Chapter 6. Control Charts for Attributes

Control Chart for Fraction Nonconforming

Fraction nonconforming is based on the binomial distribution.
n: size of population
p: probability of nonconformance
D: number of products not conforming
Successive products are independent.

$$P\{D = x\} = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, \ldots, n \qquad (6\text{-}1)$$

Mean of *D* = *np*
Variance of *D* = *np(1-p)*

Sample fraction nonconformance

$$\hat{p} = \frac{D}{n} \tag{6-2}$$

Mean of p̂: $\mu = p$ (6-3)

Variance of p̂: $\sigma_{\hat{p}}^2 = \frac{p(1-p)}{n}$ (6-4)

w: statistics for quality
Mean of *w*: μw
Variance of *w*: σ_w^2
L: distance of control limit from center line (in standard deviation units)

$$\begin{aligned} \text{UCL} &= \mu_w + L\sigma_w \\ \text{Center line} &= \mu_w \\ \text{LCL} &= \mu_w - L\sigma_w \end{aligned} \tag{6-5}$$

If *p* is the true fraction nonconformance:

Fraction Nonconforming Control Chart: Standard Given

$$\begin{aligned} \text{UCL} &= p + 3\sqrt{\frac{p(1-p)}{n}} \\ \text{Center line} &= p \\ \text{LCL} &= p - 3\sqrt{\frac{p(1-p)}{n}} \end{aligned} \tag{6-6}$$

If p is not know, we estimate it from samples.
m: samples, each with n units (or observations)
D_i: number of nonconforming units in sample i

$$\hat{p}_i = \frac{D_i}{n} \qquad i = 1, 2, \ldots, m$$

Average of all observations:

$$\overline{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m} \tag{6-7}$$

Fraction Nonconforming Control Chart: No Standard Given

$$\begin{aligned} \text{UCL} &= \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \\ \text{Center line} &= \overline{p} \\ \text{LCL} &= \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \end{aligned} \tag{6-8}$$

Example 6-1. 6-oz cardboard cans of orange juice

Table 6-1 Data for Trial Control Limits, Example 6-1, Sample Size $n = 50$

Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$	Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$
1	12	0.24	17	10	0.20
2	15	0.30	18	5	0.10
3	8	0.16	19	13	0.26
4	10	0.20	20	11	0.22
5	4	0.08	21	20	0.40
6	7	0.14	22	18	0.36
7	16	0.32	23	24	0.48
8	9	0.18	24	15	0.30
9	14	0.28	25	9	0.18
10	10	0.20	26	12	0.24
11	5	0.10	27	7	0.14
12	6	0.12	28	13	0.26
13	17	0.34	29	9	0.18
14	12	0.24	30	6	0.12
15	22	0.44		347	$\bar{p} = 0.2313$
16	8	0.16			

$$\overline{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{347}{(30)(50)} = 0.2313$$

$$\overline{p} \pm 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.2313 \pm 3\sqrt{\frac{0.2313(0.7687)}{50}}$$
$$= 0.2313 \pm 3(0.0596)$$
$$= 0.2313 \pm 0.1789$$

$$\text{UCL} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.2313 + 0.1789 = 0.4102$$

$$\text{LCL} = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.2313 - 0.1789 = 0.0524$$

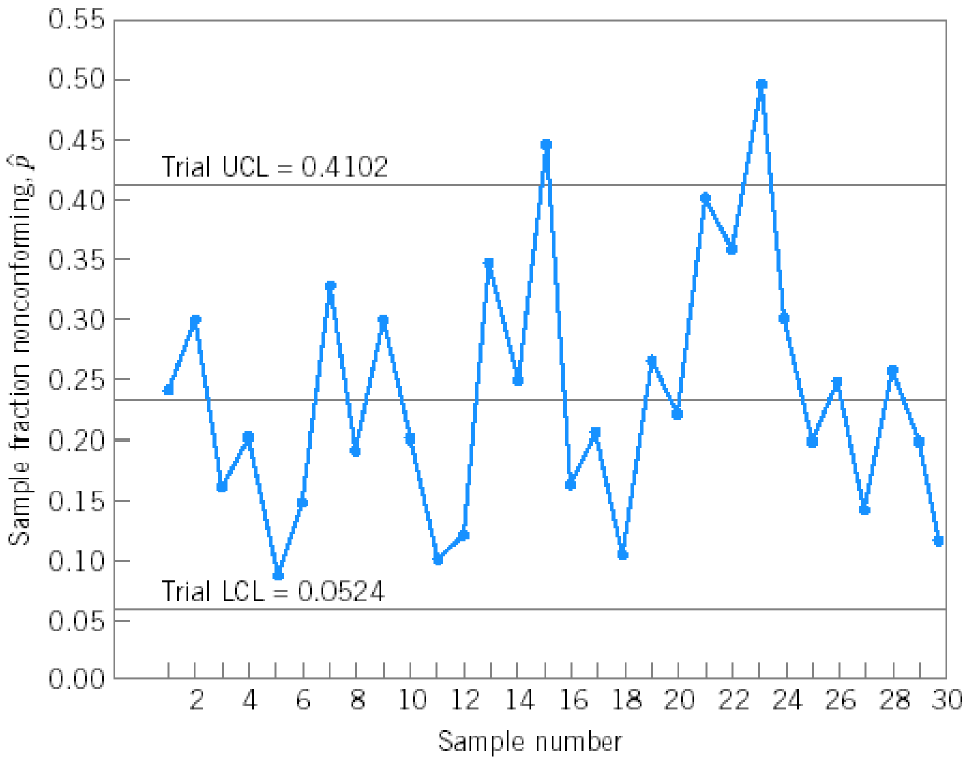

Figure 6-1 Initial phase I fraction nonconforming control chart for the data in Table 6-1.

If samples 15 and 23 are eliminated: $\bar{p} = \frac{301}{(28)(50)} = 0.2150$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2150 + 3\sqrt{\frac{0.2150(0.7850)}{50}} = 0.3893$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2150 - 3\sqrt{\frac{0.2150(0.7850)}{50}} = 0.0407$$

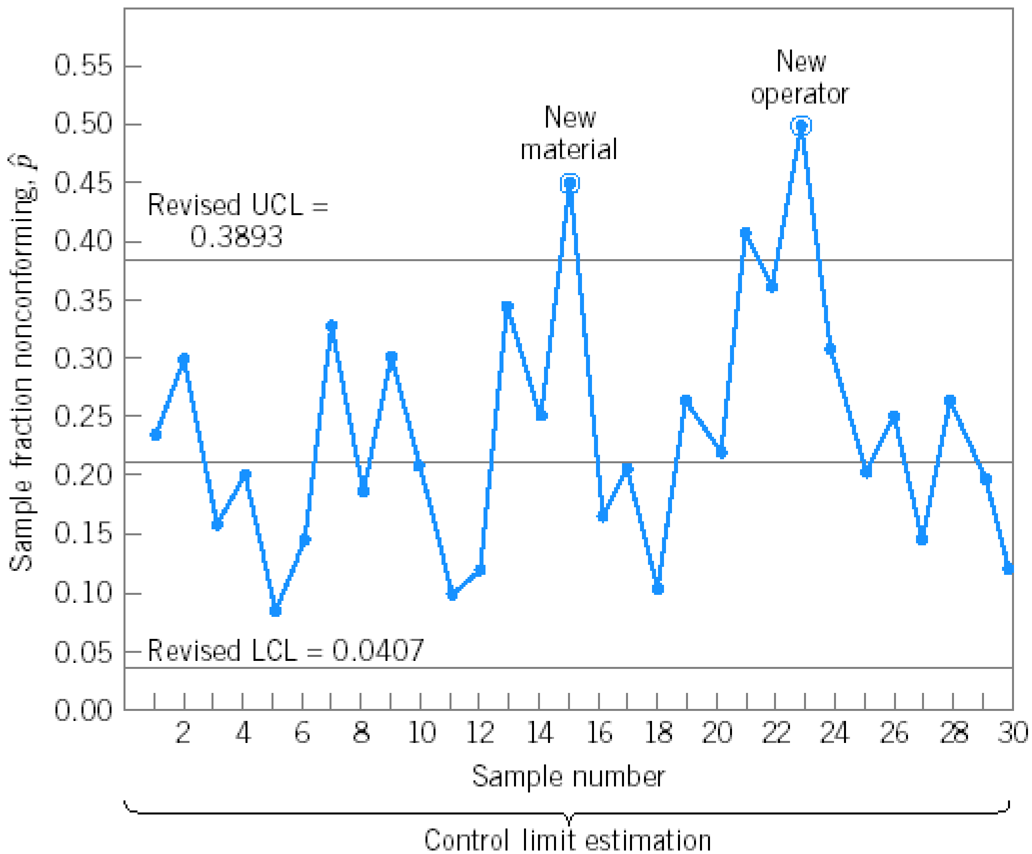

Figure 6-2 Revised control limits for the data in Table 6-1.

