.26 \text{ NT}$$

4.100 The resultant upward force on the balloon is 8000 N through B. Determine the tension cable AB.

$BC = \sqrt{30^2 + 60^2 + 30^2} = 73.48$

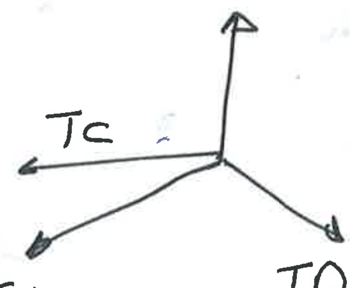
$BD = \sqrt{20^2 + 60^2 + 20^2} = 66.33$

$BA = \sqrt{60^2 + 30^2} = 67.08$



$\Sigma F_y = 0 \uparrow$

$$8000 - \left(\frac{T_D}{66.33}\right)60 - \left(\frac{T_A}{67.08}\right)60 - \left(\frac{T_C}{73.48}\right)60 = 0$$



$$8000 - 0.9T_D - 0.89\,T_A - .82\,T_C = 0 \quad (1)$$

$\Sigma F_z = 0 \swarrow +$

$$\left(\frac{T_A}{67.08}\right)30 - \left(\frac{T_C}{73.48}\right)30 - \left(\frac{T_D}{66.33}\right)20 = 0$$

$$0.45T_A - 0.4T_C - 0.3T_D = 0 \quad \text{——— (2)}$$

$$\Sigma F_x = 0 \quad \xrightarrow{+}$$

$$\left(\frac{T_D}{66.33}\right)20 - \left(\frac{T_C}{73.48}\right)30 = 0$$

$$T_D = 1.35\,T_C \quad \text{--- (3)}$$

from (2) & (3)

$$\boxed{\therefore T_C = 0.559\,T_A}$$

$$\therefore T_D = 1.35(.559\,T_A)$$

$$\boxed{T_D = 0.75\,T_A}$$

sub. T_C & T_D values into equ. (1)

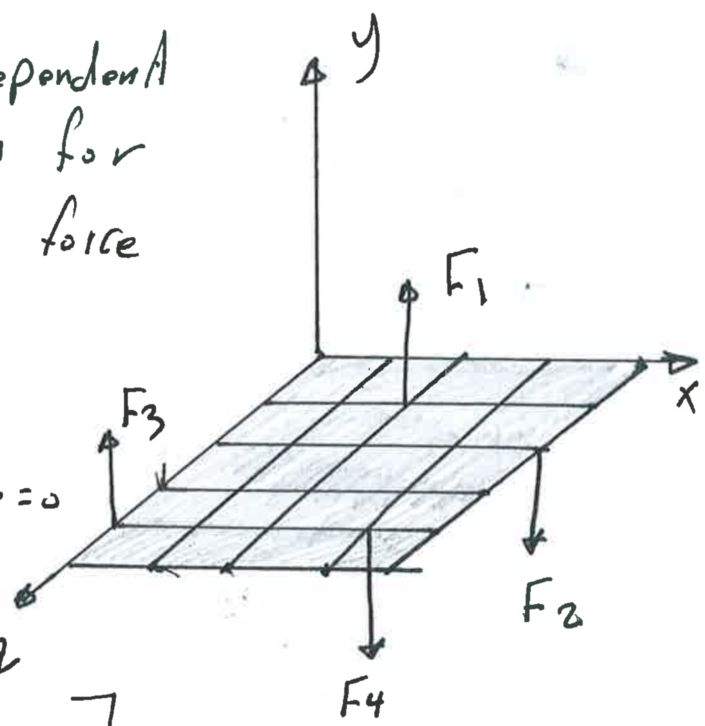
$$8000 - 0.9(.75\,T_A) - .89\,T_A - .82(.559\,T_A) = 0$$

$$\boxed{\therefore T_A = 3960\,N}$$

2. Equilibrium of Bodies Acted by parallel Non-Coplanar force systems.

The resultant of a parallel force system in space is either a single force or a couple.

A complete set of independent equations of equilibrium for a parallel, non-coplanar force system is :-



1. $\Sigma F_y = 0, \Sigma M_x = 0, \Sigma M_z = 0$

[y axis is parallel to the forces of the system]

Another set of independent equations of equil is :-

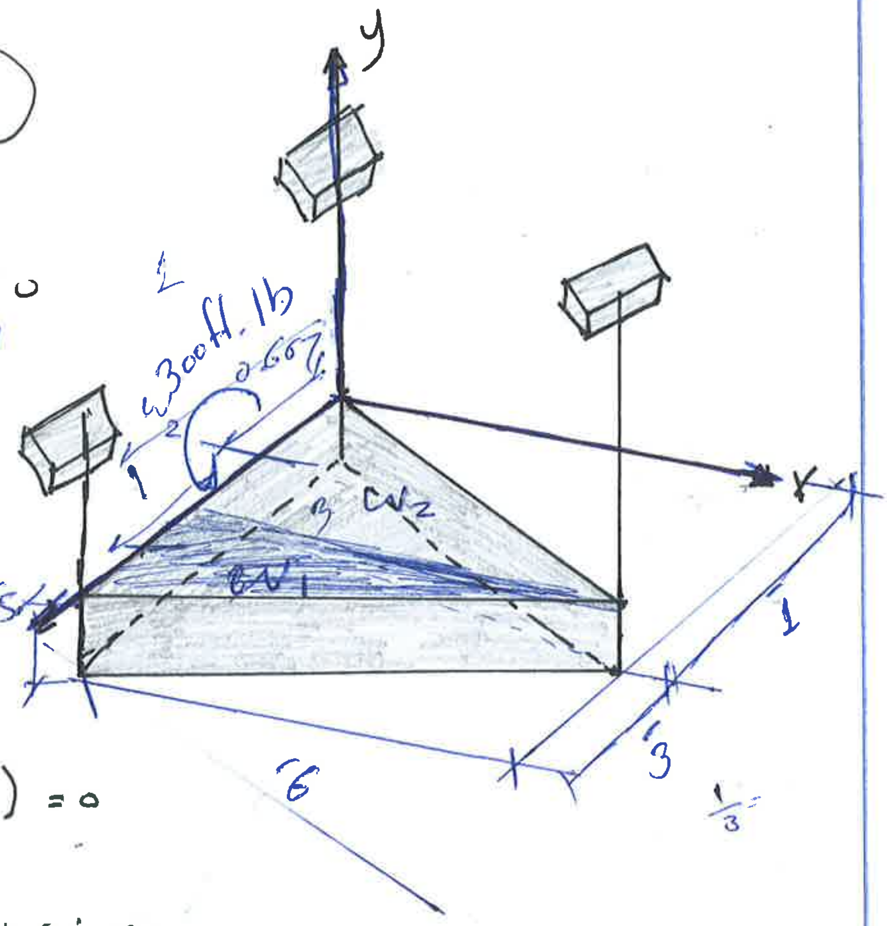
$\Sigma M_A = 0, \quad \Sigma M_x = 0, \quad \Sigma M_z = 0$

4-10 Triangular steel plate shown in fig. it is acted on by 300 ft. lb couple in yz plane. Determin the tension in each Cable. specific weight of steel is 490 lb/ft^3.

