Streamflow Measurements

قياس الجريان في المجاري المائية

1. Introduction: streamflow representing a runoff phase of the hydrologic cycle, is the most important basic data for hydrologic studies.

الجريان في المجاري المائية يمثل حالة السيح في الدورة الهيدرولوجية وهو كبير الأهمية لأنه يعد معلومات أساسية في الدراسات الهيدرولوجية .

A stream can be defined as a flow channel into which the surface runoff from a specified basin drains.

المجرى المائي هو الجريان في القناة التي تصرف السيح السطحي من جابية معلومة .

Generally, there is considerable exchange of water between an stream and underground water. Streamflow is measured by discharge units (i.e, m^3/sec). The measurement in a stream forms an important branch of Hydrometry. (What is the Hydrometry ??)

Streamflow measurement techniques can be classified into two categories: (1) Direct Methods
(2) Indirect Methods

<u>Direct Methods:</u> الطرق المباشرة

including (a) area-velocity Method — المساحة - السرعة
(b) Dilution techniques — تقنية التخفيف
(c) Electromagnetic Method — الطريقة الكهرومغناطيسية
(d) Ultrasonic method — طريقة الأمواج فوق الصوتية

<u>Indirect Methods:</u>

including (a) Hydraulic structures, such as weirs, flume, ... etc
(b) Slope-area method

In general, the direct measurement is very time-consuming and costly procedure. Hence two steps is followed;

(1) The discharge measurements should be related to water surface (stage) through series carfuel measurements.
(2) Construct a discharge-stage relationship and use it to estimate a discharge through knowing the stage.

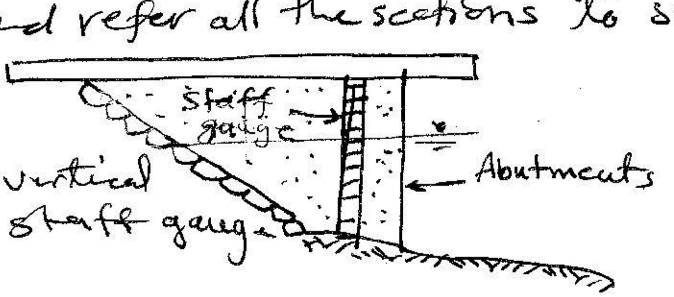
The stage المنسوب

The stage of a river is defined as the water surface elevation above a datum. هو ارتفاع سطح الماء والمقاس نسبة الى مستوى مرجعي محدد.
The datum can be the (MSL) or any arbitrary datum connect to MSL. المستوى المقارنة (datum) ممكن ان يكون متوسط منسوب سطح البحر او اي منسوب يرتبط بشكل غير مباشر الى متوسط منسوب سطح البحر

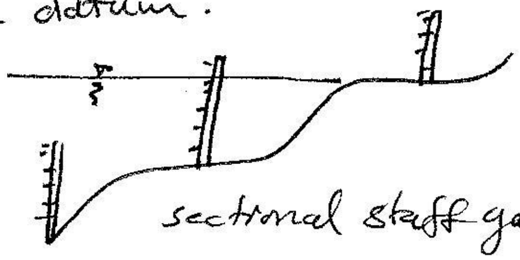
the stage measurement

مقاييس المنسوب.

1. Manual gauges المقاييس اليدوية
   (a) staff gauge: it is similar to those on surveying staff. The surface of water must in contact with staff, Sometimes The staff is fixed rigidly to struture like abutment, Pier wall, etc... The staff may be vertical or inclined with clear and accurate grduated permanent markings. In such cases, the gauge is built in section at different locations. The sectional gauges must provide an overlap between gauges. = مقاييس مقطعية and refer all the sections to same datum.

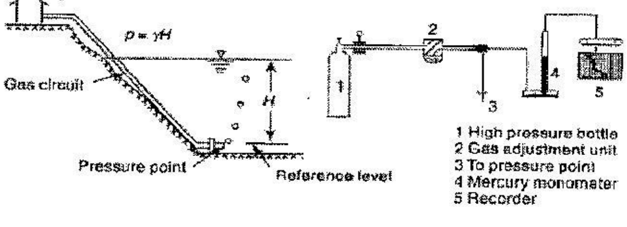


vertical staff gauge

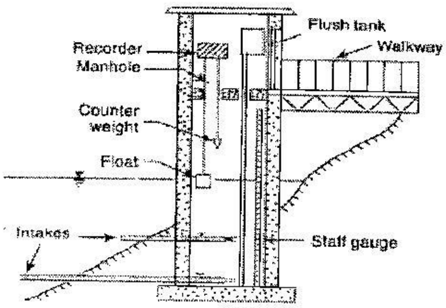

sectional staff gauge

(b) wire gauge
The gauge used to measure the water surface elevation from above surface such as bridge or similar structures. In this a weight is lowered by a reel (حبل) to touch the water surface.

2. Automatic stage recorders المقاييس ذاتية التسجيل
   (a) Float gauge recorder المقياس ذو التسجيل بالعوامة
   (b) Bubble gauge المقياس ذو الفقاعات



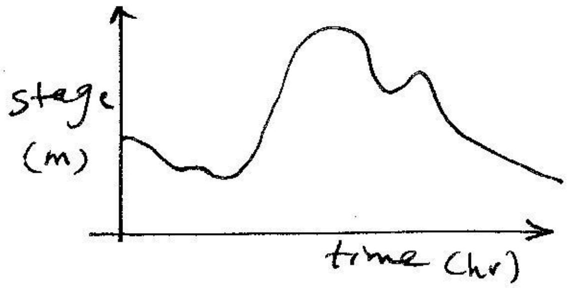
Bubble Gauge



FLOAT-GAUGE RECORDER with Stilling well Installation

Stage data   معلومات المنسوب

The stage data is often represented in the form of plot of stage against chrono-logical time, and known as stage Hydrograph.



The stage Hydrograph is of importance in design of Hydraulic structures such as bridges, weirs, ... etc, flood warning and flood-protection work.

Velocity Measurement   قياس السرعة

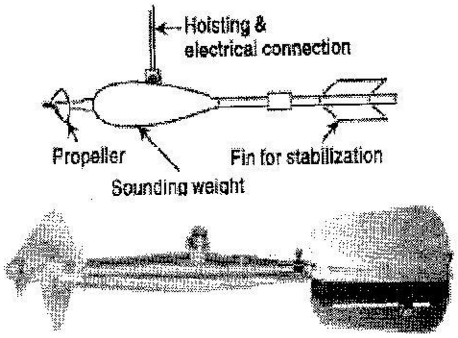
The measurement of streamflow requires pre-knowledge about velocity values across the river or channel. The most commonly method to measure the velocity is the use of mechanical device called current meter عداد التيار

Current meters   التيار

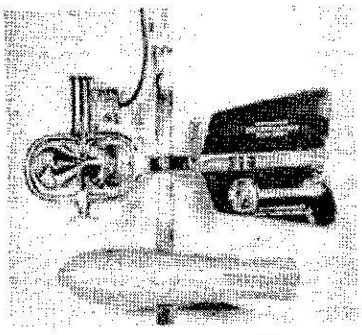
Robert Hooke (1963) invented a propeller-type current meter to measure the distance traversed by a ship. The present day cup-type instrument was invented by Henry in 1868. There are two types (mainly) of current meters:

