Stress Relaxation Behavior

Relaxation is the counterpart of creep. It is the decline in stress with time, in response to a constant applied strain, at constant temperature . Stress relaxation behavior is one of the fundamental natures of polymers. At the time of stress relaxation, polymer molecules rotate due to the applied strain. The rearrangement of molecules continues as a function of time and reduces stress but maintains the strain level same. Figure below schematically illustrates the stress relaxation behavior of three different types of materials: elastic solid, viscous fluid, and viscoelastic material. Purely elastic material holds a constant stress level for an extended period of time; purely viscous material dissipates stress immediately and will come back to zero stress level, while viscoelastic material relaxes stress with time. Relaxation behavior of polymers is more often expressed by the stress relaxation modulus which changes as a function of time.

$$E_{rel.}(t) = \frac{\sigma(t)}{\varepsilon o}$$

Where $E_{rel.}(t)$ is the relaxation modulus, $\sigma(t)$ is the time-dependent stress, and ε_o is the applied constant strain.

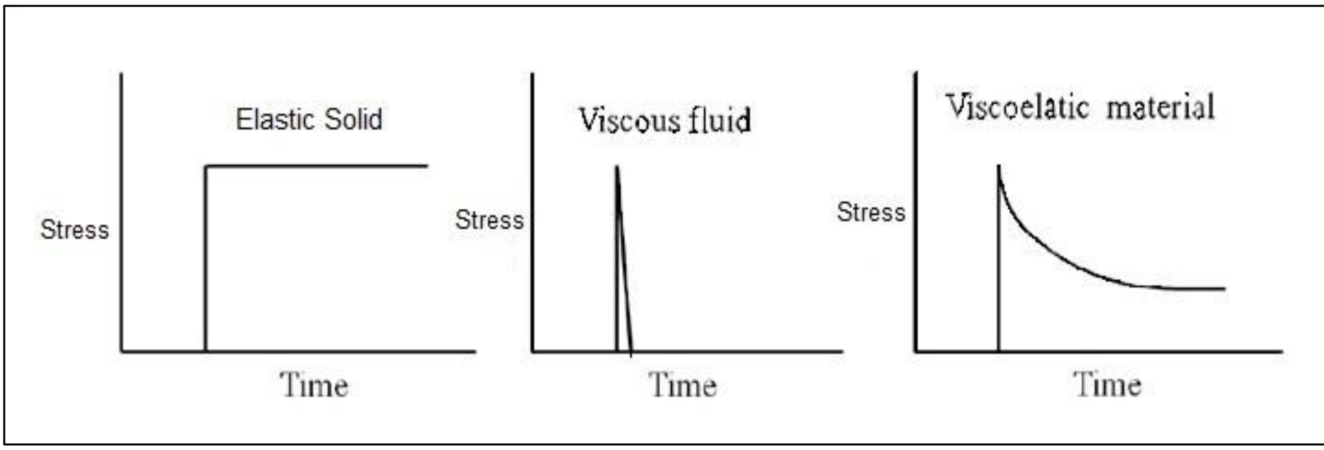

Figure ,Stress relaxation behavior of three different types of materials subjected to same amount of strain.

Viscoelastic Models

The stress strain time relation of viscoelastic material has been analyzed with the aid of mechanical models where the stress and strain instead of force and deformation of model are used. All linear viscoelasticity models are made up of linear spring and linear viscoelastic dashpot. The stress strain time relation of viscoelastic material has been analyzed with aid of mechanical models where the stress and strain instead of force and deformation of model are used.