$\Sigma M_z = 0$ ⟲+

$6T_C - 2W_1 - 2W_2 = 0$

$\boxed{T_C = 980 \text{ lb Tension}}$



$\Sigma M_x = 0$ ⟳+

$= 300 - 4T_B - 1T_C$

$+ 0.667(\overset{735}{W_2}) + 2(\overset{2205}{W_1}) = 0$

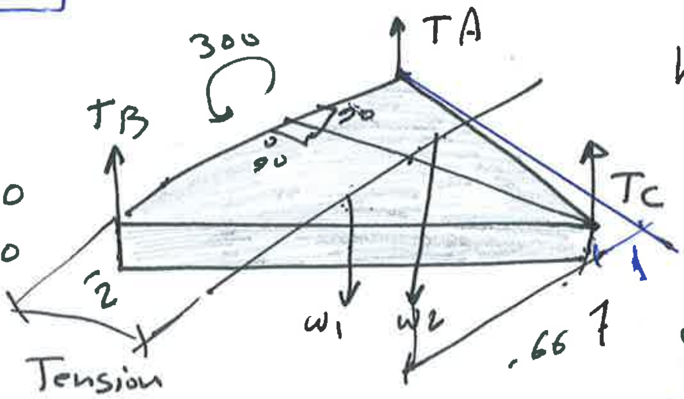
$\boxed{T_B = 1730}$ lb Tension

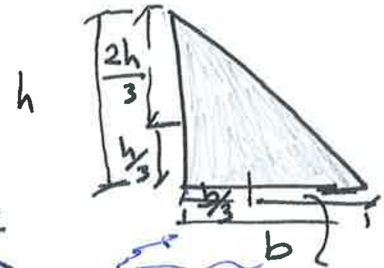
$\Sigma F_y = 0$ ↑

$T_A + 1730 + 980$

$- 2205 - 735 = 0$

$\boxed{\therefore T_A = 230 \text{ lb}}$ Tension





$\frac{2}{3}b$

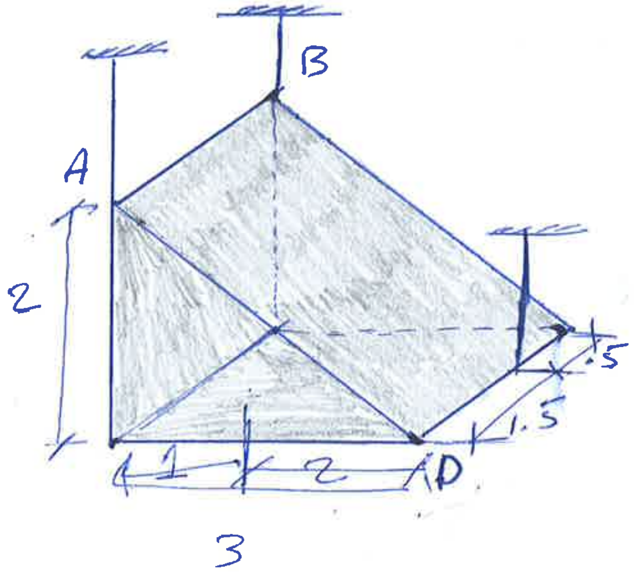
$W_1 = \frac{3 \times 6}{2}(.5) * 490$

$= 2205$

$\boxed{W_2 = \frac{1 \times 6}{2}(.5) * 490 = 735}$

(113)

4.103 Determin the tensions in the three supporting wires at A, B and C, prism weight 250 N.



$\sum M_z = 0$

$50 + 250(1) - T_c(3) = 0$

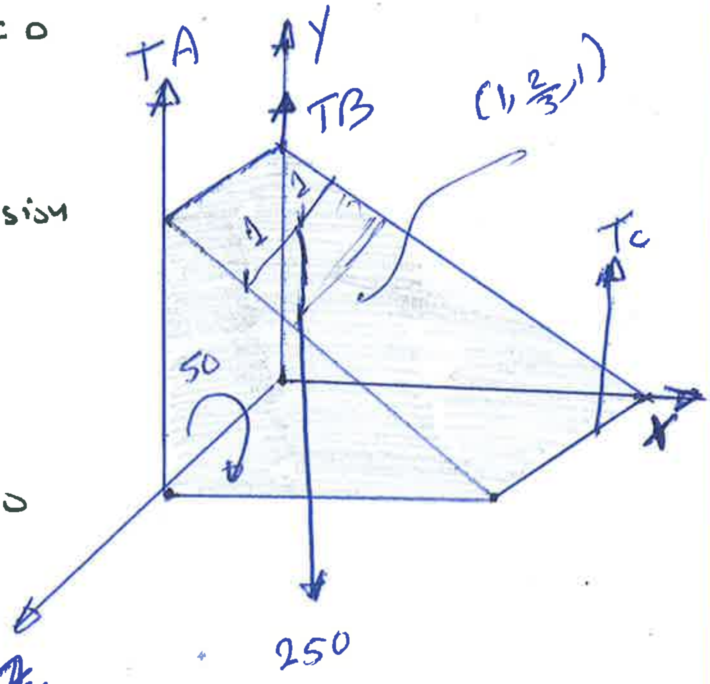
$\therefore T_c = \frac{300}{3} = 100$ N Tension

$\sum M_x = 0$

$T_c(.5) + T_A(2) - 250(1) = 0$

$50 + 2T_A - 250 = 0$

$\therefore T_A = \frac{200}{2} = 100$ N Tension



$\sum F_y = 0$

$T_A + T_B + T_c - 250 = 0$

$\therefore T_B = 250 - 100 - 100$

$\therefore T_B = 50$ N Tension

3. Equilibrium of Bodies Acted on by Non-Concurrent Non parallel, Non coplanar force systems :-

The resultant of this force system is a single force a couple, or a force and a couple.

A set of equations of equilibrium for this general force system is :

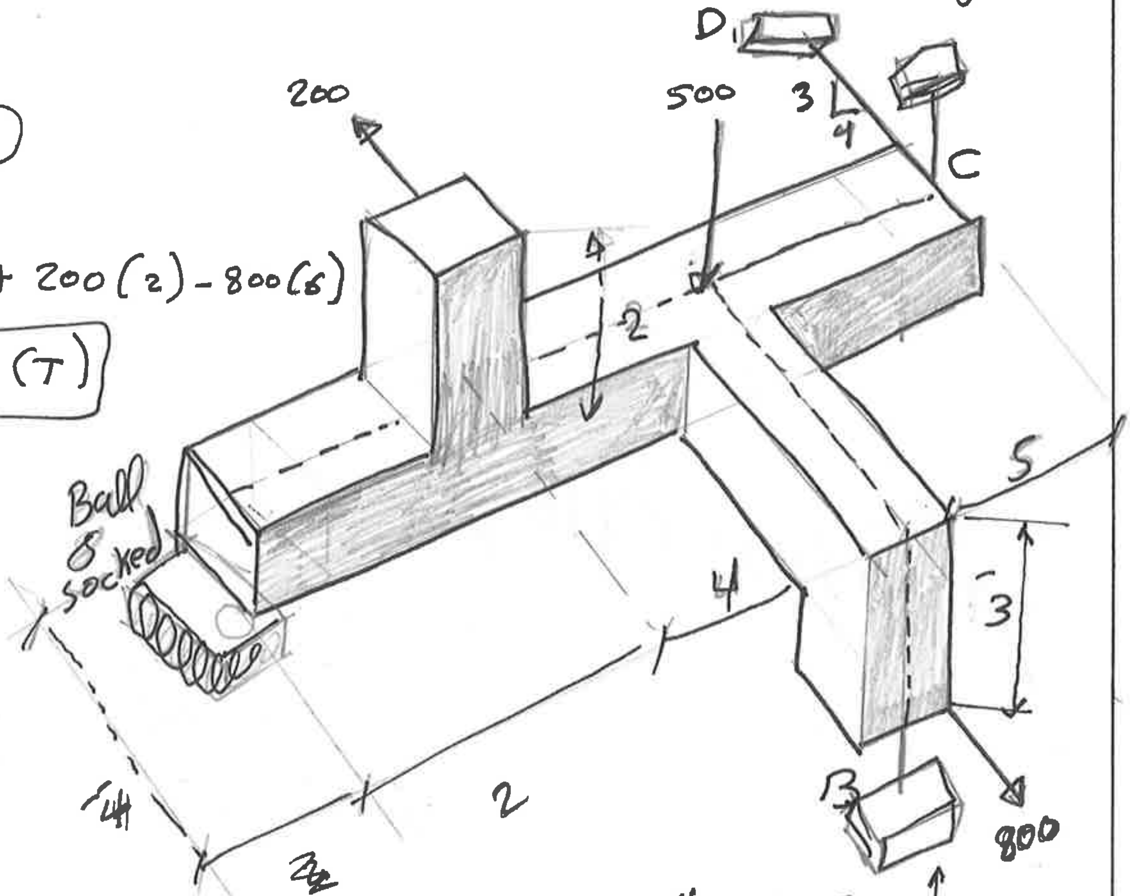
$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \qquad 1-a$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \qquad 1-b$$

one or all of the force equation (1-a) can be replaced by additional moment equations, provided the moment axes so selected that six independent equations are obtained

Note: only six unknowns (magnitudes, distances, or slopes) can be determined from one free-body diag.. acted on by a general force system.

Ex:- Determin the tension in each of the cables and the components of the reaction on the body at A.