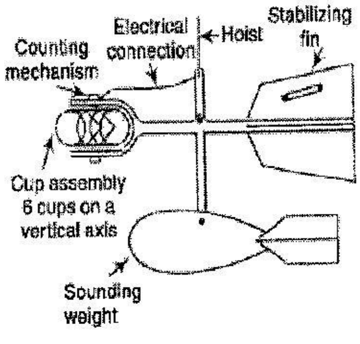
(1) Vertical-axis meter   العداد ذو المحور الرأسي
(2) Horizontal-axis meter   العداد ذو المحور الأفقي

measured velocity range 0.15 – 4 m/s
e = 1.5% to 0.3% V



Horizontal-axis Current Meter


Cup-type Current Meter



Vertical-axis Current Meter

Current meter equation   معادلة العداد

$$U = a N_s + b \qquad (1)$$

U = velocity at device point m/s
N_s = revolution per second
a, b = Constants of meter

For current meter type price (Cup-model) of size 12.5cm in diameter the standard value of $a = 0.65$ and $b = 0.03$. النوع القدحي طراز برايس
يطلق عادة على العامل b بالصحيح للسرعة الحرجة اذا كانت $N_s = 0$

Calibration: المعايرة

The relation between velocity & revolutions per second of meter as in eq(1) above, is called calibration equation. The checking of whether the meter records are correct or not is very important from time to time. The calibration is performed in Lab. of Hydraulic with so-called towing tank خزان الجر .

Feild use of current meter

(1) The velocity in vertical Line of such section in river below water surface can be measured at 0.6 of the depth and considered as for shallow river (depth ≤ 3 m) and considered as average velocity in the vertical ($\bar{v}$). This method call called sigle-point observation method.

(2) For streams depths greater than 3 m, the two-point method were considered as;

$$\bar{v} = \frac{V_{0.2} + V_{0.8}}{2} \qquad (2)$$

$V_{0.2}$ = point velocity at 0.2 depth below water surface
$V_{0.8}$ = point velocity at 0.8 depth below water surface
$\bar{v}$ = average velocity of the vertical.

3) for case of flood, only the surface velocity V_s can be measured, and the average velocity becomes:

$$\bar{v} = K V_s \qquad (3)$$

K = constant obtained from low flow measurement and varied from 0.85 to 0.95 [almost 0.85].
V_s = surface velocity m/s

Discharge Measurement:

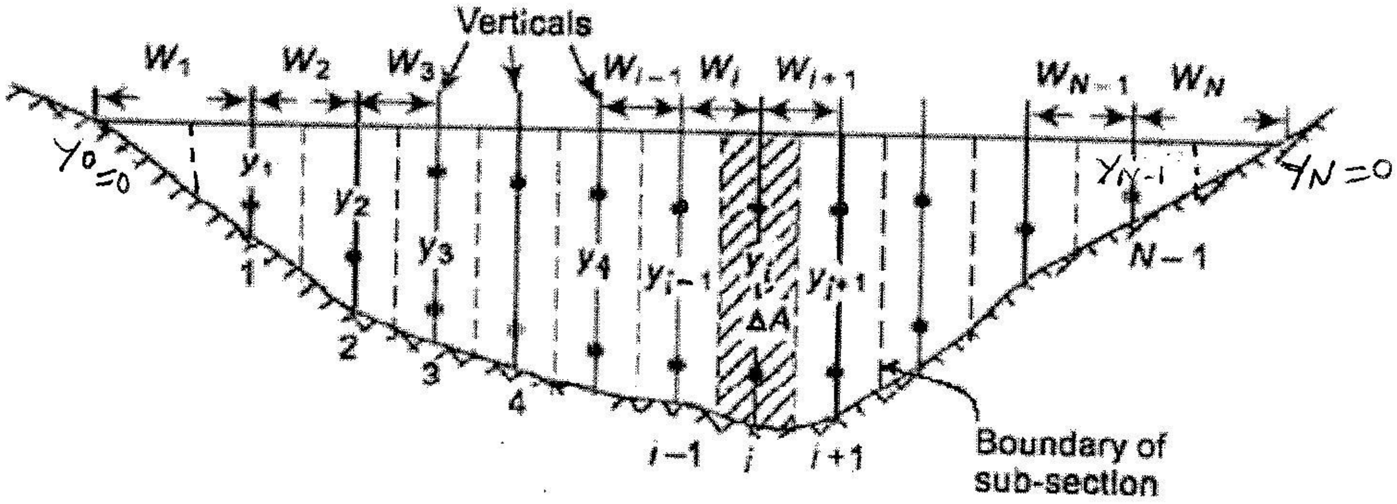
Area-Velocity Method

Measuring of velocity of flow through cross-sectional area are performing at gauging site موقع القياس. The gauging site must be selected carefully to ensure that the stage-discharge curve is reasonably constant over many years (say up to 10 years). The following criteria of this site are adopted:

* The stream has well-defined cross-section which does not change in various seasons. المجرى المائي له مقطع عرضي لا يتغير مع الفصول
* It should be easily accessible all through the year. سهل الوصول لموقع القياس
* The site should be in a straight, stable reach. موقع القياس في قطاع مستقيم ومستقر
* The site should be free from backwater effects. موقع القياس لا يتأثر بالارتداد كلياً

After the site selecting, the depths are measured for different locations by sounding rods or by electroacoustic instrument called echo-depth recorder is used.
تقاس الأعماق القليلة بعصا سبر الأعماق
والأعماق الكبيرة بجهاز صدى الأعماق حيث يعتبر الأخير مفيداً في المجاري ذات السرع العالية

The cross-section is divided into a large number of subsections by so called verticals. The average velocity of each subsection are measured by current meter. It is quite obvious that the discharge estimation accuracy increases with increase of number of subsections. However large number of segments lead to large effort.



Stream Section for Area-velocity Method

The followings are guidelines to select No. of segments:

(1) The discharge in each segment < 10% of total discharge.

(2) The difference in velocities of adjacent segement ≤ 20%

(3) The No. of segments > b ; where b = 15 − 20

i.e width of segment $< \left(\frac{1}{15} - \frac{1}{20}\right)$ of total width.

It should be noted that for natural river the verticals for velocity measurements are not necessairly equally spaced.

في الانهار الطبيعية لا يشترط ان تكون القطاعات الرأسية لقياس السرعة متساوية البعد العرضي

Calculation of Discharge

Referring to figure above ;

N = number of stations

$N-2$ = number of segments (subsections = No. Verticals of nonzero depths)

$$Q = \sum_{i=1}^{N-2} \Delta q_i \quad \text{———— (4)}$$

$$\Delta q_i = \left(\frac{W_i + W_{i+1}}{2}\right) \times y_i \times v_i \quad \text{—— (5)}$$

$$= \Delta A_i \times v_i$$

This method is called midsection technique, while the other method that called mean section technique is not used in current ($\Delta A_i = W_i \frac{d_i + d_{i+1}}{2}$) study.