Additional samples collected after adjustment of control chart:

Table 6-2 Orange Juice Concentrate Can Data in Samples of Size $n = 50$

Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$	Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$
31	9	0.18	44	6	0.12
32	6	0.12	45	5	0.10
33	12	0.24	46	4	0.08
34	5	0.10	47	8	0.16
35	6	0.12	48	5	0.10
36	4	0.08	49	6	0.12
37	6	0.12	50	7	0.14
38	3	0.06	51	5	0.10
39	7	0.14	52	6	0.12
40	6	0.12	53	3	0.06
41	2	0.04	54	5	0.10
42	4	0.08		133	$\bar{p} = 0.1108$
43	3	0.06			

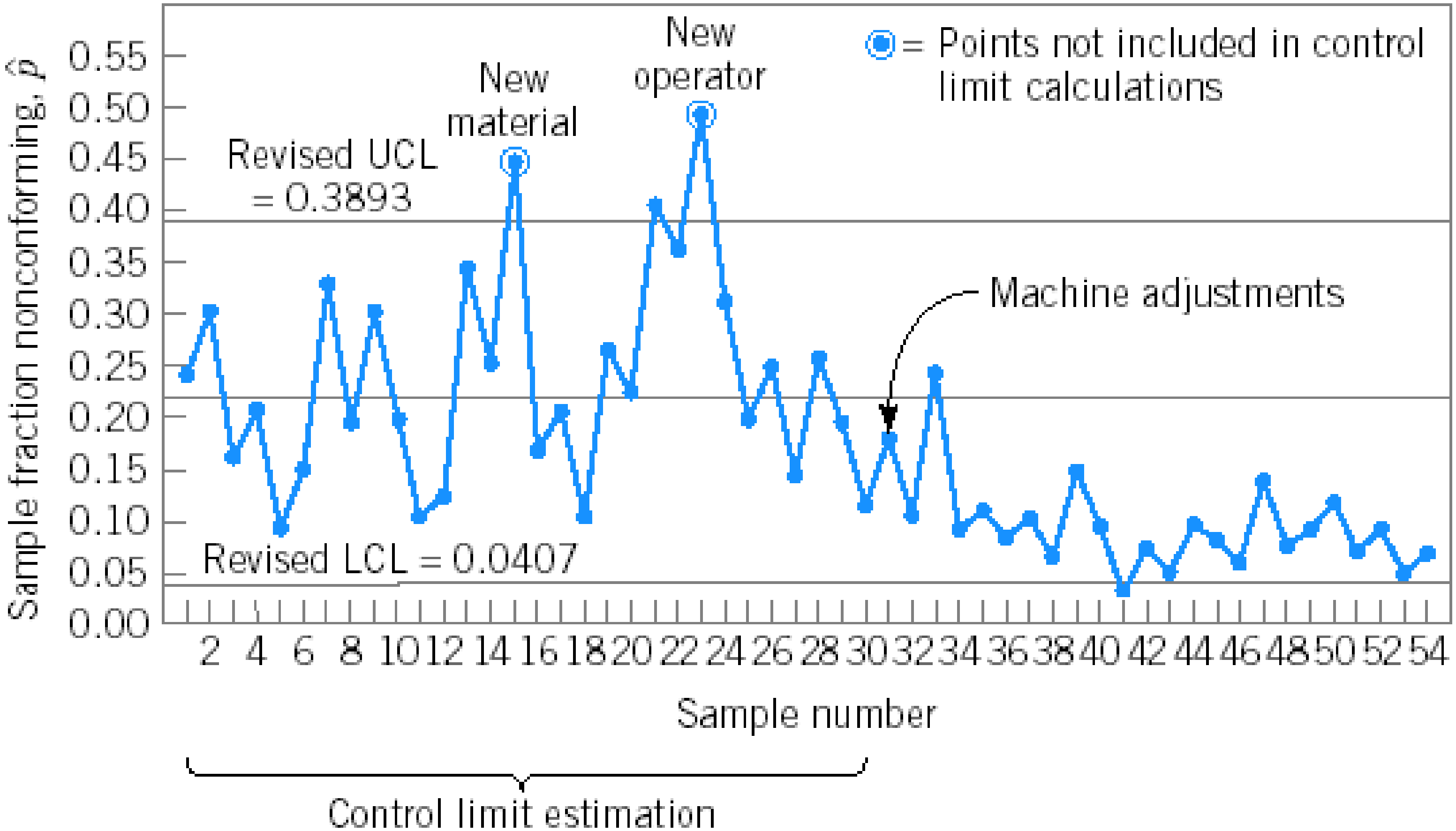

Figure 6-3 Continuation of the fraction nonconforming control chart, Example 6-1.

Control chart variables using only the recent 24 samples:

$$\text{Center line} = \overline{p} = 0.1108$$

$$\text{UCL} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.1108 + 3\sqrt{\frac{(0.1108)(0.8892)}{50}} = 0.2440$$

$$\text{LCL} = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.1108 - 3\sqrt{\frac{(0.1108)(0.8892)}{50}} = -0.0224 = 0$$

Set equal to zero for negative value

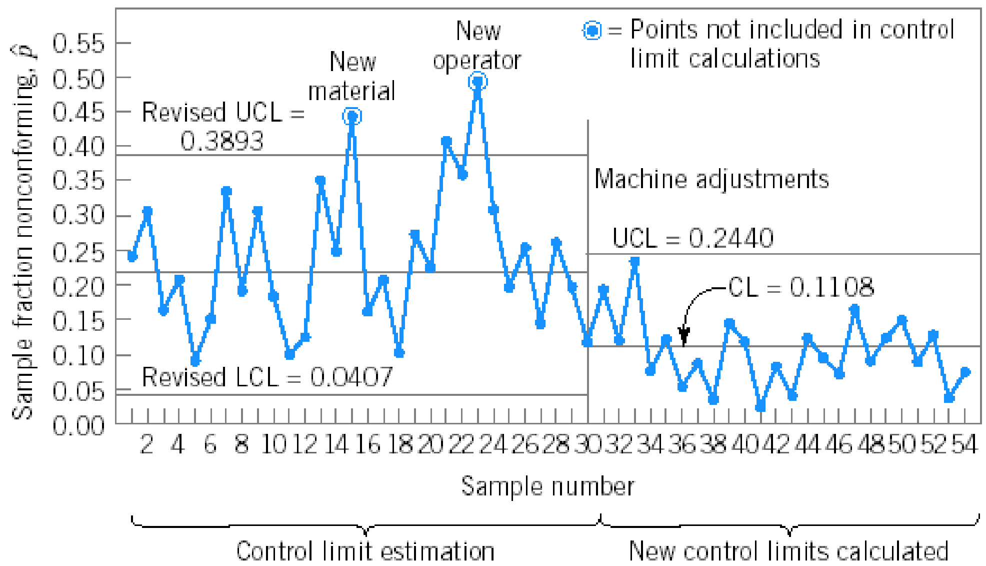

Figure 6-4 New control limits on the fraction nonconforming control chart, Example 6-1.

Table 6-3 New Data for the Fraction Nonconforming Control Chart in Fig. 6-5, $n = 50$

Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$	Sample Number	Number of Nonconforming Cans, D_i	Sample Fraction Nonconforming, $\hat{p}_i$
55	8	0.16	75	5	0.10
56	7	0.14	76	8	0.16
57	5	0.10	77	11	0.22
58	6	0.12	78	9	0.18
59	4	0.08	79	7	0.14
60	5	0.10	80	3	0.06
61	2	0.04	81	5	0.10
62	3	0.06	82	2	0.04
63	4	0.08	83	1	0.02
64	7	0.14	84	4	0.08
65	6	0.12	85	5	0.10
66	5	0.10	86	3	0.06
67	5	0.10	87	7	0.14
68	3	0.06	88	6	0.12
69	7	0.14	89	4	0.08
70	9	0.18	90	4	0.08
71	6	0.12	91	6	0.12
72	10	0.20	92	8	0.16
73	4	0.08	93	5	0.10
74	3	0.06	94	6	0.12

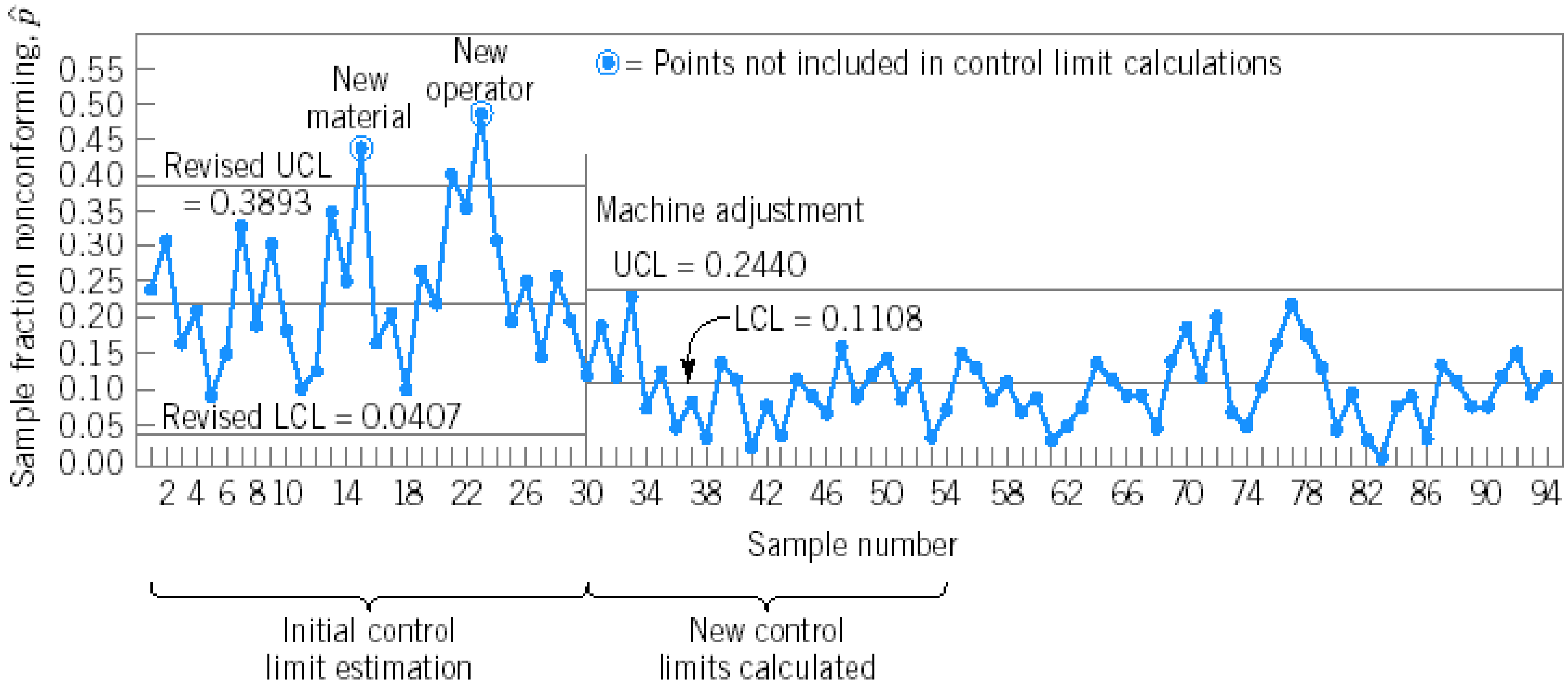

Figure 6-5 Completed fraction nonconforming control chart, Example 6-1.

Design of Fraction Nonconforming Chart

Three parameters to be specified:

1. sample size
2. frequency of sampling
3. width of control limits

Common to base chart on 100% inspection of all process output over time.