$\Sigma M_y = 0$

$= \frac{4}{5} T_D (11) + 200(2) - 800(6)$

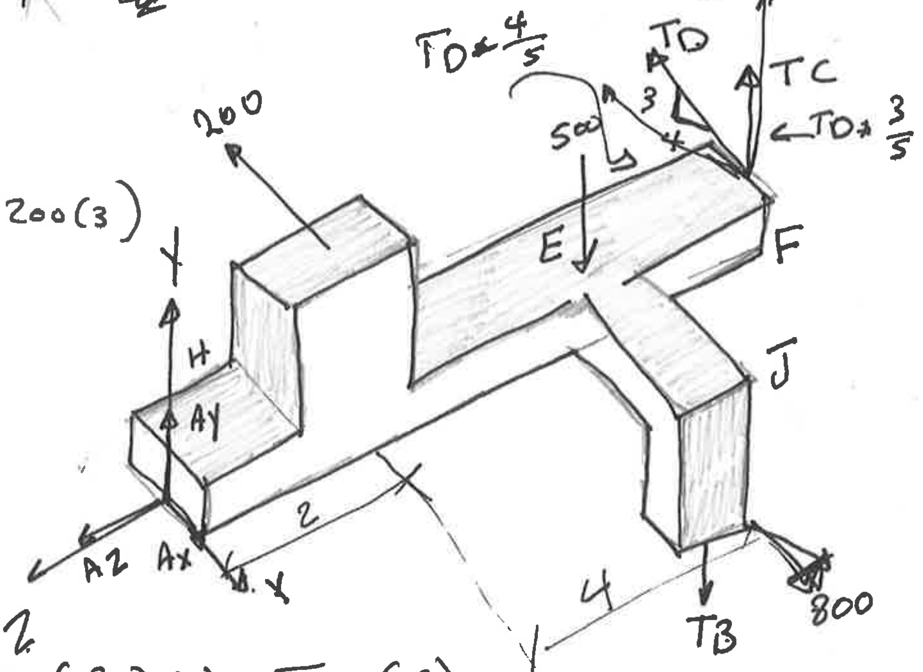
$\boxed{T_D = 500\ N\ (T)}$

$\Sigma M_z = 0$

$T_B(4) - 800(2) - 200(3)$

$-\left(\frac{4}{4} T_D\right)(1) = 0$

$T_B = 650\ T$



$\Sigma M_x = 0$

$T_C(11) - 500(6) - T_D\left(\frac{3}{5}\right)(1) - T_B(6) = 0$

$T_C = 655\ N\ (T)$

$\sum F_x = 0$ $\searrow +$

$A_x + 800 - 200 - \frac{4}{5}(500) = 0$

$A_x = -200\ N \Longrightarrow A_x = 200\ N \nwarrow$

$\swarrow + \sum F_z = 0$

$A_z - \left(\frac{3}{5}\right) 500 = 0$

$A_z = 300 \swarrow$

$\sum M_{Ej} = A_z(1) + A_y(6) - T_c(5) = 0$

$A_y = -495.8 = 495.8 \uparrow$

or $+\uparrow \sum F_y = 0$

$+ A_y + T_c - T_B - 500 = 0$

$A_y = 655 - 650 - 500$

$A_y = 495\ N \uparrow$

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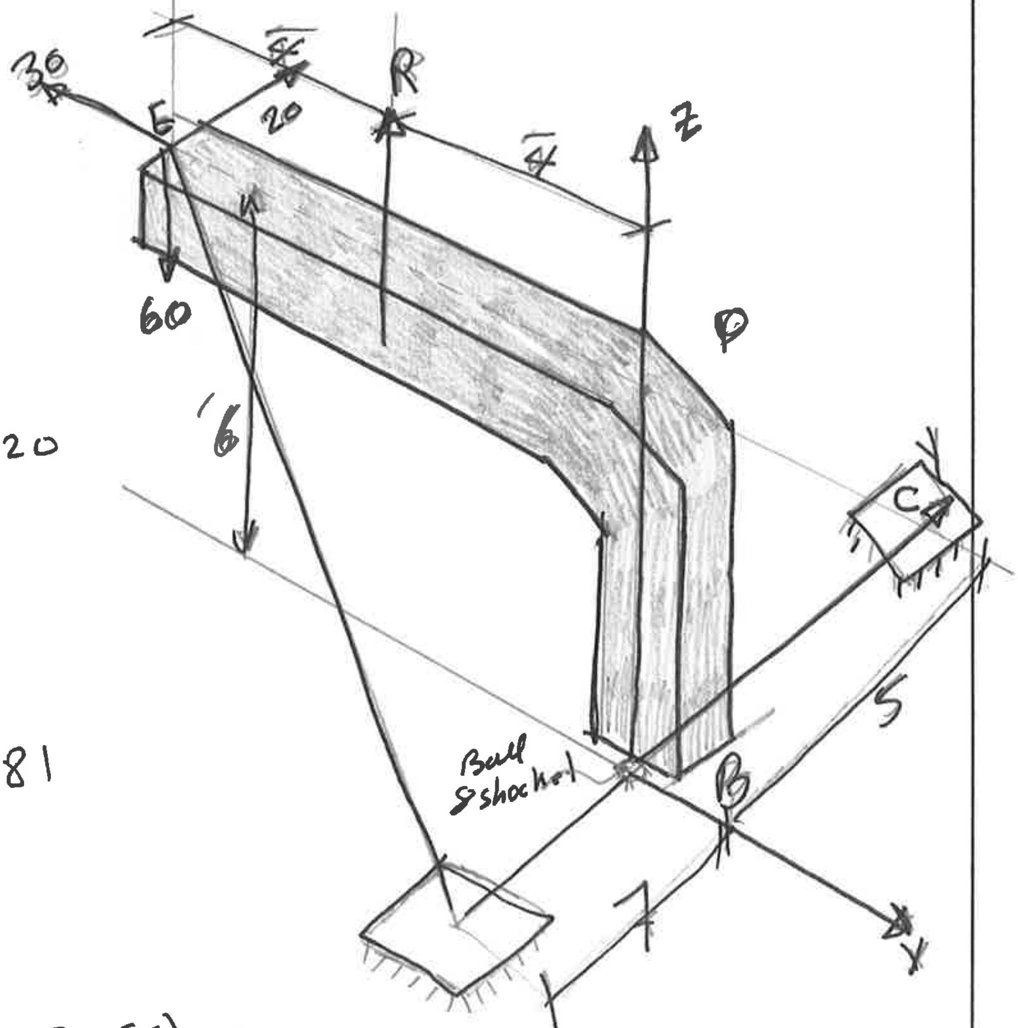
4.109 The frame EDB is in equilibrium, calculate R and the forces in the wires AE and CD.