If the depth at the edge of water or at last point is nozero, the velocity is estimated as a 65% of the adjacent vertical velocity, because it is not possible to measure the velocity by current meter.

example 1:

The data pertaining to stream gauging station given below. The equation of the current meter is $v = 0.5 N_s + 0.03$, N is the revolution per second. Calculate the Discharge of the stream & the average velocity of the river

Distance From Left bank (m):	0	1	3	5	7	9	11	12
Depth (m):	0	1.1	2	2.5	2	1.7	1.0	0
Revolutions of current meter:	0	39	58	112	90	45	30	0
Elapsed time (sec):	0	100	100	150	150	100	100	0

Solution

8 stations → 6 subsections (6 nonzero depths)

The answer must be tabulated as follow:-

Distance From left bank (m)	width (m)	$\bar{w}$ (m)	Depth (m)	Velocity m/s	Segment discharge m3/sec
0	—	—	0	—	—
1	1	1.5	1.1	0.2889 0.225	0.37768
3	2	2	2	0.3258 0.32	1.3032
5	2	2	2.5	0.4110 0.403	2.0549
7	2	2	2	0.3360 0.33	1.3440
9	2	2	1.7	0.2595 0.255	0.8823
11	2	1.5	1.0	0.1830 0.18	0.2745
12	1	—	0	—	

$V = \frac{Q}{A} = \frac{6.23658}{19.59} = 0.319\ m/s$ $\qquad Q = \sum_{1}^{6} q_i = 6.23658\ m^3/s$

Example 2:

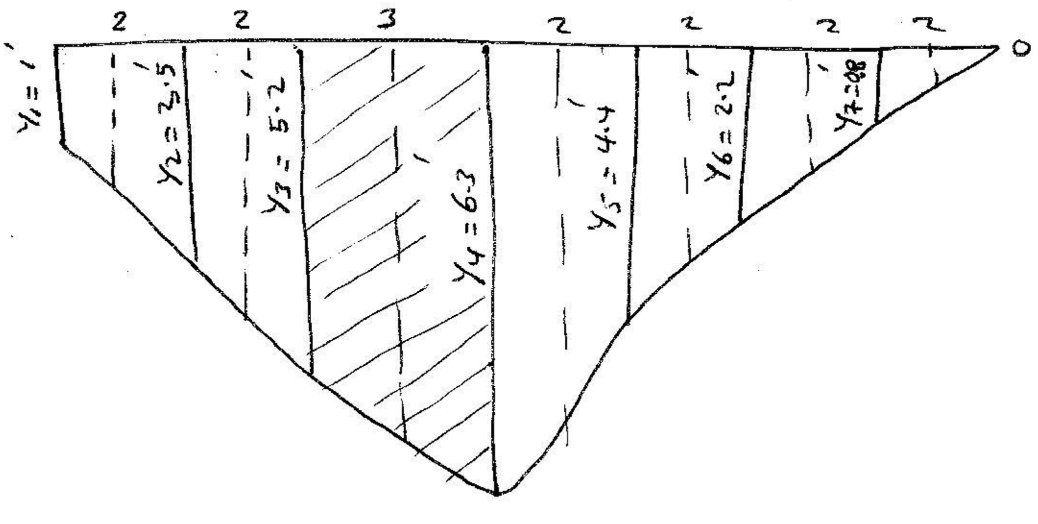
Compute the discharge by area-velocity method for the following measurement data. The current meter rating is given by:

$v = 2.2 N_s + 0.1$, wher N_s (rev/sec) and v (ft/sec

Distance From left (ft)	Depth (ft)	depth of observation	Revolution	Elapsed time (sec)
0	1	No measur.	—	—
2	3.5	0.2	35	50
		0.8	22	50
4	5.2	0.2	40	60
		0.8	30	55
7	6.3	0.2	45	60
		0.8	30	55
9	4.4	0.2	33	45
		0.8	30	50
11	2.2	0.6	22	50
13	0.8	0.6	10	45
15	0	No measur.	—	—

Solution

8 stations $\longrightarrow$ 7 subsection (7 nonzero depths).



The crosssectional area of the river.

Distance (ft)	width (ft)	$\bar{w}$ (ft)	Depth (ft)	v fps at point	$\bar{v}$ (fps)	Discharge ft^3/sec
0	—	1	1	—	0.88*	0.88
2	2	2	3.5	1.64 1.07	1.36	9.52
4	2	2.5	5.2	1.57 1.30	1.44	18.72
7	3	2.5	6.3	1.75 1.30	1.53	24.10
9	2	2	4.4	1.71 1.42	1.57	13.82
11	2	2	2.2	1.07	1.07	4.71
13	2	2	0.8	0.59	0.59	0.94
15	2	—	0	—	—	—

* $0.65 \times 1.36 = 0.88$ Fps

Area $= 51.55\ ft^2$ $\quad Q = \sum \Delta q_i = 72.69\ ft^3/sec$

$\Rightarrow V = 1.41\ ft/sec$

$V_{0.6}$, $V_{0.2}$ and $V_{0.8}$ relations

The flow in river is almost turbulent and the velocity distribution across a vertical section is logarithmic in nature, and can be expressed as;

$$\frac{v}{v_*} = 2.5 \ln\left(\frac{30y}{K_s}\right) \quad ; K_s: \text{equivalent sand-grain roughness}$$

or in a more simple relation (Blasius equation), some time call power law profile, that is

$$\frac{v}{v_a} = \left(\frac{y}{a}\right)^{1/m}$$

V = average velocity that cane be obtained from integration as

$$V = \frac{1}{Y_0}\int_0^{y_0} v\,dy = \frac{V_a}{Y_0}\int_0^{y_0}\left(\frac{y}{a}\right)^{1/m} dy = \frac{m}{m+1} V_0$$

V_0 = velocity at $y = Y_0$ (i.e, surface velocity)

proof using logarithmic Velocity distribution:

From Hydraulics The velocity profile for rough surface of turbulent flow is;

$$\frac{v}{V_*} = 2.5 \ln \frac{y}{c}$$; wher c is constant , $c = \frac{K}{30.1}$ for rough channel surface , K : roughness height

thus $\frac{v}{V_*} = 2.5 \ln \frac{30.1 y}{K}$ —(1)

From Keulegan's equation

$$\frac{V}{V_*} = 6.25 + 2.5 \ln\left(\frac{R}{K}\right)$$; where R = hydraulic radius

or $\frac{V}{V_*} = 2.5 \ln\left(\frac{12.2 R}{K}\right)$ ——(2)

Note: for wide rectangular channel $\frac{V}{V_*} = 2.5 \ln \frac{11.1 y_0}{K}$

* The position where the $v = V$ is;

$$\frac{12.2 R}{K} = \frac{30.1 y}{K} \quad \text{or} \quad y = 0.4 R$$

for wide channel $R \simeq y_0$; thus;

$\Rightarrow y = 0.4 y_0$

in other meaning $y = 0.6 y_0$ from surface of water.

