



Magnetic Circuits

11.1 INTRODUCTION

Magnetism plays an integral part in almost every electrical device used today in industry, research, or the home. Generators, motors, transformers, circuit breakers, televisions, computers, tape recorders, and telephones all employ magnetic effects to perform a variety of important tasks.

The compass, used by Chinese sailors as early as the second century A.D., relies on a **permanent magnet** for indicating direction. The permanent magnet is made of a material, such as steel or iron, that will remain magnetized for long periods of time without the need for an external source of energy.

In 1820, the Danish physicist Hans Christian Oersted discovered that the needle of a compass would deflect if brought near a current-carrying conductor. For the first time it was demonstrated that electricity and magnetism were related, and in the same year the French physicist André-Marie Ampère performed experiments in this area and developed what is presently known as **Ampère's circuital law**. In subsequent years, men such as Michael Faraday, Karl Friedrich Gauss, and James Clerk Maxwell continued to experiment in this area and developed many of the basic concepts of **electromagnetism**—magnetic effects induced by the flow of charge, or current.

There is a great deal of similarity between the analyses of electric circuits and magnetic circuits. This will be demonstrated later in this chapter when we compare the basic equations and methods used to solve magnetic circuits with those used for electric circuits.

Difficulty in understanding methods used with magnetic circuits will often arise in simply learning to use the proper set of units, not because of the equations themselves. The problem exists because three different systems of units are still used in the industry. To the extent practical, SI will be used throughout this chapter. For the CGS and English systems, a conversion table is provided in Appendix F.

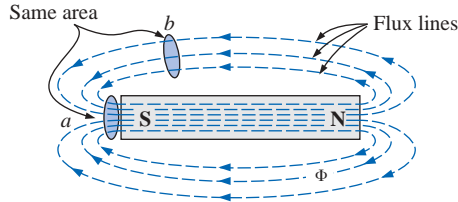


FIG. 11.1

Flux distribution for a permanent magnet.

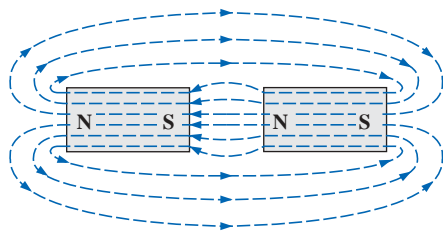


FIG. 11.2

Flux distribution for two adjacent, opposite poles.

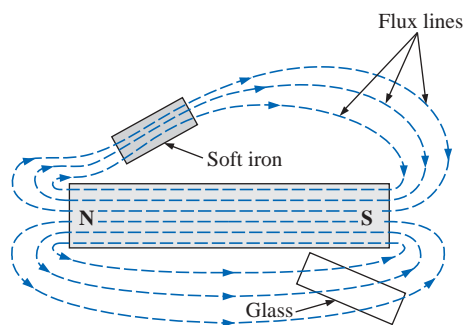


FIG. 11.4

Effect of a ferromagnetic sample on the flux distribution of a permanent magnet.

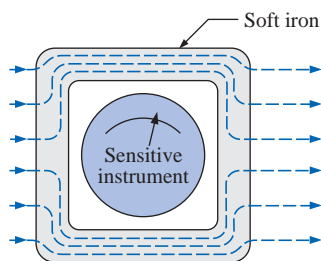


FIG. 11.5

Effect of a magnetic shield on the flux distribution.

11.2 MAGNETIC FIELDS

In the region surrounding a permanent magnet there exists a magnetic field, which can be represented by **magnetic flux lines** similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in continuous loops, as shown in Fig. 11.1. The symbol for magnetic flux is the Greek letter Φ (phi).

The magnetic flux lines radiate from the north pole to the south pole, returning to the north pole through the metallic bar. Note the equal spacing between the flux lines within the core and the symmetric distribution outside the magnetic material. These are additional properties of magnetic flux lines in homogeneous materials (that is, materials having uniform structure or composition throughout). It is also important to realize that the continuous magnetic flux line will strive to occupy as small an area as possible. This will result in magnetic flux lines of minimum length between the like poles, as shown in Fig. 11.2. The strength of a magnetic field in a particular region is directly related to the density of flux lines in that region. In Fig. 11.1, for example, the magnetic field strength at a is twice that at b since twice as many magnetic flux lines are associated with the perpendicular plane at a than at b . Recall from childhood experiments that the strength of permanent magnets was always stronger near the poles.

If unlike poles of two permanent magnets are brought together, the magnets will attract, and the flux distribution will be as shown in Fig. 11.2. If like poles are brought together, the magnets will repel, and the flux distribution will be as shown in Fig. 11.3.

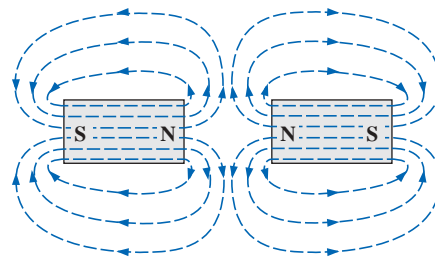


FIG. 11.3

Flux distribution for two adjacent, like poles.

If a nonmagnetic material, such as glass or copper, is placed in the flux paths surrounding a permanent magnet, there will be an almost unnoticeable change in the flux distribution (Fig. 11.4). However, if a magnetic material, such as soft iron, is placed in the flux path, the flux lines will pass through the soft iron rather than the surrounding air because flux lines pass with greater ease through magnetic materials than through air. This principle is put to use in the shielding of sensitive electrical elements and instruments that can be affected by stray magnetic fields (Fig. 11.5).

As indicated in the introduction, a magnetic field (represented by concentric magnetic flux lines, as in Fig. 11.6) is present around every wire that carries an electric current. The direction of the magnetic flux lines can be found simply by placing the thumb of the *right* hand in the direction of *conventional* current flow and noting the direction of the

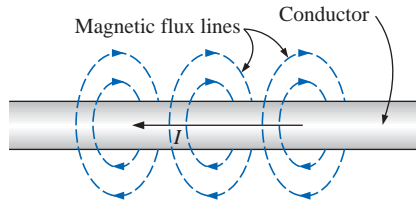


FIG. 11.6

Magnetic flux lines around a current-carrying conductor.

fingers. (This method is commonly called the *right-hand rule*.) If the conductor is wound in a single-turn coil (Fig. 11.7), the resulting flux will flow in a common direction through the center of the coil. A coil of more than one turn would produce a magnetic field that would exist in a continuous path through and around the coil (Fig. 11.8).

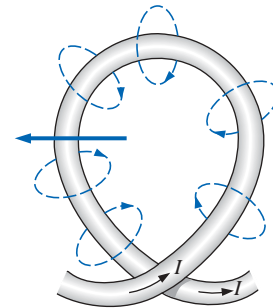


FIG. 11.7

Flux distribution of a single-turn coil.

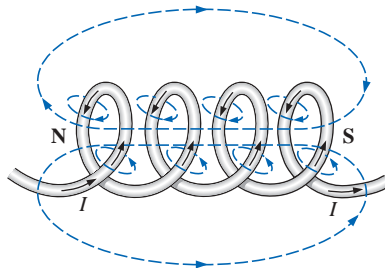


FIG. 11.8

Flux distribution of a current-carrying coil.

The flux distribution of the coil is quite similar to that of the permanent magnet. The flux lines leaving the coil from the left and entering to the right simulate a north and a south pole, respectively. The principal difference between the two flux distributions is that the flux lines are more concentrated for the permanent magnet than for the coil. Also, since the strength of a magnetic field is determined by the density of the flux lines, the coil has a weaker field strength. The field strength of the coil can be effectively increased by placing certain materials, such as iron, steel, or cobalt, within the coil to increase the flux density (defined in the next section) within the coil. By increasing the field strength with the addition of the core, we have devised an *electromagnet* (Fig. 11.9) that, in addition to having all the properties of a permanent magnet, also has a field strength that can be varied by changing one of the component values (current, turns, and so on). Of course, current must pass through the coil of the electromagnet in order for magnetic flux to be developed, whereas there is no need for the coil or current in the permanent magnet. The direction of flux lines can be determined for the electromagnet (or in any core with a wrapping of turns) by placing the fingers of the right hand in the direction of current flow around the core. The thumb will then point in the direction of the north pole of the induced magnetic flux, as demonstrated in Fig. 11.10(a). A cross section of the same electromagnet is included as Fig. 11.10(b) to introduce the convention for directions perpendicular to the page. The cross and dot refer to the tail and head of the arrow, respectively.

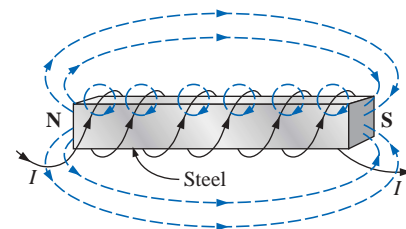
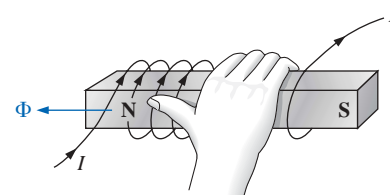
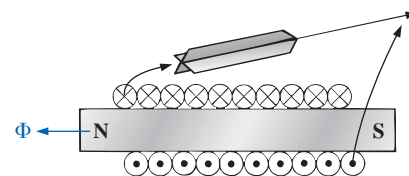


FIG. 11.9

Electromagnet.



(a)



(b)

FIG. 11.10

Determining the direction of flux for an electromagnet: (a) method; (b) notation.



Other areas of application for electromagnetic effects are shown in Fig. 11.11. The flux path for each is indicated in each figure.

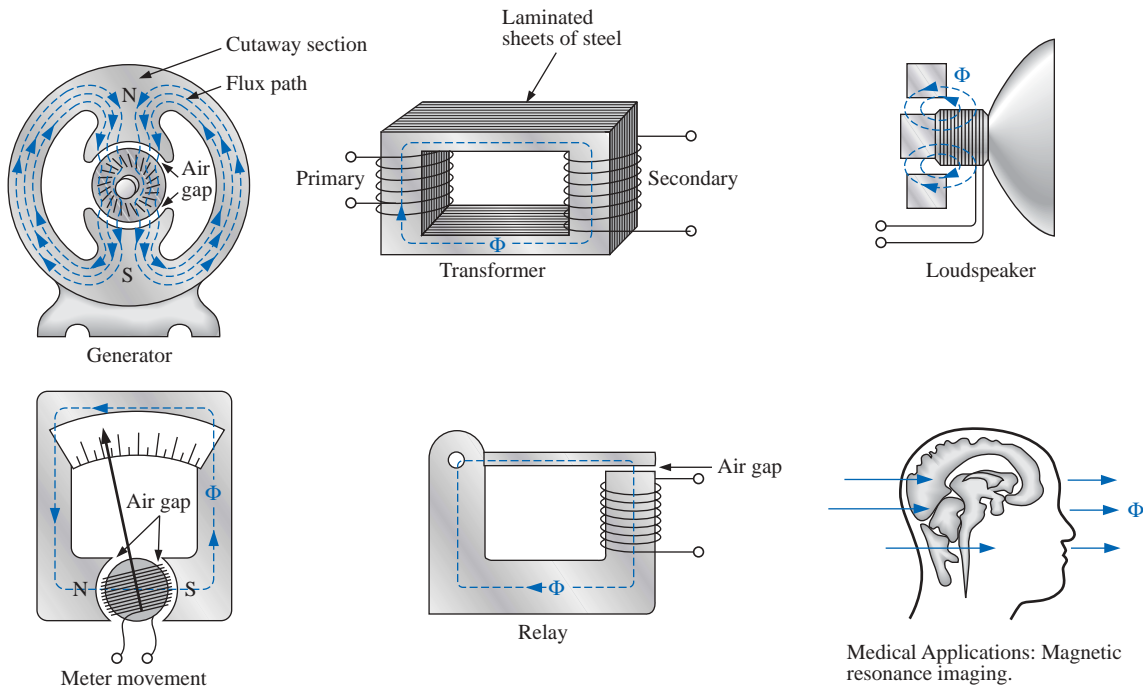


FIG. 11.11

Some areas of application of magnetic effects.

German (Wittenberg, Göttingen) (1804–91) Physicist Professor of Physics, University of Göttingen



Courtesy of the Smithsonian Institution Photo No. 52,604

An important contributor to the establishment of a system of *absolute units* for the electrical sciences, which was beginning to become a very active area of research and development. Established a definition of electric current in an electromagnetic system based on the magnetic field produced by the current. He was politically active and, in fact, was dismissed from the faculty of the University of Göttingen for protesting the suppression of the constitution by the King of Hanover in 1837. However, he found other faculty positions and eventually returned to Göttingen as director of the astronomical observatory. Received honors from England, France, and Germany, including the Copley Medal of the Royal Society.

FIG. 11.12

Wilhelm Eduard Weber.

11.3 FLUX DENSITY

In the SI system of units, magnetic flux is measured in *webers* (note Fig. 11.12) and has the symbol Φ . The number of flux lines per unit area is called the **flux density**, is denoted by the capital letter B , and is measured in *teslas* (note Fig. 11.15). Its magnitude is determined by the following equation:

$$B = \frac{\Phi}{A} \quad \begin{matrix} B = \text{teslas (T)} \\ \Phi = \text{webers (Wb)} \\ A = \text{square meters (m}^2\text{)} \end{matrix} \quad (11.1)$$

where Φ is the number of flux lines passing through the area A (Fig. 11.13). The flux density at position a in Fig. 11.1 is twice that at b because twice as many flux lines are passing through the same area. By definition,

$$1 \text{ T} = 1 \text{ Wb/m}^2$$

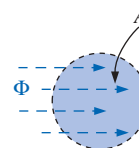


FIG. 11.13

Defining the flux density B .



EXAMPLE 11.1 For the core of Fig. 11.14, determine the flux density B in teslas.

Solution:

$$B = \frac{\Phi}{A} = \frac{6 \times 10^{-5} \text{ Wb}}{1.2 \times 10^{-3} \text{ m}^2} = 5 \times 10^{-2} \text{ T}$$

EXAMPLE 11.2 In Fig. 11.14, if the flux density is 1.2 T and the area is 0.25 in.^2 , determine the flux through the core.