Rational subgroups may also play role in determining sampling frequency.

np Control Chart

The *np* Control Chart

$$\begin{aligned}
\text{UCL} &= np + 3\sqrt{np(1-p)} \\
\text{Center line } &= np \\
\text{LCL} &= np - 3\sqrt{np(1-p)}
\end{aligned} \tag{6-13}$$

Variable Sample Size

Variable-Width Control Limits

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}} \qquad \text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

$$\bar{p} = \frac{\sum_{i=1}^{25} D_i}{\sum_{i=1}^{25} n_i} = \frac{234}{2450} = 0.096$$

$$\text{UCL} = \bar{p} + 3\hat{\sigma}_{\hat{p}} = 0.096 + 3\sqrt{\frac{(0.096)(0.904)}{n_i}}$$

$$\text{LCL} = \bar{p} - 3\hat{\sigma}_{\hat{p}} = 0.096 - 3\sqrt{\frac{(0.096)(0.904)}{n_i}}$$

Table 6-4 Data for a Control Chart for Fraction Nonconforming with Variable Sample Size

Sample Number, i	Sample Size, n_i	Number of Nonconforming Units, D_i	Sample Fraction Nonconforming, $\hat{p}_i = D_i/n_i$	Standard Deviation $\hat{\sigma}_{\hat{p}} = \sqrt{\frac{(0.096)(0.904)}{n_i}}$	Control Limits LCL	UCL
1	100	12	0.120	0.029	0.009	0.183
2	80	8	0.100	0.033	0	0.195
3	80	6	0.075	0.033	0	0.195
4	100	9	0.090	0.029	0.009	0.183
5	110	10	0.091	0.028	0.012	0.180
6	110	12	0.109	0.028	0.012	0.180
7	100	11	0.110	0.029	0.009	0.183
8	100	16	0.160	0.029	0.009	0.183
9	90	10	0.110	0.031	0.003	0.189
10	90	6	0.067	0.031	0.003	0.189
11	110	20	0.182	0.028	0.012	0.180
12	120	15	0.125	0.027	0.015	0.177
13	120	9	0.075	0.027	0.015	0.177
14	120	8	0.067	0.027	0.015	0.177
15	110	6	0.055	0.028	0.012	0.180
16	80	8	0.100	0.033	0	0.195
17	80	10	0.125	0.033	0	0.195
18	80	7	0.088	0.033	0	0.195
19	90	5	0.056	0.031	0.003	0.189
20	100	8	0.080	0.029	0.009	0.183
21	100	5	0.050	0.029	0.009	0.183
22	100	8	0.080	0.029	0.009	0.183
23	100	10	0.100	0.029	0.009	0.183
24	90	6	0.067	0.031	0.003	0.189
25	90	9	0.100	0.031	0.003	0.189
	2450	234	2.383			

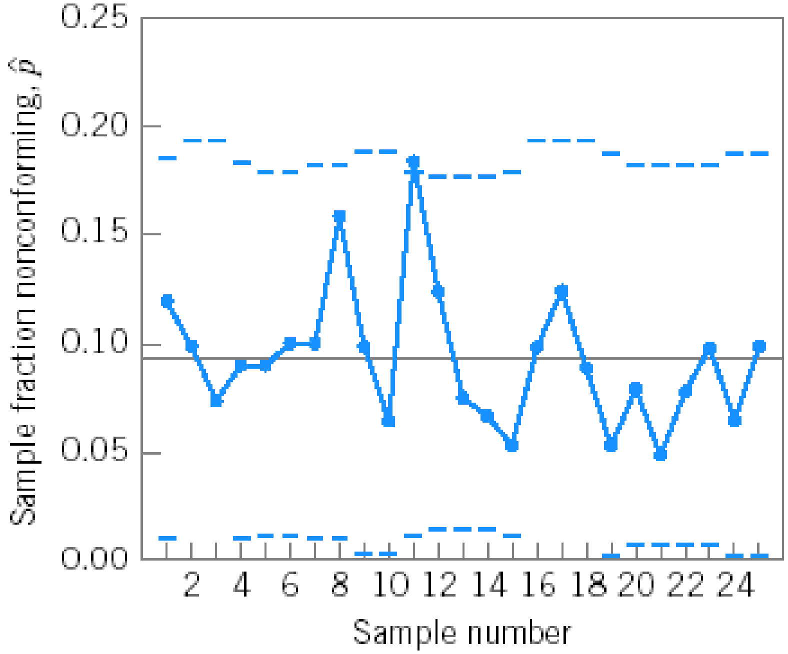

Figure 6-6 Control chart for fraction nonconforming with variable sample size.

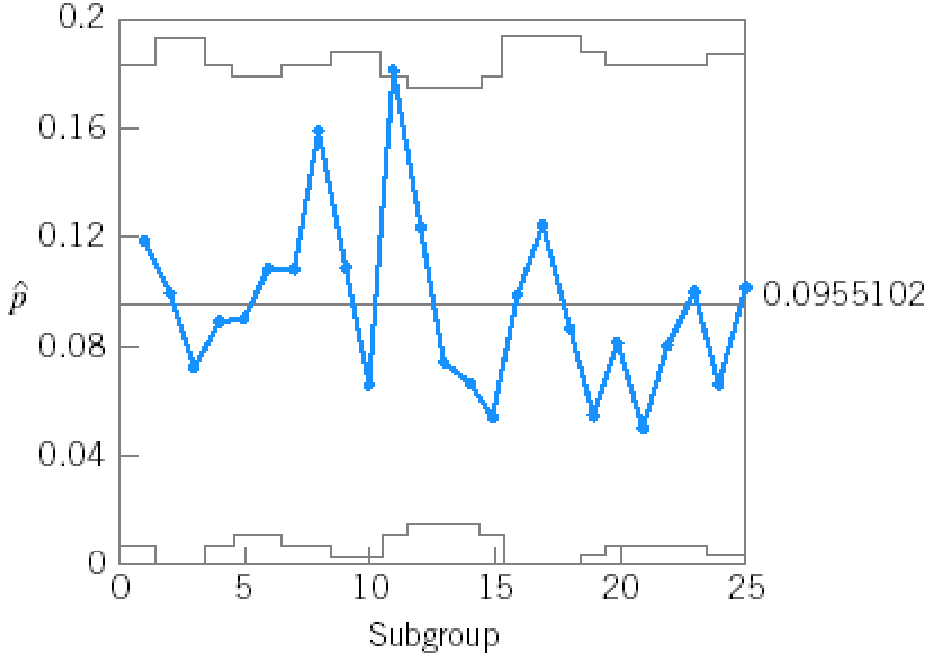

Figure 6-7 Control chart for fraction nonconforming with variable sample size using Minitab.

Variable Sample Size

Control Limits Based on an Average Sample Size

Use average sample size. For previous example:

$$\overline{n} = \frac{\sum_{i=1}^{25} n_i}{25} = \frac{2450}{25} = 98$$

$$\text{UCL} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{\overline{n}}} = 0.096 + 3\sqrt{\frac{(0.096)(0.904)}{98}} = 0.185$$

$$\text{LCL} = \overline{p} - \sqrt{\frac{\overline{p}(1-\overline{p})}{\overline{n}}} = 0.096 - 3\sqrt{\frac{(0.096)(0.904)}{98}} = 0.007$$

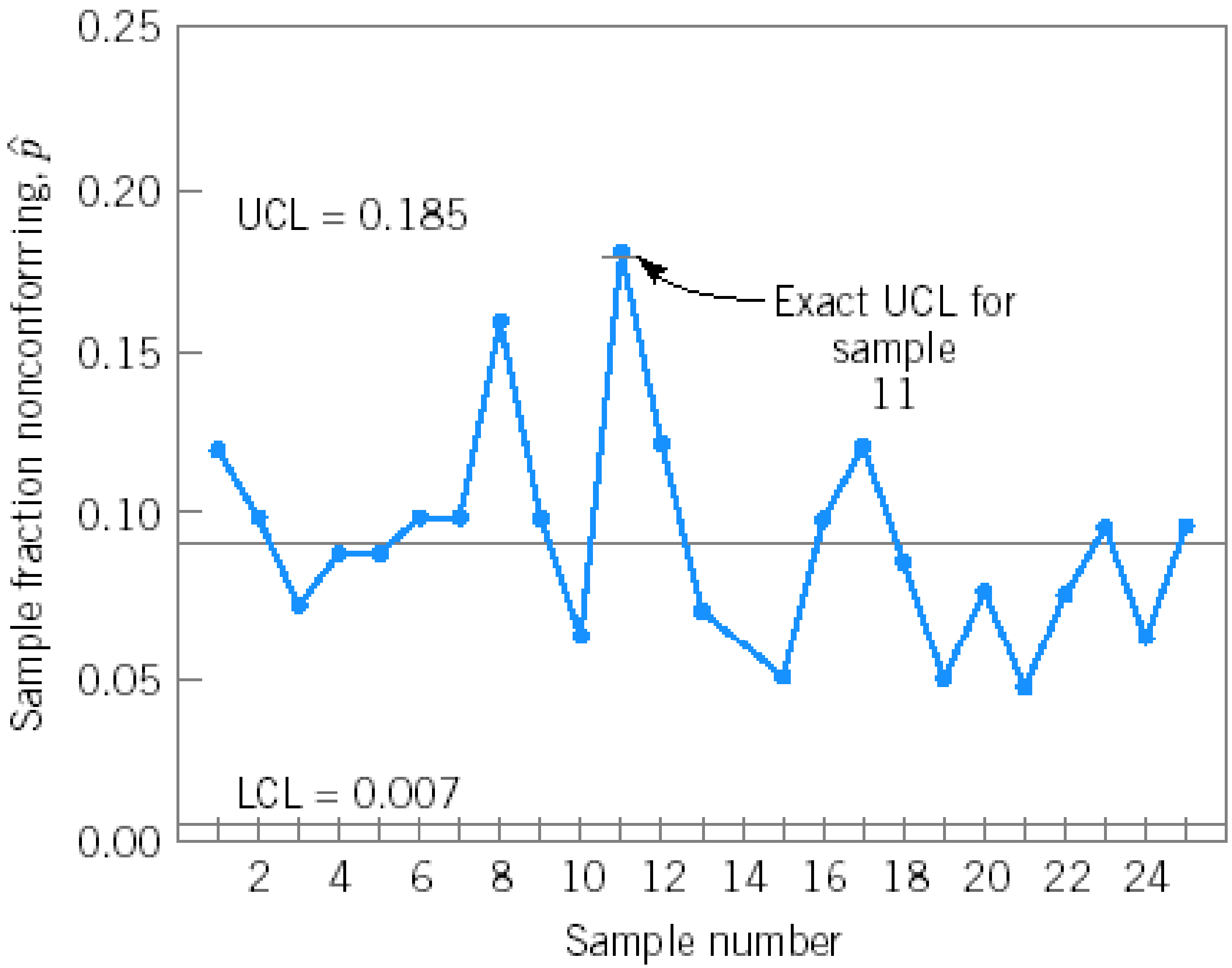

Figure 6-8 Control chart for fraction nonconforming based on average sample size.