**

$$\frac{v_{0.8} + v_{0.2}}{2 V_*} = \frac{1}{2}\left[2.5 \ln\left(\frac{30.1(0.2)y_0}{K}\right) + 2.5 \ln\left(\frac{30.1(0.8)y_0}{K}\right)\right]$$

$$= 8.51 + \frac{2.5}{2} \ln\left(\frac{0.8 y_0 * 0.2 y_0}{K^2}\right)$$

$$= 8.51 + \frac{2.5}{2} \ln \frac{0.16 y_0^2}{K^2} = 8.51 + 2.5 \ln \frac{0.4 y_0}{K}$$

$$= 2.5 \ln \frac{12.2 y_0}{K}$$

$$= \frac{V}{V_*} \quad (\text{for } R = y_0).$$

From eq.(2)

$$\Rightarrow \frac{V}{V_*} = \frac{v_{0.8} + v_{0.2}}{2 V_*}$$

$$\Rightarrow V = \frac{v_{0.2} + v_{0.8}}{2}$$

The value of m varied from 6 to 8-5, usually taken as $m=7$, therefore the Blasius equation is called one-seventh power law -

$$\Rightarrow \bar{V} = \frac{7}{8} V_o \approx 0.875 V_o$$

$$V_{0.6} = V_{y=0.4y_o} \Rightarrow V_{0.6} = V_a \left(\frac{0.4 y_o}{a}\right)^{1/m}$$

$$= V_a \left(\frac{y_o}{a}\right)^{1/m} (0.4)^{1/m}$$

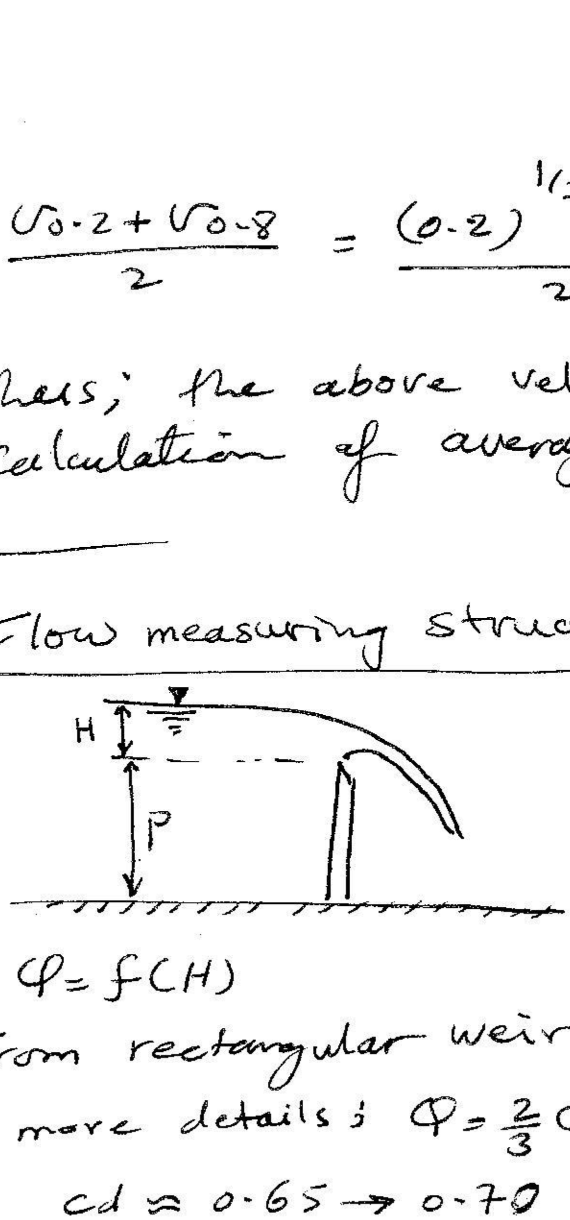
$$= V_a \frac{V_o}{V_a} (0.4)^{1/7}$$

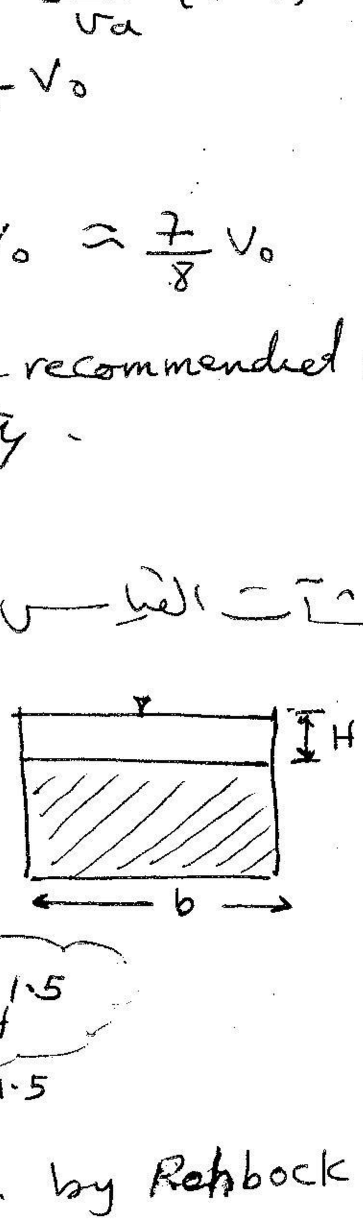
$$= \frac{7}{8} V_o$$

$$\frac{V_{0.2} + V_{0.8}}{2} = \frac{(0.2)^{1/7} + (0.8)^{1/7}}{2} V_o \approx \frac{7}{8} V_o$$

thus; the above velocities are recommended for calculation of average velocity.

Flow measuring structures منشآت القياس





$\varphi = f(H)$

From rectangular weir; $\varphi = K H^{1.5}$

in more details; $\varphi = \frac{2}{3} C_d \, b \sqrt{2g} \, H^{1.5}$

$C_d \approx 0.65 \rightarrow 0.70$, or given by Rehbock equation:

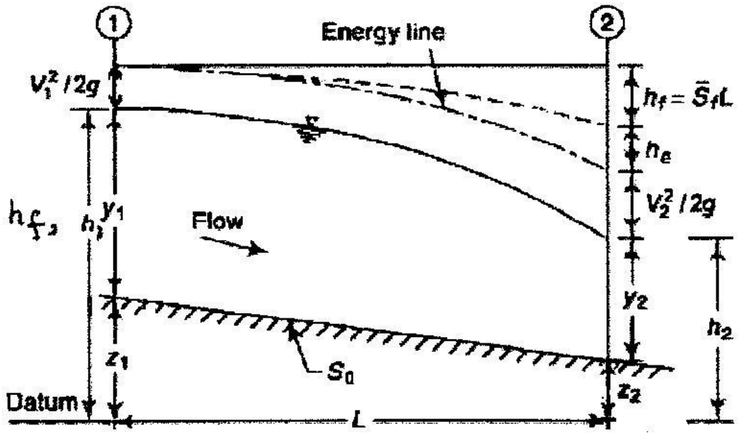
$$C_d = 0.611 + 0.08 \frac{H}{P}$$

Slope – Area Method

The Manning's equation was adopted herein as a Flow resistance equation for uniform flow in open channel, while other formulae can be used also.

Consider the reach of the river where the surface elevations at two section (1) and (2) are known.