length of AE

$\sqrt{8^2+7^2+6^2} = 12.20$

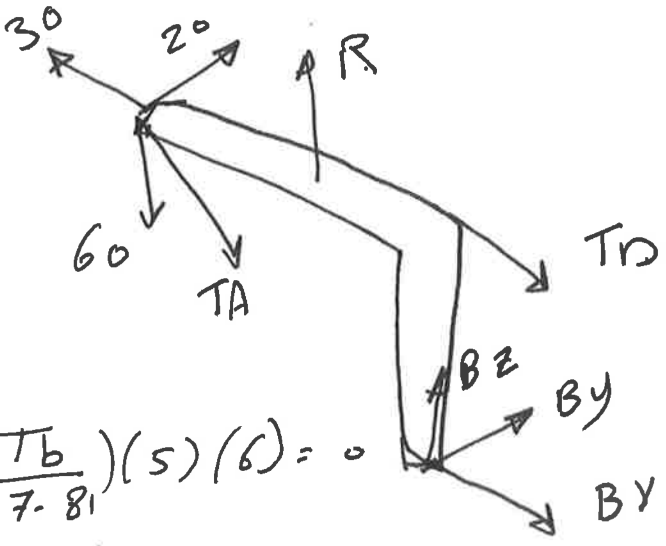
length of CD

$= \sqrt{5^2+6^2} = 7.81$

$\Sigma M_{Ac} = 0$

$R(4) - 60(8) - 30(6) = 0$

$R = 165\ N \uparrow$



$\Sigma M_z = 0$

$-20(8) + \frac{TA}{12.2}(7)(8) = 0$

$TA = 34.857$

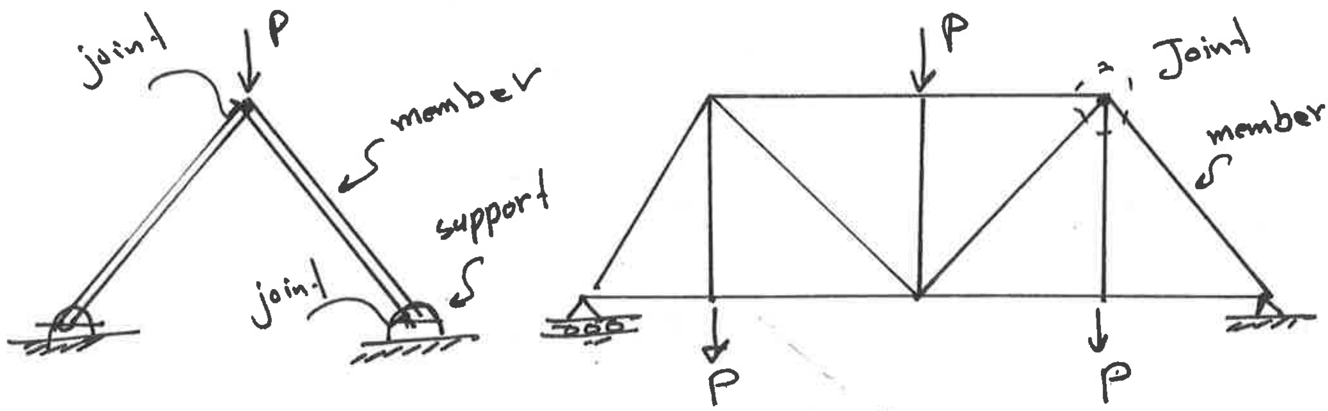
$\Sigma M_x = 0$

$20(6) - \left(\frac{TA}{12.2}\right)7(6) + \left(\frac{Tb}{7.81}\right)(5)(6) = 0$

$TD = 0$

Trusses

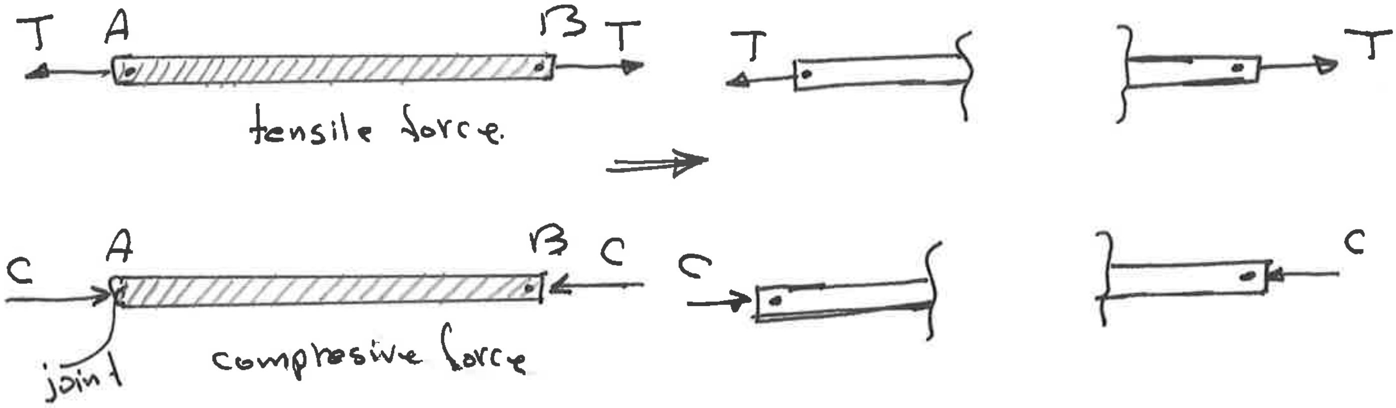
A truss is a structure made up at anumber of members fastened to gether at their ends by joint is such manner as to form a rigid body.



- The Calculation for the internal force in the members of atruss are based on the following assuptions.

1. The members of the truss are joint to gether by smooth pins.
2. The load and reactions act only at the joint.
3. The weight of the member can be neglected

- Each member of the truss is a two-force member



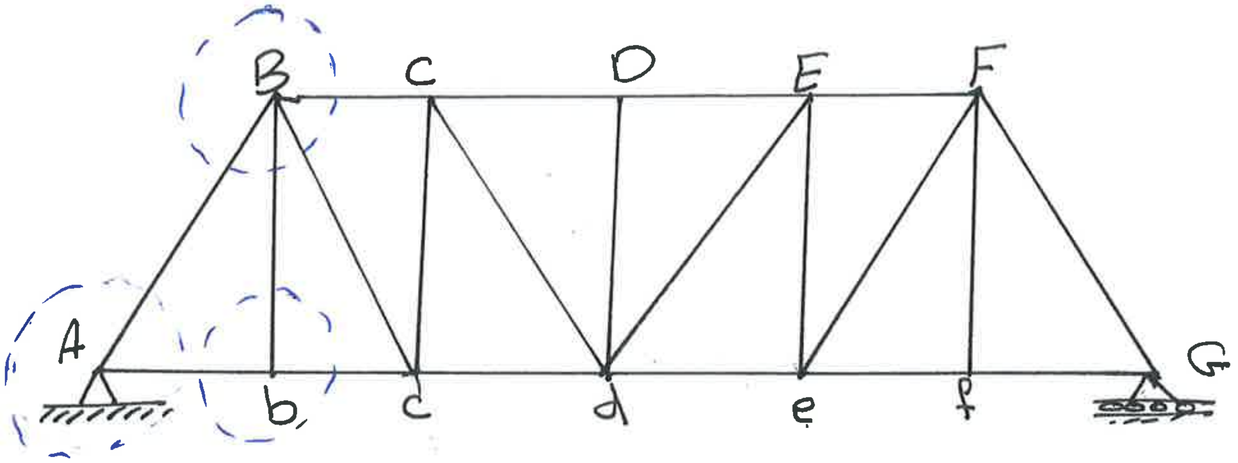
Analysis of Truss

The forces in the members can be determined by two method

1. Method of Joints
2. Method of sections.

1. Method of joints

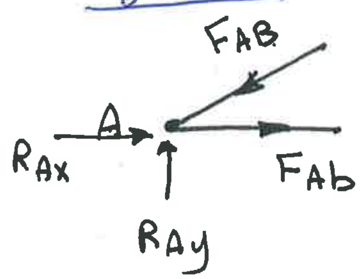
A single joint in the truss is isolated as a free body and then applying the equations of equilibrium.

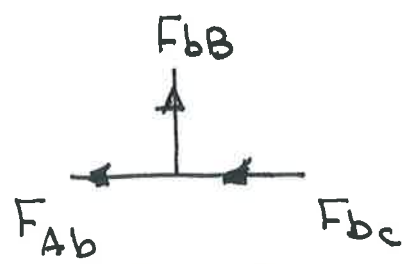


joint A

$\Sigma F_x = 0$

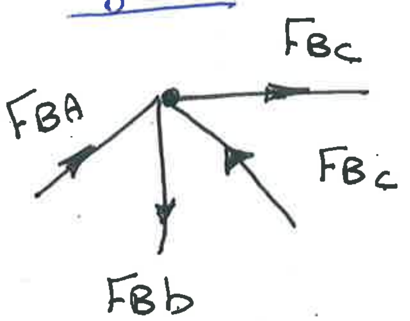
$\Sigma F_y = 0$





joint b

joint B



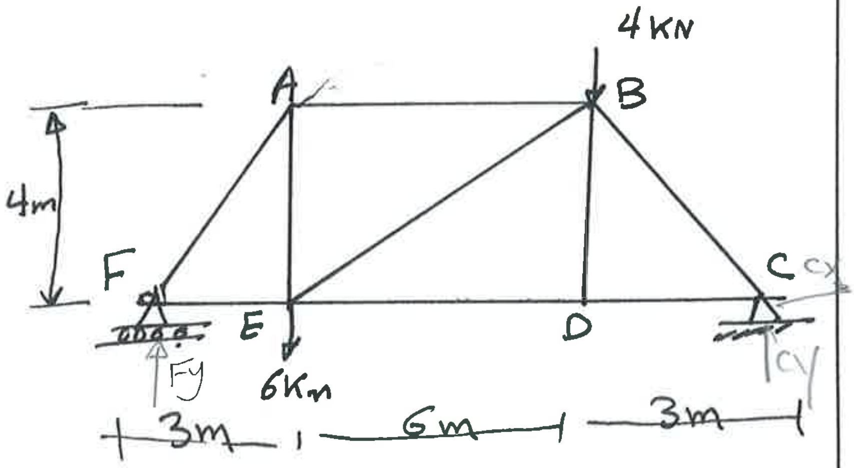
Ex: Analysis the truss shown below by joint Method

