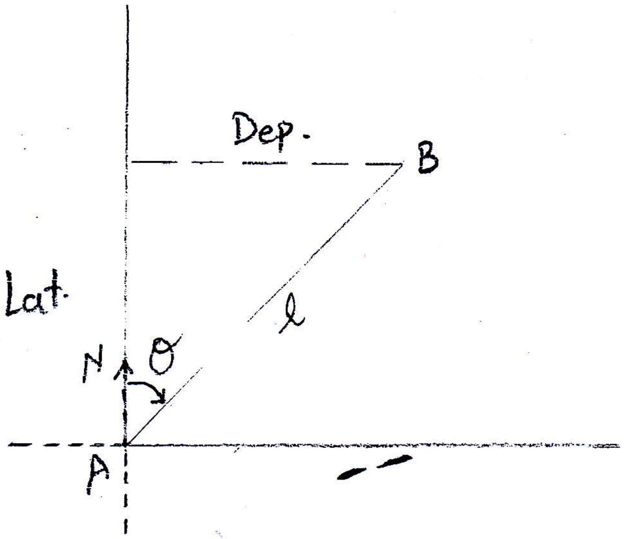
Inverse computation

To find the length and bearing of line dependent on coordinates or departure and Latitude.



$Dep. = l_{AB} \sin\theta$ where: Departure (Dep.)

$Lat. = l_{AB} \cos\theta$ Latitude (Lat.)

l_{AB}: is the length of line AB

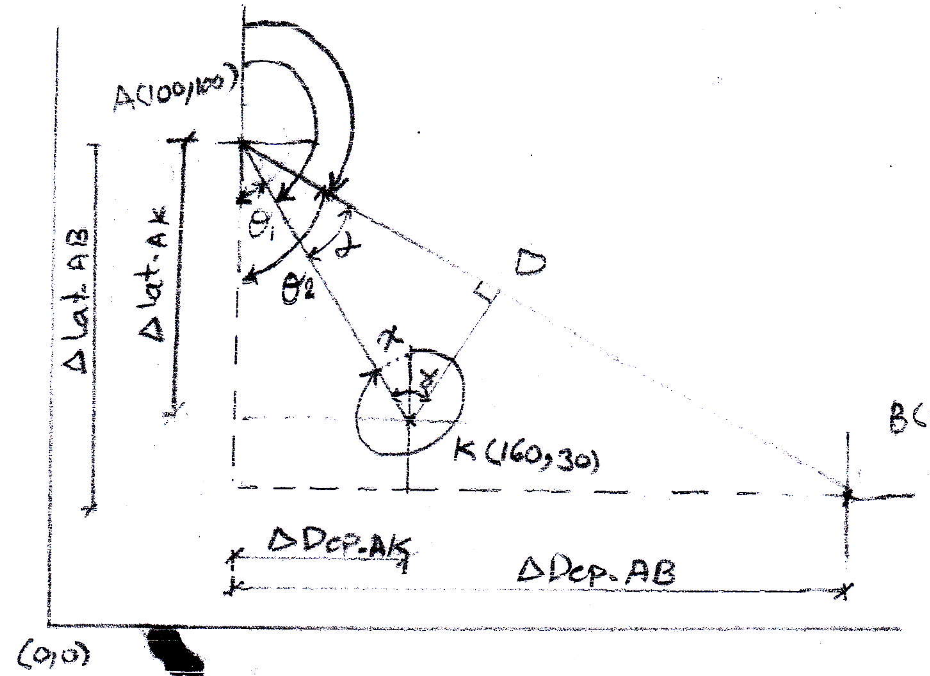
$$l_{AB} = \sqrt{(\Delta Dep.)^2 + (\Delta Lat.)^2}$$

where : $\Delta Dep._{A \to B}$ (the difference between A and B in x-axis)

$\Delta lat._{A \to B}$ (the difference between A an B in y-axis)

$$\tan\theta = \frac{\Delta Dep._{AB}}{\Delta Lat._{AB}} \Rightarrow \theta = \tan^{-1} \frac{\Delta Dep._{AB}}{\Delta lat._{AB}}$$

Example 1/ Calculate the length and the of line KD, which point D located on the line AB and the line KD was perpendicular to the line AB. The coordinate of point A was (100, 100), while the coordinate of point B was (250, 20), and the coordinate of point K was (160, 30).