Apply the energy equation for section (1) and (2);



Slope Area Method

$$Z_1 + y_1 + \frac{V_1^2}{2g} = Z_2 + y_2 + \frac{V_2^2}{2g} + h_L$$

h_L = Total headloss made up of two parts (1) frictional loss h_f, and (2) eddy loss h_e, i.e;

$$h_L = h_f + h_e$$

$Z + y$ = water surface elevation which is usually called piezometric head h, i.e;

$$h = z + y .$$

The above energy equation becomes;

$$h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + h_e + h_f \quad \text{——— (1)}$$

$$\text{or } h_f = (h_1 - h_2) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - h_e \quad \text{——— (2)}$$

Recalling Manning's equation;

$$Q = \frac{1}{n} A R^{2/3} \sqrt{S_f} \quad \text{——— (3)}$$

$$\text{or } S_f = \frac{Q^2}{K^2} \quad \text{——— (4)}$$

where $K = f(n, A, P)$; Conveyance factor معامل النقل للقناة

$$\text{or; } K = \frac{1}{n} A R^{2/3} \quad \text{——— (5)}$$

From eq. (4);

$$\frac{h_f}{L} = S_f = \frac{Q^2}{K^2} \quad \text{——— (6)}$$

The eddy loss ـــــ فاقد الطاقة للدوامات can be estimated as;

$$h_e = K_e \left| \frac{V_1^2 - V_2^2}{2g} \right| \quad \text{——— (7)}$$

where K_e = eddy coefficient having values as tabulated below.

Cross-section Characteristic	Kc values	
	Expansion توسع	تقلص Contraction
Uniform منتظم	0	0
Gradual transion انتقال تدريجي	0.3	0.1
Abrupt transion مفاجئ	0.8	0.6

Equations (2), (6) and (7) together with Continuity equation $Q = VA$ enable the discharge Q to be estimated for known values of h_1, h_2, geometry of both sections as well as the roughness coefficient n.

المعادلات (2) و(6) و(7) بالاضافة الى معادلة الاتصال للنهر ومعرفة قيم مستويا سطح الماء عند المقطعين 1 و 2 ومعرفة الخصائص الهندسية (خصائص الشكل) ومعامل الخشونة (n) تمكن منها حساب التصريف بطريقة المحاولة التكرارية .

The discharge Q is calculated using iterative method according the following algorithm:

1- Assume $V_1 = V_2$; From equation (2)

$$h_f = F \quad , \text{where } F = \text{fall in water surface} = h_1 - h_2$$

or; $h_f = h_1 - h_2$; From eq (6) $S_f = \frac{h_f}{L} = \frac{Q^2}{K^2}$ or

$Q = \bar{K}\sqrt{S_f}$; where $\bar{K} = \sqrt{K_1 K_2}$; معدل هندسي

2- Calculate $V_1 = \frac{Q}{A_1}$, $V_2 = \frac{Q}{A_2}$, h_e from eq (7)

Find $h_{f_{new}}$ by full equation (2) ; estimate $Q = \bar{K}\sqrt{\frac{h_{f_{new}}}{L}}$

3- Repeat the steps (2) until obtaining equal successive values of h_f, then reported Q.

<u>Note:</u> The item used to stop iteration is h_f rather than Q, because it is more suitable for accuracy.

استعمل الـ h_f بدلاً من Q لايقاف المعادلة التكرارية للحل لانها ذات دقة أفضل في الحل

Example 1

During a flood flow, the depth of water in a 10m width of rectangular channel was found to be 3.0 m and 2.9m at two sections of 200m apart. The drop in water surface elevation was observed to be 0.12 m. Manning "n" is 0.025. Estimate the flood discharge Q ?

Solution:

section ①

$Y_1 = 3\,m$

$A_1 = 3\times 10 = 30\,m^2$

$P_1 = 3\times 2 + 10 = 16\,m$

$R_1 = \frac{A_1}{P_1} = 1.875\,m$

$K_1 = \frac{1}{0.025} \times 30 \times 1.875^{2/3}$

$K_1 = 1824.7$

section ②

$Y_2 = 2.9\,m$

$A_2 = 10\times 2.9 = 29\,m^2$

$P_2 = 2(2.9) + 10 = 15.8\,m$

$R_2 = \frac{A_2}{P_2} = 1.835$

$K_2 = \frac{1}{0.025} \times 29 \times 1.835^{2/3}$

$K_2 = 1738.9$

$$\bar{K} = \sqrt{K_1 K_2} = \sqrt{1824.7\,(1738.9)} = 1781.3$$

procedure steps

① $Q = 1781.3\sqrt{S_f}$

② $S_f = \frac{h_f}{200}$

③ $h_f = 0.12 + \left(\frac{V_1^2 - V_2^2}{2g}\right) - \cancel{h_e}$ neglect

ملاحظة: اذا لم تعط معلومات
عن Ke عندها تهمل he ، ويعلم
من مقارنة
التقصر أو التوسع
الحاصل ، بدءا المقطع
المقطع للقناة فيعرف من تغير
عرض القناة تقصراً أو توسعاً.

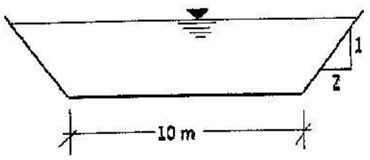
iteration	h_f (m)	S_f m/m	Q m³/s	$\frac{V_1^2}{2g}$ m	$\frac{V_2^2}{2g}$ m	h_f m	Verifying
1	0.12	0.000600	43.63	0.1078	0.1154	0.1124	NO
2	0.1124	0.0005614	42.21	0.1009	0.1080	0.1129	NO
3	0.1129	0.0005645	42.32	0.1014	0.1085	0.1129	Yes

thus ; Q = 42.32 m3/sec.

Example 2

A channel of trapezoidal section having base width (b=10m)and side slope Z=2(1V:2H). During flood period ,the following data are recorded for the two sections apart of 5km:

Section	Bed Level	Water Level
U/S	23.5 m	26.2 m
D/S	22.1 m	24.5 m
Manning Roughness coeff. n=0.025		
k_e=0.35 for expansion , k_e=0.12 for contraction		



Estimate the discharge of the channel using Slope- Area method.

Solution;

U/s	D/s
y_1= 2.7 m	y_2= 2.4 m
A_1=41.58 m^2	A_2=35.52 m^2
P_1= 22.074 m	P_2= 20.7331m
R_1= 1.8836 m	R_2= 1.7132 m
K_1= (1/0.025) A_1 $R_1^{(2/3)}$=2536.70	K_2= (1/0.025) A_1 $R_1^{(2/3)}$=2034.25
$\overline{K}=\sqrt{K_1K_2}$ =2271.625 ;	*k_e=0.12 (contraction)*

$$h_f = fall + \left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right) - ke\left|\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right| \quad ; L=5000\ m$$

$$h_f = 1.7 + \left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right) - 0.12\left|\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right| \quad ; \quad Q=\overline{K}\sqrt{S_f} \quad ; S_f=\frac{h_f}{L}$$

Trial	h_f(m)	Sx10^{-4}	Q (m^3/s)	h_{v1} (m)	h_{v2} (m)	h_f(m)	Verification
1	**1.7000**	3.4000	41.8867	0.0517	0.0709	**1.67855**	No
2	**1.67855**	3.3571	41.6216	0.0511	0.0700	**1.67882**	No
3	**1.67882**	3.35764	41.6249	0.0511	0.0700	**1.67881**	Yes*

(*) *Stop iteration when the difference vanished at fourth decimal.*

Q=41.6249 m^3/s

Stage - Discharge relationship علاقة المنسوب-التصريف

The measured discharges when plotted against the Corresponding stage gives effets of wide range of channel & flow parameters. The combined effect of these parameters is called <u>Control</u>.