$\Sigma F_x = 0 \Longrightarrow \therefore C_x = 0$

$\Sigma M_c = 0$ +↻

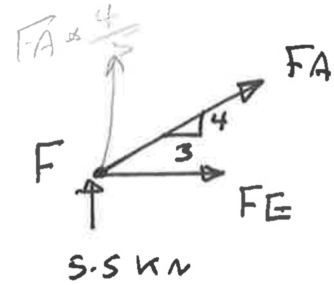
$F_y * 12 - 6 * 9 - 4 * 3 = 0$

$\therefore F_y = 5.5\ KN \uparrow$



$\sum F_y = 0 \Longrightarrow 5.5 - 4 - 6 + C_y = 0$

$C_y = 4.5 \text{ kN} \uparrow$



joint F

$+\uparrow \sum F_y = 0 \Longrightarrow 5.5 + F_A * \frac{4}{5} = 0$

$\therefore F_A = -6.87 \text{ kN} \nearrow = \boxed{6.87 \text{ kN} \swarrow \text{ (C)}}$

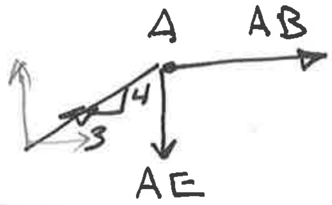
* دائماً قوى الانضغاط تفرضها سشد

$\xrightarrow{+} \sum F_x = 0$

$-F_A \cdot \frac{3}{5} + F_E = 0$

$\therefore \boxed{F_E = 4.125 \text{ kN} \longrightarrow}$ (T.)

joint A



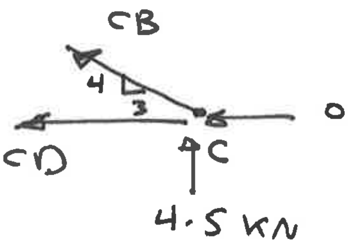
$+\downarrow \sum F_y = 0 \Longrightarrow AE - F_A * \frac{4}{5} = 0$

$AE - 6.87 * \frac{4}{5} = 0 \Longrightarrow \boxed{AE = 5.5 \text{ kN}}$

$\xrightarrow{+} \sum F_x = 0 \Longrightarrow AB + 6.87 + \frac{3}{5} = 0$

$\therefore AB = -4.125 \text{ kN (T.)} = 4.125 \text{ kN} \longleftarrow \text{(C.)}$

joint C



$\sum F_y = 0 \uparrow +$

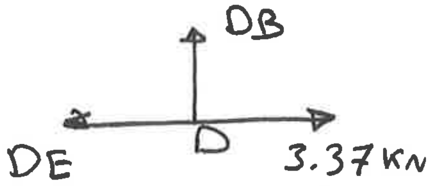
$4.5 + CB * \frac{4}{5} = 0$

$\therefore CB = -5.62 \text{ kN} \uparrow = 5.62 \text{ kN} \downarrow \text{ (C.)}$

$\sum F_x = 0 \; \xleftarrow{+}$

$CD - 5.62 * \frac{3}{5} = 0 \Longrightarrow CD = 3.37 \text{ kN} \xleftarrow{+} \text{(T.)}$

Joint D



$+\uparrow \Sigma F_y = 0$

$DB = 0$

$\Sigma F_x = 0 \xrightarrow{+}$

$3.37 - DE = 0$

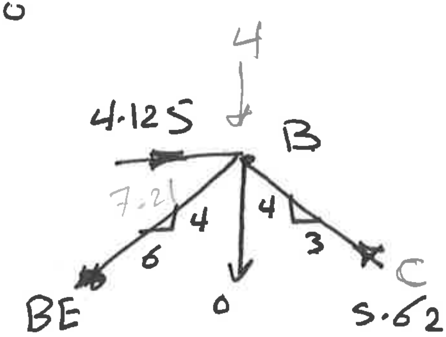
$\therefore DE = 3.37 \text{ kN } (T.) \overset{+}{\leftarrow}$

Joint B

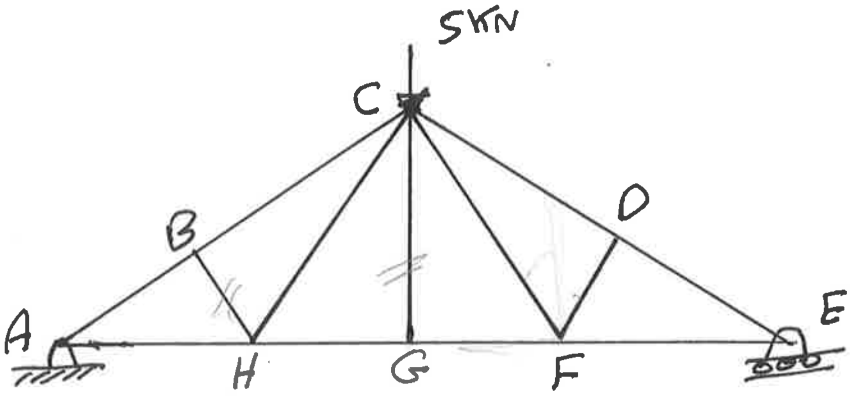
$+\downarrow \Sigma F_y = 0$

$4 - \underset{CB}{5.62} * \frac{4}{5} + BE * \frac{4}{7.21} = 0$

$\boxed{\therefore BE = 0.9 \text{ kN } (T.)}$



Ex: Using the method of joints, determine all the members of the truss shown in fig.



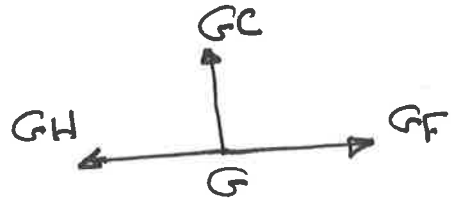
Solution

(Joint G)

$+\uparrow \sum F_y = 0 \Longrightarrow GC = 0$

$\rightarrow \sum F_x = 0 \Longrightarrow GF - GH = 0$

$\boxed{\therefore GF = GH}$

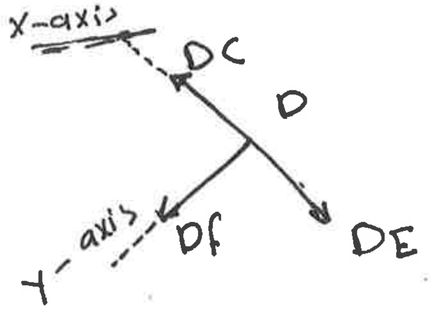


(Joint D)

$+\swarrow \sum F_y = 0 \Longrightarrow DF = 0$

$+\searrow \sum F_x = 0 \Longrightarrow DE - D_c = 0$

$\boxed{\therefore DE = D_c}$



(Joint F)

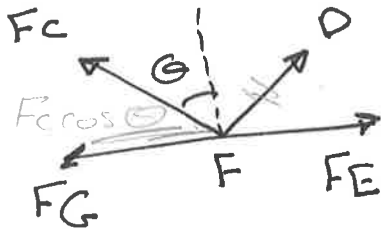
$\uparrow \sum F_y = 0 \Longrightarrow F_c * \cos\theta = 0$

since $\cos\theta \neq 0$

$\therefore f_c = 0$

$\therefore f_c \sin\theta = 0$

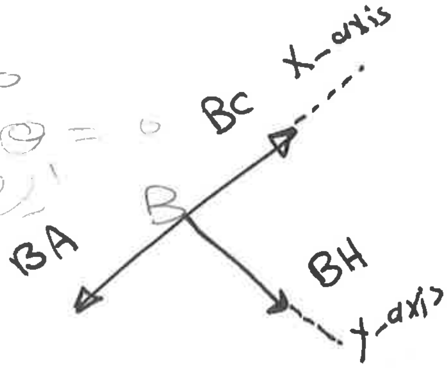
$\boxed{\therefore F_c = 0}$



~~Joint (B)~~

$+\rightarrow \sum F_x = 0 \Longrightarrow F_c * \sin\theta + FG - FE = 0$

$\therefore \boxed{FG = FE}$



(Joint B)

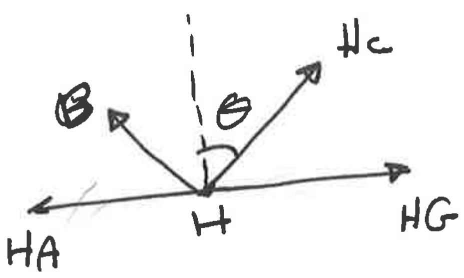
$\overset{+}{\downarrow} \sum F_y = 0 \Longrightarrow BH = 0$

