12

Inductors

12.1 INTRODUCTION

We have examined the resistor and the capacitor in detail. In this chapter we shall consider a third element, the **inductor,** which has a number of response characteristics similar in many respects to those of the capacitor. In fact, some sections of this chapter will proceed parallel to those for the capacitor to emphasize the similarity that exists between the two elements.

12.2 FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

If a conductor is moved through a magnetic field so that it cuts magnetic lines of flux, a voltage will be induced across the conductor, as shown in Fig. 12.1. The greater the number of flux lines cut per unit time (by increasing the speed with which the conductor passes through the field), or the stronger the magnetic field strength (for the same tra-

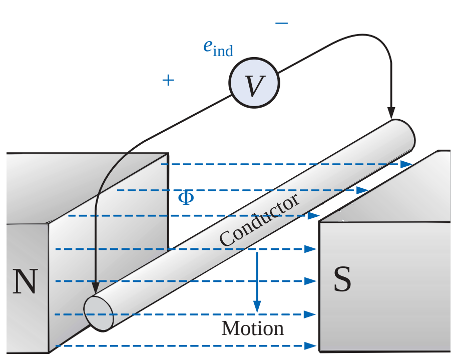

FIG. 12.1
Generating an induced voltage by moving a conductor through a magnetic field.

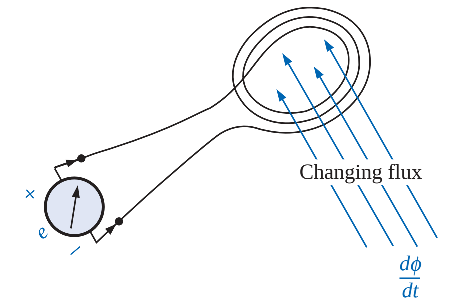

FIG. 12.2
Demonstrating Faraday's law.

versing speed), the greater will be the induced voltage across the conductor. If the conductor is held fixed and the magnetic field is moved so that its flux lines cut the conductor, the same effect will be produced.

If a coil of N turns is placed in the region of a changing flux, as in Fig. 12.2, a voltage will be induced across the coil as determined by **Faraday's law:**

$$e = N\frac{d\phi}{dt} \qquad \text{(volts, V)} \tag{12.1}$$

where N represents the number of turns of the coil and $d\phi/dt$ is the instantaneous change in flux (in webers) linking the coil. The term *linking* refers to the flux within the turns of wire. The term *changing* simply indicates that either the strength of the field linking the coil changes in magnitude or the coil is moved through the field in such a way that the number of flux lines through the coil changes with time.

If the flux linking the coil ceases to change, such as when the coil simply sits still in a magnetic field of fixed strength, $d\phi/dt = 0$, and the induced voltage $e = N(d\phi/dt) = N(0) = 0$.

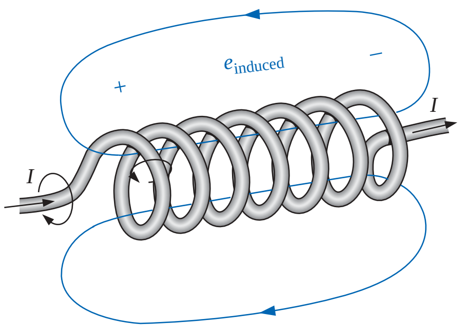

FIG. 12.3
Demonstrating the effect of Lenz's law.

12.3 LENZ'S LAW

In Section 11.2 it was shown that the magnetic flux linking a coil of N turns with a current I has the distribution of Fig. 12.3.

If the current increases in magnitude, the flux linking the coil also increases. It was shown in Section 12.2, however, that a changing flux linking a coil induces a voltage across the coil. For this coil, therefore, an induced voltage is developed *across* the coil due to the change in current *through* the coil. The polarity of this induced voltage tends to establish a current in the coil that produces a flux that will oppose any change in the original flux. In other words, the induced effect (e_{ind}) is a result of the increasing current through the coil. However, the resulting induced voltage will tend to establish a current that will oppose the increasing change in current through the coil. Keep in mind that this is all occurring simultaneously. The instant the current begins to increase in magnitude, there will be an opposing effect trying to limit the change. It is "choking" the change in current through the coil. Hence, the term **choke** is often applied to the inductor or coil. In fact, we will find shortly that the current through a coil cannot change instantaneously. A period of time determined by the coil and the resistance of the circuit is required before the inductor discontinues its opposition to a momentary change in current. Recall a similar situation for the voltage across a capacitor in Chapter 10. The reaction above is true for increasing or decreasing levels of current through the coil. This effect is an example of a general principle known as **Lenz's law,** which states that

an induced effect is always such as to oppose the cause that produced it.