Variable Sample Size

Standard Control Chart

- Points are plotted in standard deviation units.

  UCL = 3

  Center line = 0

  LCL = -3

$$Z_i = \frac{\hat{p}_i - p}{\sqrt{\frac{p(1-p)}{n_i}}} \qquad (6\text{-}14)$$

Table 6-5 Calculations for the Standardized Control Chart in Fig. 6-9, $\bar{p} = 0.096$

Sample Number, i	Sample Size, n_i	Number of Nonconforming Units, D_i	Sample Fraction Nonconforming, $\hat{p}_i = D_i/n_i$	Standard Deviation $\hat{\sigma}_p = \sqrt{\frac{(0.096)(0.904)}{n_i}}$	$Z_i = \frac{\hat{p}_i - \bar{p}}{\sqrt{\frac{(0.096)(0.904)}{n_i}}}$
1	100	12	0.120	0.029	0.83
2	80	8	0.100	0.033	0.12
3	80	6	0.075	0.033	−0.64
4	100	9	0.090	0.029	−0.21
5	110	10	0.091	0.028	−0.18
6	110	12	0.109	0.028	0.46
7	100	11	0.110	0.029	0.48
8	100	16	0.160	0.029	2.21
9	90	10	0.110	0.031	0.45
10	90	6	0.067	0.031	−0.94
11	110	20	0.182	0.028	3.07
12	120	15	0.125	0.027	1.07
13	120	9	0.075	0.027	−0.78
14	120	8	0.067	0.027	−1.07
15	110	6	0.055	0.028	−1.46
16	80	8	0.100	0.033	0.12
17	80	10	0.125	0.033	0.88
18	80	7	0.088	0.033	−0.24
19	90	5	0.056	0.031	−1.29
20	100	8	0.080	0.029	−0.55
21	100	5	0.050	0.029	−1.59
22	100	8	0.080	0.029	−0.55
23	100	10	0.100	0.029	0.14
24	90	6	0.067	0.031	−0.94
25	90	9	0.100	0.031	0.13

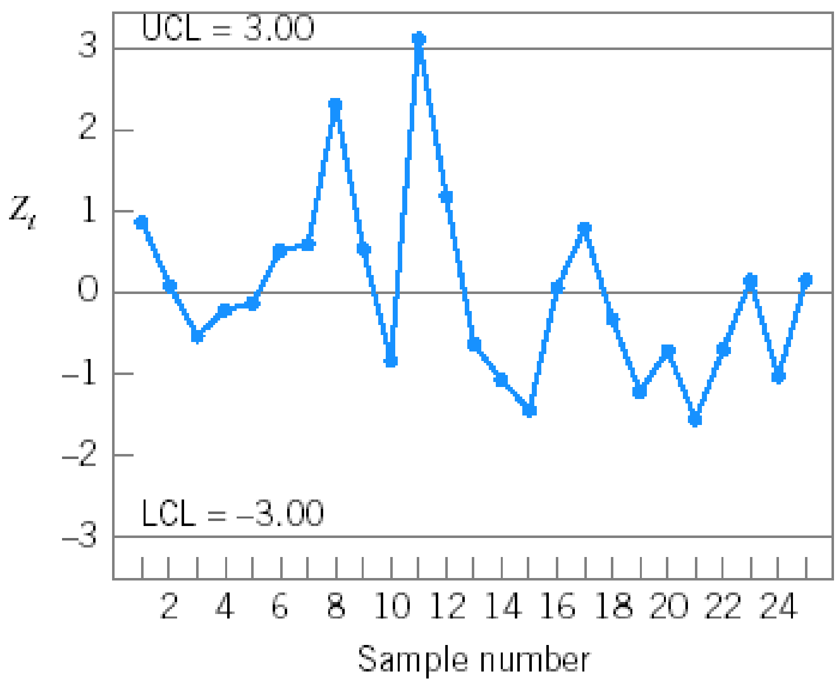

Figure 6-9 Standardized control chart for fraction nonconforming.

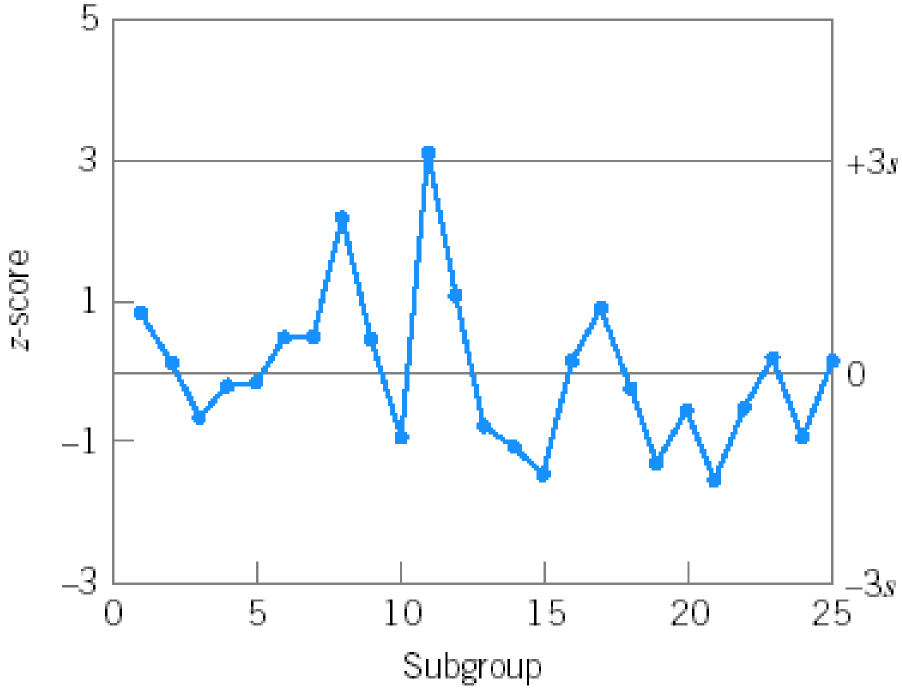

Figure 6-10 Standardized control chart from Minitab for fraction nonconforming, Table 6-4.

Skip Section 6.2.3 pages 284 - 285

Operating Characteristic Function and Average Run Length Calculations

Probability of type II error

$$\beta = P\left\{\hat{p} < \text{UCL} \mid p\right\} - P\left\{\hat{p} \le \text{LCL} \mid p\right\}$$
$$= P\left\{D < n\text{UCL} \mid p\right\} - P\left\{D \le n\text{LCL} \mid p\right\}$$

Table 6-6 Calculations[a] for Constructing the OC Curve for a Control Chart for Fraction Nonconforming with $n = 50$, LCL = 0.0303, and UCL = 0.3697

p	$P\{D \leq 18 \mid p\}$	$P\{D \leq 1 \mid p\}$	$\beta = P\{D \leq 18 \mid p\} - P\{D \leq 1 \mid p\}$
0.01	1.0000	0.9106	0.0894
0.03	1.0000	0.5553	0.4447
0.05	1.0000	0.2794	0.7206
0.10	1.0000	0.0338	0.9662
0.15	0.9999	0.0029	0.9970
0.20	0.9975	0.0002	0.9973
0.25	0.9713	0.0000	0.9713
0.30	0.8594	0.0000	0.8594
0.35	0.6216	0.0000	0.6216
0.40	0.3356	0.0000	0.3356
0.45	0.1273	0.0000	0.1273
0.50	0.0325	0.0000	0.0325
0.55	0.0053	0.0000	0.0053

[a]The probabilities in this table were found by evaluating the cumulative binomial distribution. For small p ($p < 0.1$, say) the Poisson approximation could be used, and for larger values of p the normal approximation could be used.

$$\beta = P\{D < (50)(0.3697)|p\} - P\{D \le (50)(0.0303)|p\}$$
$$= P\{D < 18.49|p\} - P\{D \le 1.52|p\}$$

Since *D* is an integer, $\beta = P\{D \le 18|p\} - P\{D \le 1|p\}$

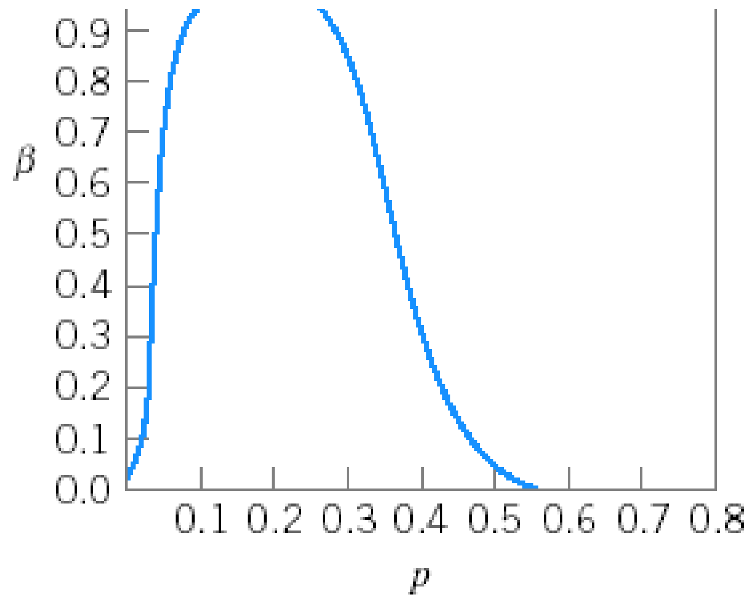

Figure 6-11 Operating-characteristic curve for the fraction nonconforming control chart with $\bar{p} = 0.20$, LCL = 0.0303, and UCL = 0.3697.