Solution: By Eq. (11.1),

$$\Phi = BA$$

However, converting 0.25 in.^2 to metric units,

$$A = 0.25 \text{ in.}^2 \left(\frac{1 \text{ m}}{39.37 \text{ in.}} \right) \left(\frac{1 \text{ m}}{39.37 \text{ in.}} \right) = 1.613 \times 10^{-4} \text{ m}^2$$

and

$$\begin{aligned} \Phi &= (1.2 \text{ T})(1.613 \times 10^{-4} \text{ m}^2) \\ &= \mathbf{1.936 \times 10^{-4} \text{ Wb}} \end{aligned}$$

An instrument designed to measure flux density in gauss (CGS system) appears in Fig. 11.16. Appendix F reveals that $1 \text{ T} = 10^4 \text{ gauss}$. The magnitude of the reading appearing on the face of the meter in Fig. 11.16 is therefore

$$1.964 \text{ gauss} \left(\frac{1 \text{ T}}{10^4 \text{ gauss}} \right) = 1.964 \times 10^{-4} \text{ T}$$

11.4 PERMEABILITY

If cores of different materials with the same physical dimensions are used in the electromagnet described in Section 11.2, the strength of the magnet will vary in accordance with the core used. This variation in strength is due to the greater or lesser number of flux lines passing through the core. Materials in which flux lines can readily be set up are said to be *magnetic* and to have *high permeability*. The **permeability** (μ) of a material, therefore, is a measure of the ease with which magnetic flux lines can be established in the material. It is similar in many respects to conductivity in electric circuits. The permeability of free space μ_o (vacuum) is

$$\mu_o = 4\pi \times 10^{-7} \frac{\text{Wb}}{\text{A} \cdot \text{m}}$$

As indicated above, μ has the units of $\text{Wb}/\text{A} \cdot \text{m}$. Practically speaking, the permeability of all nonmagnetic materials, such as copper, aluminum, wood, glass, and air, is the same as that for free space. Materials that have permeabilities slightly less than that of free space are said to be **diamagnetic**, and those with permeabilities slightly greater than that of free space are said to be **paramagnetic**. Magnetic materials, such as iron, nickel, steel, cobalt, and alloys of these metals, have permeabilities hundreds and even thousands of times that of free space. Materials with these very high permeabilities are referred to as **ferromagnetic**.



$$\begin{aligned} \Phi &= 6 \times 10^{-5} \text{ Wb} \\ A &= 1.2 \times 10^{-3} \text{ m}^2 \end{aligned}$$

FIG. 11.14
Example 11.1.

Croatian-American
(Smiljan, Paris,
Colorado Springs,
New York City)
(1856–1943)
Electrical Engineer
and Inventor
Recipient of the
Edison Medal in
1917



Courtesy of the
Smithsonian Institution
Photo No. 52,223

Often regarded as one of the most innovative and inventive individuals in the history of the sciences. He was the first to introduce the *alternating-current machine*, removing the need for commutator bars of dc machines. After emigrating to the United States in 1884, he sold a number of his patents on *ac machines*, *transformers*, and *induction coils* (including the *Tesla coil* as we know it today) to the Westinghouse Electric Company. Some say that his most important discovery was made at his laboratory in Colorado Springs, where in 1900 he discovered *terrestrial stationary waves*. The range of his discoveries and inventions is too extensive to list here but extends from *lighting systems* to *polyphase power systems* to a *wireless world broadcasting system*.

FIG. 11.15
Nikola Tesla.



FIG. 11.16
Digital display gaussmeter. (Courtesy of LDJ Electronics, Inc.)



The ratio of the permeability of a material to that of free space is called its **relative permeability**; that is,

$$\mu_r = \frac{\mu}{\mu_o} \quad (11.2)$$

In general, for ferromagnetic materials, $\mu_r \geq 100$, and for nonmagnetic materials, $\mu_r = 1$.

Since μ_r is a variable, dependent on other quantities of the magnetic circuit, values of μ_r are not tabulated. Methods of calculating μ_r from the data supplied by manufacturers will be considered in a later section.

11.5 RELUCTANCE

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho \frac{l}{A} \quad (\text{ohms, } \Omega)$$

The **reluctance** of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\mathcal{R} = \frac{l}{\mu A} \quad (\text{rels, or At/Wb}) \quad (11.3)$$

where \mathcal{R} is the reluctance, l is the length of the magnetic path, and A is the cross-sectional area. The t in the units At/Wb is the number of turns of the applied winding. More is said about ampere-turns (At) in the next section. Note that the resistance and reluctance are inversely proportional to the area, indicating that an increase in area will result in a reduction in each and an *increase* in the desired result: current and flux. For an increase in length the opposite is true, and the desired effect is reduced. The reluctance, however, is inversely proportional to the permeability, while the resistance is directly proportional to the resistivity. The larger the μ or the smaller the ρ , the smaller the reluctance and resistance, respectively. Obviously, therefore, materials with high permeability, such as the ferromagnetics, have very small reluctances and will result in an increased measure of flux through the core. There is no widely accepted unit for reluctance, although the *rel* and the At/Wb are usually applied.

11.6 OHM'S LAW FOR MAGNETIC CIRCUITS

Recall the equation

$$\text{Effect} = \frac{\text{cause}}{\text{opposition}}$$

appearing in Chapter 4 to introduce Ohm's law for electric circuits. For magnetic circuits, the effect desired is the flux Φ . The cause is the **magnetomotive force (mmf)** \mathcal{F} , which is the external force (or "pressure") required to set up the magnetic flux lines within the magnetic material. The opposition to the setting up of the flux Φ is the reluctance \mathcal{R} .



Substituting, we have

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} \quad (11.4)$$

The magnetomotive force \mathcal{F} is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire (Fig. 11.17). In equation form,

$$\mathcal{F} = NI \quad (\text{ampere-turns, At}) \quad (11.5)$$

This equation clearly indicates that an increase in the number of turns or the current through the wire will result in an increased “pressure” on the system to establish flux lines through the core.

Although there is a great deal of similarity between electric and magnetic circuits, one must continue to realize that the flux Φ is not a “flow” variable such as current in an electric circuit. Magnetic flux is established in the core through the alteration of the atomic structure of the core due to external pressure and is not a measure of the flow of some charged particles through the core.

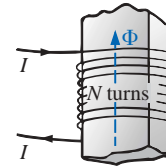


FIG. 11.17
Defining the components of a magnetomotive force.

11.7 MAGNETIZING FORCE

The magnetomotive force per unit length is called the **magnetizing force** (H). In equation form,

$$H = \frac{\mathcal{F}}{l} \quad (\text{At/m}) \quad (11.6)$$

Substituting for the magnetomotive force will result in

$$H = \frac{NI}{l} \quad (\text{At/m}) \quad (11.7)$$

For the magnetic circuit of Fig. 11.18, if $NI = 40 \text{ At}$ and $l = 0.2 \text{ m}$, then

$$H = \frac{NI}{l} = \frac{40 \text{ At}}{0.2 \text{ m}} = 200 \text{ At/m}$$

In words, the result indicates that there are 200 At of “pressure” per meter to establish flux in the core.

Note in Fig. 11.18 that the direction of the flux Φ can be determined by placing the fingers of the right hand in the direction of current around the core and noting the direction of the thumb. It is interesting to realize that *the magnetizing force is independent of the type of core material*—it is determined solely by the number of turns, the current, and the length of the core.

The applied magnetizing force has a pronounced effect on the resulting permeability of a magnetic material. As the magnetizing force increases, the permeability rises to a maximum and then drops to a minimum, as shown in Fig. 11.19 for three commonly employed magnetic materials.

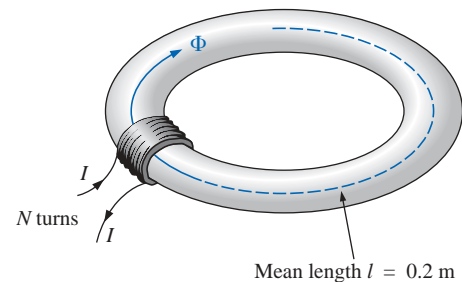


FIG. 11.18
Defining the magnetizing force of a magnetic circuit.

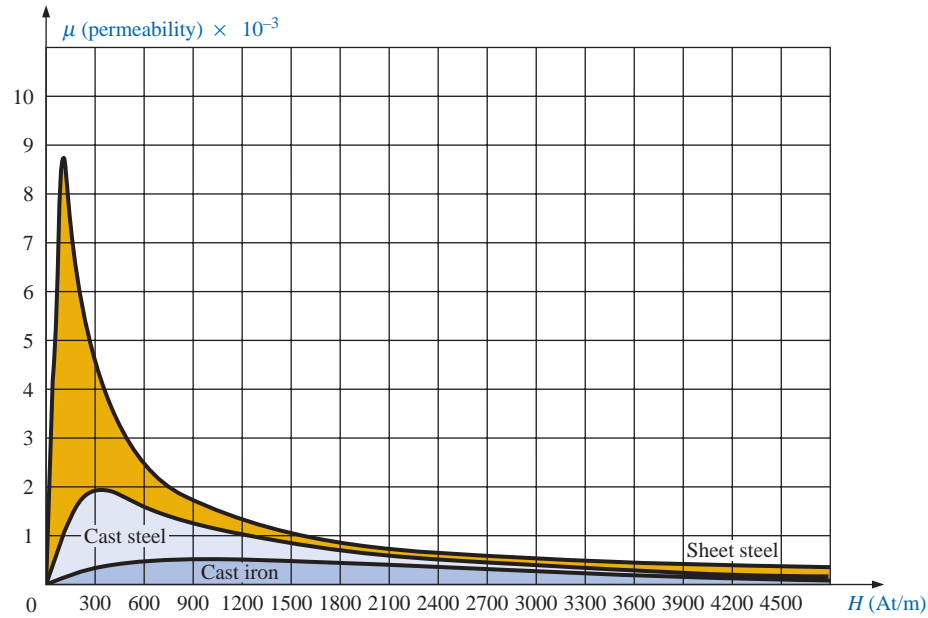


FIG. 11.19
Variation of μ with the magnetizing force.

The flux density and the magnetizing force are related by the following equation:

$$B = \mu H \tag{11.8}$$

This equation indicates that for a particular magnetizing force, the greater the permeability, the greater will be the induced flux density.

Since henries (H) and the magnetizing force (H) use the same capital letter, it must be pointed out that all units of measurement in the text, such as henries, use roman letters, such as H, whereas variables such as the magnetizing force use italic letters, such as H .

11.8 HYSTERESIS

A curve of the flux density B versus the magnetizing force H of a material is of particular importance to the engineer. Curves of this type can usually be found in manuals, descriptive pamphlets, and brochures published by manufacturers of magnetic materials. A typical B - H curve for a ferromagnetic material such as steel can be derived using the setup of Fig. 11.20.

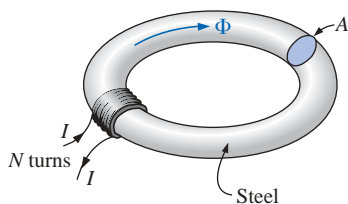


FIG. 11.20
Series magnetic circuit used to define the hysteresis curve.

The core is initially unmagnetized and the current $I = 0$. If the current I is increased to some value above zero, the magnetizing force H will increase to a value determined by

$$H \uparrow = \frac{NI \uparrow}{l}$$

The flux Φ and the flux density B ($B = \Phi/A$) will also increase with the current I (or H). If the material has no residual magnetism, and the magnetizing force H is increased from zero to some value H_a , the B - H curve will follow the path shown in Fig. 11.21 between o and a . If the

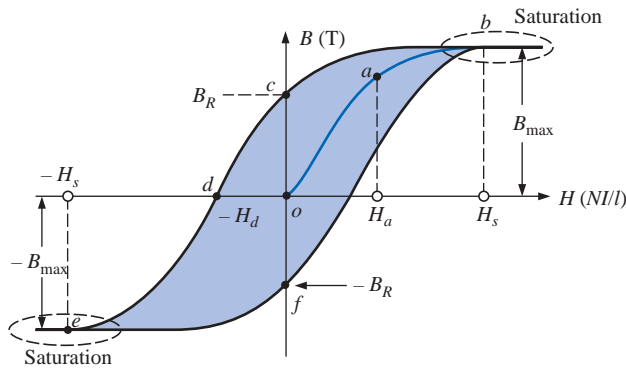


FIG. 11.21
Hysteresis curve.

magnetizing force H is increased until saturation (H_s) occurs, the curve will continue as shown in the figure to point b . When saturation occurs, the flux density has, for all practical purposes, reached its maximum value. Any further increase in current through the coil increasing $H = NI/l$ will result in a very small increase in flux density B .

If the magnetizing force is reduced to zero by letting I decrease to zero, the curve will follow the path of the curve between b and c . The flux density B_R , which remains when the magnetizing force is zero, is called the *residual flux density*. It is this residual flux density that makes it possible to create permanent magnets. If the coil is now removed from the core of Fig. 11.20, the core will still have the magnetic properties determined by the residual flux density, a measure of its “retentivity.” If the current I is reversed, developing a magnetizing force, $-H$, the flux density B will decrease with an increase in I . Eventually, the flux density will be zero when $-H_d$ (the portion of curve from c to d) is reached. The magnetizing force $-H_d$ required to “coerce” the flux density to reduce its level to zero is called the *coercive force*, a measure of the coercivity of the magnetic sample. As the force $-H$ is increased until saturation again occurs and is then reversed and brought back to zero, the path def will result. If the magnetizing force is increased in the positive direction ($+H$), the curve will trace the path shown from f to b . The entire curve represented by $bcdefb$ is called the **hysteresis** curve for the ferromagnetic material, from the Greek *hysterein*, meaning “to lag behind.” The flux density B lagged behind the magnetizing force H during the entire plotting of the curve. When H was zero at c , B was not zero but had only begun to decline. Long after H had passed through zero and had become equal to $-H_d$ did the flux density B finally become equal to zero.

If the entire cycle is repeated, the curve obtained for the same core will be determined by the maximum H applied. Three hysteresis loops for the same material for maximum values of H less than the saturation value are shown in Fig. 11.22. In addition, the saturation curve is repeated for comparison purposes.

Note from the various curves that for a particular value of H , say, H_x , the value of B can vary widely, as determined by the history of the core. In an effort to assign a particular value of B to each value of H , we compromise by connecting the tips of the hysteresis loops. The resulting curve, shown by the heavy, solid line in Fig. 11.22 and for various

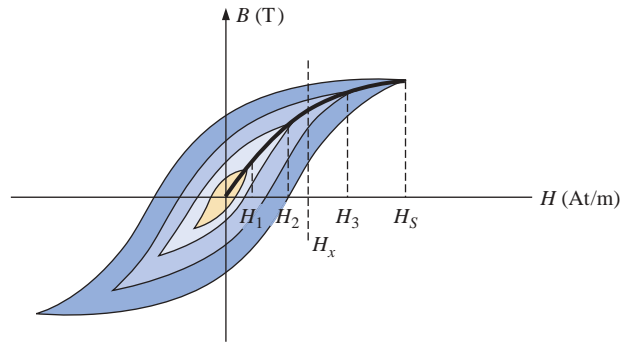


FIG. 11.22

Defining the normal magnetization curve.