12.4 SELF-INDUCTANCE

The ability of a coil to oppose any change in current is a measure of the **self-inductance** L of the coil. For brevity, the prefix *self* is usually dropped. Inductance is measured in henries (H), after the American physicist Joseph Henry (Fig. 12.4).

American (Albany, NY; Princeton, NJ)
(1797–1878)
Physicist and Mathematician
Professor of Natural Philosophy, Princeton University

Courtesy of the Smithsonian Institution Photo No. 59,054

In the early 1800s the title Professor of Natural Philosophy was applied to educators in the sciences. As a student and teacher at the Albany Academy, Henry performed extensive research in the area of electromagnetism. He improved the design of *electromagnets* by insulating the coil wire to permit a tighter wrap on the core. One of his earlier designs was capable of lifting 3600 pounds. In 1832 he discovered and delivered a paper on *self-induction.* This was followed by the construction of an effective *electric telegraph transmitter and receiver* and extensive research on the oscillatory nature of lightning and discharges from a *Leyden jar.* In 1845 he was appointed the first Secretary of the Smithsonian.

FIG. 12.4
Joseph Henry.

Inductors are coils of various dimensions designed to introduce specified amounts of inductance into a circuit. The inductance of a coil varies directly with the magnetic properties of the coil. Ferromagnetic materials, therefore, are frequently employed to increase the inductance by increasing the flux linking the coil.

A close approximation, in terms of physical dimensions, for the inductance of the coils of Fig. 12.5 can be found using the following equation:

$$L = \frac{N^2 \mu A}{l} \quad \text{(henries, H)} \tag{12.2}$$

where N represents the number of turns; μ, the permeability of the core (as introduced in Section 11.4; recall that μ is not a constant but depends on the level of B and H since $\mu = B/H$); A, the area of the core in square meters; and l, the mean length of the core in meters.

Substituting $\mu = \mu_r \mu_o$ into Eq. (12.2) yields

$$L = \frac{N^2 \mu_r \mu_o A}{l} = \mu_r \frac{N^2 \mu_o A}{l}$$

and

$$L = \mu_r L_o \tag{12.3}$$

where L_o is the inductance of the coil with an air core. In other words, the inductance of a coil with a ferromagnetic core is the relative permeability of the core times the inductance achieved with an air core.

Equations for the inductance of coils different from those shown above can be found in reference handbooks. Most of the equations are more complex than those just described.

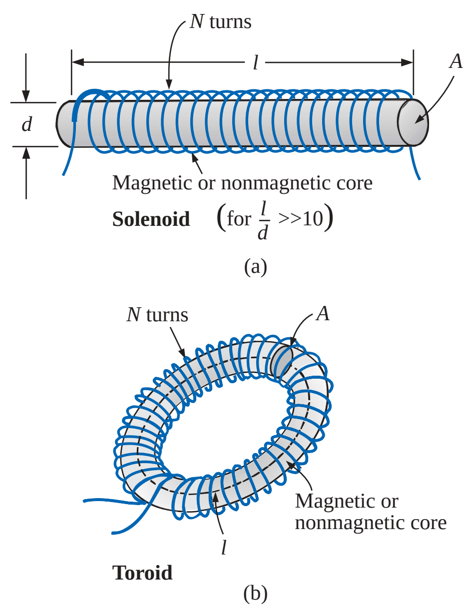

FIG. 12.5
Inductor configurations for which Equation (12.2) is appropriate.

EXAMPLE 12.1 Find the inductance of the air-core coil of Fig. 12.6.

Solution:

$$\mu = \mu_r \mu_o = (1)(\mu_o) = \mu_o$$

$$A = \frac{\pi d^2}{4} = \frac{(\pi)(4 \times 10^{-3}\ \text{m})^2}{4} = 12.57 \times 10^{-6}\ \text{m}^2$$

$$L_o = \frac{N^2 \mu_o A}{l} = \frac{(100\ \text{t})^2(4\pi \times 10^{-7}\ \text{Wb/A·m})(12.57 \times 10^{-6}\ \text{m}^2)}{0.1\ \text{m}}$$

$$= \mathbf{1.58\ \mu H}$$

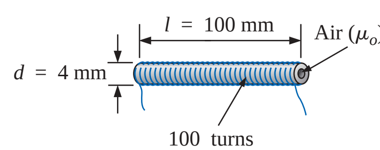

FIG. 12.6
Example 12.1.