In the linear spring shown in Figure. A spring models a perfectly elastic solid. The following relation can be written[1] :

$$\sigma = E\,\varepsilon$$

where: σ :stress , E : linear spring constant or Young's modulus, ε :strain

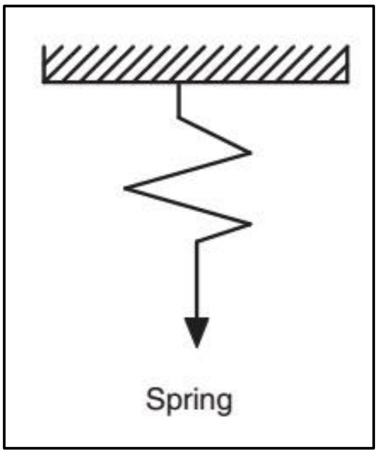

Figure linear spring

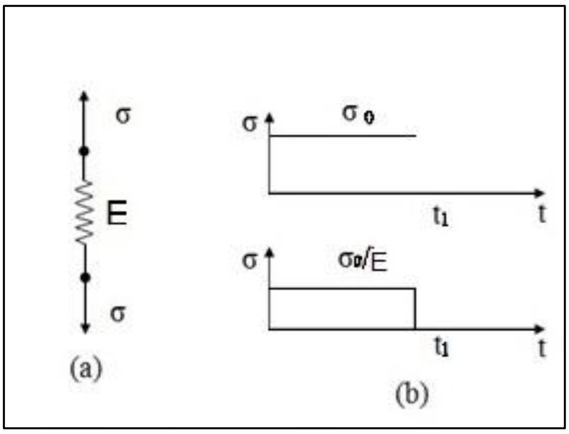

Figure behavior of linear spring

The spring element exhibits instantaneous elasticity and instantaneous recovery as shown in Figure. A linear dashpot element is shown in Figure . A dashpot model is a perfectly viscous material. where :

$$\sigma = \eta \frac{d\varepsilon}{dt} = \eta\,\dot{\varepsilon}$$

Where:

η = coefficient of viscoelasticity , $\dot{\varepsilon}$ =strain rate.

Dashpot will be deformed continuously at constant rate when it is subjected to a step of constant stress as shown in Figure

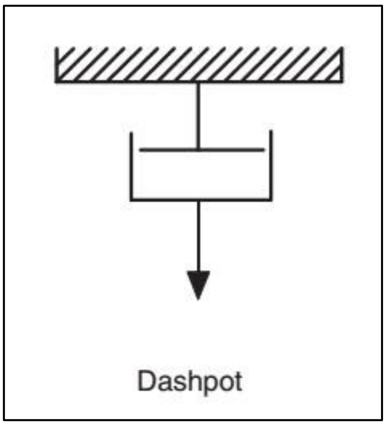

Figure Linear dashpot

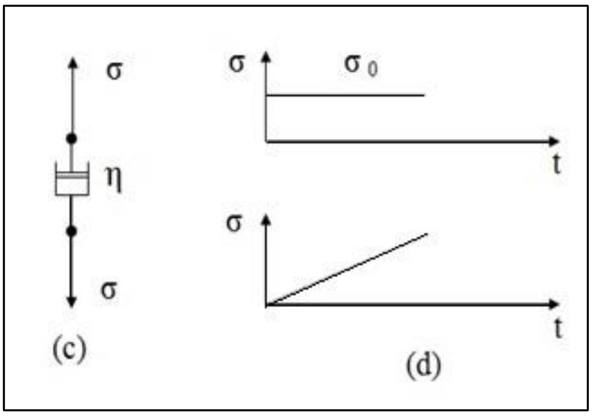

Figure Behavior of linear dashpot

Maxwell Model

The Maxwell model represents a viscoelastic material in which a mechanical model would be represented as a spring and a dash-pot in series. This model is intended to model a combination of elastic deformation and viscous flow The Maxwell model is a two element model consisting of linear spring and linear dashpot element connected in series as shown in Figure . By examining the Maxwell model, it can be seen that the stress throughout the model is constant, while the strain would equal the sum of the two strain components, the Maxwell model may be expressed in terms of stress

The stress strain relation of spring and dashpot is derived from Eq.

as follows:

$\sigma = E\,\varepsilon_2$ Or $\varepsilon_2 = \frac{\sigma}{E}$

$\sigma = \eta\,\dot{\varepsilon}$ Or $\dot{\varepsilon}_1 = \frac{\sigma}{\eta}$

Where: ε_2 = the strain in spring ، $\dot{\varepsilon}_1$= the strain in dashpot.