Average run length

$$\text{ARL} = \frac{1}{P(\text{sample point plots out of control})}$$

If the process is in control:

$$\text{ARL}_0 = \frac{1}{\alpha}$$

If the process is out of control

$$\text{ARL}_1 = \frac{1}{1-\beta}$$

For Table 6-6: $n = 50$, UCL $= 0.3698$, LCL $= 0.0303$, center line $\bar{p} = 0.20$.

If process is in control with $p = \bar{p}$, probability of point plotting in control = 0.9973.

$\Rightarrow \alpha = 1 - \beta = 0.0027$.

$$\text{ARL}_0 = \frac{1}{\alpha} = \frac{1}{0.0027} \simeq 370$$

If process shifts out of control to $p = 0.3$, $\beta = 0.8594$.

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1-0.8594} \simeq 7$$

Control Charts for Nonconformities (or Defects)

Procedures with Constant Sample Size

x: number of nonconformities

c > 0: parameter of Poisson distribution

$$p(x) = \frac{e^{-c}c^x}{x!} \qquad x = 0,\ 1,\ 2\ ,\ \ldots$$

Control Chart for Nonconformities: Standard Given

$$\begin{aligned} \text{UCL} &= c + 3\sqrt{c} \\ \text{Center line} &= c \\ \text{LCL} &= c - 3\sqrt{c} \end{aligned} \qquad (6\text{-}16)$$

Set to zero if negative

If no standard is given, estimate c then use the following parameters:

Control Chart for Nonconformities: No Standard Given

$$\begin{aligned} \text{UCL} &= \bar{c} + 3\sqrt{\bar{c}} \\ \text{Center line} &= \bar{c} \\ \text{LCL} &= \bar{c} - 3\sqrt{\bar{c}} \end{aligned} \tag{6-17}$$

Set to zero if negative

EXAMPLE 6-3

Table 6-7 presents the number of nonconformities observed in 26 successive samples of 100 printed circuit boards. Note that, for reasons of convenience, the inspection unit is defined as 100 boards.

Table 6-7 Data on the Number of Nonconformities in Samples of 100 Printed Circuit Boards

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	21	14	19
2	24	15	10
3	16	16	17
4	12	17	13
5	15	18	22
6	5	19	18
7	28	20	39
8	20	21	30
9	31	22	24
10	25	23	16
11	20	24	19
12	24	25	17
13	16	26	15

There are 516 defects in total of 26 samples. Thus.

$$\overline{c} = \frac{516}{26} = 19.85$$

$$\text{UCL} = \overline{c} + 3\sqrt{\overline{c}} = 19.85 + 3\sqrt{19.85} = 33.22$$

$$\text{Center line } = \overline{c} = 19.85$$

$$\text{LCL} = \overline{c} - 3\sqrt{\overline{c}} = 19.85 - 3\sqrt{19.85} = 6.48$$

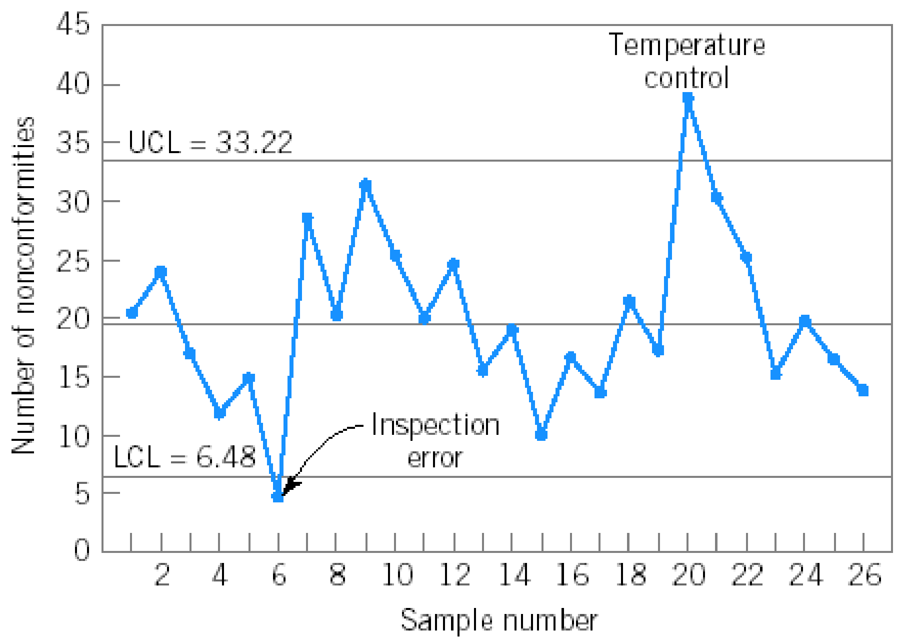

Figure 6-12 Control chart for nonconformities for Example 6-3.

There are 516 defects in total of 26 samples. Thus.

Sample 6 was due to inspection error.
Sample 20 was due to a problem in wave soldering machine.
Eliminate these two samples, and recalculate the control parameters.

$$\bar{c} = \frac{472}{24} = 19.67$$

New control limits:

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 19.67 + 3\sqrt{19.67} = 32.97$$

$$\text{Center line } = \bar{c} = 19.67$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 19.67 - 3\sqrt{19.67} = 6.36$$

Additional samples collected.

Table 6-8 Additional Data for the Control Chart for Nonconformities, Example 6-3

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
27	16	37	18
28	18	38	21
29	12	39	16
30	15	40	22
31	24	41	19
32	21	42	12
33	28	43	14
34	20	44	9
35	25	45	16
36	19	46	21

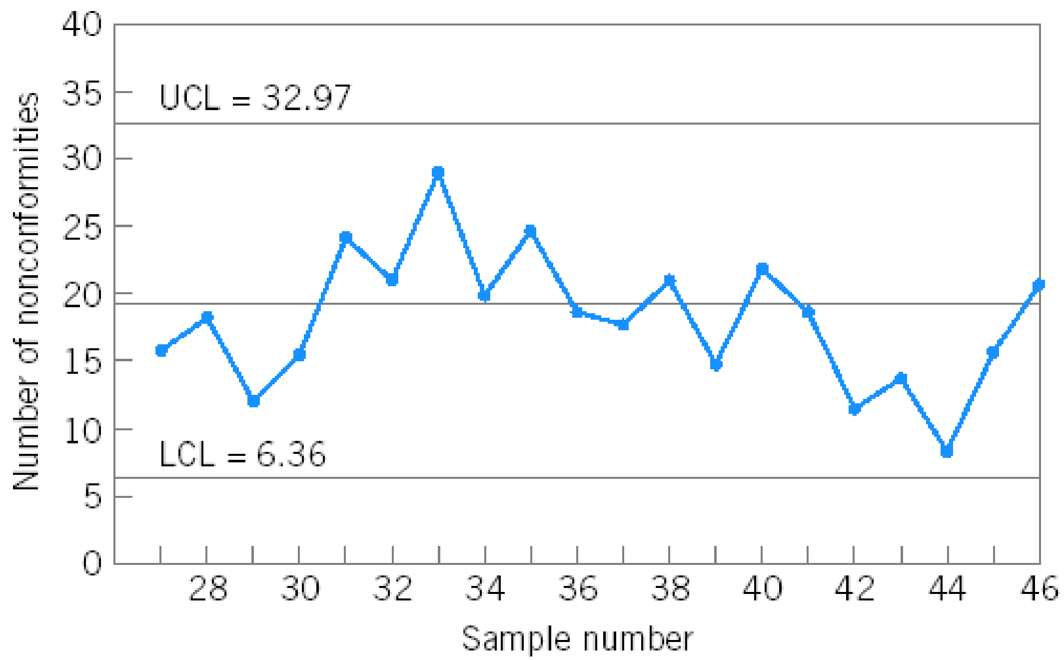

Figure 6-13 Continuation of the control chart for nonconformities, Example 6-3.

Further Analysis of Nonconformities

Defect code		Freq.	Cum. freq.	Percent	Cum. percent
Sold. Insufficie	**	40	40	40.82	40.82
Sold.cold joint	********************	20	60	20.41	61.23
Sold. opens/dewe	*******	7	67	7.14	68.37
Comp. improper 1	******	6	73	6.12	74.49
Sold. splatter/w	*****	5	78	5.10	79.59
Tst. mark ec mark	***	3	81	3.06	82.65
Tst. mark white m	***	3	84	3.06	85.71
Raw cd shroud re	***	3	87	3.06	88.78
Comp. extra part	**	2	89	2.04	90.82
Comp. damaged	**	2	91	2.04	92.86
Comp. missing	**	2	93	2.04	94.90
Wire incorrect s	*	1	94	1.02	95.92
Stamping oper id	*	1	95	1.02	96.94
Stamping missing	*	1	96	1.02	97.96
Sold. short	*	1	97	1.02	98.98
Raw cd damaged	*	1	98	1.02	100.00
	1 10 20 30 40				
	Number of defects				

Figure 6-14 Pareto analysis of nonconformities for the printed circuit board process.