EXAMPLE 12.2 Repeat Example 12.1, but with an iron core and conditions such that $\mu_r = 2000$.

Solution: By Eq. (12.3),

$$L = \mu_r L_o = (2000)(1.58 \times 10^{-6}\ \text{H}) = \mathbf{3.16\ mH}$$

12.5 TYPES OF INDUCTORS

Practical Equivalent

Inductors, like capacitors, are not ideal. Associated with every inductor are a resistance equal to the resistance of the turns and a stray capaci-

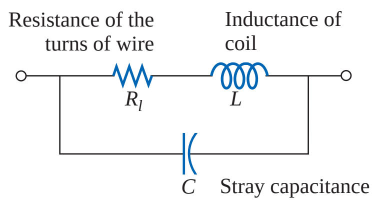

FIG. 12.7
Complete equivalent model for an inductor.

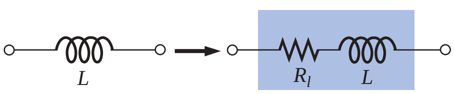

FIG. 12.8
Practical equivalent model for an inductor.

tance due to the capacitance between the turns of the coil. To include these effects, the equivalent circuit for the inductor is as shown in Fig. 12.7. However, for most applications considered in this text, the stray capacitance appearing in Fig. 12.7 can be ignored, resulting in the equivalent model of Fig. 12.8. The resistance R_l can play an important role in the analysis of networks with inductive elements. For most applications, we have been able to treat the capacitor as an ideal element and maintain a high degree of accuracy. For the inductor, however, R_l must often be included in the analysis and can have a pronounced effect on the response of a system (see Chapter 20, "Resonance"). The level of R_l can extend from a few ohms to a few hundred ohms. Keep in mind that the longer or thinner the wire used in the construction of the inductor, the greater will be the dc resistance as determined by $R = \rho l / A$. Our initial analysis will treat the inductor as an ideal element. Once a general feeling for the response of the element is established, the effects of R_l will be included.

Symbols

The primary function of the inductor, however, is to introduce inductance—not resistance or capacitance—into the network. For this reason, the symbols employed for inductance are as shown in Fig. 12.9.

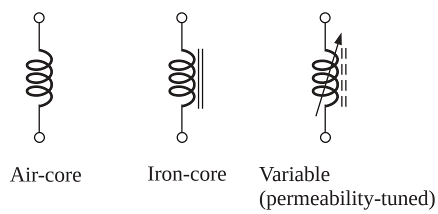

FIG. 12.9
Inductor symbols.

Appearance

All inductors, like capacitors, can be listed under two general headings: *fixed* and *variable.* The fixed air-core and iron-core inductors were described in the last section. The permeability-tuned variable coil has a ferromagnetic shaft that can be moved within the coil to vary the flux linkages of the coil and thereby its inductance. Several fixed and variable inductors appear in Fig. 12.10.

Testing

The primary reasons for inductor failure are shorts that develop between the windings and open circuits in the windings due to factors such as excessive currents, overheating, and age. The open-circuit condition can be checked easily with an ohmmeter (∞ ohms indication), but the short-circuit condition is harder to check because the resistance of many good inductors is relatively small and the shorting of a few windings will not adversely affect the total resistance. Of course, if one is aware of the typical resistance of the coil, it can be compared to the