$\overset{+}{\nearrow} \sum F_x = 0 \Longrightarrow \boxed{BC = BA}$

(Joint H)

$\because BH = 0$

$\therefore HC = 0$



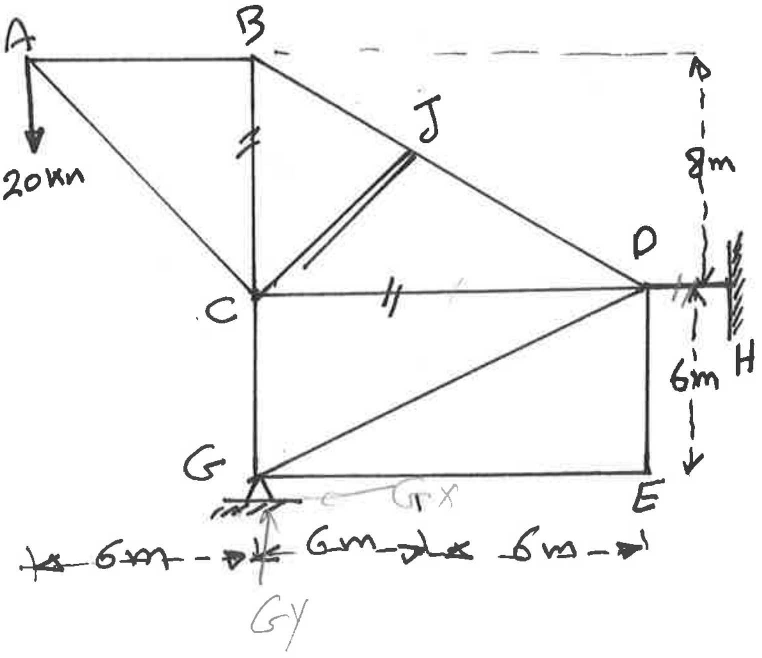
$+\uparrow \sum F_y = 0 \Longrightarrow H_C * \cos\theta = 0$

since $\cos\theta \neq 0$

$\boxed{\therefore H_c = 0}$

$\overset{+}{\rightarrow} \sum F_x = 0 \Longrightarrow HG = HA$

Ex: Determine the bar forces in member CD, CB and DH for the truss shown below



Solution

$\sum M_G = 0 \circlearrowright^{+}$

$DH * 6 - 20 * 6 = 0$

$\boxed{\therefore DH = 20\ kN\ (T)}$

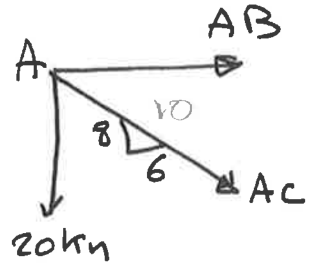
$\overset{+}{\rightarrow} \sum F_x = 0 \Longrightarrow DH + G_x = 0$

$20 + G_x = 0 \Longrightarrow \therefore \boxed{G_x = -20\ kn} = 20\ kn \overset{+}{\leftarrow}$

$+\uparrow \Sigma F_y = 0$

$G_y - 20 = 0 \Longrightarrow \boxed{G_y = 20 \text{ kn} \uparrow +}$

(Joint A)



$+\uparrow \Sigma F_y = 0$

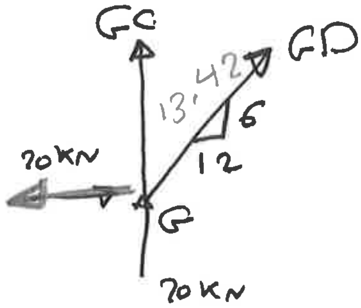
$-20 - AC \times \frac{8}{10} = 0$

$\boxed{\therefore AC = -25 \text{ kn} = 25 \text{ kn} \;\overset{+}{\nwarrow}\; (C.)}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \Longrightarrow AB - AC \times \frac{6}{10} = 0$

$\boxed{\therefore AB = 15 \text{ kn} \;\overset{+}{\rightarrow}\; (T.)}$

(Joint G)



$\overset{+}{\rightarrow} \Sigma F_x = 0 \Longrightarrow -20 + GD \times \frac{12}{13.42} = 0$

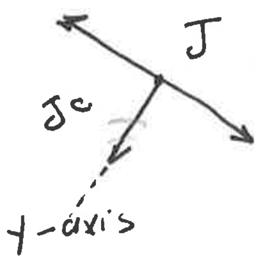
$\boxed{\therefore GD = 22.36 \text{ KN} \overset{+}{\nearrow} (T.)}$

$\uparrow \Sigma F_y = 0 \Longrightarrow 20 + GD \times \frac{6}{13.42} + GC = 0$

$\therefore GC = -20 - 22.36 \times \frac{6}{13.42} = -30 \text{ kN} \uparrow$

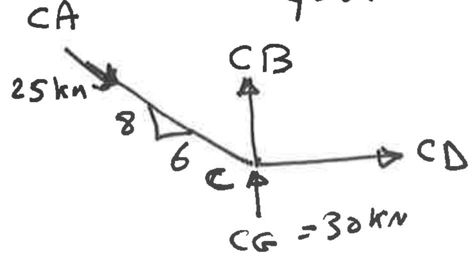
$\boxed{= 30 \text{ kn} \overset{+}{\downarrow} (C.)}$

(Joint J)



$\Sigma F_y = 0 \overset{+}{\swarrow}$

$\therefore JC = 0$



(Joint C)

$\overset{+}{\rightarrow} \sum F_x = 0 \Longrightarrow CD + CA * \frac{6}{10} = 0$

$\therefore CD = -15\ kn = 15\ kn \overset{+}{\leftarrow}$ (C.)

$+\uparrow \sum F_y = 0$

$CG + CB - CA * \frac{8}{10} = 0$

$\Longrightarrow 30 + CB - 25 * \frac{8}{10} = 0$

$\therefore CB = -10^{kn} = 10^{kn} \downarrow^{+}$ (C.)

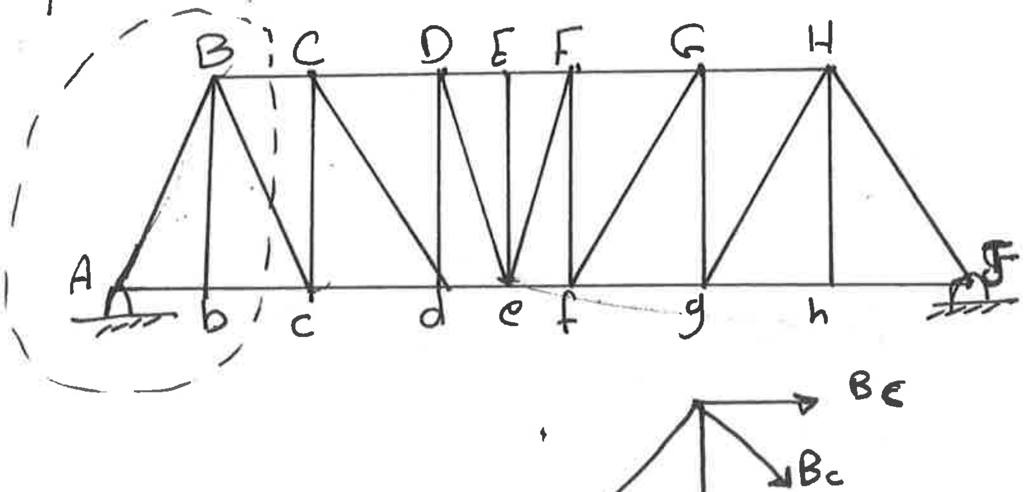
2. Method of Sections

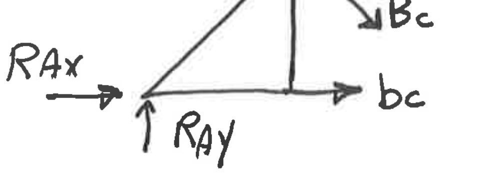
Two or more non-concurrent members are cut to obtain a free body diagram and make applying equation of equilibrium.