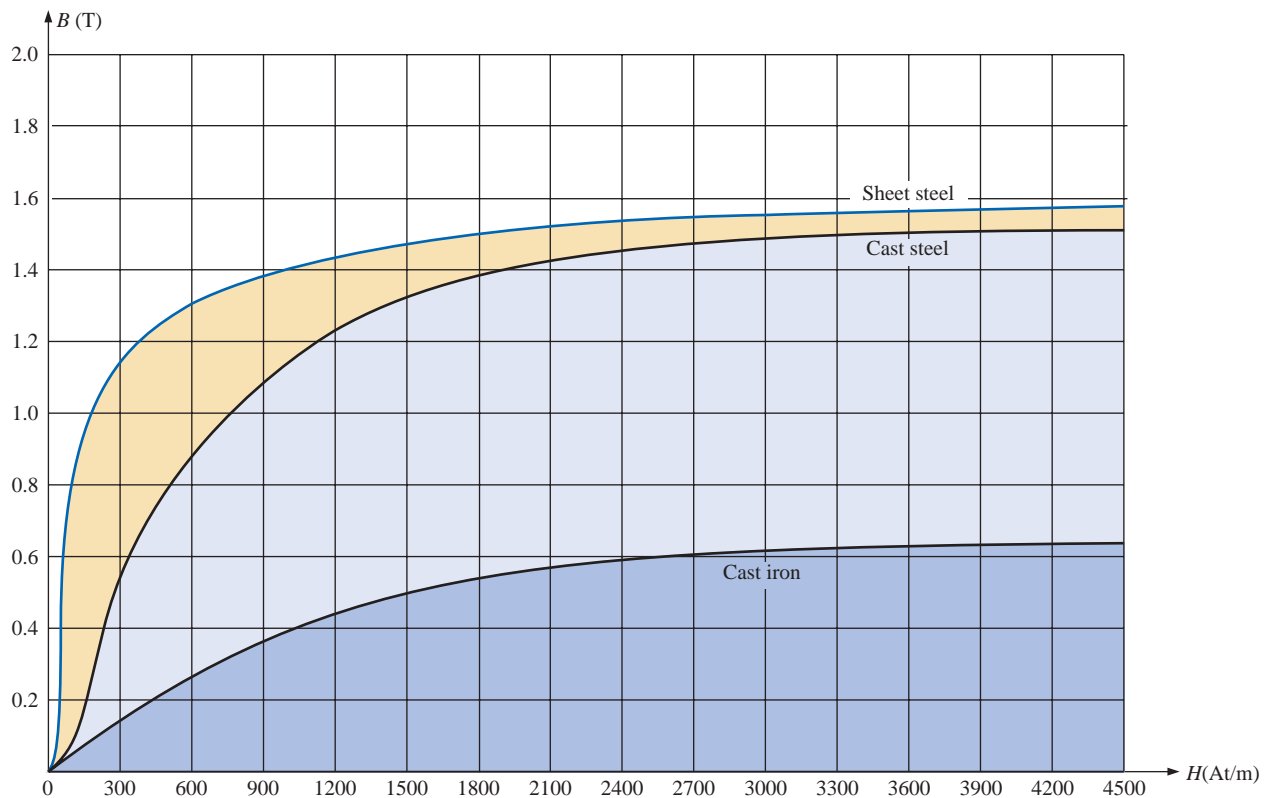


FIG. 11.23

Normal magnetization curve for three ferromagnetic materials.

materials in Fig. 11.23, is called the *normal magnetization curve*. An expanded view of one region appears in Fig. 11.24.

A comparison of Figs. 11.19 and 11.23 shows that for the same value of H , the value of B is higher in Fig. 11.23 for the materials with the higher μ in Fig. 11.19. This is particularly obvious for low values of H . This correspondence between the two figures must exist since $B = \mu H$. In fact, if in Fig. 11.23 we find μ for each value of H using the equation $\mu = B/H$, we will obtain the curves of Fig. 11.19. An instrument that will provide a plot of the B - H curve for a magnetic sample appears in Fig. 11.25.

It is interesting to note that the hysteresis curves of Fig. 11.22 have a *point symmetry* about the origin; that is, the inverted pattern to the left of the vertical axis is the same as that appearing to the right of the ver-

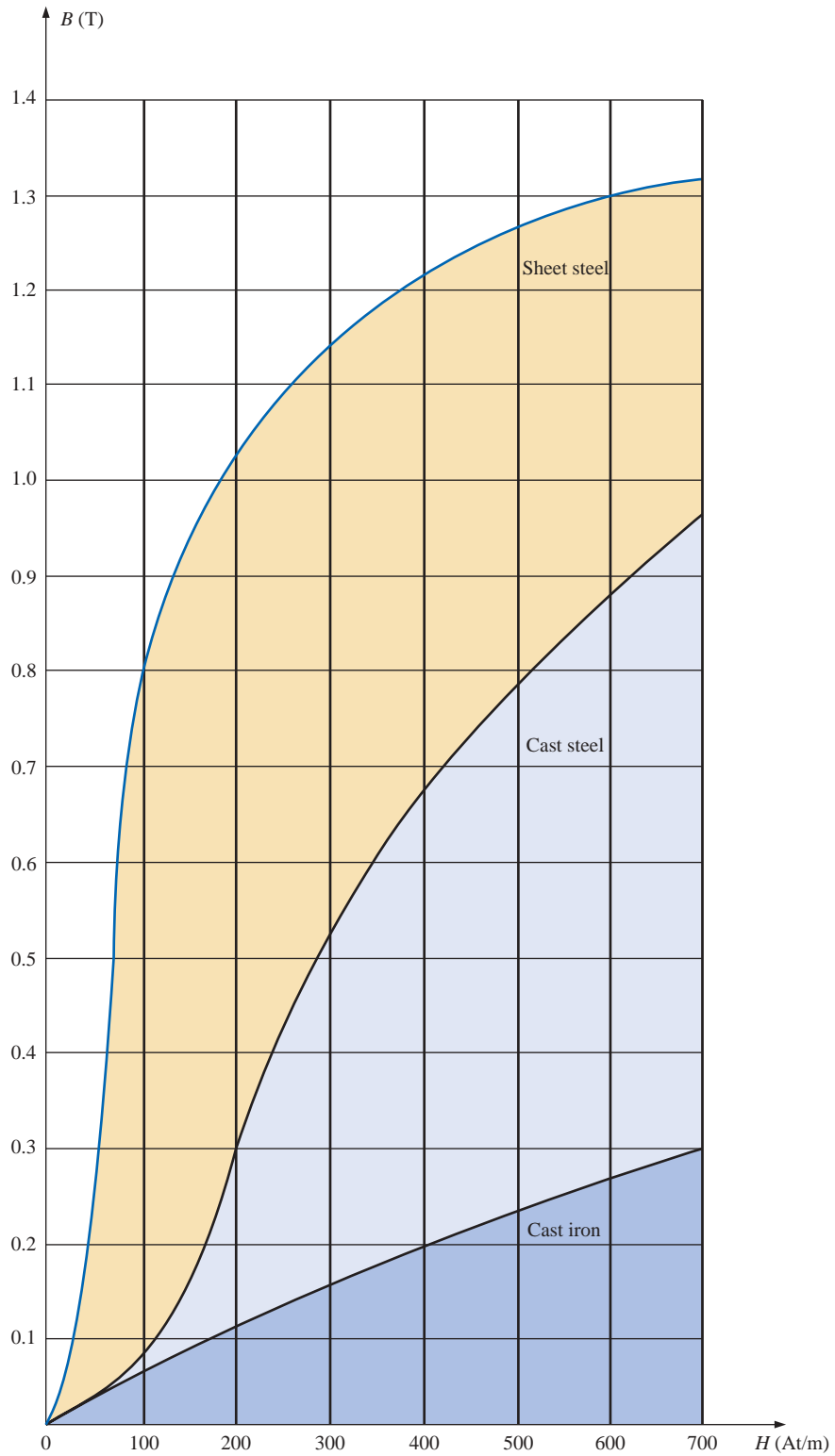


FIG. 11.24

Expanded view of Fig. 11.23 for the low magnetizing force region.



FIG. 11.25

Model 9600 vibrating sample magnetometer. (Courtesy of LDJ Electronics, Inc.)

tical axis. In addition, you will find that a further application of the same magnetizing forces to the sample will result in the same plot. For a current I in $H = NI/l$ that will move between positive and negative maximums at a fixed rate, the same B - H curve will result during each cycle. Such will be the case when we examine ac (sinusoidal) networks in the later chapters. The reversal of the field (Φ) due to the changing current direction will result in a loss of energy that can best be described by first introducing the *domain theory of magnetism*.

Within each atom, the orbiting electrons (described in Chapter 2) are also spinning as they revolve around the nucleus. The atom, due to its spinning electrons, has a magnetic field associated with it. In nonmagnetic materials, the net magnetic field is effectively zero since the magnetic fields due to the atoms of the material oppose each other. In magnetic materials such as iron and steel, however, the magnetic fields of groups of atoms numbering in the order of 10^{12} are aligned, forming very small bar magnets. This group of magnetically aligned atoms is called a **domain**. Each domain is a separate entity; that is, each domain is independent of the surrounding domains. For an unmagnetized sample of magnetic material, these domains appear in a random manner, such as shown in Fig. 11.26(a). The net magnetic field in any one direction is zero.

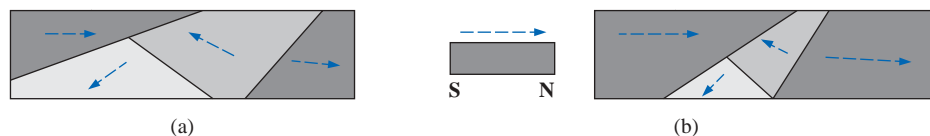


FIG. 11.26

Demonstrating the domain theory of magnetism.

When an external magnetizing force is applied, the domains that are nearly aligned with the applied field will grow at the expense of the less favorably oriented domains, such as shown in Fig. 11.26(b). Eventually, if a sufficiently strong field is applied, all of the domains will have the orientation of the applied magnetizing force, and any further increase in external field will not increase the strength of the magnetic flux through the core—a condition referred to as *saturation*. The elasticity of the



above is evidenced by the fact that when the magnetizing force is removed, the alignment will be lost to some measure, and the flux density will drop to B_R . In other words, the removal of the magnetizing force will result in the return of a number of misaligned domains within the core. The continued alignment of a number of the domains, however, accounts for our ability to create permanent magnets.

At a point just before saturation, the opposing unaligned domains are reduced to small cylinders of various shapes referred to as *bubbles*. These bubbles can be moved within the magnetic sample through the application of a *controlling* magnetic field. These magnetic bubbles form the basis of the recently designed bubble memory system for computers.

11.9 AMPÈRE'S CIRCUITAL LAW

As mentioned in the introduction to this chapter, there is a broad similarity between the analyses of electric and magnetic circuits. This has already been demonstrated to some extent for the quantities in Table 11.1.

TABLE 11.1

	Electric Circuits	Magnetic Circuits
Cause	E	\mathcal{F}
Effect	I	Φ
Opposition	R	\mathcal{R}

If we apply the “cause” analogy to Kirchhoff’s voltage law ($\sum_C V = 0$), we obtain the following:

$$\sum_C \mathcal{F} = 0 \quad (\text{for magnetic circuits}) \quad (11.9)$$

which, in words, states that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop.

Equation (11.9) is referred to as **Ampère’s circuital law**. When it is applied to magnetic circuits, sources of mmf are expressed by the equation

$$\mathcal{F} = NI \quad (\text{At}) \quad (11.10)$$

The equation for the mmf drop across a portion of a magnetic circuit can be found by applying the relationships listed in Table 11.1; that is, for electric circuits,

$$V = IR$$

resulting in the following for magnetic circuits:

$$\mathcal{F} = \Phi \mathcal{R} \quad (\text{At}) \quad (11.11)$$

where Φ is the flux passing through a section of the magnetic circuit and \mathcal{R} is the reluctance of that section. The reluctance, however, is sel-

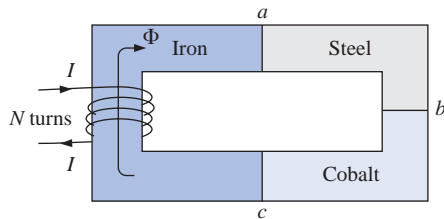


FIG. 11.27
Series magnetic circuit of three different materials.

dom calculated in the analysis of magnetic circuits. A more practical equation for the mmf drop is

$$\mathcal{F} = Hl \quad (\text{At}) \quad (11.12)$$

as derived from Eq. (11.6), where H is the magnetizing force on a section of a magnetic circuit and l is the length of the section.

As an example of Eq. (11.9), consider the magnetic circuit appearing in Fig. 11.27 constructed of three different ferromagnetic materials. Applying Ampère’s circuital law, we have

$$\begin{aligned} \sum_c \mathcal{F} &= 0 \\ +NI - \underbrace{H_{ab}l_{ab}}_{\text{Drop}} - \underbrace{H_{bc}l_{bc}}_{\text{Drop}} - \underbrace{H_{ca}l_{ca}}_{\text{Drop}} &= 0 \\ \underbrace{NI}_{\text{Impressed mmf}} &= \underbrace{H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}}_{\text{mmf drops}} \end{aligned}$$

All the terms of the equation are known except the magnetizing force for each portion of the magnetic circuit, which can be found by using the B - H curve if the flux density B is known.

11.10 THE FLUX Φ

If we continue to apply the relationships described in the previous section to Kirchhoff’s current law, we will find that the sum of the fluxes entering a junction is equal to the sum of the fluxes leaving a junction; that is, for the circuit of Fig. 11.28,

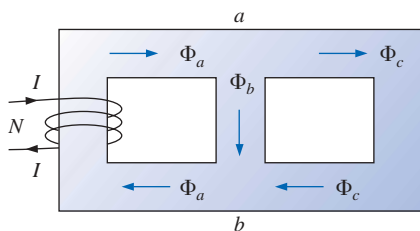


FIG. 11.28
Flux distribution of a series-parallel magnetic network.

$$\Phi_a = \Phi_b + \Phi_c \quad (\text{at junction } a)$$

or
$$\Phi_b + \Phi_c = \Phi_a \quad (\text{at junction } b)$$

both of which are equivalent.

11.11 SERIES MAGNETIC CIRCUITS: DETERMINING NI

We are now in a position to solve a few magnetic circuit problems, which are basically of two types. In one type, Φ is given, and the impressed mmf NI must be computed. This is the type of problem encountered in the design of motors, generators, and transformers. In the other type, NI is given, and the flux Φ of the magnetic circuit must be found. This type of problem is encountered primarily in the design of magnetic amplifiers and is more difficult since the approach is “hit or miss.”

As indicated in earlier discussions, the value of μ will vary from point to point along the magnetization curve. This eliminates the possibility of finding the reluctance of each “branch” or the “total reluctance” of a network, as was done for electric circuits where ρ had a fixed value for any applied current or voltage. If the total reluctance could be determined, Φ could then be determined using the Ohm’s law analogy for magnetic circuits.

For magnetic circuits, the level of B or H is determined from the other using the B - H curve, and μ is seldom calculated unless asked for.



An approach frequently employed in the analysis of magnetic circuits is the *table* method. Before a problem is analyzed in detail, a table is prepared listing in the extreme left-hand column the various sections of the magnetic circuit. The columns on the right are reserved for the quantities to be found for each section. In this way, the individual doing the problem can keep track of what is required to complete the problem and also of what the next step should be. After a few examples, the usefulness of this method should become clear.