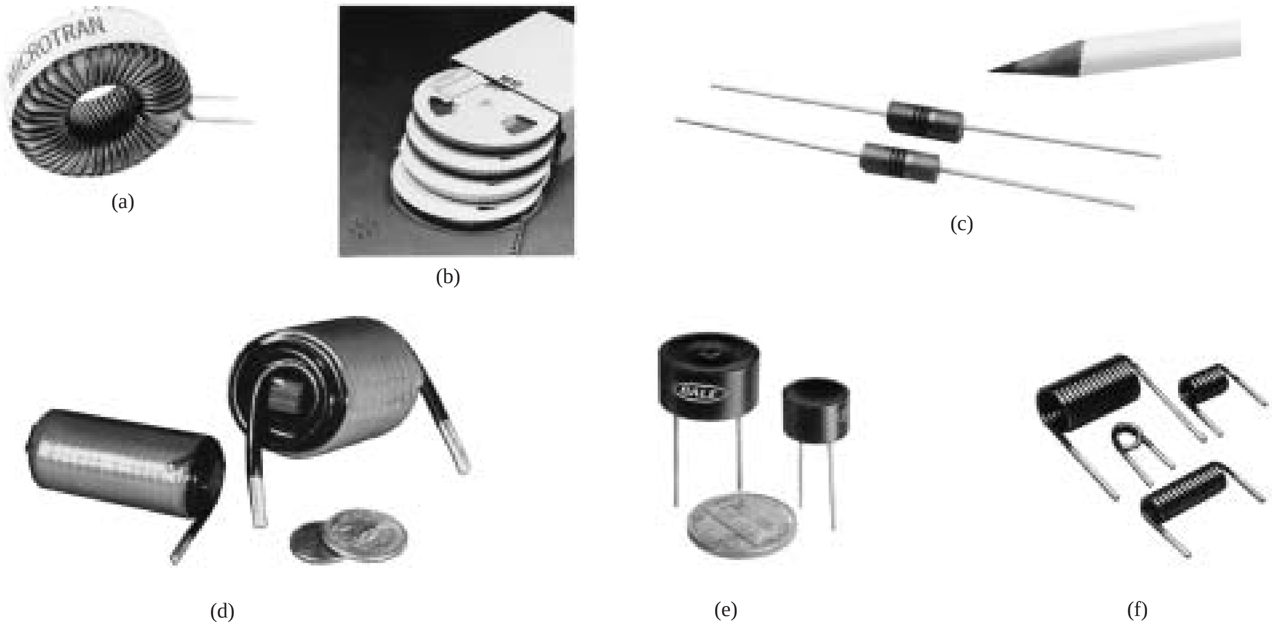

FIG. 12.10
Various types of inductors: (a) toroidal power inductor (1.4 μH to 5.6 mH) (courtesy of Microtan Co., Inc.); (b) surface-mount inductors on reels (0.1 μH through 1000 μH on 500-piece reels in 46 values) (courtesy of Bell Industries); (c) molded inductors (0.1 μH to 10 μH); (d) high-current filter inductors (24 μH at 60 A to 500 μH at 15 A); (e) toroid filter inductors (40 μH to 5 H); (f) air-core inductors (1 to 32 turns) for high-frequency applications. [*Parts (c) through (f) courtesy of Dale Electronics, Inc.*]

measured value. A short between the windings and the core can be checked by simply placing one lead of the meter on one wire (terminal) and the other on the core itself. An indication of zero ohms reflects a short between the two because the wire that makes up the winding has an insulation jacket throughout. The universal LCR meter of Fig. 10.20 can be used to check the inductance level.

Standard Values and Recognition Factor

The standard values for inductors employ the same numerical multipliers used with resistors and inductors. Like the capacitor, the most common employ the same numerical multipliers as the most common resistors, that is, those with the full range of tolerances (5%, 10%, and 20%), as appearing in Table 3.8. However, inductors are also readily available with the multipliers associated with the 5% and 10% resistors of Table 3.8. In general, therefore, expect to find inductors with the following multipliers: **0.1** μH, **0.12** μH, **0.15** μH, **0.18** μH, **0.22** μH, **0.27** μH, **0.33** μH, **0.39** μH, **0.47** μH, **0.56** μH, **0.68** μH, and **0.82** μH, and then **1** mH, **1.2** mH, **1.5** mH, **1.8** mH, **2.2** mH, **2.7** mH, and so on.

Figure 12.11 was developed to establish a recognition factor when it comes to the various types and uses for inductors—in other words, to help the reader develop the skills to identify types of inductors, their typical range of values, and some of the most common applications. Figure 12.11 is certainly not all-inclusive, but it does offer a first step in establishing a sense for what to expect for various applications.