التأثير المشترك لخصائص القناة والجريان يطلق عليه المتحكم. إذا كانت علاقة (Q-G) ثابتة مع الزمن فإن مقطع القياس يسمى بالمقطع المتحكم الدائم وإذا تغير مع الوقت تسمى مقطع التحكم المتنقل.

If the (Q-G) relationship for gauging station section is constant (does not cheang with time), the control is said to be <u>Permanent</u>. If it changed with time, it was called <u>Shifting control</u>.

(1) Permanent control المقطع المتحكم الدائم

Most of streams and rivers, especially nonalluvial rivers exhibits permanent control and a single-valued relation cane be expressed as

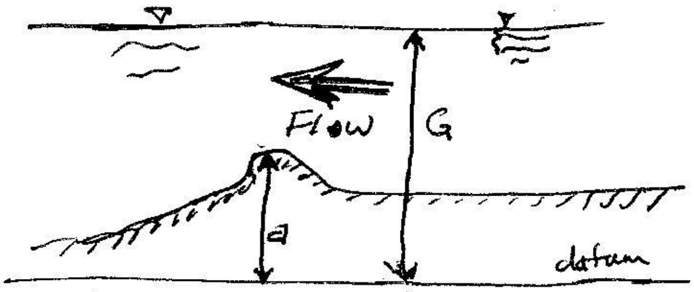
$$Q = \alpha (G - G_0)^{\beta} \qquad (1)$$

Q: discharge, m^3/s

G: gauge height (stage), m

G_0: a constant which represent gauge reading corresponding to zero discharge

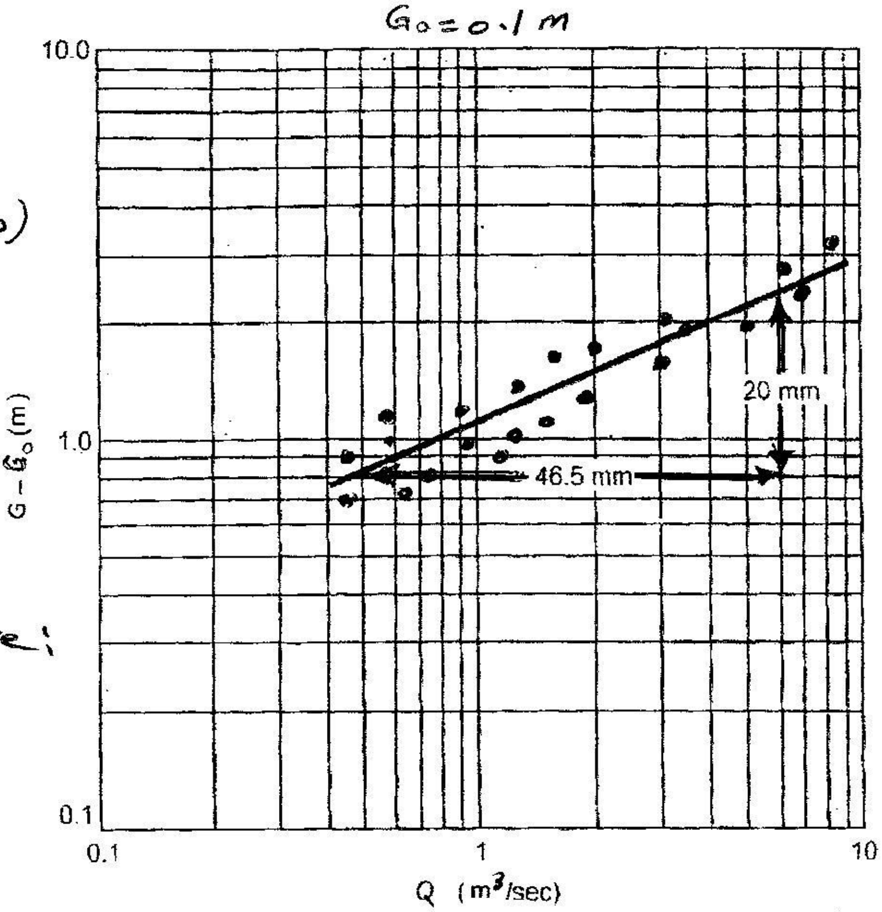
α, β = are constants of the rating curve (Q-G retionship)



The constant α and β can be computed by either graphical representation or by the least-square-erron method.

① Graphical method

plot $(G-G_0)$ against Q on log-Log paper



The linearized equation of equation (1) is:

$$\log Q = \log\alpha + \beta \log(G-G_0)$$

β = slope of the best fit line of plotted values.

in example shown in figure;

$$\beta = \frac{\text{horizontal distance}}{\text{Vertical distance}}$$

$$\beta = \frac{46.5}{20} = 2.325$$

for $(G-0.1) = 1.0$, the corresponding $Q = 0.8$

or $\log Q = \log\alpha + \log(G-G_0)$

$\log 0.8 = \log\alpha + \log(1.0)$

$\log\alpha = -0.097$

or $\alpha = 0.8$

thus;

$$Q = 0.8(G-0.1)^{2.325}$$

L. Regression Method

The regression involves two variables one dependent (Q) and one independent (G−G₀). IF the equation of the generated curve relates to a straight line, the regression is called linear.

Consider the rating curve equation $Q = \alpha (G - G_0)^{\beta}$. The first step is to apply the linearization to this power equation;

$$\log Q = \log \alpha + \beta \log (G - G_0)$$

$$\left.\begin{aligned} \text{set } \log Q &= y \\ \log (G - G_0) &= x \\ \beta &= b \\ \log \alpha &= a \end{aligned}\right\} \Rightarrow y = a + bx \quad (\text{linear equation})$$

The regression procedure is applied to this linear equation

$$\Sigma y = na + b\,\Sigma x \quad \text{——— (1)}$$

$$xy = ax + bx^2$$

$$\Sigma xy = a\,\Sigma x + b\,\Sigma x^2 \quad \text{——— (2)}$$

From both equations above, one can get;

$$\left.\begin{aligned} a &= \frac{\Sigma y - b\,\Sigma x}{n} \\ b &= \frac{n\,\Sigma(xy) - (\Sigma x)(\Sigma y)}{n\,\Sigma x^2 - (\Sigma x)^2} \end{aligned}\right\} \quad \text{——— (3)}$$

$$r = \sqrt{\frac{\Sigma (y_{ob} - y_{cal})^2}{\Sigma (y_{ob} - \bar{y})^2}} = b \sqrt{\frac{N(\Sigma x^2) - (\Sigma x)^2}{N(\Sigma y^2) - (\Sigma y)^2}} \quad \text{——— (4)}$$

or; $$r = \frac{n\,\Sigma xy - \Sigma x\,\Sigma y}{\sqrt{n\,\Sigma x^2 - (\Sigma x)^2 / (n\,\Sigma y^2 - (\Sigma y)^2)}}$$; r = is regression coefficient

The regression coefficient ranged between 0.6 to 1.0. In Hydrologic representati $r<0.7$ is unacceptable.