This section will consider only *series* magnetic circuits in which the flux Φ is the same throughout. In each example, the magnitude of the magnetomotive force is to be determined.

EXAMPLE 11.3 For the series magnetic circuit of Fig. 11.29:

- Find the value of I required to develop a magnetic flux of $\Phi = 4 \times 10^{-4}$ Wb.
- Determine μ and μ_r for the material under these conditions.

Solutions: The magnetic circuit can be represented by the system shown in Fig. 11.30(a). The electric circuit analogy is shown in Fig. 11.30(b). Analogies of this type can be very helpful in the solution of magnetic circuits. Table 11.2 is for part (a) of this problem. The table is fairly trivial for this example, but it does define the quantities to be found.

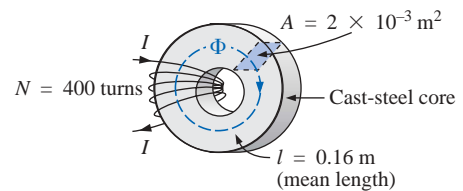
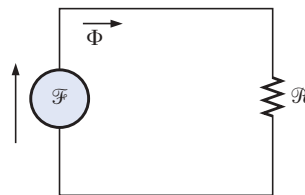
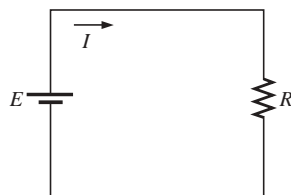


FIG. 11.29
Example 11.3.



(a)



(b)

FIG. 11.30

(a) Magnetic circuit equivalent and
(b) electric circuit analogy.

TABLE 11.2

Section	Φ (Wb)	A (m ²)	B (T)	H (At/m)	l (m)	HI (At)
One continuous section	4×10^{-4}	2×10^{-3}			0.16	

- The flux density B is

$$B = \frac{\Phi}{A} = \frac{4 \times 10^{-4} \text{ Wb}}{2 \times 10^{-3} \text{ m}^2} = 2 \times 10^{-1} \text{ T} = 0.2 \text{ T}$$



Using the B - H curves of Fig. 11.24, we can determine the magnetizing force H :

$$H \text{ (cast steel)} = 170 \text{ At/m}$$

Applying Ampère's circuital law yields

$$NI = Hl$$

and
$$I = \frac{Hl}{N} = \frac{(170 \text{ At/m})(0.16 \text{ m})}{400 \text{ t}} = \mathbf{68 \text{ mA}}$$

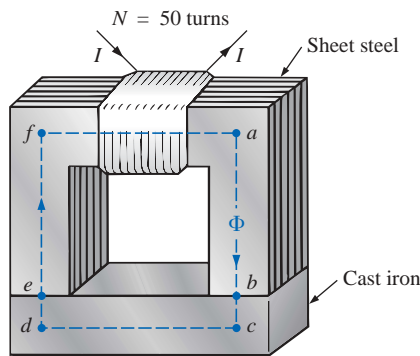
(Recall that t represents turns.)

b. The permeability of the material can be found using Eq. (11.8):

$$\mu = \frac{B}{H} = \frac{0.2 \text{ T}}{170 \text{ At/m}} = \mathbf{1.176 \times 10^{-3} \text{ Wb/A}\cdot\text{m}}$$

and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = \mathbf{935.83}$$



$l_{ab} = l_{cd} = l_{ef} = l_{fa} = 4 \text{ in.}$
 $l_{bc} = l_{de} = 0.5 \text{ in.}$
 Area (throughout) = 1 in.^2
 $\Phi = 3.5 \times 10^{-4} \text{ Wb}$

FIG. 11.31

Electromagnet for Example 11.4.

EXAMPLE 11.4 The electromagnet of Fig. 11.31 has picked up a section of cast iron. Determine the current I required to establish the indicated flux in the core.

Solution: To be able to use Figs. 11.23 and 11.24, we must first convert to the metric system. However, since the area is the same throughout, we can determine the length for each material rather than work with the individual sections:

$$l_{efab} = 4 \text{ in.} + 4 \text{ in.} + 4 \text{ in.} = 12 \text{ in.}$$

$$l_{bcde} = 0.5 \text{ in.} + 4 \text{ in.} + 0.5 \text{ in.} = 5 \text{ in.}$$

$$12 \text{ in.} \left(\frac{1 \text{ m}}{39.37 \text{ in.}} \right) = 304.8 \times 10^{-3} \text{ m}$$

$$5 \text{ in.} \left(\frac{1 \text{ m}}{39.37 \text{ in.}} \right) = 127 \times 10^{-3} \text{ m}$$

$$1 \text{ in.}^2 \left(\frac{1 \text{ m}}{39.37 \text{ in.}} \right) \left(\frac{1 \text{ m}}{39.37 \text{ in.}} \right) = 6.452 \times 10^{-4} \text{ m}^2$$

The information available from the specifications of the problem has been inserted in Table 11.3. When the problem has been completed, each space will contain some information. Sufficient data to complete the problem can be found if we fill in each column from left to right. As the various quantities are calculated, they will be placed in a similar table found at the end of the example.

TABLE 11.3

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	HI (At)
efab	3.5×10^{-4}	6.452×10^{-4}			304.8×10^{-3}	
bcde	3.5×10^{-4}	6.452×10^{-4}			127×10^{-3}	

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{3.5 \times 10^{-4} \text{ Wb}}{6.452 \times 10^{-4} \text{ m}^2} = 0.542 \text{ T}$$



and the magnetizing force is

$$H \text{ (sheet steel, Fig. 11.24)} \cong 70 \text{ At/m}$$

$$H \text{ (cast iron, Fig. 11.23)} \cong 1600 \text{ At/m}$$

Note the extreme difference in magnetizing force for each material for the required flux density. In fact, when we apply Ampère’s circuital law, we will find that the sheet steel section could be ignored with a minimal error in the solution.

Determining HI for each section yields

$$H_{efab}l_{efab} = (70 \text{ At/m})(304.8 \times 10^{-3} \text{ m}) = 21.34 \text{ At}$$

$$H_{bcde}l_{bcde} = (1600 \text{ At/m})(127 \times 10^{-3} \text{ m}) = 203.2 \text{ At}$$

Inserting the above data in Table 11.3 will result in Table 11.4.

TABLE 11.4

Section	Φ (Wb)	A (m ²)	B (T)	H (At/m)	l (m)	HI (At)
<i>efab</i>	3.5×10^{-4}	6.452×10^{-4}	0.542	70	304.8×10^{-3}	21.34
<i>bcde</i>	3.5×10^{-4}	6.452×10^{-4}	0.542	1600	127×10^{-3}	203.2

The magnetic circuit equivalent and the electric circuit analogy for the system of Fig. 11.31 appear in Fig. 11.32.

Applying Ampère’s circuital law,

$$NI = H_{efab}l_{efab} + H_{bcde}l_{bcde} = 21.34 \text{ At} + 203.2 \text{ At} = 224.54 \text{ At}$$

and $(50 \text{ t})I = 224.54 \text{ At}$

so that $I = \frac{224.54 \text{ At}}{50 \text{ t}} = 4.49 \text{ A}$

EXAMPLE 11.5 Determine the secondary current I_2 for the transformer of Fig. 11.33 if the resultant clockwise flux in the core is $1.5 \times 10^{-5} \text{ Wb}$.

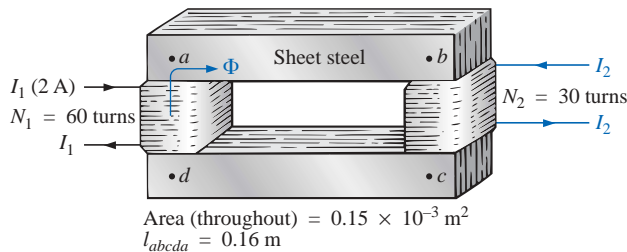


FIG. 11.33 Transformer for Example 11.5.

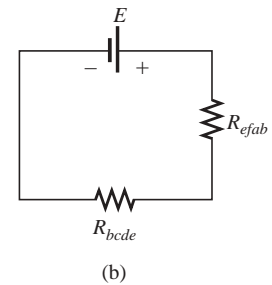
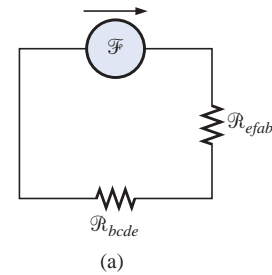


FIG. 11.32 (a) Magnetic circuit equivalent and (b) electric circuit analogy for the electromagnet of Fig. 11.31.

Solution: This is the first example with two magnetizing forces to consider. In the analogies of Fig. 11.34 you will note that the resulting flux of each is opposing, just as the two sources of voltage are opposing in the electric circuit analogy.

The structural data appear in Table 11.5.

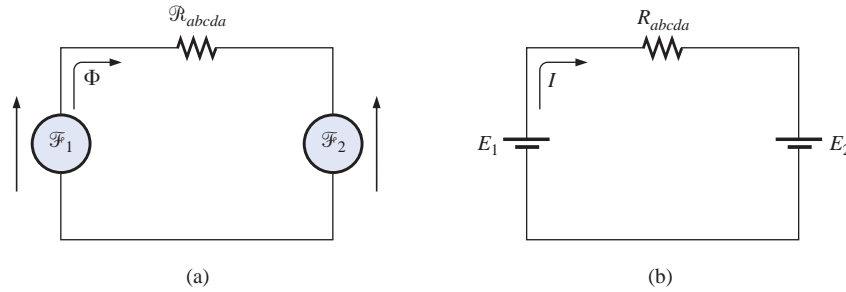


FIG. 11.34
 (a) Magnetic circuit equivalent and (b) electric circuit analogy for the transformer of Fig. 11.33.

TABLE 11.5

Section	Φ (Wb)	A (m ²)	B (T)	H (At/m)	l (m)	HI (At)
abcd	1.5 × 10 ⁻⁵	0.15 × 10 ⁻³			0.16	

The flux density throughout is

$$B = \frac{\Phi}{A} = \frac{1.5 \times 10^{-5} \text{ Wb}}{0.15 \times 10^{-3} \text{ m}^2} = 10 \times 10^{-2} \text{ T} = 0.10 \text{ T}$$

and

$$H \text{ (from Fig. 11.24)} \cong \frac{1}{5}(100 \text{ At/m}) = 20 \text{ At/m}$$

Applying Ampère's circuital law,

$$\begin{aligned} N_1 I_1 - N_2 I_2 &= H_{abcd} l_{abcd} \\ (60 \text{ t})(2 \text{ A}) - (30 \text{ t})(I_2) &= (20 \text{ At/m})(0.16 \text{ m}) \\ 120 \text{ At} - (30 \text{ t})I_2 &= 3.2 \text{ At} \end{aligned}$$

and

$$(30 \text{ t})I_2 = 120 \text{ At} - 3.2 \text{ At}$$

or

$$I_2 = \frac{116.8 \text{ At}}{30 \text{ t}} = \mathbf{3.89 \text{ A}}$$

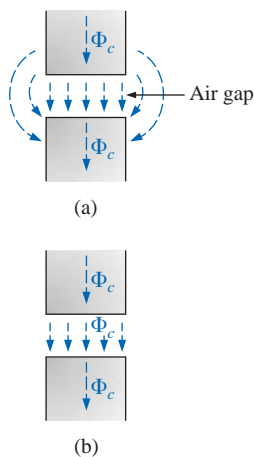


FIG. 11.35
 Air gaps: (a) with fringing; (b) ideal.

For the analysis of most transformer systems, the equation $N_1 I_1 = N_2 I_2$ is employed. This would result in 4 A versus 3.89 A above. This difference is normally ignored, however, and the equation $N_1 I_1 = N_2 I_2$ considered exact.

Because of the nonlinearity of the B - H curve, *it is not possible to apply superposition to magnetic circuits*; that is, in Example 11.5, we cannot consider the effects of each source independently and then find the total effects by using superposition.

11.12 AIR GAPS

Before continuing with the illustrative examples, let us consider the effects that an air gap has on a magnetic circuit. Note the presence of air gaps in the magnetic circuits of the motor and meter of Fig. 11.11. The spreading of the flux lines outside the common area of the core for the air gap in Fig. 11.35(a) is known as *fringing*. For our purposes, we shall neglect this effect and assume the flux distribution to be as in Fig. 11.35(b).



The flux density of the air gap in Fig. 11.35(b) is given by

$$B_g = \frac{\Phi_g}{A_g} \tag{11.13}$$

where, for our purposes,

$$\Phi_g = \Phi_{\text{core}}$$

and

$$A_g = A_{\text{core}}$$

For most practical applications, the permeability of air is taken to be equal to that of free space. The magnetizing force of the air gap is then determined by

$$H_g = \frac{B_g}{\mu_o} \tag{11.14}$$

and the mmf drop across the air gap is equal to $H_g l_g$. An equation for H_g is as follows:

$$H_g = \frac{B_g}{\mu_o} = \frac{B_g}{4\pi \times 10^{-7}}$$

and
$$H_g = (7.96 \times 10^5) B_g \quad (\text{At/m}) \tag{11.15}$$

EXAMPLE 11.6 Find the value of I required to establish a magnetic flux of $\Phi = 0.75 \times 10^{-4}$ Wb in the series magnetic circuit of Fig. 11.36.

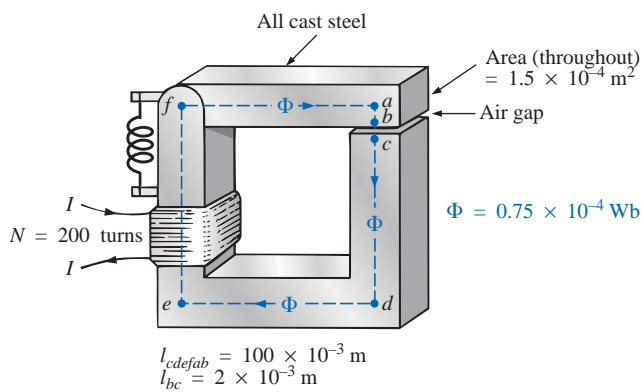
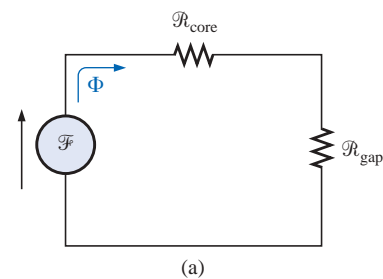


FIG. 11.36
Relay for Example 11.6.

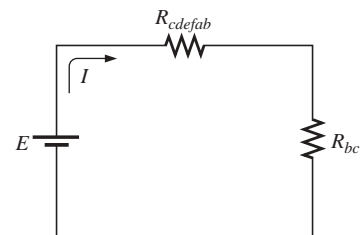
Solution: An equivalent magnetic circuit and its electric circuit analogy are shown in Fig. 11.37.