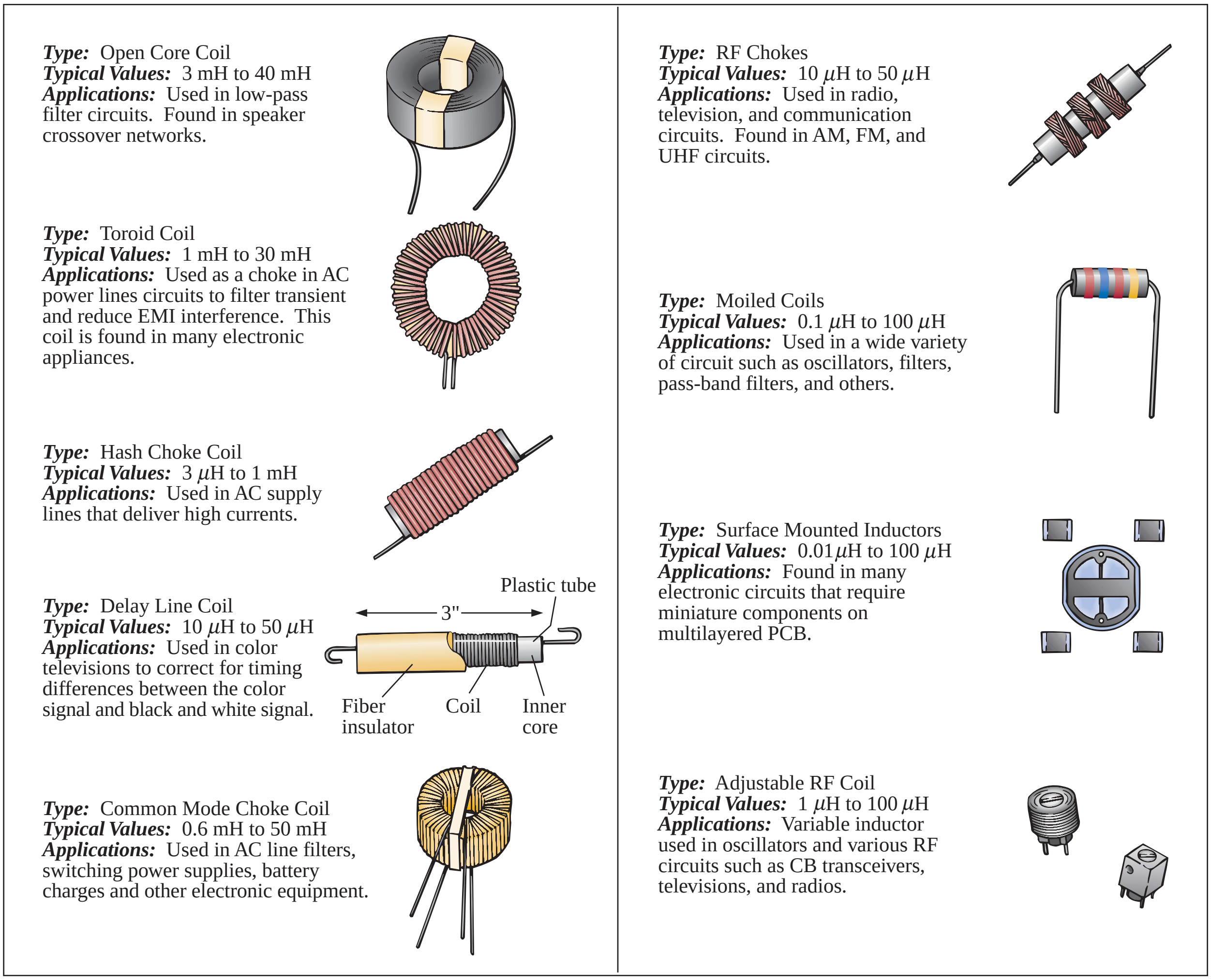

FIG. 12.11
Typical areas of application for inductive elements.

12.6 INDUCED VOLTAGE

The inductance of a coil is also a measure of the change in flux linking a coil due to a change in current through the coil; that is,

$$L = N\frac{d\phi}{di} \quad \text{(H)} \qquad \textbf{(12.4)}$$

where N is the number of turns, ϕ is the flux in webers, and i is the current through the coil. If a change in current through the coil fails to result in a significant change in the flux linking the coil through its center, the resulting inductance level will be relatively small. For this reason the inductance of a coil is sensitive to the point of operation on the hysteresis curve (described in detail in Section 11.8). If the coil is operating on the steep slope, the change in flux will be relatively high for a change in current through the coil. If the coil is operating near or in saturation, the change in flux will be relatively small for the same change in current, resulting in a reduced level of inductance. This effect is particularly important when we examine ac circuits since a dc level associated with the applied ac signal may put the coil at or near saturation, and the resulting inductance level for the applied ac signal will be significantly less than expected. You will find that the maximum dc current is normally provided in supply manuals and data sheets to ensure avoidance of the saturation region.