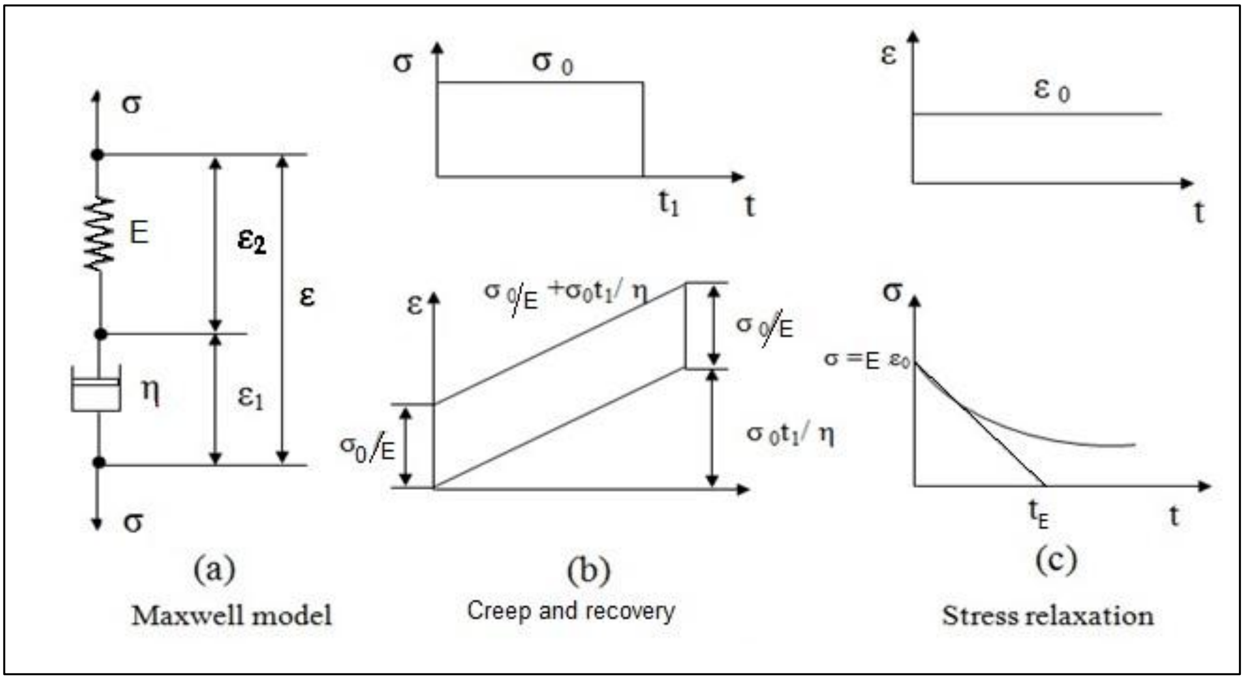

Figure Behavior of Maxwell model

Since both elements are connected in series, the total strain is:

$\varepsilon = \varepsilon_2 + \varepsilon_1$

Or the rate of strain is:

$$\dot{\varepsilon} = \dot{\varepsilon}_2 + \dot{\varepsilon}_1$$

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} \qquad ..... \quad *$$

The equation of strain during time can be found by solving (*) for the following boundary initial condition: At $t=0$, $\sigma = \sigma_o$

After solving the first order differential equation (3.8), the strain time relation can be obtained as follows:

$$\varepsilon(t) = \frac{\sigma o}{E} + \frac{\sigma o}{\eta} t$$

Also if the Maxwell is subjected to a constant strain (ε_o at t=0.0) with initial stress value, the stress response can be obtained as :

$$\sigma(t) = \sigma_0\ e^{-Et/\eta}$$

Kelvin Model

The Kelvin model (Voigt Model) used to model a damped elastic response is a parallel arrangement of the spring and dash-pot elements. In this arrangement, the strain in each individual component is equivalent to the total strain, and the total stress is the sum of the stresses in the individual components. By analyzing the creep modulus and relaxation modulus, further insight may be gained regarding the viscoelastic behavior of polymers.