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{0.75 \times 10^{-4} \text{ Wb}}{1.5 \times 10^{-4} \text{ m}^2} = 0.5 \text{ T}$$



(a)



(b)

FIG. 11.37
(a) Magnetic circuit equivalent and
(b) electric circuit analogy for the relay of Fig. 11.36.



From the B - H curves of Fig. 11.24,

$$H \text{ (cast steel)} \cong 280 \text{ At/m}$$

Applying Eq. (11.15),

$$H_g = (7.96 \times 10^5)B_g = (7.96 \times 10^5)(0.5 \text{ T}) = 3.98 \times 10^5 \text{ At/m}$$

The mmf drops are

$$H_{\text{core}}l_{\text{core}} = (280 \text{ At/m})(100 \times 10^{-3} \text{ m}) = 28 \text{ At}$$

$$H_g l_g = (3.98 \times 10^5 \text{ At/m})(2 \times 10^{-3} \text{ m}) = 796 \text{ At}$$

Applying Ampère's circuital law,

$$NI = H_{\text{core}}l_{\text{core}} + H_g l_g$$

$$= 28 \text{ At} + 796 \text{ At}$$

$$(200 \text{ t})I = 824 \text{ At}$$

$$I = \mathbf{4.12 \text{ A}}$$

Note from the above that the air gap requires the biggest share (by far) of the impressed NI due to the fact that air is nonmagnetic.

11.13 SERIES-PARALLEL MAGNETIC CIRCUITS

As one might expect, the close analogies between electric and magnetic circuits will eventually lead to series-parallel magnetic circuits similar in many respects to those encountered in Chapter 7. In fact, the electric circuit analogy will prove helpful in defining the procedure to follow toward a solution.

EXAMPLE 11.7 Determine the current I required to establish a flux of $1.5 \times 10^{-4} \text{ Wb}$ in the section of the core indicated in Fig. 11.38.

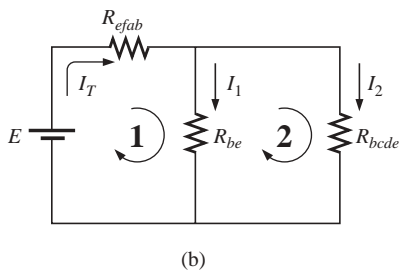
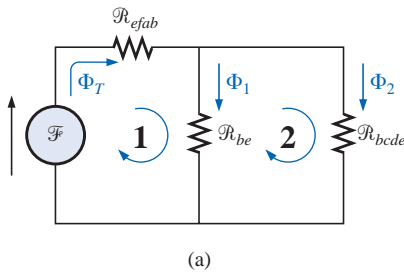


FIG. 11.39

(a) Magnetic circuit equivalent and
(b) electric circuit analogy for the series-parallel system of Fig. 11.38.

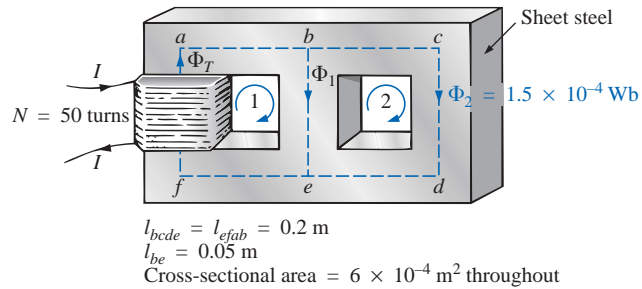


FIG. 11.38

Example 11.7.

Solution: The equivalent magnetic circuit and the electric circuit analogy appear in Fig. 11.39. We have

$$B_2 = \frac{\Phi_2}{A} = \frac{1.5 \times 10^{-4} \text{ Wb}}{6 \times 10^{-4} \text{ m}^2} = 0.25 \text{ T}$$

From Fig. 11.24,

$$H_{bcde} \cong 40 \text{ At/m}$$

Applying Ampère's circuital law around loop 2 of Figs. 11.38 and 11.39,



$$\begin{aligned}\sum_{\mathcal{C}} \mathcal{F} &= 0 \\ H_{be}l_{be} - H_{bcde}l_{bcde} &= 0 \\ H_{be}(0.05 \text{ m}) - (40 \text{ At/m})(0.2 \text{ m}) &= 0 \\ H_{be} &= \frac{8 \text{ At}}{0.05 \text{ m}} = 160 \text{ At/m}\end{aligned}$$

From Fig. 11.24,

$$B_1 \cong 0.97 \text{ T}$$

and

$$\Phi_1 = B_1 A = (0.97 \text{ T})(6 \times 10^{-4} \text{ m}^2) = 5.82 \times 10^{-4} \text{ Wb}$$

The results are entered in Table 11.6.

TABLE 11.6

Section	Φ (Wb)	A (m ²)	B (T)	H (At/m)	l (m)	HI (At)
<i>bcde</i>	1.5×10^{-4}	6×10^{-4}	0.25	40	0.2	8
<i>be</i>	5.82×10^{-4}	6×10^{-4}	0.97	160	0.05	8
<i>efab</i>		6×10^{-4}			0.2	

The table reveals that we must now turn our attention to section *efab*:

$$\begin{aligned}\Phi_T &= \Phi_1 + \Phi_2 = 5.82 \times 10^{-4} \text{ Wb} + 1.5 \times 10^{-4} \text{ Wb} \\ &= 7.32 \times 10^{-4} \text{ Wb} \\ B &= \frac{\Phi_T}{A} = \frac{7.32 \times 10^{-4} \text{ Wb}}{6 \times 10^{-4} \text{ m}^2} \\ &= 1.22 \text{ T}\end{aligned}$$

From Fig. 11.23,

$$H_{efab} \cong 400 \text{ At}$$

Applying Ampère's circuital law,

$$\begin{aligned}+NI - H_{efab}l_{efab} - H_{be}l_{be} &= 0 \\ NI &= (400 \text{ At/m})(0.2 \text{ m}) + (160 \text{ At/m})(0.05 \text{ m}) \\ (50 \text{ t})I &= 80 \text{ At} + 8 \text{ At} \\ I &= \frac{88 \text{ At}}{50 \text{ t}} = \mathbf{1.76 \text{ A}}\end{aligned}$$

To demonstrate that μ is sensitive to the magnetizing force H , the permeability of each section is determined as follows. For section *bcde*,

$$\mu = \frac{B}{H} = \frac{0.25 \text{ T}}{40 \text{ At/m}} = 6.25 \times 10^{-3}$$

and

$$\mu_r = \frac{\mu}{\mu_o} = \frac{6.25 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{4972.2}$$

For section *be*,

$$\mu = \frac{B}{H} = \frac{0.97 \text{ T}}{160 \text{ At/m}} = 6.06 \times 10^{-3}$$



$$\text{and} \quad \mu_r = \frac{\mu}{\mu_o} = \frac{6.06 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{4821}$$

For section *efab*,

$$\mu = \frac{B}{H} = \frac{1.22 \text{ T}}{400 \text{ At/m}} = 3.05 \times 10^{-3}$$

$$\text{and} \quad \mu_r = \frac{\mu}{\mu_o} = \frac{3.05 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{2426.41}$$

11.14 DETERMINING Φ

The examples of this section are of the second type, where NI is given and the flux Φ must be found. This is a relatively straightforward problem if only one magnetic section is involved. Then

$$H = \frac{NI}{l} \quad H \rightarrow B \quad (B-H \text{ curve})$$

$$\text{and} \quad \Phi = BA$$

For magnetic circuits with more than one section, there is no set order of steps that will lead to an exact solution for every problem on the first attempt. In general, however, we proceed as follows. We must find the impressed mmf for a *calculated guess* of the flux Φ and then compare this with the specified value of mmf. We can then make adjustments to our guess to bring it closer to the actual value. For most applications, a value within $\pm 5\%$ of the actual Φ or specified NI is acceptable.

We can make a reasonable guess at the value of Φ if we realize that the maximum mmf drop appears across the material with the smallest permeability if the length and area of each material are the same. As shown in Example 11.6, if there is an air gap in the magnetic circuit, there will be a considerable drop in mmf across the gap. As a starting point for problems of this type, therefore, we shall assume that the total mmf (NI) is across the section with the lowest μ or greatest \mathcal{R} (if the other physical dimensions are relatively similar). This assumption gives a value of Φ that will produce a calculated NI greater than the specified value. Then, after considering the results of our original assumption very carefully, we shall *cut* Φ and NI by introducing the effects (reluctance) of the other portions of the magnetic circuit and *try* the new solution. For obvious reasons, this approach is frequently called the *cut and try* method.

EXAMPLE 11.8 Calculate the magnetic flux Φ for the magnetic circuit of Fig. 11.40.

Solution: By Ampère's circuital law,

$$\begin{aligned} NI &= H_{abcd} l_{abcd} \\ \text{or} \quad H_{abcd} &= \frac{NI}{l_{abcd}} = \frac{(60 \text{ t})(5 \text{ A})}{0.3 \text{ m}} \\ &= \frac{300 \text{ At}}{0.3 \text{ m}} = 1000 \text{ At/m} \end{aligned}$$

$$\text{and} \quad B_{abcd} \text{ (from Fig. 11.23)} \cong 0.39 \text{ T}$$

Since $B = \Phi/A$, we have

$$\Phi = BA = (0.39 \text{ T})(2 \times 10^{-4} \text{ m}^2) = \mathbf{0.78 \times 10^{-4} \text{ Wb}}$$

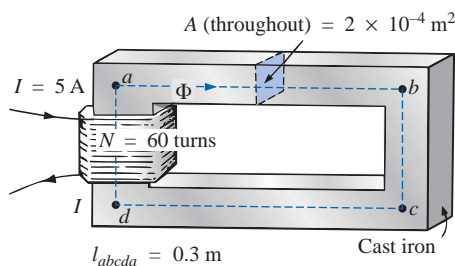


FIG. 11.40
Example 11.8.



EXAMPLE 11.9 Find the magnetic flux Φ for the series magnetic circuit of Fig. 11.41 for the specified impressed mmf.

Solution: Assuming that the total impressed mmf NI is across the air gap,

$$NI = H_g l_g$$

or
$$H_g = \frac{NI}{l_g} = \frac{400 \text{ At}}{0.001 \text{ m}} = 4 \times 10^5 \text{ At/m}$$

and
$$B_g = \mu_o H_g = (4\pi \times 10^{-7})(4 \times 10^5 \text{ At/m}) = 0.503 \text{ T}$$

The flux

$$\begin{aligned} \Phi_g &= \Phi_{\text{core}} = B_g A \\ &= (0.503 \text{ T})(0.003 \text{ m}^2) \\ \Phi_{\text{core}} &= 1.51 \times 10^{-3} \text{ Wb} \end{aligned}$$

Using this value of Φ , we can find NI . The data are inserted in Table 11.7.

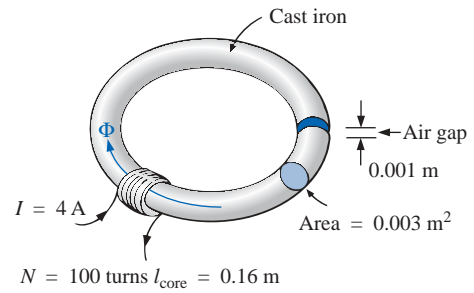


FIG. 11.41
Example 11.9.

TABLE 11.7

Section	Φ (Wb)	A (m ²)	B (T)	H (At/m)	l (m)	HI (At)
Core	1.51×10^{-3}	0.003	0.503	1500 (<i>B-H</i> curve)	0.16	
Gap	1.51×10^{-3}	0.003	0.503	4×10^5	0.001	400

$$H_{\text{core}} l_{\text{core}} = (1500 \text{ At/m})(0.16 \text{ m}) = 240 \text{ At}$$

Applying Ampère's circuital law results in

$$\begin{aligned} NI &= H_{\text{core}} l_{\text{core}} + H_g l_g \\ &= 240 \text{ At} + 400 \text{ At} \\ &= 640 \text{ At} > 400 \text{ At} \end{aligned}$$

Since we neglected the reluctance of all the magnetic paths but the air gap, the calculated value is greater than the specified value. We must therefore reduce this value by including the effect of these reluctances. Since approximately $(640 \text{ At} - 400 \text{ At})/640 \text{ At} = 240 \text{ At}/640 \text{ At} \cong 37.5\%$ of our calculated value is above the desired value, let us reduce Φ by 30% and see how close we come to the impressed mmf of 400 At:

$$\begin{aligned} \Phi &= (1 - 0.3)(1.51 \times 10^{-3} \text{ Wb}) \\ &= 1.057 \times 10^{-3} \text{ Wb} \end{aligned}$$

See Table 11.8.

TABLE 11.8

Section	Φ (Wb)	A (m ²)	B (T)	H (At/m)	l (m)	HI (At)
Core	1.057×10^{-3}	0.003			0.16	
Gap	1.057×10^{-3}	0.003			0.001	



$$B = \frac{\Phi}{A} = \frac{1.057 \times 10^{-3} \text{ Wb}}{0.003 \text{ m}^2} \cong 0.352 \text{ T}$$

$$H_g l_g = (7.96 \times 10^5) B_g l_g$$

$$= (7.96 \times 10^5)(0.352 \text{ T})(0.001 \text{ m})$$

$$\cong 280.19 \text{ At}$$

From the B - H curves,

$$H_{\text{core}} \cong 850 \text{ At/m}$$

$$H_{\text{core}} l_{\text{core}} = (850 \text{ At/m})(0.16 \text{ m}) = 136 \text{ At}$$

Applying Ampère's circuital law yields

$$NI = H_{\text{core}} l_{\text{core}} + H_g l_g$$

$$= 136 \text{ At} + 280.19 \text{ At}$$

$$= \mathbf{416.19 \text{ At}} > 400 \text{ At} \quad (\text{but within } \pm 5\%$$

and therefore acceptable)

The solution is, therefore,

$$\Phi \cong \mathbf{1.057 \times 10^{-3} \text{ Wb}}$$

11.15 APPLICATIONS

Recording Systems

The most common application of magnetic material is probably in the increasing number of recording instruments used every day in the office and the home. For instance, the VHS tape and the 8-mm cassette of Fig. 11.42 are used almost daily by every family with a VCR or cassette player. The basic recording process is not that difficult to understand and will be described in detail in the section to follow on computer hard disks.



(a)



(b)

FIG. 11.42

Magnetic tape: (a) VHS and 8-mm cassette (Courtesy of Maxell Corporation of America); (b) manufacturing process (Courtesy of Ampex Corporation).



Speakers and Microphones

Electromagnetic effects are the moving force in the design of speakers such as the one shown in Fig. 11.43. The shape of the pulsating waveform of the input current is determined by the sound to be reproduced by the speaker at a high audio level. As the current peaks and returns to the valleys of the sound pattern, the strength of the electromagnet varies in exactly the same manner. This causes the cone of the speaker to vibrate at a frequency directly proportional to the pulsating input. The higher the pitch of the sound pattern, the higher the oscillating frequency between the peaks and valleys and the higher the frequency of vibration of the cone.