To calculate α, and β, the following statistical parameter must be found :-

① Σx ② Σx^2 ③ Σy ④ Σxy ⑤ Σy^2 and may be $\bar{Y}$ and $\bar{X}$ (the mean values of each x and y values). since from eq (3) ($a = \bar{Y} + b\bar{X}$). Note that "n" is the number of given data.

Example:

The following are the coordinates of smooth curve that well-represent the stage-discharge data:

Stage(G), m	30	33.5	35	37	39.5	41	42.5
Discharge(Q), (m^3/s)	80	120	190	220	250	300	320

- *The zero discharge elevation "G_o" is equal to 20 m.*
- *Find the regression equation $Q = \alpha(G - G_o)^\beta$,*
- *What is the discharge at G=50m ?*

Go = 20.0	n = 7

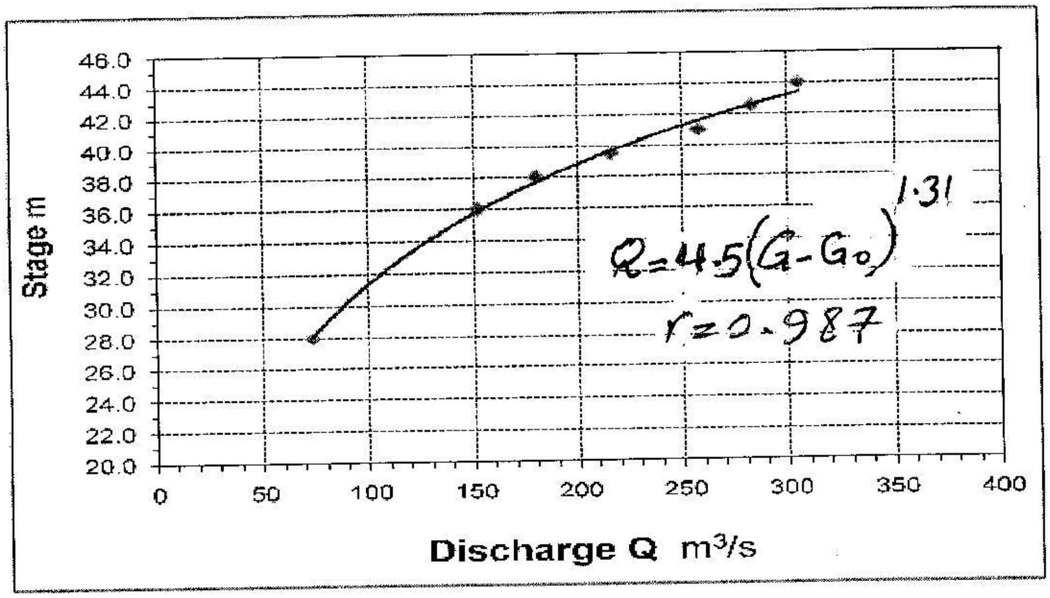
X= G m	y = Q m^3/s	x=G-Go	x_1=log x	y_1=log y	$x_1 y_1$	x_1^2	y_1^2
28.0	73	8.0	0.90309	1.86332	1.68275	0.81557	3.47197
36.0	152	16.0	1.20412	2.18184	2.62720	1.44990	4.76044
38.0	180	18.0	1.25527	2.25527	2.83098	1.57571	5.08625
39.5	216	19.5	1.29003	2.33445	3.01153	1.66419	5.44967
41.0	258	21.0	1.32222	2.41162	3.18869	1.74826	5.81591
42.5	283	22.5	1.35218	2.45179	3.31526	1.82840	6.01126
44.0	305	24.0	1.38021	2.48430	3.42886	1.90498	6.17175

Calculated parameters:

Σy = 15.98260 Σx= 8.70713 Σxy= 20.08527 Σx^2= 10.98702

Σy^2= 36.76725   b=β= 1.30985285   a= 0.653934 α= 4.50748

r= 0.9873 $Q_{G=50} = \alpha(G-Go)^\beta =$ 387.923 m^3/s



Stage for Zero discharge (G_o)

The stage height for zero discharge is a hypothetical parameter that cannot be measured in the field. The alternative methods used to evaluate G_o are:

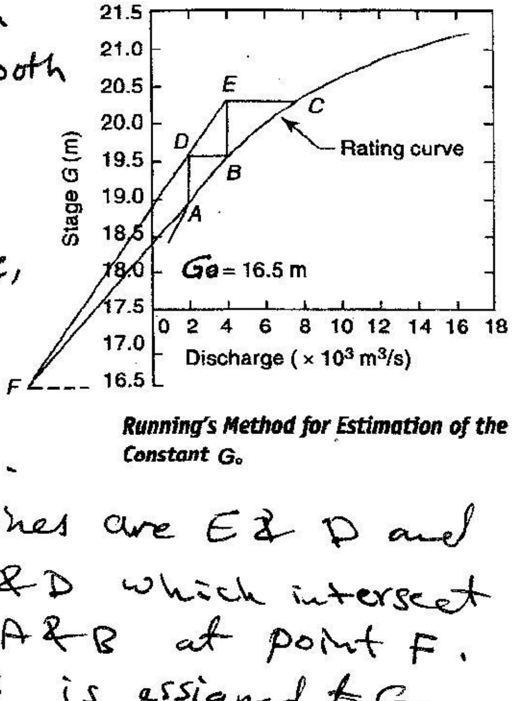
① Trial-and-error method:

plot Q vs. G on an arithmetic paper, then draw the best fit curve. Extrapolate the curve (by: eye judgment) to find the value of G corresponding to $Q = 0$, i.e. G_o. Using this value of G_o to plot on log-log paper Q vs. $(G - G_o)$. Verify whether the data plots as straight line or not. If not, attempt with neighbourhood value close to previous value of G_o, above or below until getting a straight line of graph.

② Running Method:

The Q vs. G data are plotted to an arithmetic scale and draw a smooth curve through the plotted data. Three points A, B, C are selected such that their discharges are in geometric relationship; i.e,

$Q_B = \sqrt{Q_A Q_C}$, or $\frac{Q_A}{Q_B} = \frac{Q_B}{Q_C}$.



Running's Method for Estimation of the Constant G_o

At A & B draw vertical lines
At C & B draw horizontal lines.
Two intersection between these lines are E & D and draw a line passes through E & D which intersect with other line passes through A & B at point F. The value of F at vertical axis is assigned to G_o.
This method assumes that the lower part of the curve is parabola.

