-Mustansiriyah University
Faculty of Engineering
Mat. Eng. Dept.
Exam

Subject: Engineering Analysis
Max. Time: 90 min.
Class: 3th year
Date: 24 / 12 / 2018

2018– 2019

**Max.Mark: 15 %**

Q1/ Find the real form of the Fourier series for the function:

$f(x) = |\sinh ax|\,, \quad -\pi < x < \pi$ , and then obtain complex form.

Q2/ Determine the Eigen values and Eigen vectors for $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and then find $A^{-2}$?

Q3/ A) Find K if the following system has infinitely many solutions:

$6x - y + z = 13$ , $x + y + z = 9$ , $10x + y - k\,z = 19$

B) Solve this system if $k = 1$ ?

**Examiner: Baydaa Jaber** **Head of Department:**

حلول اسئلة امتحان الفصل الاول

Q2/

$\because f(x) = |\sinh ax| = \begin{cases} \sinh ax & \sinh ax \geq 0 \quad [0,\pi) \\ -\sinh ax & \sinh ax < 0 \quad (-\pi, 0) \end{cases}$

الفترة متناظرة .

$\therefore f(-x) = |-\sinh ax| = |\sinh ax| \Longrightarrow$ even

$\therefore b_n = 0$

$\therefore a_0 = \frac{2}{\pi}\int_0^{\pi} \sinh ax \cdot \frac{a}{a} = \frac{2}{a\pi} \cosh ax \Big]_0^{\pi}$

$= \frac{2}{a\pi}\left[\cosh a\pi - 1\right]$

$a_n = \frac{2}{\pi}\int_0^{\pi} \sinh ax \cos \frac{n\pi x}{\pi}\, dx$

$= \frac{\cancel{2}}{\pi}\int_0^{\pi} \left(\frac{e^{ax} - e^{-ax}}{\cancel{2}}\right) \cos nx \, dx$

$\left\{ \int e^{ax} \cos bn = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b \sin bx\right] \right.$

$= \frac{1}{\pi}\left[\int_0^{\pi} e^{ax} \cos nx - \int_0^{\pi} e^{-ax} \cos nx\right] dx$

$= \frac{1}{\pi}\left[\frac{e^{ax}}{a^2+n^2}\left(a\cos nx + n \sin nx\right]_0^{\pi} - \frac{e^{-ax}}{a^2+n^2}\left[-a\cos nx + n \sin nx\right]_0^{\pi}\right.$

$$= \frac{1}{\pi(a^2+n^2)}\left[e^{a\pi}\left(a(-1)^n\right) - a - \left[e^{-a\pi}\left(-a(-1)^n\right) + a\right]\right.$$

$$= \frac{1}{\pi(a^2+n^2)}\left[ae^{a\pi}(-1)^n - a + ae^{-a\pi}(-1)^n - a\right]$$

$$= \frac{1}{\pi(a^2+n^2)}\left[a(-1)^n\left[e^{a\pi} + e^{-a\pi}\right] - 2a\right]$$

$$a_n = \frac{2a}{\pi(a^2+n^2)}\left[(-1)^n \cosh a\pi - 1\right]$$

$$C_0 = \frac{a_0}{2} \Rightarrow C_0 = \frac{1}{a\pi}\left[\cosh a\pi - 1\right]$$

$$C_n = \frac{a_0}{2} = C_{-n} = \frac{a}{\pi(a^2+n^2)}\left[(-1)^n \cosh a\pi - 1\right]$$

Find the eigen Values and eigen vectors for

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \text{ and then find } A^{-2}$$

$$\therefore |A-\lambda I| = 0 \Rightarrow \begin{bmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{bmatrix}$$

$$\Rightarrow (2-\lambda)[(1-\lambda)(2-\lambda)] - (1-\lambda) = 0$$

$$\therefore (2-\lambda)[2-3\lambda+\lambda^2] - 1 + \lambda = 0 \Rightarrow 4 - 6\lambda + 2\lambda^2 - 2\lambda + 3\lambda^2 - \lambda^3 - 1 + \lambda = 0$$

$$-\lambda^3 + 5\lambda^2 - 7\lambda + 3 = 0 \Rightarrow \lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

$$(\lambda-1)(\lambda^2-4\lambda+3) = 0 \Rightarrow (\lambda-1)(\lambda-3)(\lambda-1) = 0$$

$$\begin{array}{r} \lambda^2 - 4\lambda + 3 \\ (\lambda-1)\ \overline{)\ \lambda^3 - 5\lambda^2 + 7\lambda - 3} \\ -\lambda^3 \pm \lambda^2 \\ \hline -4\lambda^2 + 7\lambda - 3 \\ \pm 4\lambda^2 \mp 4\lambda \\ \hline 3\lambda - 3 \\ 3\lambda - 3 \\ \hline 0 \quad 0 \end{array}$$