A second design used more frequently in more expensive speaker systems appears in Fig. 11.44. In this case the permanent magnet is fixed and the input is applied to a movable core within the magnet, as shown in the figure. High peaking currents at the input produce a strong flux pattern in the voice coil, causing it to be drawn well into the flux pattern of the permanent magnet. As occurred for the speaker of Fig. 11.43, the core then vibrates at a rate determined by the input and provides the audible sound.

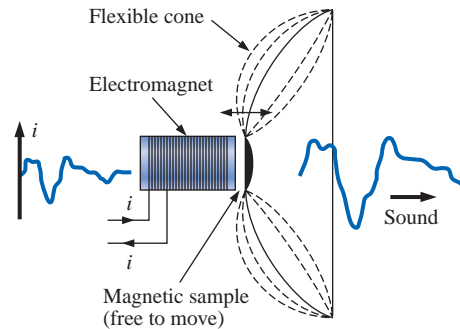


FIG. 11.43
Speaker.

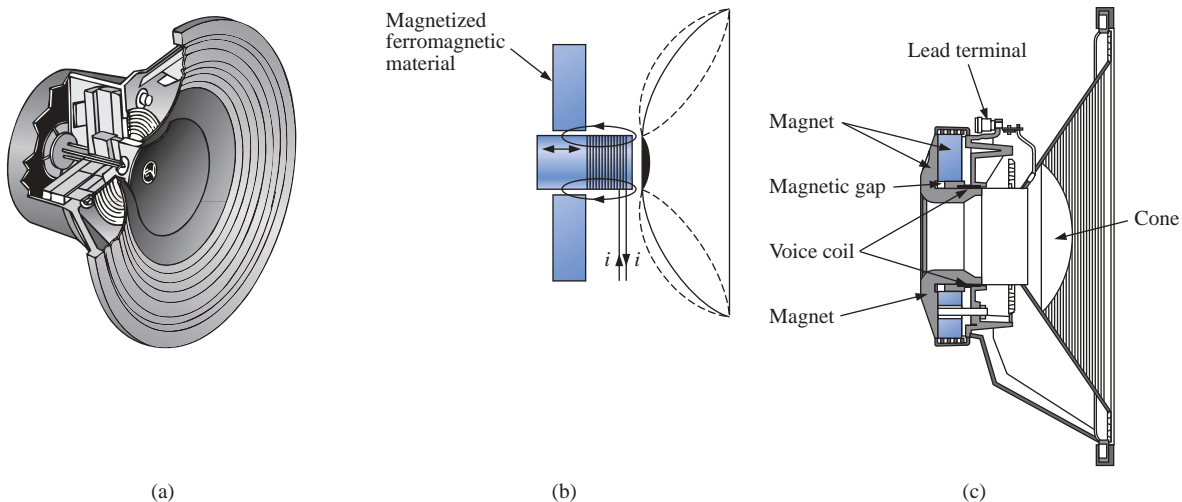


FIG. 11.44

Coaxial high-fidelity loudspeaker: (a) construction; (b) basic operation; (c) cross section of actual unit. (Courtesy of Electro-Voice, Inc.)

Microphones such as those in Fig. 11.45 also employ electromagnetic effects. The sound to be reproduced at a higher audio level causes the core and attached moving coil to move within the magnetic field of the permanent magnet. Through Faraday's law ($e = N d\phi/dt$), a voltage is induced across the movable coil proportional to the speed with which it is moving through the magnetic field. The resulting induced voltage pattern can then be amplified and reproduced at a much higher audio level through the use of speakers, as described earlier. Microphones of this type are the most frequently employed, although other types that use capacitive, carbon granular, and piezoelectric* effects are available. This particular design is commercially referred to as a *dynamic* microphone.

*Piezoelectricity is the generation of a small voltage by exerting pressure across certain crystals.



FIG. 11.45

Dynamic microphone. (Courtesy of Electro-Voice, Inc.)

Computer Hard Disks

The computer *hard disk* is a sealed unit in a computer that stores data on a magnetic coating applied to the surface of circular platters that spin like a record. The platters are constructed on a base of aluminum or glass (both nonferromagnetic), which makes them rigid—hence the term *hard disk*. Since the unit is sealed, the internal platters and components are inaccessible, and a “crash” (a term applied to the loss of data from a disk or the malfunction thereof) usually requires that the entire unit be replaced. Hard disks are currently available with diameters from less than 1 in. to 5¼ in., with the 3½ in. the most popular for today’s desktop units. Lap-top units typically use 2½ in. All hard disk drives are often referred to as *Winchester drives*, a term first applied in the 1960s to an IBM drive that had 30 MB [a byte is a series of binary bits (0s and 1s) representing a number, letter, or symbol] of fixed (nonaccessible) data storage and 30 MB of accessible data storage. The term *Winchester* was applied because the 30-30 data capacity matched the name of the popular 30-30 Winchester rifle.

The magnetic coating on the platters is called the *media* and is of either the *oxide* or the *thin-film* variety. The oxide coating is formed by first coating the platter with a gel containing iron-oxide (ferromagnetic) particles. The disk is then spun at a very high speed to spread the material evenly across the surface of the platter. The resulting surface is then covered with a protective coating that is made as smooth as possible. The thin-film coating is very thin, but durable, with a surface that is smooth and consistent throughout the disk area. In recent years the trend has been toward the thin-film coating because the read/write heads (to be described shortly) must travel closer to the surface of the platter, requiring a consistent coating thickness. Recent techniques have resulted in thin-film magnetic coatings as thin as one-millionth of an inch.

The information on a disk is stored around the disk in circular paths called *tracks* or *cylinders*, with each track containing so many bits of information per inch. The product of the number of bits per inch and the number of tracks per inch is the *Areal* density of the disk, which provides an excellent quantity for comparison with early systems and reveals how far the field has progressed in recent years. In the 1950s the first drives had an Areal density of about 2 kbits/in.² compared to today’s typical 4 Gbits/in.², an incredible achievement; consider



4,000,000,000,000 bits of information on an area the size of the face of your watch. **Electromagnetism** is the key element in the *writing* of information on the disk and the *reading* of information off the disk.

In its simplest form the *write/read head* of a hard disk (or floppy disk) is a U-shaped electromagnet with an air gap that rides just above the surface of the disk, as shown in Fig. 11.46. As the disk rotates, information in the form of a voltage with changing polarities is applied to the winding of the electromagnet. For our purposes we will associate a positive voltage level with a 1 level (of binary arithmetic) and a negative voltage level with a 0 level. Combinations of these 0 and 1 levels can be used to represent letters, numbers, or symbols. If energized as shown in Fig. 11.46 with a 1 level (positive voltage), the resulting magnetic flux pattern will have the direction shown in the core. When the flux pattern encounters the air gap of the core, it jumps to the magnetic material (since magnetic flux always seeks the path of least reluctance and air has a high reluctance) and establishes a flux pattern, as shown on the disk, until it reaches the other end of the core air gap, where it returns to the electromagnet and completes the path. As the head then moves to the next bit sector, it leaves behind the magnetic flux pattern just established from the left to the right. The next bit sector has a 0 level input (negative voltage) that reverses the polarity of the applied voltage and the direction of the magnetic flux in the core of the head. The result is a flux pattern in the disk opposite that associated with a 1 level. The next bit of information is also a 0 level, resulting in the same pattern just generated. In total, therefore, information is stored on the disk in the form of small magnets whose polarity defines whether they are representing a 0 or a 1.

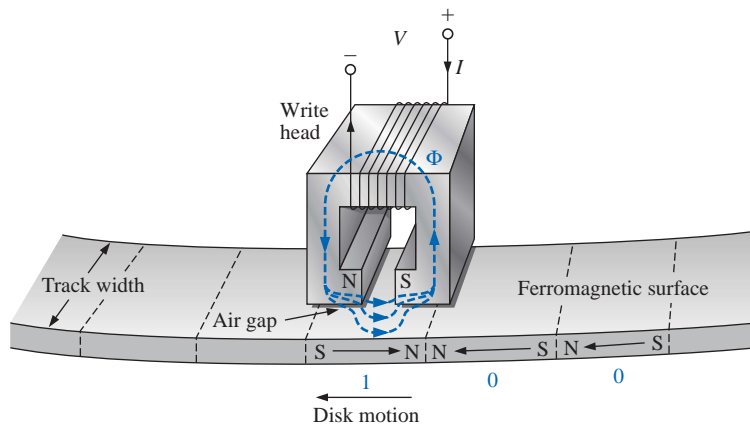


FIG. 11.46

Hard disk storage using a U-shaped electromagnet write head.

Now that the data have been stored, we must have some method to retrieve the information when desired. The first few hard disks used the same head for both the write and the read functions. In Fig. 11.47(a), the U-shaped electromagnet in the read mode simply picks up the flux pattern of the current bit of information. *Faraday's law of electromagnetic induction* states that a voltage is induced across a coil if exposed to a changing magnetic field. The change in flux for the core in Fig. 11.47(a) is minimal as it passes over the induced bar magnet on the surface of the disk. A flux pattern is established in the core because of the

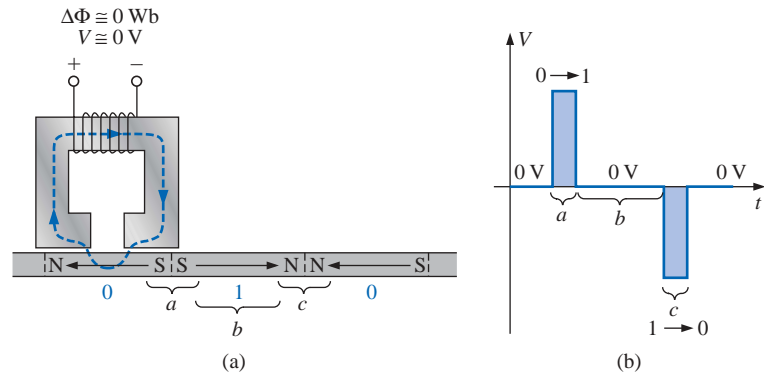


FIG. 11.47

Reading the information off a hard disk using a U-shaped electromagnet.

bar magnet on the disk, but the lack of a significant change in flux level results in an induced voltage at the output terminals of the pickup of approximately 0 V, as shown in Fig. 11.47(b) for the readout waveform. A significant change in flux occurs when the head passes over the transition region so marked in Fig. 11.47(a). In region *a* the flux pattern changes from one direction to the other—a significant change in flux occurs in the core as it reverses direction, causing a measurable voltage to be generated across the terminals of the pickup coil as dictated by Faraday's law and indicated in Fig. 11.47(b). In region *b* there is no significant change in the flux pattern from one bit area to the next, and a voltage is not generated, as also revealed in Fig. 11.47(b). However, when region *c* is reached, the change in flux is significant but opposite that occurring in region *a*, resulting in another pulse but of opposite polarity. In total, therefore, the output bits of information are in the form of pulses that have a shape totally different from the read signals but that are certainly representative of the information being stored. In addition, note that the output is generated at the transition regions and not in the constant flux region of the bit storage.

In the early years, the use of the same head for the read and write functions was acceptable; but as the tracks became narrower and the seek time (the average time required to move from one track to another a random distance away) had to be reduced, it became increasingly difficult to construct the coil or core configuration in a manner that was sufficiently thin with minimum weight. In the late 1970s IBM introduced the *thin-film inductive head*, which was manufactured in much the same way as the small integrated circuits of today. The result is a head having a length typically less than $\frac{1}{10}$ in., a height less than $\frac{1}{50}$ in., and minimum mass and high durability. The average seek time has dropped from a few hundred milliseconds to 6 ms to 8 ms for very fast units and 8 ms to 10 ms for average units. In addition, production methods have improved to the point that the head can “float” above the surface (to minimize damage to the disk) at a height of only 5 microinches or 0.000005 in. Using a typical lap-top hard disk speed of 3600 rpm (as high as 7200 rpm for desktops) and an average diameter of 1.75 in. for a 3.5-in. disk, the speed of the head over the track is about 38 mph. Scaling the floating height up to $\frac{1}{4}$ in. (multiplying by a factor of 50,000), the speed would increase to about 1.9×10^6 mph. In other words, the speed of the head over the surface of the platter is analogous to a mass traveling $\frac{1}{4}$ in. above a surface at 1.9 million miles per hour, all the while ensuring that the head never touches the surface of the disk—quite a technical achievement and amaz-



ingly enough one that perhaps will be improved by a factor of 10 in the next decade. Incidentally, the speed of rotation of floppy disks is about $\frac{1}{10}$ that of the hard disk, or 360 rpm. In addition, the head touches the magnetic surface of the floppy disk, limiting the storage life of the unit. The typical magnetizing force needed to lay down the magnetic orientation is 400 mA-turn (peak-to-peak). The result is a write current of only 40 mA for a 10-turn, thin-film inductive head.

Although the thin-film inductive head could also be used as a read head, the *magnetoresistive* (MR) head has improved reading characteristics. The MR head depends on the fact that the resistance of a soft ferromagnetic conductor such as permalloy is sensitive to changes in external magnetic fields. As the hard disk rotates, the changes in magnetic flux from the induced magnetized regions of the platter change the terminal resistance of the head. A constant current passed through the sensor displays a terminal voltage sensitive to the magnitude of the resistance. The result is output voltages with peak values in excess of 300 V, which exceeds that of typical inductive read heads by a factor of 2 or 3:1.

Further investigation will reveal that the best write head is of the thin-film inductive variety and that the optimum read head is of the MR variety. Each has particular design criteria for maximum performance, resulting in the increasingly common *dual-element head*, with each head containing separate conductive paths and different gap widths. The Areal density of the new hard disks will essentially require the dual-head assembly for optimum performance.