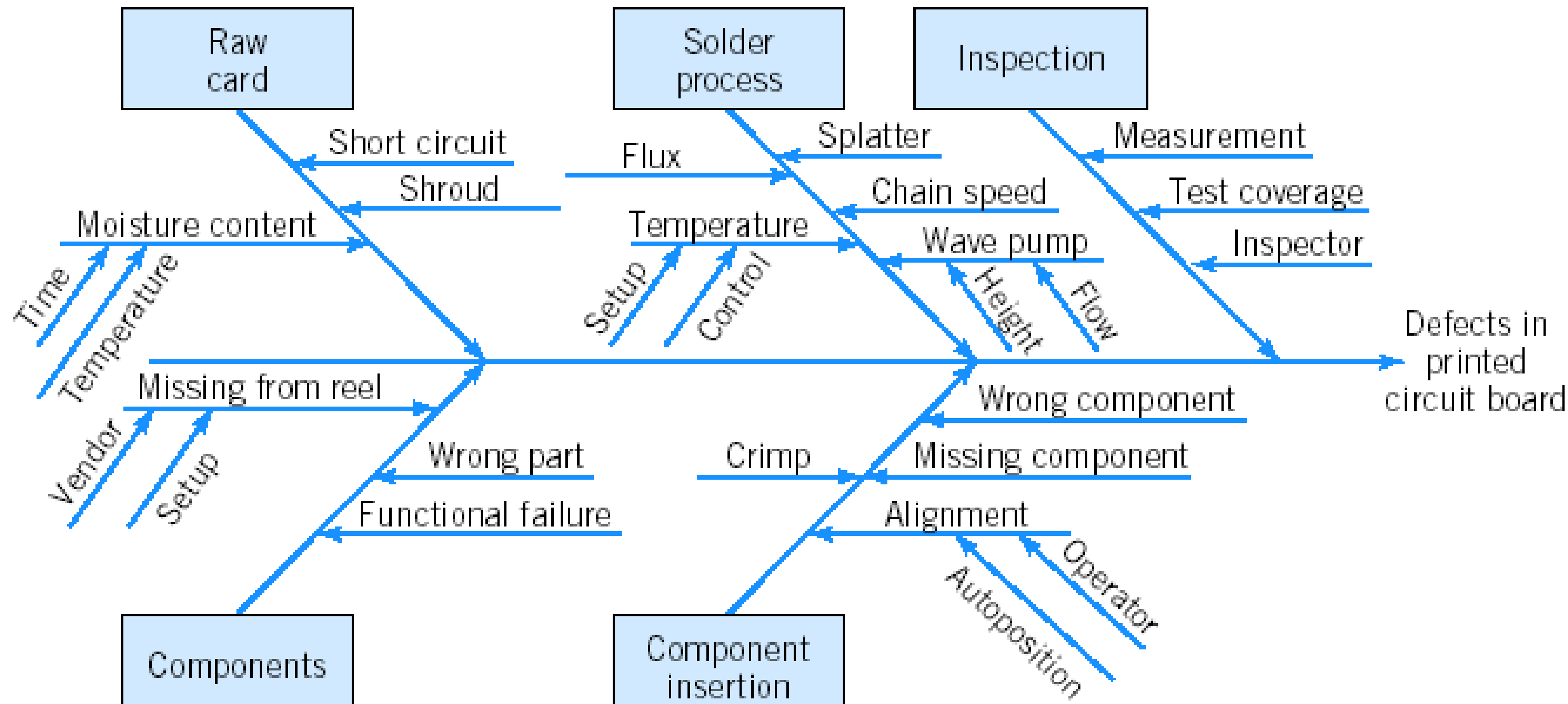

Figure 6-15 Cause-and-effect diagram.

Choice of Sample Size: *μ* Chart

x: total nonconformities in *n* inspection units
u: average number of nonconformities per inspection unit

$$u = \frac{x}{n} \qquad (6\text{-}18)$$

Control Chart for Average Number of Nonconformities per Unit

$$\begin{aligned} \text{UCL} &= \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} \\ \text{Center line} &= \bar{u} \\ \text{LCL} &= \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} \end{aligned} \qquad (6\text{-}19)$$

$\bar{u}$: obserd average number of nonconformities per inspection unit

EXAMPLE 6-4

A supply chain engineering group monitors shipments of materials through the company distribution network. Errors on either the delivered material or the accompanying documentation are tracked on a weekly basis. Fifty randomly selected shipments are examined and the errors recorded. Data for 20 weeks are shown in Table 6-10.

Table 6-10 Data on Number of Shipping Errors in a Supply Chain Network

Sample Number (week), i	Sample Size, n	Total Number of Errors (Nonconformities), x_i	Average Number of Errors (Nonconformities) per Unit, $u_i = x_i/n$
1	50	2	0.04
2	50	3	0.06
3	50	8	0.16
4	50	1	0.02
5	50	1	0.02
6	50	4	0.08
7	50	1	0.02
8	50	4	0.08
9	50	5	0.10
10	50	1	0.02
11	50	8	0.16
12	50	2	0.04
13	50	4	0.08
14	50	3	0.06
15	50	4	0.08
16	50	1	0.02
17	50	8	0.16
18	50	3	0.06
19	50	7	0.14
20	50	4	0.08
		74	1.48

$$\bar{u} = \frac{\sum_{i=1}^{20} u_i}{20} = \frac{1.48}{20} = 0.0740$$

$$\text{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 + 3\sqrt{\frac{0.0740}{50}} = 0.1894$$

$$\text{Center line } = \bar{u} = 1.93$$

$$\text{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 0.0740 - 3\sqrt{\frac{0.0740}{50}} = -0.0414$$

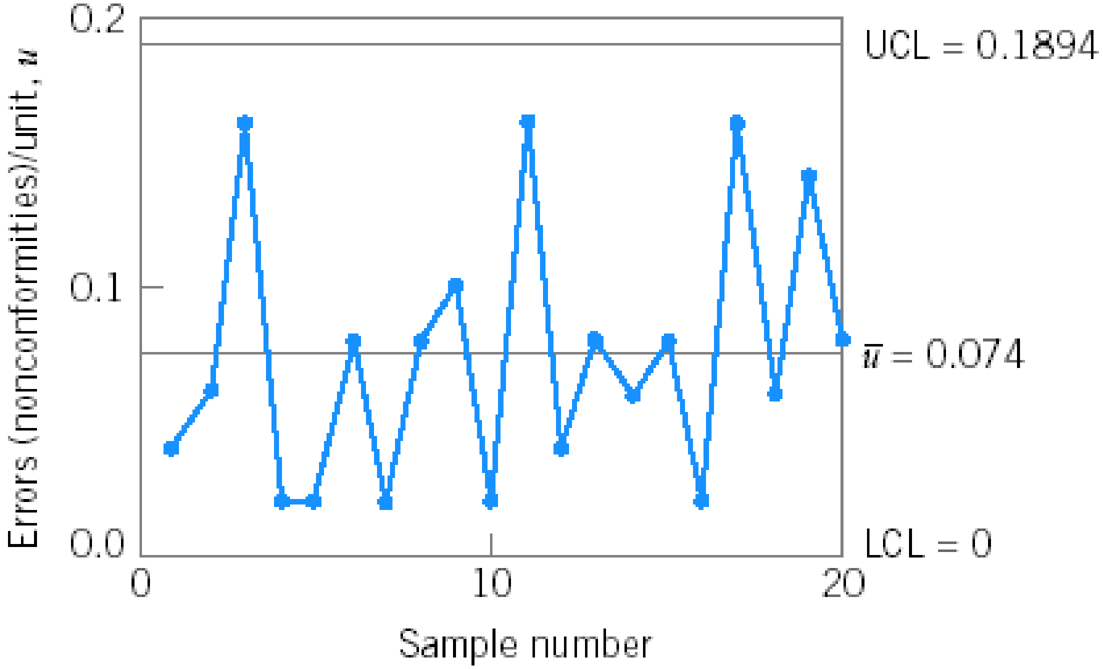

Figure 6-16 The control chart for nonconformities per unit from Minitab for Example 6-4.

Control Charts for Nonconformities

Procedure with Variable Sample Size

EXAMPLE 6-5

In a textile finishing plant, dyed cloth is inspected for the occurrence of defects per 50 square meters. The data on ten rolls of cloth are shown in Table 6-11. We will use these data to set up a control chart for nonconformities per unit.

Table 6-11 Occurrence of Nonconformities in Dyed Cloth

Roll Number	Number of Square Meters	Total Number of Nonconformities	Number of Inspection Units in Roll, n	Number of Nonconformities per Inspection Unit
1	500	14	10.0	1.40
2	400	12	8.0	1.50
3	650	20	13.0	1.54
4	500	11	10.0	1.10
5	475	7	9.5	0.74
6	500	10	10.0	1.00
7	600	21	12.0	1.75
8	525	16	10.5	1.52
9	600	19	12.0	1.58
10	625	23	12.5	1.84
		153	107.50	$\bar{u} = \frac{153}{107.5} = 1.42$

Table 6-12 Calculation of Control Limits, Example 6-5

Roll Number, i	n_i	UCL $= \bar{u} + 3\sqrt{\bar{u}/n_i}$	LCL $= \bar{u} - 3\sqrt{\bar{u}/n_i}$
1	10.0	2.55	0.29
2	8.0	2.68	0.16
3	13.0	2.41	0.43
4	10.0	2.55	0.29
5	9.5	2.58	0.26
6	10.0	2.55	0.29
7	12.0	2.45	0.39
8	10.5	2.52	0.32
9	12.0	2.45	0.39
10	12.5	2.43	0.41

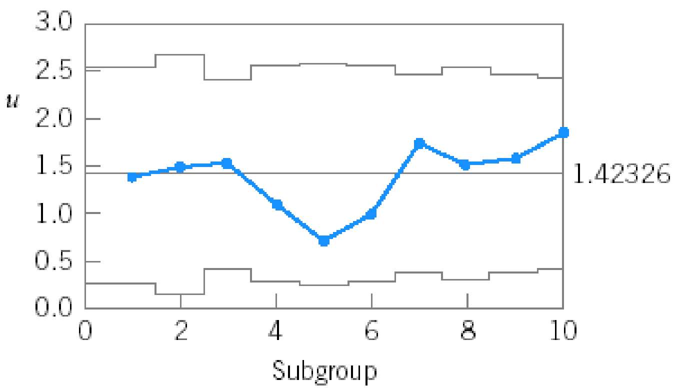

Figure 6-17 Computer-generated (Minitab) control chart for Example 6-5.

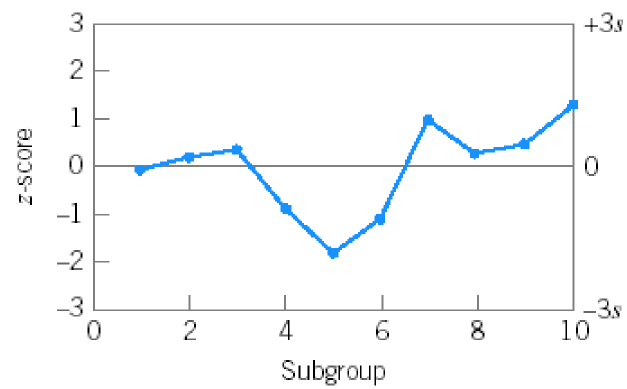

Figure 6-18 Standardized control chart for nonconformities per unit, Example 6-5.