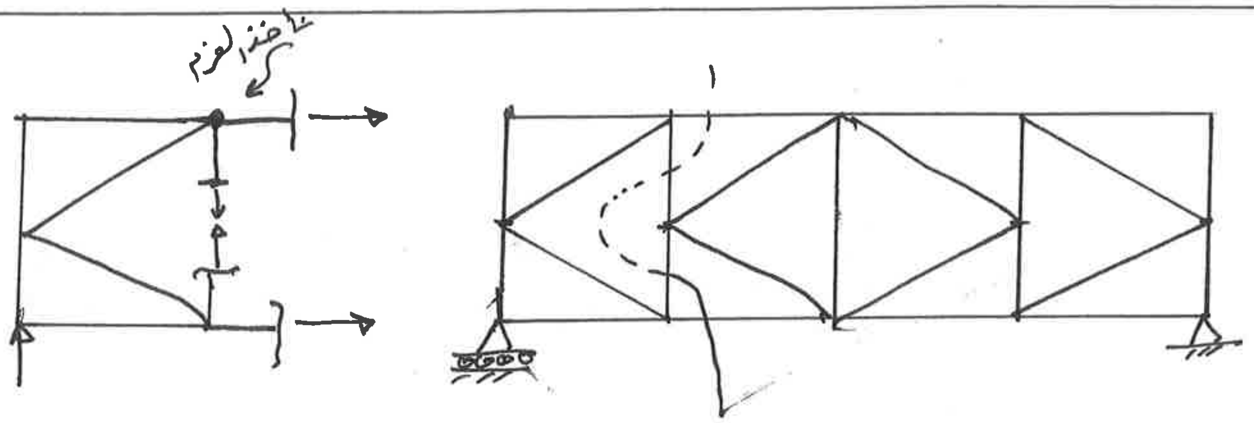
$\sum F_x = 0$

$\sum F_y = 0$

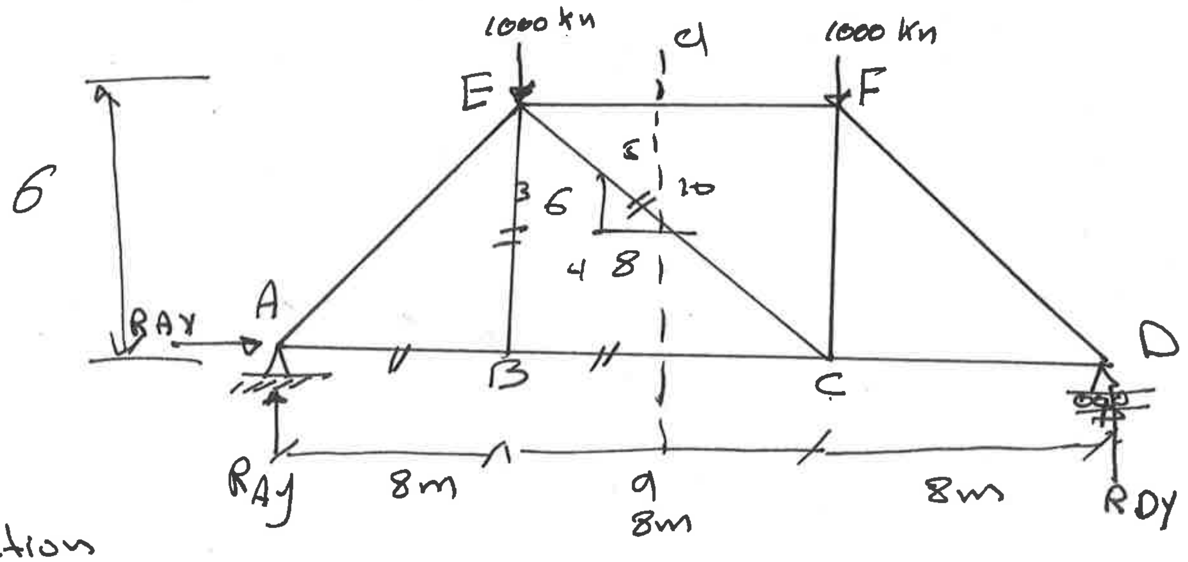
$\sum M_a = 0$







EX: For the truss shown in fig. Determine the force in member EC, BC, EB and AB



Solution

$\sum M_A = 0$ (+)

$$1000(8) + 1000(16) - R_{Dy}(24) = 0$$

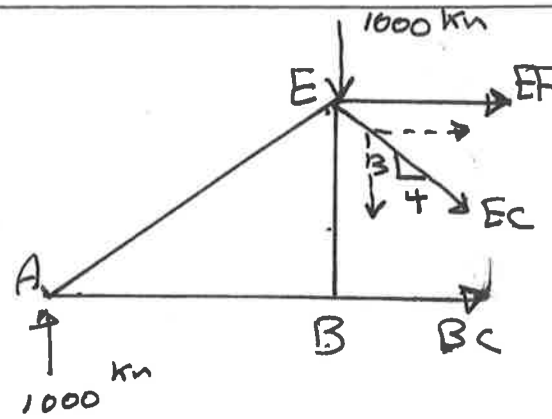
$\therefore R_{Dy} = 1000 \text{ kN} \uparrow$

$\sum F_x = 0 \xrightarrow{+}$ $\therefore R_{Ax} = 0$

$\sum F_y = 0 \; +\uparrow \Longrightarrow R_{Dy} - 1000 - 1000 + R_{Ay} = 0$

$\therefore R_{Ay} = 1000 \text{ kN} \uparrow$

From sec. a-a



$+\uparrow \sum F_y = 0$

$\cancel{1000} - \cancel{1000} - Ec * \frac{3}{5} = 0$

since $\frac{3}{5} \neq 0$

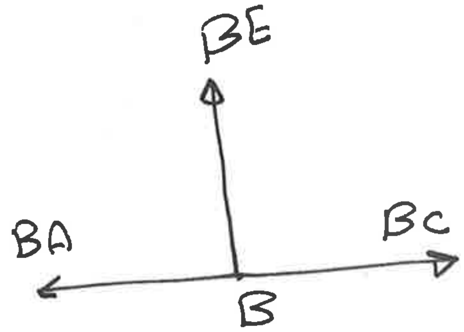
$\boxed{\therefore Ec = 0}$

$\sum M_E = 0$ +

$1000(8) - Bc(6) = 0$

$\boxed{\therefore Bc = 1333.33^{\,kn} \, (T.)}$

(Joint B)

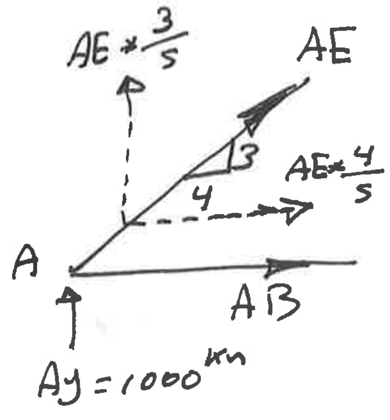


$+\uparrow \sum F_y = 0 \implies \boxed{BE = 0}$

$\overset{+}{\rightarrow} \sum F_x = 0 \implies Bc - BA = 0$

$\therefore BA = 1333.33^{\,kn} \, (T.)$

or (Joint A)



$+\uparrow \sum F_y = 0 \implies 1000 + AE * \frac{3}{5} = 0$

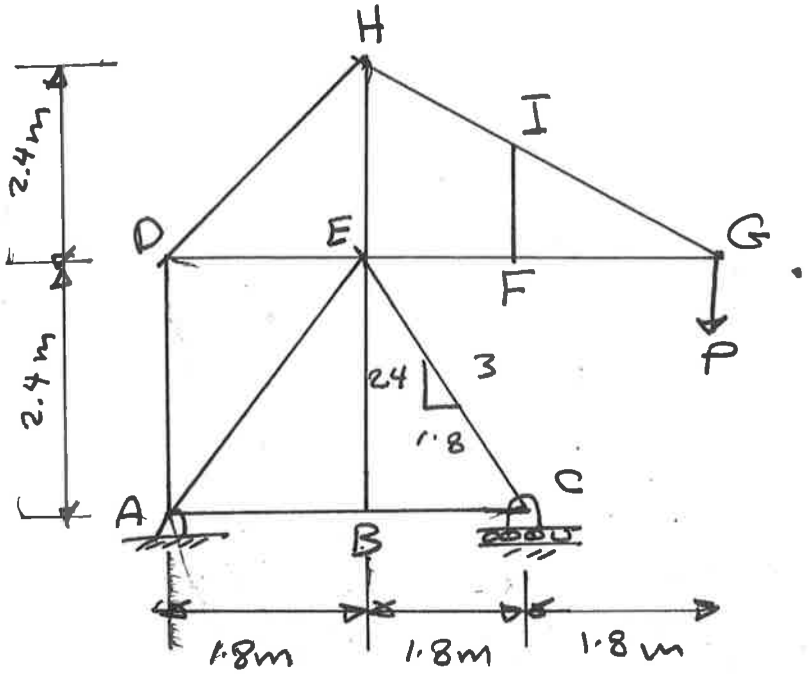
$\therefore AE = -1666.66^{\,kn} \, (T.)$
$= +1666.66^{\,kn} \, (C.)$

$\sum F_x = 0 \longrightarrow AB - AE * \frac{4}{5} = 0 \implies$

$AB - 1666.66 * \frac{4}{5} = 0$

$\boxed{\therefore AB = 1333.33^{\,kn} \, (T.)}$

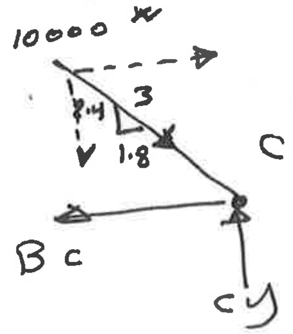
EX: Determin the load (P) which can be supported by the truss shown in fig, and product a force of 10000 N comp. in member CE?