As the density of the disk increases, the width of the tracks or cylinders will decrease accordingly. The net result will be smaller heads for the read/write function, an arm supporting the head that must be able to move into and out of the rotating disk in smaller increments, and an increased sensitivity to temperature effects which can cause the disk itself to contract or expand. At one time the mechanical system with its gears and pulleys was sensitive enough to perform the task. However, today's density requires a system with less play and with less sensitivity to outside factors such as temperature and vibration. A number of modern drives use a voice coil and ferromagnetic arm as shown in Fig. 11.48. The current through the coil will determine the magnetic field strength within the coil and will cause

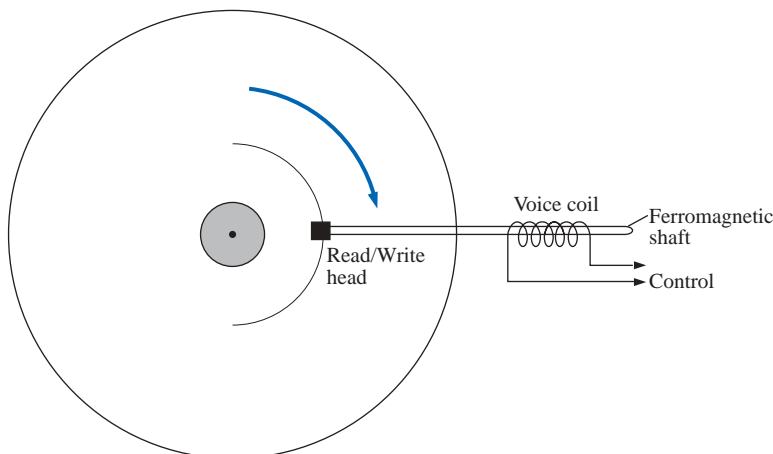


FIG. 11.48

Disk drive with voice coil and ferromagnetic arm.



the supporting arm for the head to move in and out, thereby establishing a rough setting for the extension of the arm over the disk. It would certainly be possible to relate the position of the arm to the applied voltage to the coil, but this would lack the level of accuracy required for high-density disks. For the desired accuracy, a laser beam has been added as an integral part of the head. Circular strips placed around the disk (called *track indicators*) ensure that the laser beam homes in and keeps the head in the right position. Assuming that the track is a smooth surface and the surrounding area a rough texture, a laser beam will be reflected back to the head if it's on the track, whereas the beam will be scattered if it hits the adjoining areas. This type of system permits continuous recalibration of the arm by simply comparing its position with the desired location—a maneuver referred to as “recalibration on the fly.”

As with everything, there are limits to any design. However, in this case, it is not because larger disks cannot be made or that more tracks cannot be put on the disk. The limit to the size of hard drives in PCs is set by the BIOS (Basic Input Output System) drive that is built into all PCs. When first developed years ago, it was designed around a maximum storage possibility of 8.4 gigabytes. At that time this number seemed sufficiently large to withstand any new developments for many years to come. However, 8-gigabyte drives and larger are now becoming commonplace, with lap-tops averaging 20 gigabytes and desktops averaging 40 gigabytes. The result is that mathematical methods had to be developed to circumvent the designed maximum for each component of the BIOS system. Fundamentally, the maximum values for the BIOS drive are the following:

Cylinders (or tracks)	1024
Heads	128
Sectors	128
Bytes per sector	512

Multiplying through all the factors results in a maximum of 8.59 gigabytes, but the colloquial reference is normally 8.4 gigabytes.

Most modern drives use a BIOS translation technique whereby they play a mathematical game in which they make the drive appear different to the BIOS system than it actually is. For instance, the drive may have 2048 tracks and 16 heads, but through the mathematical link with the BIOS system it will appear to have 1024 tracks and 32 heads. In other words, there was a trade-off between numbers in the official maximum listing. This is okay for certain combinations, but the total combination of figures for the design still cannot exceed 8.4 gigabytes. Also be aware that this mathematical manipulation is possible only if the operating system has BIOS translation built in. By implementing new enhanced IDE controllers, BIOS can have access drives greater than 8.4 gigabytes.

The above is clear evidence of the importance of magnetic effects in today's growing industrial, computer-oriented society. Although research continues to maximize the Areal density, it appears certain that the storage will remain magnetic for the write/read process and will not be replaced by any of the growing alternatives such as the optic laser variety used so commonly in CD-ROMs.

A 3.5-in. full-height disk drive, which is manufactured by the Seagate Corporation and has a formatted capacity of 17.2 gigabytes (GB) with an average search time of 9 ms, appears in Fig. 11.49.



FIG. 11.49

A 3.5-in. hard disk drive with a capacity of 17.2 GB and an average search time of 9 ms. (Courtesy of Seagate Corporation.)



Hall Effect Sensor

The Hall effect sensor is a semiconductor device that generates an output voltage when exposed to a magnetic field. The basic construction consists of a slab of semiconductor material through which a current is passed, as shown in Fig. 11.50(a). If a magnetic field is applied as shown in the figure perpendicular to the direction of the current, a voltage V_H will be generated between the two terminals, as indicated in Fig. 11.50(a). The difference in potential is due to the separation of charge established by the Lorentz force first studied by Professor Hendrick Lorentz in the early eighteenth century. He found that electrons in a magnetic field are subjected to a force proportional to the velocity of the electrons through the field and the strength of the magnetic field. The direction of the force is determined by the left-hand rule. Simply place the index finger of the left hand in the direction of the magnetic field, with the second finger at right angles to the index finger in the direction of conventional current through the semiconductor material, as shown in Fig. 11.50(b). The thumb, if placed at right angles to the index finger, will indicate the direction of the force on the electrons. In Fig. 11.50(b), the force causes the electrons to accumulate in the bottom region of the semiconductor (connected to the negative terminal of the voltage V_H), leaving a net positive charge in the upper region of the material (connected to the positive terminal of V_H). The stronger the current or strength of the magnetic field, the greater the induced voltage V_H .

In essence, therefore, the Hall effect sensor can reveal the strength of a magnetic field or the level of current through a device if the other determining factor is held fixed. Two applications of the sensor are therefore apparent—to measure the strength of a magnetic field in the vicinity of a sensor (for an applied fixed current) and to measure the level of current through a sensor (with knowledge of the strength of the magnetic field linking the sensor). The gaussmeter in Fig. 11.16 employs a Hall effect sensor. Internal to the meter, a fixed current is passed through the sensor with the voltage V_H indicating the relative strength of the field. Through amplification, calibration, and proper scaling, the meter can display the relative strength in gauss.

The Hall effect sensor has a broad range of applications that are often quite interesting and innovative. The most widespread is as a trigger for an alarm system in large department stores, where theft is often a difficult problem. A magnetic strip attached to the merchandise sounds an alarm when a customer passes through the exit gates without paying for the product. The sensor, control current, and monitoring system are housed in the exit fence and react to the presence of the magnetic field as the product leaves the store. When the product is paid for, the cashier removes the strip or demagnetizes the strip by applying a magnetizing force that reduces the residual magnetism in the strip to essentially zero.

The Hall effect sensor is also used to indicate the speed of a bicycle on a digital display conveniently mounted on the handlebars. As shown in Fig. 11.51(a), the sensor is mounted on the frame of the bike, and a small permanent magnet is mounted on a spoke of the front wheel. The magnet must be carefully mounted to be sure that it passes over the proper region of the sensor. When the magnet passes over the sensor, the flux pattern in Fig. 11.51(b) results, and a voltage with a sharp peak is developed by the sensor. Assuming a bicycle with a 26-in.-diameter

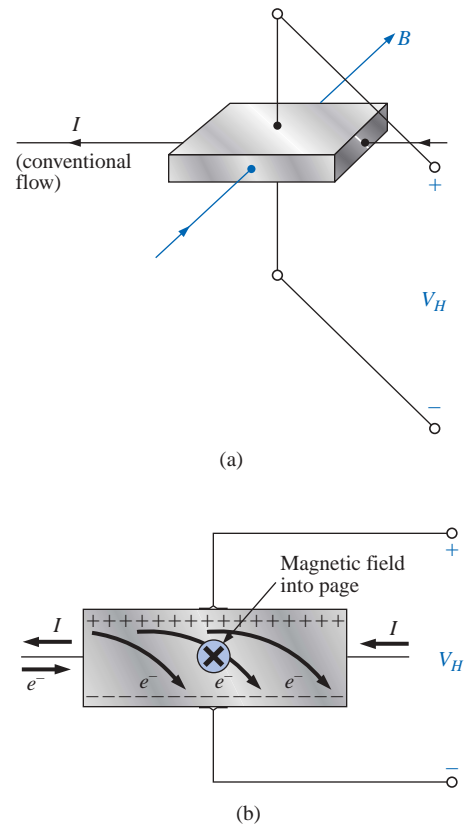


FIG. 11.50
Hall effect sensor: (a) orientation of controlling parameters; (b) effect on electron flow.

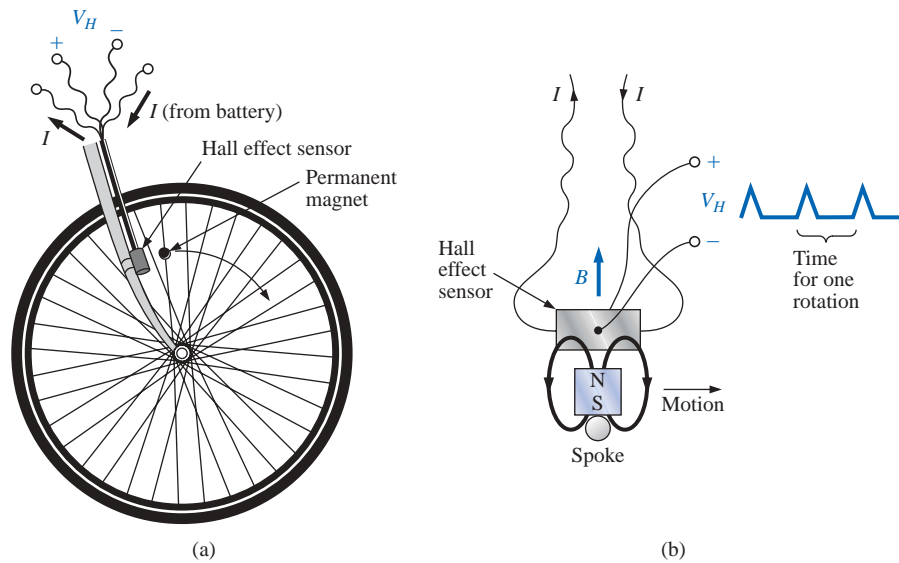


FIG. 11.51

Obtaining a speed indication for a bicycle using a Hall effect sensor: (a) mounting the components; (b) Hall effect response.

wheel, the circumference will be about 82 in. Over 1 mi, the number of rotations is

$$5280 \text{ ft} \left(\frac{12 \text{ in.}}{1 \text{ ft}} \right) \left(\frac{1 \text{ rotation}}{82 \text{ in.}} \right) \cong 773 \text{ rotations}$$

If the bicycle is traveling at 20 mph, an output pulse will occur at a rate of 4.29 per second. It is interesting to note that at a speed of 20 mph, the wheel is rotating at more than 4 revolutions per second, and the total number of rotations over 20 mi is 15,460.

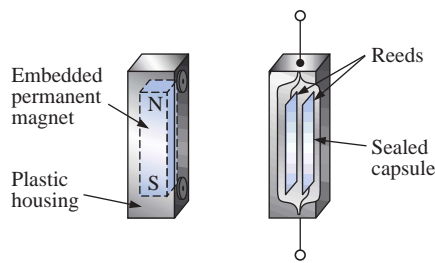


FIG. 11.52

Magnetic reed switch.

Magnetic Reed Switch

One of the most frequently employed switches in alarm systems is the *magnetic reed switch* shown in Fig. 11.52. As shown by the figure, there are two components of the reed switch—a permanent magnet embedded in one unit that is normally connected to the movable element (door, window, and so on) and a reed switch in the other unit that is connected to the electrical control circuit. The reed switch is constructed of two iron-alloy (ferromagnetic) reeds in a hermetically sealed capsule. The cantilevered ends of the two reeds do not touch but are in very close proximity to one another. In the absence of a magnetic field the reeds remain separated. However, if a magnetic field is introduced, the reeds will be drawn to each other because flux lines seek the path of least reluctance and, if possible, exercise every alternative to establish the path of least reluctance. It is similar to placing a ferromagnetic bar close to the ends of a U-shaped magnet. The bar is drawn to the poles of the magnet, establishing a magnetic flux path without air gaps and with minimum reluctance. In the open-circuit state the resistance between reeds is in excess of 100 MΩ, while in the on state it drops to less than 1 Ω.

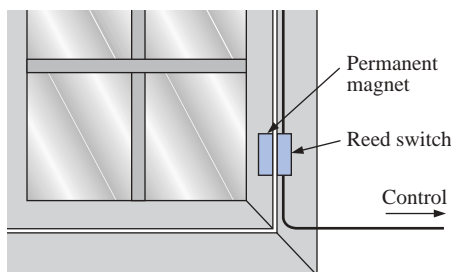


FIG. 11.53

Using a magnetic reed switch to monitor the state of a window.

In Fig. 11.53 a reed switch has been placed on the fixed frame of a window and a magnet on the movable window unit. When the window is closed as shown in Fig. 11.53, the magnet and reed switch are suffi-



ciently close to establish contact between the reeds, and a current is established through the reed switch to the control panel. In the armed state the alarm system accepts the resulting current flow as a normal secure response. If the window is opened, the magnet will leave the vicinity of the reed switch, and the switch will open. The current through the switch will be interrupted, and the alarm will react appropriately.

One of the distinct advantages of the magnetic reed switch is that the proper operation of any switch can be checked with a portable magnetic element. Simply bring the magnet to the switch and note the output response. There is no need to continually open and close windows and doors. In addition, the reed switch is hermetically enclosed so that oxidation and foreign objects cannot damage it, and the result is a unit that can last indefinitely. Magnetic reed switches are also available in other shapes and sizes, allowing them to be concealed from obvious view. One is a circular variety that can be set into the edge of a door and door jam, resulting in only two small visible disks when the door is open.

Magnetic Resonance Imaging

Magnetic resonance imaging [MRI, also called *nuclear magnetic resonance* (NMR)] is receiving more and more attention as we strive to improve the quality of the cross-sectional images of the body so useful in medical diagnosis and treatment. MRI does not expose the patient to potentially hazardous X rays or injected contrast materials such as those employed to obtain computerized axial tomography (CAT) scans.

The three major components of an MRI system are a huge magnet that can weigh up to 100 tons, a table for transporting the patient into the circular hole in the magnet, and a control center, as shown in Fig. 11.54. The image is obtained by placing the patient in the tube to a precise depth depending on the cross section to be obtained and applying a strong magnetic field that causes the nuclei of certain atoms in the body to line up. Radio waves of different frequencies are then applied to the patient in the region of interest, and if the frequency of the wave matches the natural frequency of the atom, the nuclei will be set into a state of resonance and will absorb energy from the applied signal. When the signal is removed, the nuclei release the acquired energy in the form of weak but detectable signals. The strength and duration of the energy emission vary from one tissue of the body to another. The weak signals are then amplified, digitized, and translated to provide a cross-sectional image such as the one shown in Fig. 11.55.

MRI units are very expensive and therefore are not available at all locations. In recent years, however, their numbers have grown, and one is available in almost every major community. For some patients the claustrophobic feeling they experience while in the circular tube is difficult to contend with. Today, however, a more open unit has been developed, as shown in Fig. 11.56, that has removed most of this discomfort.

Patients who have metallic implants or pacemakers or those who have worked in industrial environments where minute ferromagnetic particles may have become lodged in open, sensitive areas such as the eyes, nose, and so on, may have to use a CAT scan system because it does not employ magnetic effects. The attending physician is well trained in such areas of concern and will remove any unfounded fears or suggest alternative methods.



FIG. 11.54

Magnetic resonance imaging equipment. (Courtesy of Siemens Medical Systems, Inc.)



FIG. 11.55

Magnetic resonance image. (Courtesy of Siemens Medical Systems, Inc.)