③ Arithmetic Method:

Choose the three points as pointed out in Running method

$$G_o = \frac{G_A G_C - G_B^2}{(G_A + G_C) - 2G_B}$$

(2) Shifting Control المقطع المتحكم المتنقل

The control of gauging station can be change due to:

(i) changing characteristic caused by weed growth ترابي ج
(ii) aggradation or degradation phenomenon in alluvial channel.
النحر والترسيب في القنوات الرسوبية.
(iii) Variable backwater effect; and . تأثير الماء الخلفي
(iv) unsteady flow effects of rapid change of stage.
تأثير الجريان غير الثابت على المناسيب عند تغير المنسوب بسرعة كبيرة

There are no permenant corrective measure due to (i) & (ii), but the causes (iii) & (iv), the shifting control can be correct for the situations as below:

Backwater effects. تأثير الماء الخلفي

If the shifting control is due to backwater curve, the same stage will indicate different discharges depend on backwater effects. to remedy the problem a secondary gauge is installed D/S the gauging station and readings of both gauges are taken (stages) - The difference is called fall (F) of the water within channel reach.

$Q = f(G, F)$ as in Fig (1)

plot the three values Q, G, and F and interpolat a single value for F_o called Standard fall, then

$$\frac{Q}{Q_o} = \left(\frac{F}{F_o}\right)^m$$

Q_o = normalized discharge
Q = actual discharge
on can construct a plot as in Fig (2) between Q/Q_o & F/F_o to make the computation easy to do. ($m \approx 1/2$).

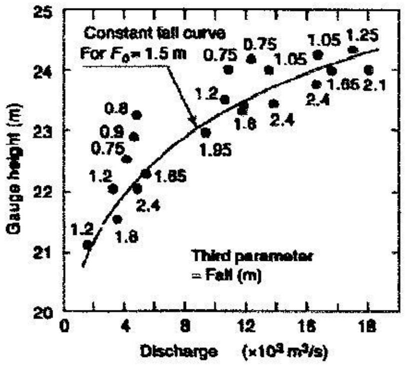


Fig. 1 Backwater Effect on a Rating Curve—Normalised Curve

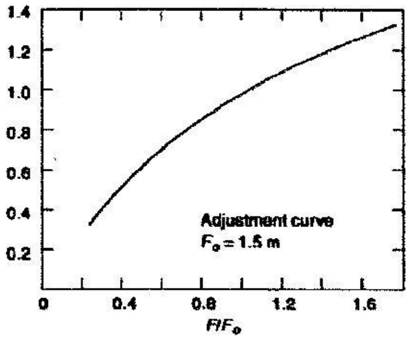


Fig. 2 Backwater Effect on a Rating Curve – Adjustment Curve

Example

A secondary gauge was used D/S the main gauge to provide corrections to the gauge-discharge relationship due to backwater effects. The following data are given:

main gauge level	Secondary gauge level	Q m^3/s
86.0	85.0	275
86.0	84.8	600

If the main gauge is still 86.0 but secondary gauge is 85.3 estimate the real discharge.

Sol.

$F_1 = 68 - 85.5 = 0.5\,m \Rightarrow Q = 275\ m^3/sec$

$F_2 = 86 - 84.8 = 1.2\,m \Rightarrow Q = 600\ m^3/sec$

$\frac{Q_1}{Q_2} = \left(\frac{F_1}{F_2}\right)^m \; ; \; \frac{275}{600} = \left(\frac{0.5}{1.2}\right)^m \Rightarrow m = 0.891$

$\frac{Q}{Q_2} = \left(\frac{F}{F_2}\right)^{0.891} \Rightarrow Q = 600\left(\frac{0.7}{1.2}\right)^{0.891} = 371\ m^3/sec.$

unsteady Flow effects تأثير الجريان غير الثابت

when flood wave advanced to gauge station, the velocity is higher than the velocity of steady flow. Thus, higher value of discharge was expected for same stage over the steady discharge. If the wave converses (retarding phase) the velocity of wave become low and discharge is lower than equivalent steady flow. Therefore the relationship of stage-discharge for unsteady flow is not single-valued as in steady flow. To calculate unsteady flow Q in term of steady flow Q_n (normal), the following equation can be used:

$$\frac{Q}{Q_n} = \left(1 + \frac{dh/dt}{V_w S_o}\right)^{1/2}$$

S_o = channel slope (water surface slope was taken, usually)

$\frac{dh}{dt}$ = rate of change of stage w.r.t. time.

V_w = flood velocity (usually assumed to be $1.4\,V$, where V is calculated from Manning equation at estimated Q).

Extrapolation of Rating Curve تمديد منحنى التصرف

There are two main methods;

1. Logarithmic expression created by least square method as reviewed previously. تم شرحه سابقا.
2. Conveyance method طريقة النقل

ⓐ Calculate $K = \frac{1}{n} A R^{2/3}$ for different stage values

ⓑ plot K vs. G

ⓒ plot a smooth curve fitted the plotted points

ⓓ Calculate $S_f = \frac{Q^2}{K^2}$

ⓔ plot a smooth curve between $\frac{1}{n}\sqrt{S_f}$ & stage values.

ⓕ the curve can be extrapolate in trend, and remember that S_f approaches constant value for high stage.

Now; to calculate Q for extended value at certain stage

ⓐ from Fig(3) Find K

ⓑ from Fig(4) Find $\sqrt{S_f}$

$\Big\}$ then $Q = K\sqrt{S_f}$

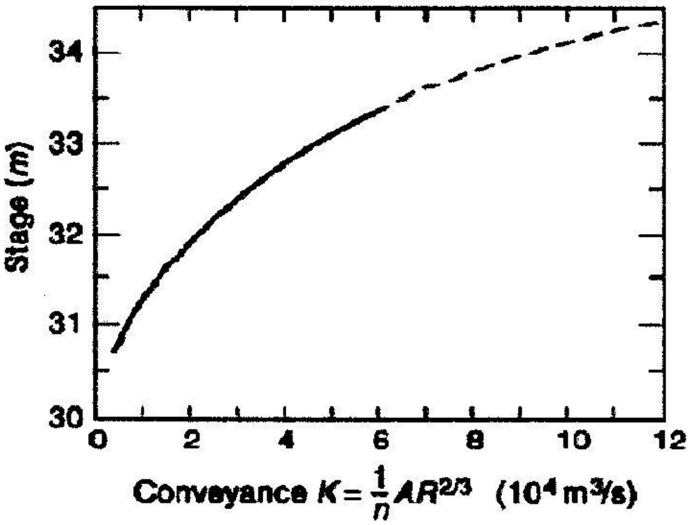


Fig. 3 Conveyance Method of Rating Curve Extension: K vs Stage

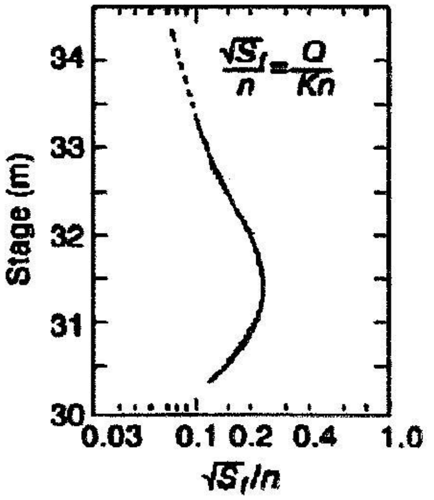


Fig. 4 Conveyance Method of Rating Curve Extension: S_f vs Stage