$\left.\begin{matrix} \lambda = 1 \\ \lambda = 3 \end{matrix}\right\}$ If $\lambda = 1 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{R_3 = R_3 - R_1}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow x_1 + x_2 + x_3 = 0 \Rightarrow x_1 = -\alpha_1 - \alpha_2$$

$$x_2 = \alpha_1, \ \alpha_3 = \alpha_2$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\alpha_1 - \alpha_2 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \alpha_1 \underline{\underline{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}}} + \alpha_2 \underline{\underline{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}}$$

If $\lambda = 3 \Rightarrow \begin{bmatrix} -1 & 1 & 1 \\ 0 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix} \xrightarrow{R_3 = R_3 + R_1} \begin{bmatrix} -1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & 2 & 0 \end{bmatrix} \Rightarrow \begin{matrix} -x_1 + x_2 + x_3 = 0 \\ -2x_2 = 0 \\ 2x_2 = 0 \end{matrix}$

If $x_3 = \alpha$
$x_2 = 0$

$$\therefore -x_1 + 0 + \alpha = 0 \Rightarrow x_1 = \alpha \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore \lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

$\therefore A^3 - 5A^2 + 7A - 3I = 0 \} A^{-1}$

$\Rightarrow A^2 - 5A + 7I - 3A^{-1} = 0 \Rightarrow A^{-1} = \frac{1}{3}[A^2 - 5A + 7I] * A^{-1}$

$\therefore A^{-2} = \frac{1}{3}[A - 5I + 7A^{-1}] = \frac{1}{3}[A - 5I + 7[\frac{1}{3}[A^2 - 5A + 7I]]$

$\Rightarrow A^{-2} = \frac{1}{3}[A - \overset{\frac{15}{3}}{5}I + \frac{7}{3}A^2 - \frac{35}{3}A + \frac{49}{3}I]$

$A^{-2} = \frac{1}{3}[\frac{7}{3}A^2 - \frac{32}{3}A + \frac{34}{3}I] \Rightarrow A^{-2} = [\frac{7}{9}A^2 - 8A + \frac{34}{3}I]$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+1 & 2+1+1 & 2+2 \\ 0 & 1 & 0 \\ 2+2 & 1+1+2 & 1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 5 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix}$$

$$\Rightarrow A^{-2} = \frac{7}{9}\begin{bmatrix} 5 & 5 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} - 8\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \frac{34}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-2} = \begin{bmatrix} \frac{35}{9} - 16 + \frac{34}{3} & \frac{35}{9} - 8. & \frac{28}{9} - 8 \\ 0 & \frac{7}{9} - 8 + \frac{34}{3} & 0 \\ \frac{28}{9} - 8 & \frac{28}{9} - 8 & \frac{35}{9} - 16 + \frac{34}{3} \end{bmatrix} =$$

$|A| = 0$

Q3

ⓐ
$$\begin{bmatrix} 6 & -1 & 1 \\ 1 & 1 & 1 \\ 10 & 1 & -k \end{bmatrix} = 0$$

$$6[-k-1] + (-k-10) + (1-10) = 0$$

$$-6k - 6 - k - 10 - 9 = 0 \Rightarrow -7k = +25 \quad \boxed{k = \frac{-25}{7}}$$

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ⓑ
$$\left[\begin{array}{ccc|c} 6 & -1 & 1 & 13 \\ 1 & 1 & 1 & 9 \\ 10 & 1 & -1 & 19 \end{array}\right] \xrightarrow{R_3 = R_3 - 10R_2} \left[\begin{array}{ccc|c} 6 & -1 & 1 & 13 \\ 1 & 1 & 1 & 9 \\ 0 & -9 & -11 & -71 \end{array}\right]$$

$$\xrightarrow{R_2 = 6R_2 - R_1} \left[\begin{array}{ccc|c} 6 & -1 & 1 & 13 \\ 0 & 7 & 5 & 41 \\ 0 & -9 & -11 & -71 \end{array}\right] \xrightarrow{R_3 = 7R_3 + 9R_2}$$

$-77 + 45$

$32$

54
13
41

$$\left[\begin{array}{ccc|c} 6 & -1 & 1 & 13 \\ 0 & 7 & 5 & 41 \\ 0 & 0 & -32 & -128 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 6 & -1 & 1 & 13 \\ 0 & 7 & 5 & 41 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$z = 4$

$7y + 20 = 41$

$7y = 21$

$y = 3$

$6x - 3 + 4 = 13$

$6x = 12$

$x = 2$