FIG. 11.56

Magnetic resonance imaging equipment (open variety). (Courtesy of Siemens Medical Systems, Inc.)



PROBLEMS

SECTION 11.3 Flux Density

- Using Appendix F, fill in the blanks in the following table. Indicate the units for each quantity.

	Φ	B
SI	5×10^{-4} Wb	8×10^{-4} T
CGS	_____	_____
English	_____	_____

- Repeat Problem 1 for the following table if area = 2 in.²:

	Φ	B
SI	_____	_____
CGS	60,000 maxwells	_____
English	_____	_____

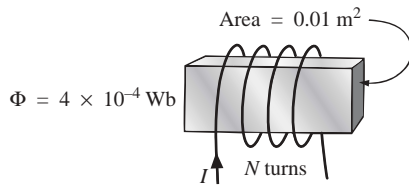


FIG. 11.57
Problem 3.

- For the electromagnet of Fig. 11.57:
 - Find the flux density in the core.
 - Sketch the magnetic flux lines and indicate their direction.
 - Indicate the north and south poles of the magnet.

SECTION 11.5 Reluctance

- Which section of Fig. 11.58—(a), (b), or (c)—has the largest reluctance to the setting up of flux lines through its longest dimension?

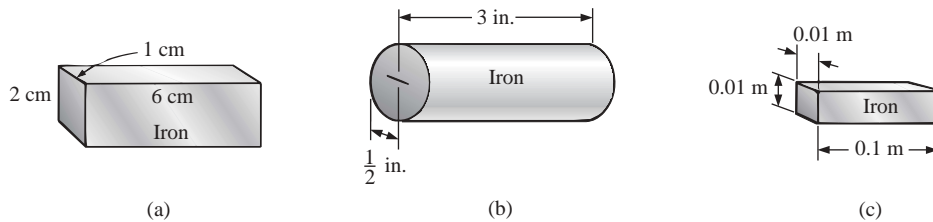


FIG. 11.58
Problem 4.

SECTION 11.6 Ohm's Law for Magnetic Circuits

- Find the reluctance of a magnetic circuit if a magnetic flux $\Phi = 4.2 \times 10^{-4}$ Wb is established by an impressed mmf of 400 At.
- Repeat Problem 5 for $\Phi = 72,000$ maxwells and an impressed mmf of 120 gilberts.

SECTION 11.7 Magnetizing Force

- Find the magnetizing force H for Problem 5 in SI units if the magnetic circuit is 6 in. long.
- If a magnetizing force H of 600 At/m is applied to a magnetic circuit, a flux density B of 1200×10^{-4} Wb/m² is established. Find the permeability μ of a material that will produce twice the original flux density for the same magnetizing force.



SECTION 11.8 Hysteresis

- 9. For the series magnetic circuit of Fig. 11.59, determine the current I necessary to establish the indicated flux.
- 10. Find the current necessary to establish a flux of $\Phi = 3 \times 10^{-4}$ Wb in the series magnetic circuit of Fig. 11.60.

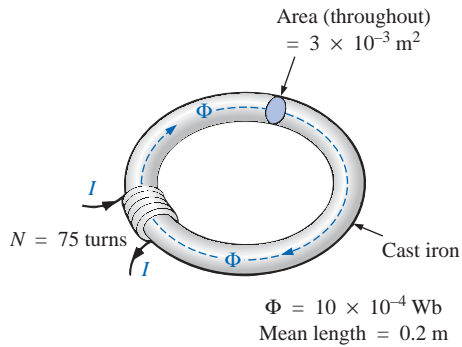


FIG. 11.59
Problem 9.

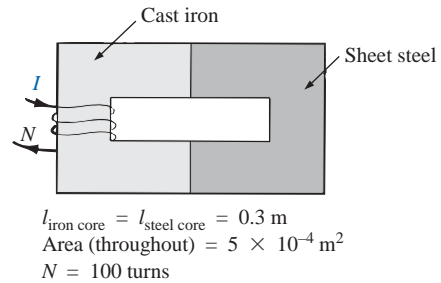


FIG. 11.60
Problem 10.

- 11. a. Find the number of turns N_1 required to establish a flux $\Phi = 12 \times 10^{-4}$ Wb in the magnetic circuit of Fig. 11.61.
- b. Find the permeability μ of the material.
- 12. a. Find the mmf (NI) required to establish a flux $\Phi = 80,000$ lines in the magnetic circuit of Fig. 11.62.
- b. Find the permeability of each material.

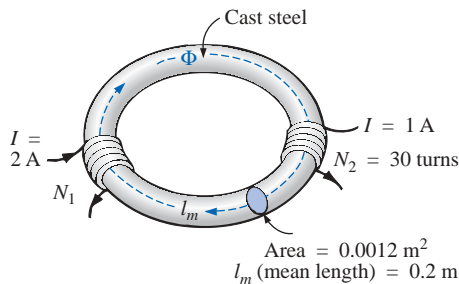


FIG. 11.61
Problem 11.

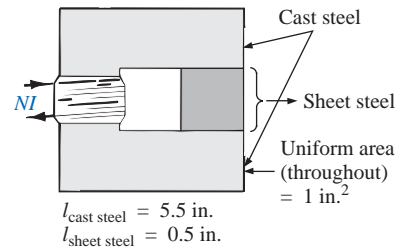


FIG. 11.62
Problem 12.

- *13. For the series magnetic circuit of Fig. 11.63 with two impressed sources of magnetic “pressure,” determine the current I . Each applied mmf establishes a flux pattern in the clockwise direction.

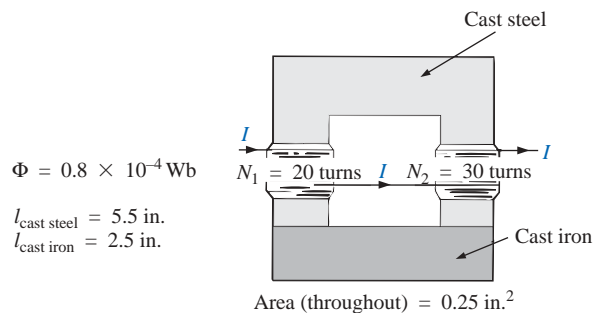


FIG. 11.63
Problem 13.

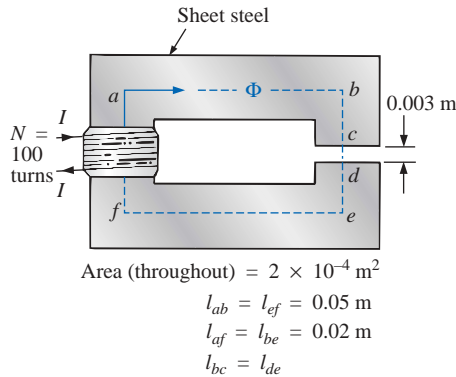


FIG. 11.64
 Problem 14.

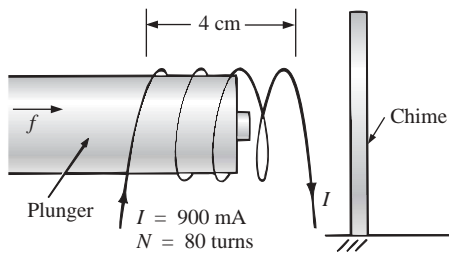


FIG. 11.65
 Door chime for Problem 15.

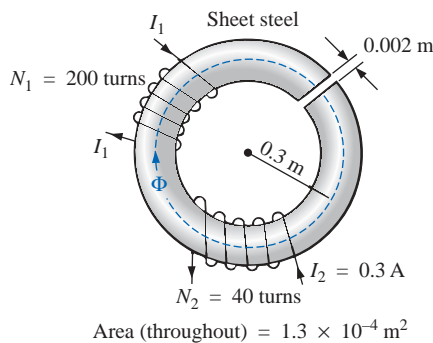


FIG. 11.66
 Problem 16.

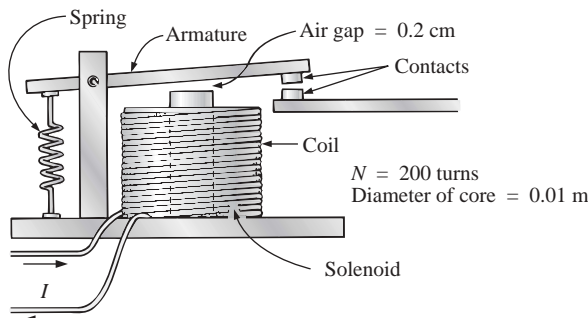


FIG. 11.67
 Relay for Problem 17.

SECTION 11.12 Air Gaps

- 14. a.** Find the current I required to establish a flux $\Phi = 2.4 \times 10^{-4} \text{ Wb}$ in the magnetic circuit of Fig. 11.64.
- b.** Compare the mmf drop across the air gap to that across the rest of the magnetic circuit. Discuss your results using the value of μ for each material.

- *15.** The force carried by the plunger of the door chime of Fig. 11.65 is determined by

$$f = \frac{1}{2} NI \frac{d\phi}{dx} \quad (\text{newtons})$$

where $d\phi/dx$ is the rate of change of flux linking the coil as the core is drawn into the coil. The greatest rate of change of flux will occur when the core is $1/4$ to $3/4$ the way through. In this region, if Φ changes from $0.5 \times 10^{-4} \text{ Wb}$ to $8 \times 10^{-4} \text{ Wb}$, what is the force carried by the plunger?

- 16.** Determine the current I_1 required to establish a flux of $\Phi = 2 \times 10^{-4} \text{ Wb}$ in the magnetic circuit of Fig. 11.66.

- *17. a.** A flux of $0.2 \times 10^{-4} \text{ Wb}$ will establish sufficient attractive force for the armature of the relay of Fig. 11.67 to close the contacts. Determine the required current to establish this flux level if we assume that the total mmf drop is across the air gap.
- b.** The force exerted on the armature is determined by the equation

$$F (\text{newtons}) = \frac{1}{2} \cdot \frac{B_g^2 A}{\mu_0}$$

where B_g is the flux density within the air gap and A is the common area of the air gap. Find the force in newtons exerted when the flux Φ specified in part (a) is established.



- *18. For the series-parallel magnetic circuit of Fig. 11.68, find the value of I required to establish a flux in the gap of $\Phi_g = 2 \times 10^{-4}$ Wb.

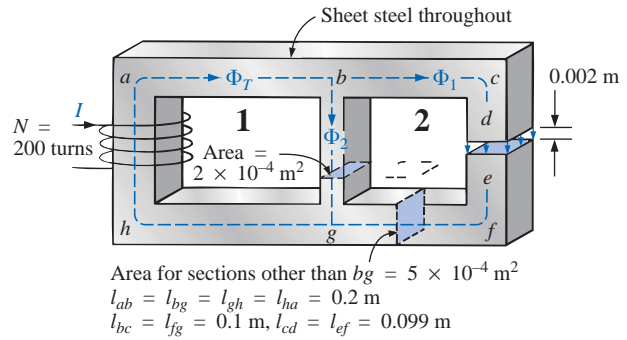


FIG. 11.68
Problem 18.

SECTION 11.14 Determining Φ

19. Find the magnetic flux Φ established in the series magnetic circuit of Fig. 11.69.

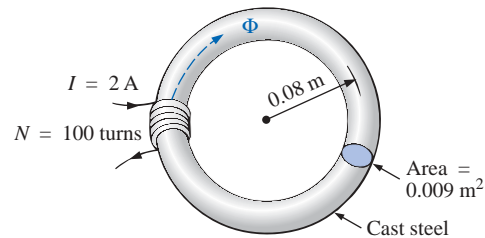


FIG. 11.69
Problem 19.

- *20. Determine the magnetic flux Φ established in the series magnetic circuit of Fig. 11.70.

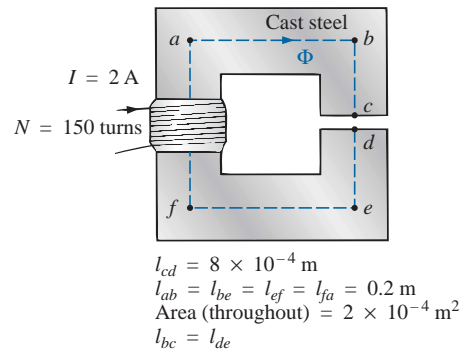


FIG. 11.70
Problem 20.

- *21. Note how closely the B - H curve of cast steel in Fig. 11.23 matches the curve for the voltage across a capacitor as it charges from zero volts to its final value.
- Using the equation for the charging voltage as a guide, write an equation for B as a function of H [$B = f(H)$] for cast steel.
 - Test the resulting equation at $H = 900 \text{ At/m}$, 1800 At/m , and 2700 At/m .
 - Using the equation of part (a), derive an equation for H in terms of B [$H = f(B)$].
 - Test the resulting equation at $B = 1 \text{ T}$ and $B = 1.4 \text{ T}$.
 - Using the result of part (c), perform the analysis of Example 11.3, and compare the results for the current I .

COMPUTER ANALYSIS Programming Language (C++, QBASIC, Pascal, etc.)

- *22. Using the results of Problem 21, write a program to perform the analysis of a core such as that shown in Example 11.3; that is, let the dimensions of the core and the applied turns be input variables requested by the program.
- *23. Using the results of Problem 21, develop a program to perform the analysis appearing in Example 11.9 for cast steel. A test routine will have to be developed to determine whether the results obtained are sufficiently close to the applied ampere-turns.



GLOSSARY

- Ampère's circuital law** A law establishing the fact that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero.
- Diamagnetic materials** Materials that have permeabilities slightly less than that of free space.
- Domain** A group of magnetically aligned atoms.
- Electromagnetism** Magnetic effects introduced by the flow of charge or current.
- Ferromagnetic materials** Materials having permeabilities hundreds and thousands of times greater than that of free space.
- Flux density (B)** A measure of the flux per unit area perpendicular to a magnetic flux path. It is measured in teslas (T) or webers per square meter (Wb/m^2).
- Hysteresis** The lagging effect between the flux density of a material and the magnetizing force applied.
- Magnetic flux lines** Lines of a continuous nature that reveal the strength and direction of a magnetic field.
- Magnetizing force (H)** A measure of the magnetomotive force per unit length of a magnetic circuit.
- Magnetomotive force (mmf) (\mathcal{F})** The "pressure" required to establish magnetic flux in a ferromagnetic material. It is measured in ampere-turns (At).
- Paramagnetic materials** Materials that have permeabilities slightly greater than that of free space.
- Permanent magnet** A material such as steel or iron that will remain magnetized for long periods of time without the aid of external means.
- Permeability (μ)** A measure of the ease with which magnetic flux can be established in a material. It is measured in Wb/Am .
- Relative permeability (μ_r)** The ratio of the permeability of a material to that of free space.
- Reluctance (\mathcal{R})** A quantity determined by the physical characteristics of a material that will provide an indication of the "reluctance" of that material to the setting up of magnetic flux lines in the material. It is measured in rels or At/Wb .