Control Charts for Nonconformities

Demerit Systems: not all defects are of equal importance

Class A Defects—Very Serious. The unit is either completely unfit for service, or will fail in service in such a manner that cannot be easily corrected in the field, or will cause personal injury or property damage.

Class B Defects—Serious. The unit will possibly suffer a Class A operating failure, or will certainly cause somewhat less serious operating problems, or will certainly have reduced life or increased maintenance cost.

Class C Defects—Moderately Serious. The unit will possibly fail in service, or cause trouble that is less serious than operating failure, or possibly have reduced life or increased maintenance costs, or have a major defect in finish, appearance, or quality of work.

Class D Defects—Minor. The unit will not fail in service but has minor defects in finish, appearance, or quality of work.

c_{iA}: number of Class A defects in i^{th} inspection units
Similarly for c_{iB}, c_{iC}, and c_{iD} for Classes B, C, and D.
d_i: number of demerits in inspection unit i

$$d_i = 100c_{iA} + 50c_{iB} + 10c_{iC} + c_{iD} \tag{6-21}$$

Constants *100, 50, 10,* and *1* are demerit weights.

n: inspection units

u_i: number of demerits per unit

$$u_i = \frac{D}{n} \quad \text{where } D = \sum_{i=1}^{n} d_i$$

μ_i: linear combination of independent Poisson variables

$$\begin{aligned} \text{UCL} &= \bar{u} + 3\hat{\sigma}_u \\ \text{Center line } &= \bar{u} \\ \text{LCL} &= \bar{u} - 3\hat{\sigma}_u \end{aligned} \quad (6\text{-}23)$$

where

$$\bar{u} = 100\bar{u}_{\text{A}} + 50\bar{u}_{\text{B}} + 10\bar{u}_{\text{C}} + \bar{u}_{\text{D}} \quad (6\text{-}24)$$

$\bar{\mu}_A$ is average number of Class A defects per unit, etc.

and

$$\hat{\sigma}_u = \left[\frac{(100)^2 \bar{u}_{\text{A}} + (50)^2 \bar{u}_{\text{B}} + (10)^2 \bar{u}_{\text{C}} + \bar{u}_{\text{D}}}{n} \right]^{1/2} \quad (6\text{-}25)$$

Control Charts for Nonconformities

Operating Characteristic Function

- x: Poisson random variable
- c: true mean value
- β: type II error probability

$$\beta = P\{x < \text{UCL} | c\} - P\{x \le \text{LCL} | c\} \tag{6-26}$$

For example 6-23

$$\beta = P\{x < 33.22 | c\} - P\{x \le 6.48 | c\}$$

Number of nonconformities is integer.

$$\beta = P\{x \le 33 | c\} - P\{x \le 6 | c\}$$

Table 6-13 Calculation of the OC Curve for a c Chart with UCL = 33.22 and LCL = 6.48

c	$P\{x \le 33 \mid c\}$	$P\{x \le 6 \mid c\}$	$\beta = P\{x \le 33 \mid c\} - P\{x \le 6 \mid c\}$
1	1.000	0.999	0.001
3	1.000	0.966	0.034
5	1.000	0.762	0.238
7	1.000	0.450	0.550
10	1.000	0.130	0.870
15	0.999	0.008	0.991
20	0.997	0.000	0.997
25	0.950	0.000	0.950
30	0.744	0.000	0.744
33	0.546	0.000	0.546
35	0.410	0.000	0.410
40	0.151	0.000	0.151
45	0.038	0.000	0.038

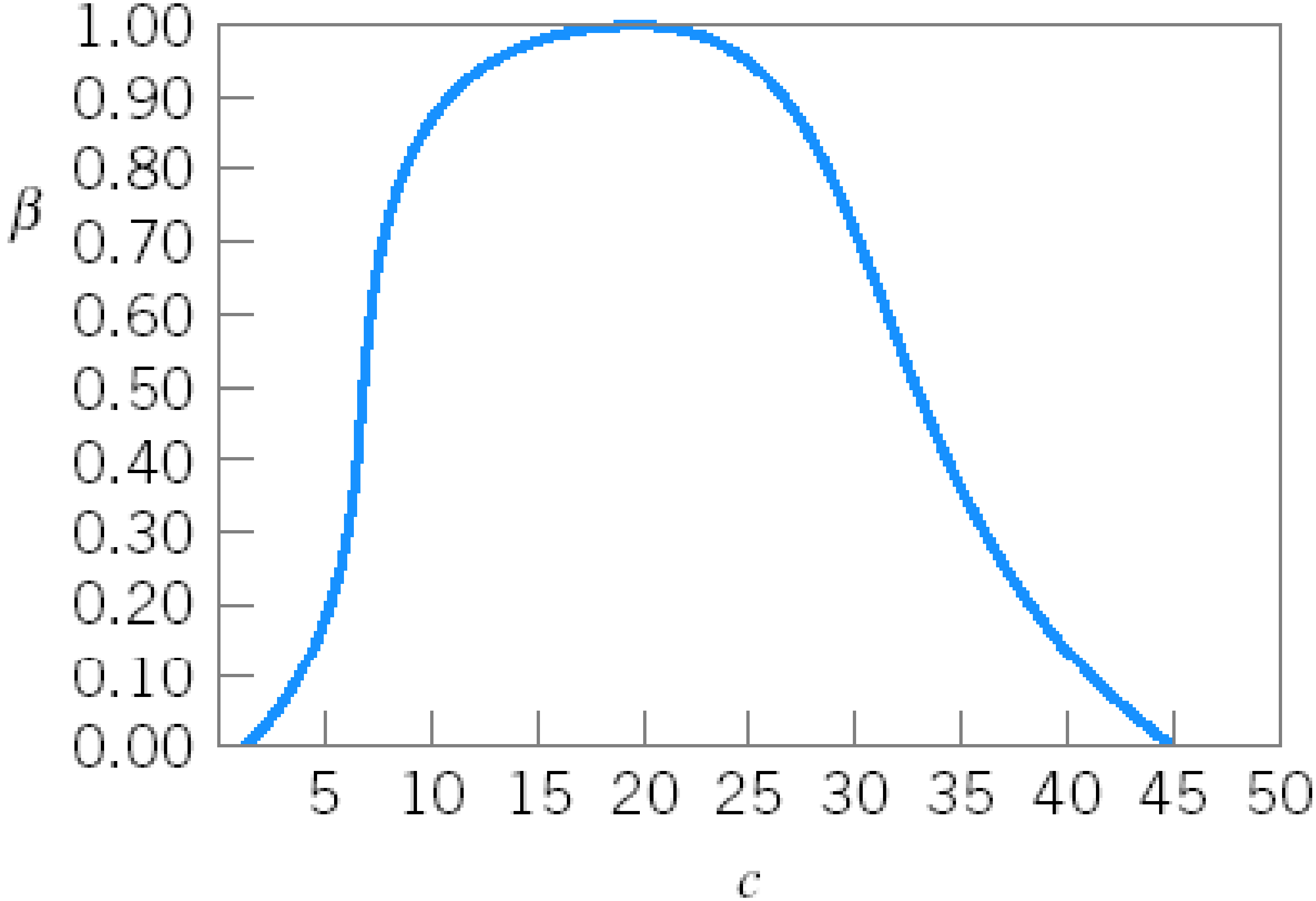

Figure 6-19 OC curve of a c chart with LCL = 6.48 and UCL = 33.22.

Control Charts for Nonconformities

Dealing with Low Defect Levels

- If defect level is low, <1000 per million, *c* and *u* charts become ineffective.
- The time-between-events control chart is more effective.
- If the defects occur according to a Poisson distribution, the probability distribution of the time between events is the exponential distribution.
- Constructing a time-between-events control chart is essentially equivalent to control charting an exponentially distributed variable.
- To use normal approximation, translate exponential distribution to Weibull distribution and then approximate with normal variable

$$x: \text{ normal approximation for exponential variable } y$$

$$x = y^{\frac{1}{3.6}} = y^{0.2777}$$

EXAMPLE 6-6

A chemical engineer wants to set up a control chart for monitoring the occurrence of failures of an important valve. She has decided to use the number of hours between failures as the variable to monitor. Table 6-14 shows the number of hours between failures for the last 20 failures of this valve. Figure 6-20 is a normal probability plot of the time between failures. Clearly, time between failures is not normally distributed.

Table 6-14 also shows the values of the transformed time between events, computed from equation 6-27. Figure 6-21 is a normal probability plot of the transformed time between failures. Note that the plot indicates that the distribution of this transformed variable is well approximated by the normal.

Figure 6-22 is a control chart for individuals and a moving range control chart for the transformed time between failures. Note that the control charts indicate a state of control, implying that the failure mechanism for this valve is constant. If a process change is made that improves the failure rate (such as a different type of maintenance action), then we would expect to see the mean time between failures get longer. This would result in points plotting above the upper control limit on the individuals control chart in Fig. 6-22.

Table 6-14 Time between Failure Data, Example 6-6

Failure	Time between Failures, y (hr)	Transformed Value of Time between Failures, $x = y^{0.2777}$
1	286	4.80986
2	948	6.70903
3	536	5.72650
4	124	3.81367
5	816	6.43541
6	729	6.23705
7	4	1.46958
8	143	3.96768
9	431	5.39007
10	8	1.78151
11	2837	9.09619
12	596	5.89774
13	81	3.38833
14	227	4.51095
15	603	5.91690
16	492	5.59189
17	1199	7.16124
18	1214	7.18601
19	2831	9.09083
20	96	3.55203

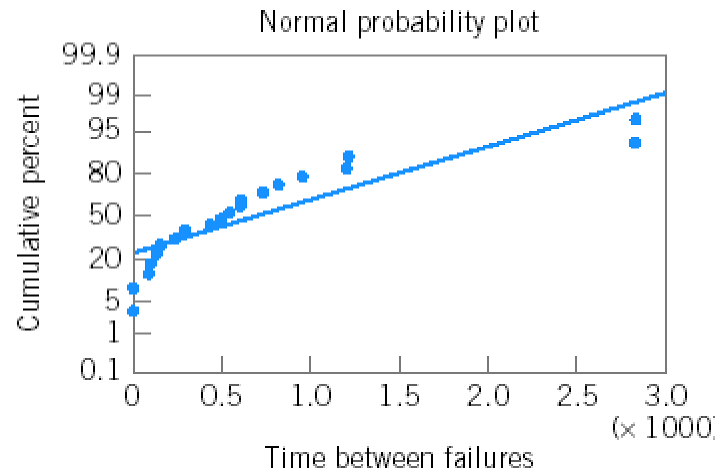

Figure 6-20 Normal probability plot of time between failures, Example 6-6.