In this model, spring and dashpot element are connected in parallel as shown in Figure.According to Eq. , the strain relation is :

$$\sigma_1 = E\,\varepsilon$$

$$\sigma_2 = \eta\,\dot{\varepsilon}$$

The total stress in the Kelvin model is:

$$\sigma = \sigma_1 + \sigma_2$$

$$\dot{\varepsilon} + \frac{E}{\eta}\varepsilon = \frac{\sigma}{\eta}$$

Equation can be solved as a first order differential equation for creep test for the following initial condition:

At t=0.0 and $\sigma = \sigma_o$. The solution being:

$$\varepsilon(t) = \frac{\sigma o}{E}\left(1 - e^{-Et/\eta}\right)$$

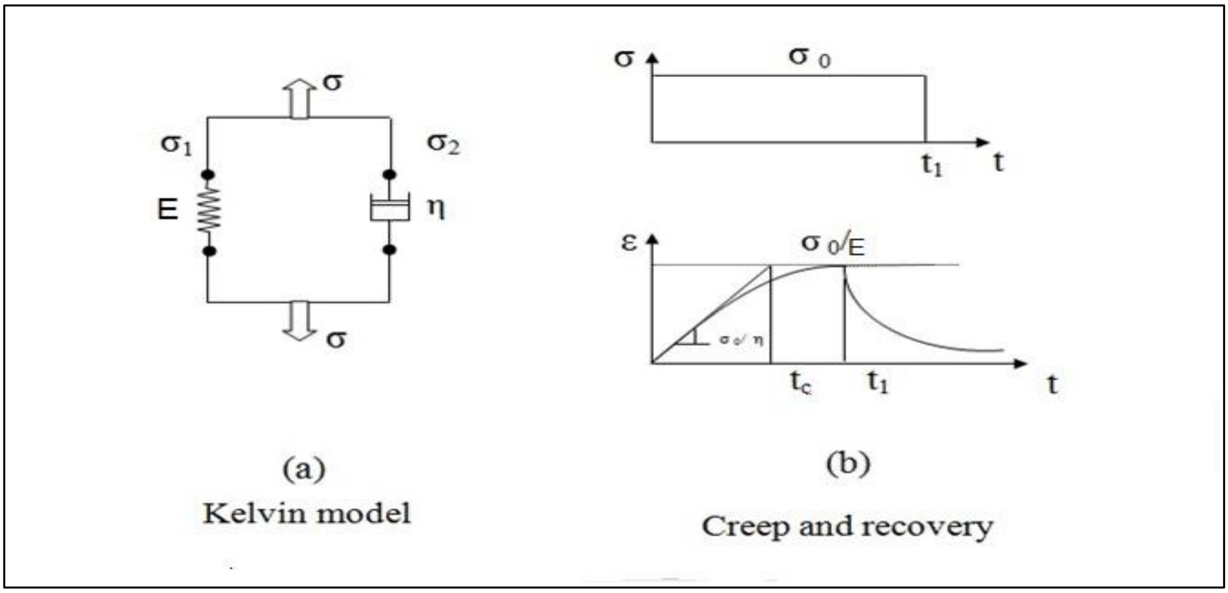

Figure Behavior of Kelvin model

Burger Four – Element Model

The burgers model is shown in Figure where the Maxwell and Kelvin model are connected in series.