Sol.

$$\theta_1 = \tan^{-1} \frac{\Delta Dep.\ AK}{\Delta lat.\ AK} = \tan^{-1} \frac{160-100}{100-30} = 40^\circ 36' 05''$$

$$Az\ AK = 180^\circ - 40^\circ 36' 05'' = 139^\circ 23' 55''$$

$$l_{AK} = \sqrt{(\Delta Dep.AK)^2 + (\Delta Lat.AK)^2} = \sqrt{(60)^2 + (70)^2} = 92.2\ m$$

$$\theta_2 = \tan^{-1} \frac{\Delta Dep.\ AB}{\Delta Lat.\ AB} = \tan^{-1} \frac{250-100}{100-20} = 61^\circ 55' 39''$$

$$Az_{AB} = 180^\circ - 61^\circ 55' 39'' = 118^\circ 04' 21''$$

$$\alpha = \theta_2 - \theta_1 = 21^\circ 19' 34''$$

$$\gamma = 180^\circ - \alpha - 90^\circ = 180^\circ - 21^\circ\,19'\,34'' - 90^\circ$$
$$= 68^\circ\,40'\,26''$$

$$\frac{AK}{\sin 90^\circ} = \frac{KD}{\sin \alpha} \Rightarrow \frac{92.2}{\sin 90} = \frac{KD}{\sin 21^\circ\,19'\,34''}$$

$$\Rightarrow KD = 33.53\ \text{m}$$

$$BAz\ AK = 139^\circ\,23'\,55'' + 180^\circ = 319^\circ\,23'\,55''$$
$$x = 360^\circ - 319^\circ\,23'\,55'' = 40^\circ\,36'\,05''$$
$$Az\ KD = \gamma - x = 68^\circ\,40'\,26'' - 40^\circ\,36'\,05''$$
$$= 28^\circ\,04'\,21''$$

H.W/ Find the coordinate of point D in example 1.

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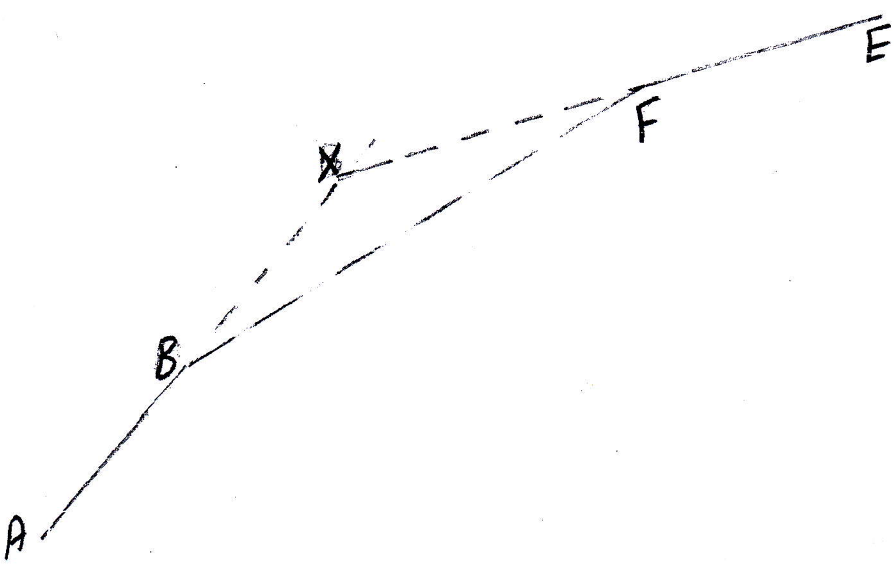
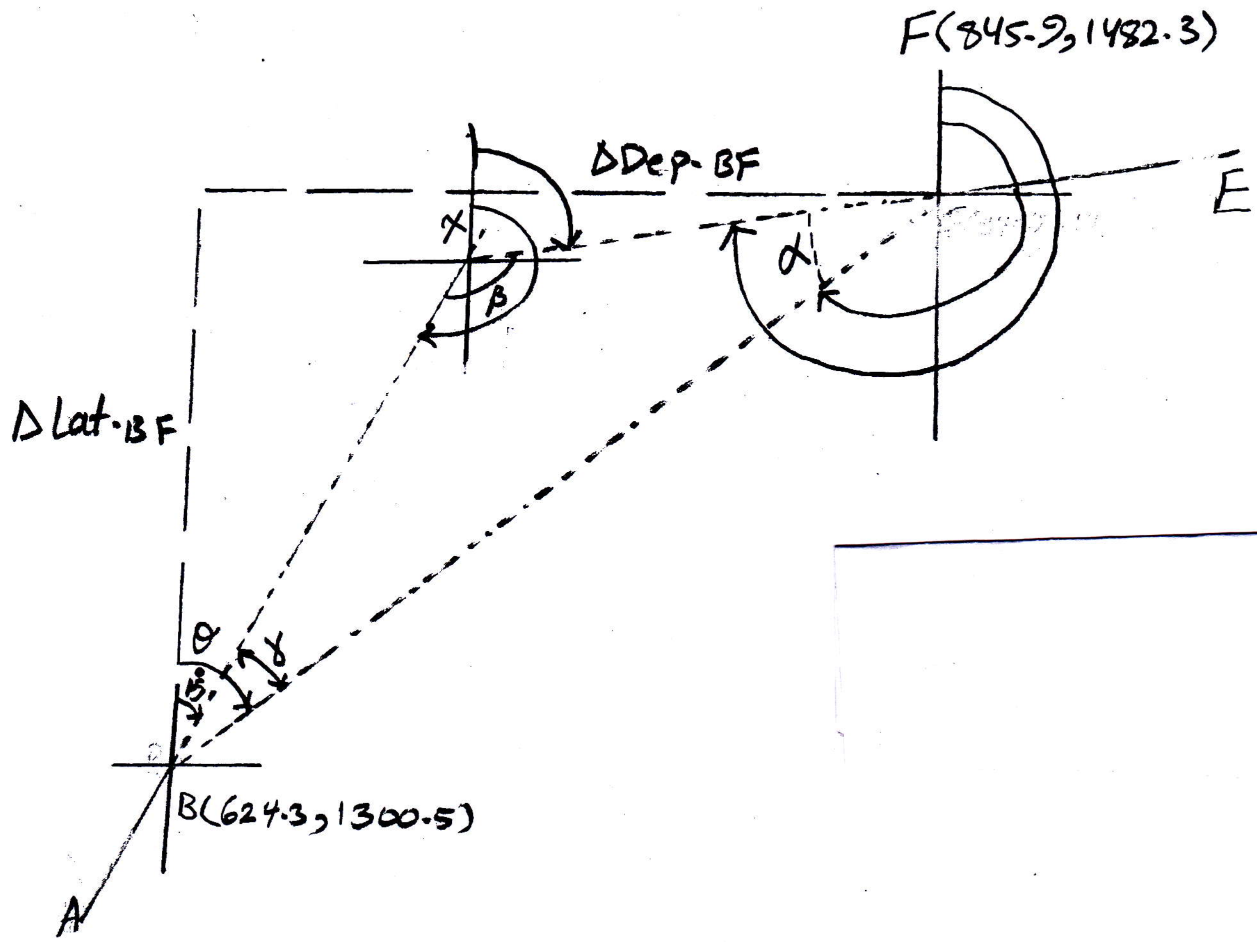
Example 2/ Through excavation in ground of specified location, the telephone cable from A to B and from E to F were placed. For technical purpose the line of cable AB was extended to point x and it connected from this point to the line of cable EF, as shown in figure below. The bearing of AB was $15^\circ 00'$, while the bearing of EF was $265^\circ 00'$. Besides to, the coordinates of B and F were (624.30, 1300.5), and (845.90, 1482.3), respectively. Find the length of excavation from x to F and from B to x.