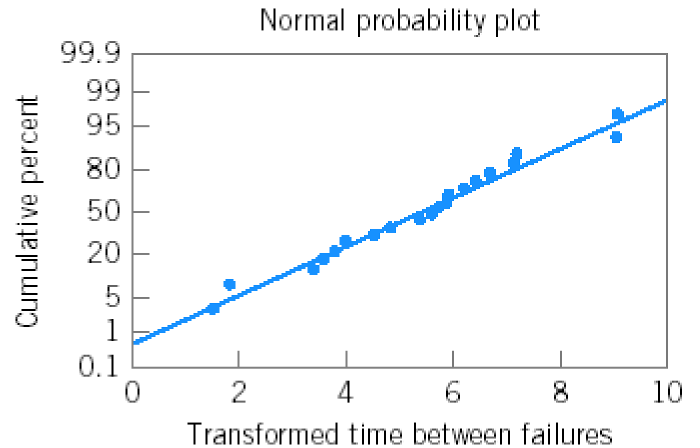

Figure 6-21 Normal probability plot for the transformed failure data.

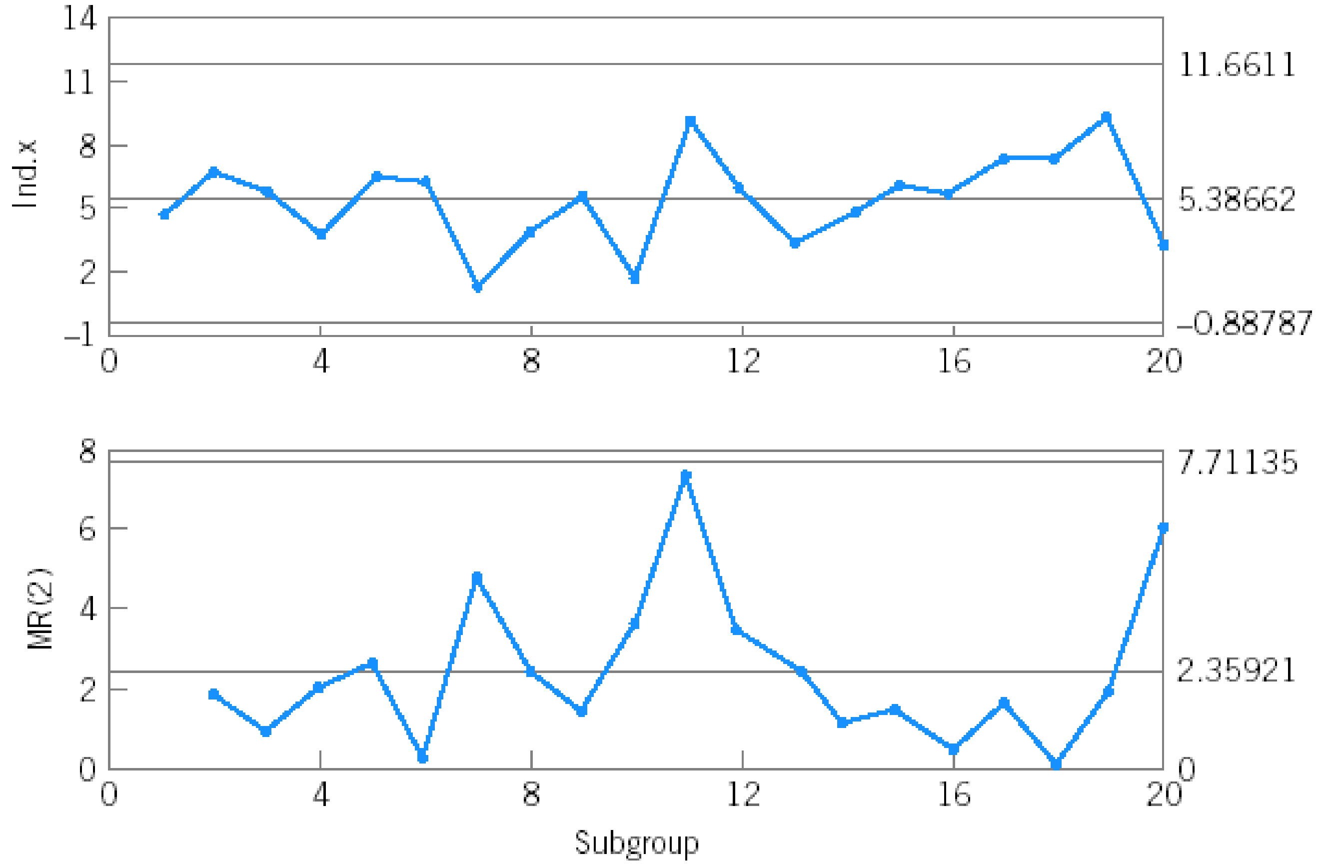

Figure 6-22 Control charts for individuals and moving-range control chart for the transformed time between failures, Example 6-6.

Guidelines for Implementing Control Charts

Applicable for both variable and attribute control

1. Determining **which** process characteristics to control
2. Determining **where** the charts should be implemented in the process
3. Choosing the proper **type** of control charts
4. Taking **actions** to improve processes as the result of SPC/control chart analysis
5. Selecting data-collection systems and computer software

Determining Which Characteristics and Where to Put Control Charts

1. At the beginning of a control chart program, control charts should be applied to any product characteristics or manufacturing operations believed to be important. The charts will provide immediate feedback as to whether they are actually needed.
2. The control charts found to be unnecessary should be removed, and others that engineering and operator judgment indicates may be required should be added. More control charts will usually be employed at the beginning than after the process has stabilized.
3. Information on the number and types of control charts on the process should be kept current. It is best to keep separate records on the variables and attributes charts. In general, after the control charts are first installed, we often find that the number of control charts tends to increase rather steadily. After that, it will usually decrease. When the process stabilizes, we typically find that it has the same number of charts from one year to the next. However, they are not necessarily the same charts.

4. If control charts are being used effectively and if new knowledge is being gained about the key process variables, we should find that the number of $\bar{x}$ and R charts increases and the number of attributes control charts decreases.
5. At the beginning of a control chart program there will usually be more attributes control charts, applied to semifinished or finished units near the *end* of the manufacturing process. As we learn more about the process, these charts will be replaced with $\bar{x}$ and R charts applied *earlier* in the process to the critical parameters and operations that result in nonconformities in the finished product. Generally, **the earlier that process control can be established, the better.** In a complex assembly process, this may imply that process controls need to be implemented at the vendor or supplier level.
6. Control charts are an on-line, process-monitoring procedure. They should be implemented and maintained as close to the work center as possible, so that feedback will be rapid. Furthermore, the process operators and process engineering should have direct responsibility for collecting the process data, maintaining the charts, and interpreting the results. The operators and engineers have the detailed knowledge of the process required to correct process upsets and use the control chart to improve process performance. Microcomputers can speed up the feedback and should be an integral part of any modern, on-line, process-control procedure.
7. The out-of-control-action plan (OCAP) is a vital part of the control chart. Operating and engineering personnel should strive to keep OCAPs up-to-date and valid.

Choosing Proper Type of Control Chart

A. $\bar{x}$ and R (or $\bar{x}$ and s) charts. Consider using variables control charts in these situations:

1. A new process is coming on stream, or a new product is being manufactured by an existing process.
2. The process has been in operation for some time, but it is chronically in trouble or unable to hold the specified tolerances.
3. The process is in trouble, and the control chart can be useful for diagnostic purposes (troubleshooting).
4. Destructive testing (or other expensive testing procedures) is required.
5. It is desirable to reduce acceptance-sampling or other downstream testing to a minimum when the process can be operated in control.
6. Attributes control charts have been used, but the process is either out of control or in control but the yield is unacceptable.
7. There are very tight specifications, overlapping assembly tolerances, or other difficult manufacturing problems.
8. The operator must decide whether or not to adjust the process, or when a setup must be evaluated.
9. A change in product specifications is desired.
10. Process stability and capability must be continually demonstrated, such as in regulated industries.

B. Attributes Charts (*p* charts, *c* charts, and *u* charts). Consider using attributes control charts in these situations:

1. Operators control the assignable causes, and it is necessary to reduce process fall-out.
2. The process is a complex assembly operation and product quality is measured in terms of the occurrence of nonconformities, successful or unsuccessful product function, and so forth. (Examples include computers, office automation equipment, automobiles, and the major subsystems of these products.)
3. Process control is necessary, but measurement data cannot be obtained.
4. A historical summary of process performance is necessary. Attributes control charts, such as p charts, c charts, and u charts, are very effective for summarizing information about the process for management review.
5. Remember that attributes charts are generally inferior to charts for variables. Always use $\bar{x}$ and R or $\bar{x}$ and s charts whenever possible.

C. Control Charts for Individuals. Consider using the control chart for individuals in conjunction with a moving-range chart in these situations:

1. It is inconvenient or impossible to obtain more than one measurement per sample, or repeat measurements will only differ by laboratory or analysis error. Examples often occur in chemical processes.
2. Automated testing and inspection technology allow measurement of every unit produced. In these cases, also consider the cumulative sum control chart and the exponentially weighted moving average control chart discussed in Chapter 7.
3. The data become available very slowly, and waiting for a larger sample will be impractical or make the control procedure too slow to react to problems. This often happens in nonproduct situations; for example, accounting data may become available only monthly.
4. Generally, once we are in phase II, individuals charts have poor performance in shift detection and can be very sensitive to departures from normality. Always use the EWMA and cusum charts of Chapter 8 in phase II instead of individuals charts whenever possible.

Actions Taken to Improve Process

		IS THE PROCESS CAPABLE?	
		Yes	No
IS THE PROCESS IN CONTROL?	Yes	SPC	SPC Experimental design Investigate specifications Change process
	No	SPC	SPC Experimental design Investigate specifications Change process