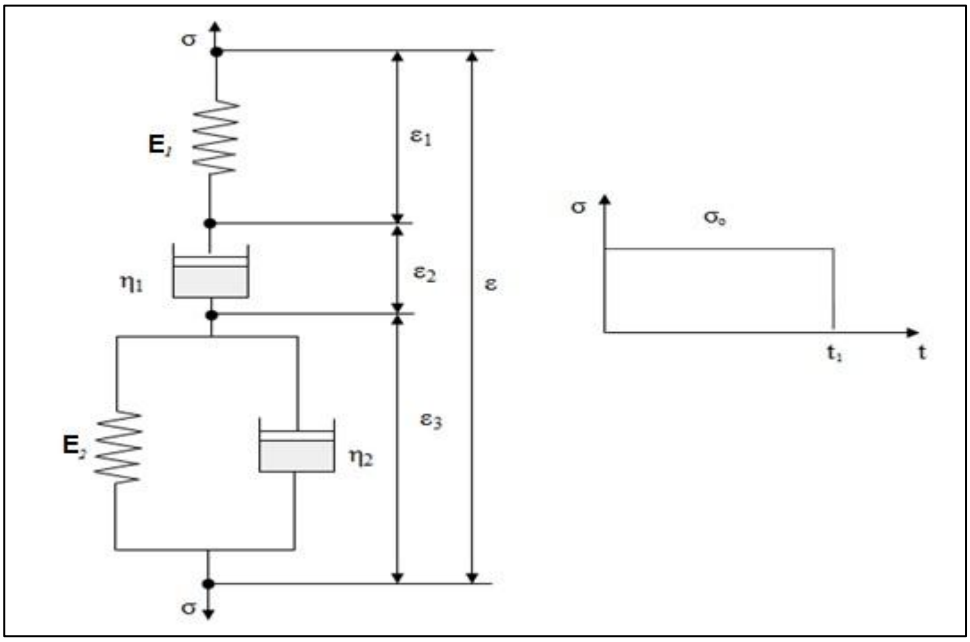

Figure Burger Model

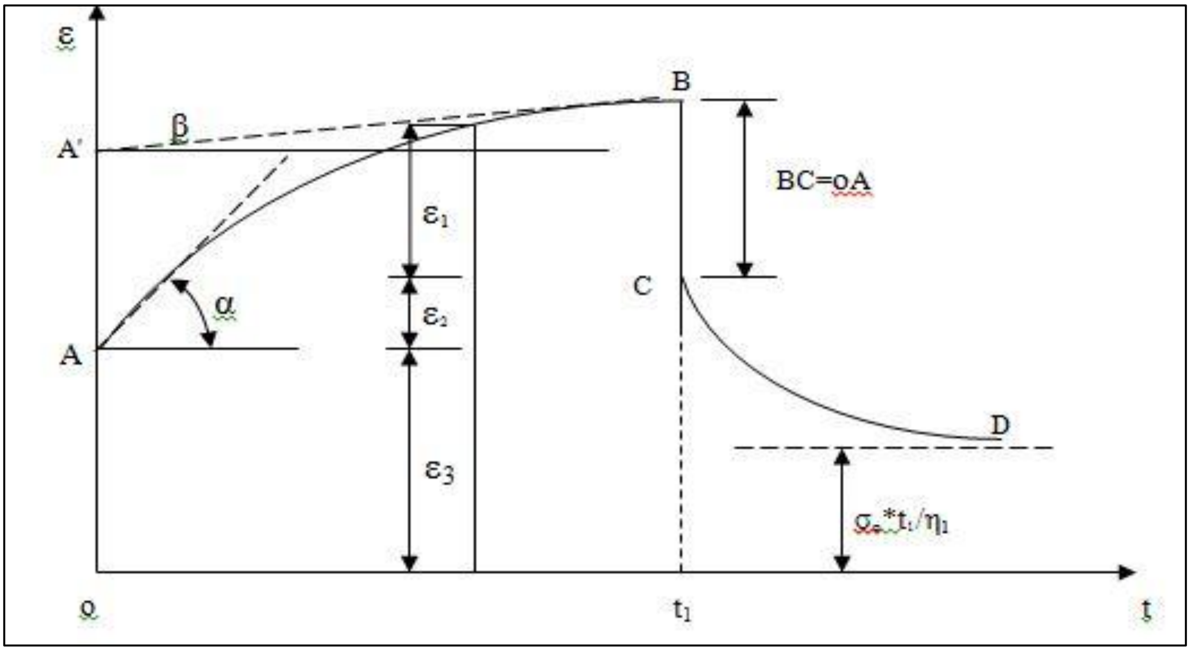

Figure Behavior of a Burger model (creep and recovery)

Standard Linear Solid

Most polymers do not exhibit the unrestricted flow permitted by the Maxwell model. Placing a spring in parallel with the Maxwell unit gives a very useful model known as the "Standard Linear Solid" (S.L.S.) shown in Figure . This spring has stiffness, so named because it provides an "equilibrium" or rubbery stiffness that remains after the stresses in the Maxwell arm have relaxed away as the dashpot extends.