Solu:

(Joint c)

$\uparrow \sum F_y = 0 \longrightarrow C_y - 10000\left(\frac{2.4}{3}\right) = 0$



$$\boxed{\therefore C_y = 8000 \text{ N} \uparrow}$$

For all structure

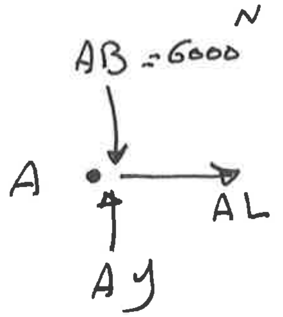
$\sum M_A = 0$ ↻+

$P(5.4) - C_y(3.6) = 0$

$$\boxed{\therefore P = \frac{8000(3.6)}{5.4} = 5333.33 \text{ N} \downarrow}$$

Ex:- Determine the force (P) and the force in (FE) when the force in AB is 6000 N compression

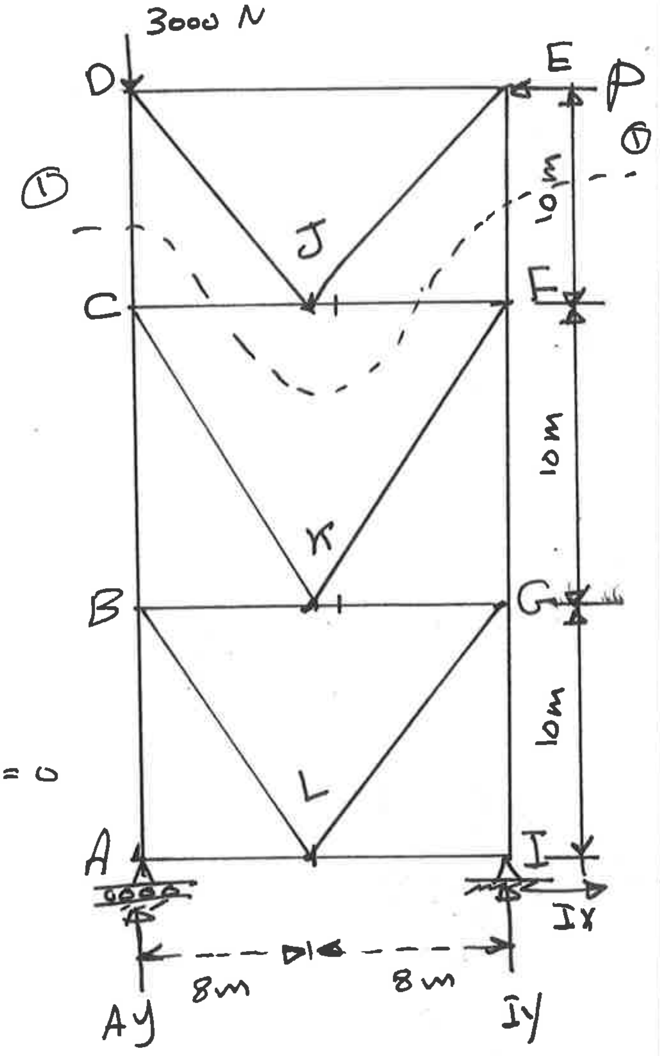
(Join A)



$+\uparrow \Sigma F_y = 0$

$\therefore Ay - 6000 = 0$

$\boxed{\therefore Ay = 6000\ N \uparrow}$



For all truss

$\Sigma M_I = 0$

$Ay(16) - 3000(16) - P(30) = 0$

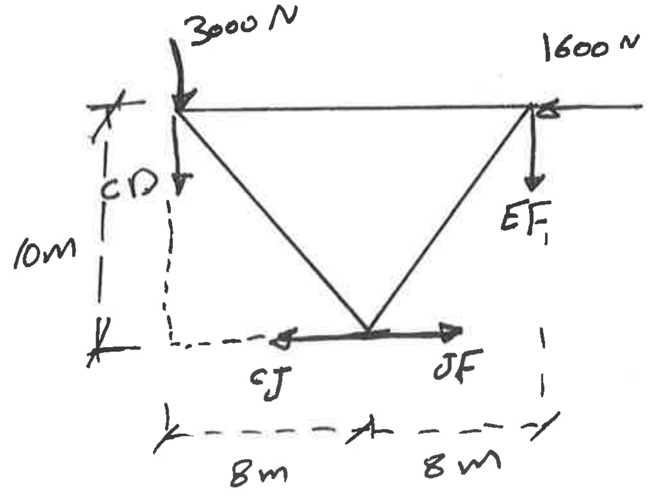
$\boxed{\therefore P = 1600\ N}$

Sec ① -- ①

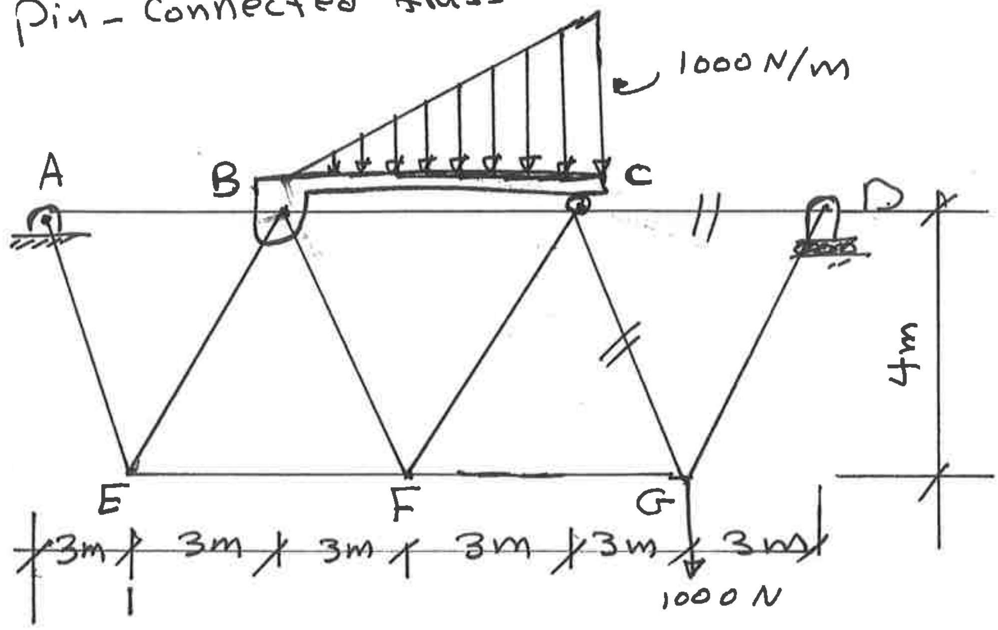
$\Sigma M_C = 0$

$-1600(10) + EF(16) = 0$

$\boxed{\therefore EF = 1000\ N\ (T.)}$



H.W Determine the forces in member CD and CG of the pin-connected truss



$\overset{+}{\curvearrowright} \Sigma M_B = 0$

$3000(4) - R_C * 6 = 0$

$\therefore R_C = 2000\,N \uparrow$

$\uparrow^+ \Sigma F_y = 0$

$2000 - 3000 + R_{By} = 0$

$\therefore R_{By} = 1000\,N \uparrow$

$\Sigma F_x = 0 \Longrightarrow R_{Bx} = 0$

for the truss

$\frac{1000 \times 6}{2} = 3000\,N$