$$Az\ BF \Rightarrow \theta = \tan^{-1}\frac{\Delta Dep._{BF}}{\Delta lat._{BF}} = \tan^{-1}\frac{845.9-624.3}{1482.3-1300.5} = 50^\circ 38' 05''$$

$$\gamma = 50^\circ 38' 05'' - 15^\circ 00' = 35^\circ 38' 05''$$

$$l_{BF} = \sqrt{(\Delta Dep._{BF})^2 + (\Delta Lat._{BF})^2} = \sqrt{(221.6)^2 + (181.8)^2}$$
$$= 286.63\ m.$$

$$B Az\ BF = 50^\circ 38' 05'' + 180^\circ = 230^\circ 38' 05''$$

$$\therefore Az\ EF = 265^\circ$$

$$\therefore \alpha = Az\ EF - B Az\ BF = 265^\circ 00' - 230^\circ 38' 05''$$
$$= 34^\circ 21' 55''$$

$B\,Az\,Ax = 15^\circ 00' + 180^\circ 00' = 195^\circ 00'$

$Az\,XF = 265^\circ 00' - 180^\circ 00' = 85^\circ 00'$

$\beta = 195^\circ 00' - 85^\circ 00' = 110^\circ 00'$

Must be the sum of traingle is 180°; therefore

$\gamma + \beta + \alpha = 180^\circ$.

For checking

$35^\circ 38' 05'' + 110^\circ 00' + 34^\circ 21' 55'' = 180^\circ 00$ o.k

$$\frac{XF}{\sin\gamma} = \frac{BF}{\sin\beta} \Rightarrow XF = \frac{BF \sin\gamma}{\sin\beta}$$

$$= \frac{286.63 \sin 35^\circ 38' 05''}{\sin 110^\circ 00'}$$

$$= 177.712\,m$$

$$\frac{Bx}{\sin\alpha} = \frac{BF}{\sin\beta} \Rightarrow Bx = \frac{BF \sin\alpha}{\sin\beta}$$

$$= \frac{286.63 \sin 34^\circ 21' 55''}{\sin 110^\circ 00'}$$

$$= 172.176\,m$$