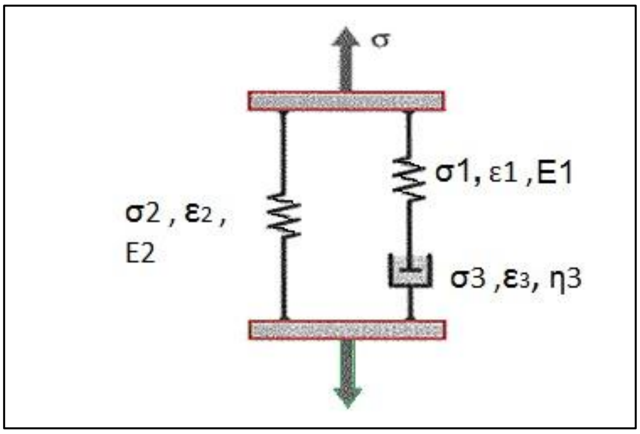

Figure the Maxwell form of the Standard Linear Solid .

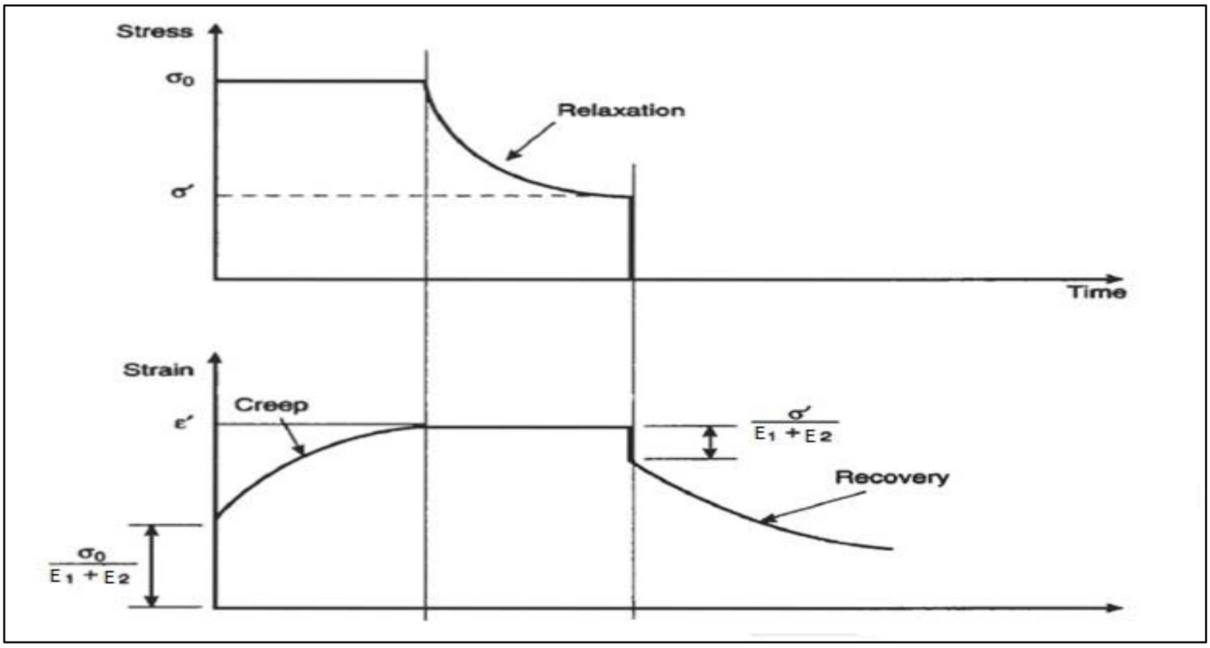

Figure. response of standard linear solid