Equation (12.4) also reveals that the larger the inductance of a coil (with N fixed), the larger will be the instantaneous change in flux linking the coil due to an instantaneous change in current through the coil.

If we write Eq. (12.1) as

$$e_L = N\frac{d\phi}{dt} = \left(N\frac{d\phi}{di}\right)\left(\frac{di}{dt}\right)$$

and substitute Eq. (12.4), we then have

$$\boxed{e_L = L\frac{di}{dt}} \quad \text{(V)} \qquad \textbf{(12.5)}$$

revealing that the magnitude of the voltage across an inductor is directly related to the inductance L and the instantaneous rate of change of current through the coil. Obviously, therefore, the greater the *rate* of change of current through the coil, the greater will be the induced voltage. This certainly agrees with our earlier discussion of Lenz's law.

When induced effects are employed in the generation of voltages such as those available from dc or ac generators, the symbol e is appropriate for the induced voltage. However, in network analysis the voltage across an inductor will always have a polarity such as to oppose the source that produced it, and therefore the following notation will be used throughout the analysis to come:

$$\boxed{v_L = L\frac{di}{dt}} \qquad \textbf{(12.6)}$$

If the current through the coil fails to change at a particular instant, the induced voltage across the coil will be zero. For dc applications, after the transient effect has passed, $di/dt = 0$, and the induced voltage is

$$v_L = L\frac{di}{dt} = L(0) = 0\text{ V}$$

Recall that the equation for the current of a capacitor is the following:

$$i_C = C\frac{dv_C}{dt}$$

Note the similarity between this equation and Eq. (12.6). In fact, if we apply the duality $v \leftrightarrows i$ (that is, interchange the two) and $L \leftrightarrows C$ for capacitance and inductance, each equation can be derived from the other.

The average voltage across the coil is defined by the equation

$$\boxed{v_{L_{av}} = L\frac{\Delta i}{\Delta t}} \quad \text{(V)} \qquad \textbf{(12.7)}$$

where Δ signifies finite change (a measurable change). Compare this to $i_C = C(\Delta v/\Delta t)$, and the meaning of Δ and application of this equation should be clarified from Chapter 10. An example follows.

EXAMPLE 12.3 Find the waveform for the average voltage across the coil if the current through a 4-mH coil is as shown in Fig. 12.12.

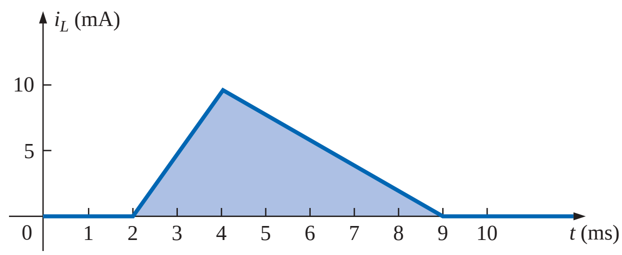

FIG. 12.12
Example 12.3.

Solutions:

a. *0 to 2 ms:* Since there is no change in current through the coil, there is no voltage induced across the coil; that is,

$$v_L = L\frac{\Delta i}{\Delta t} = L\frac{0}{\Delta t} = \mathbf{0}$$

b. *2 ms to 4 ms:*

$$v_L = L\frac{\Delta i}{\Delta t} = (4 \times 10^{-3}\text{ H})\left(\frac{10 \times 10^{-3}\text{ A}}{2 \times 10^{-3}\text{ s}}\right) = 20 \times 10^{-3}\text{ V}$$
$$= \mathbf{20\ mV}$$

c. *4 ms to 9 ms:*

$$v_L = L\frac{\Delta i}{\Delta t} = (-4 \times 10^{-3}\text{ H})\left(\frac{10 \times 10^{-3}\text{ A}}{5 \times 10^{-3}\text{ s}}\right) = -8 \times 10^{-3}\text{ V}$$
$$= \mathbf{-8\ mV}$$

d. *9 ms to ∞:*

$$v_L = L\frac{\Delta i}{\Delta t} = L\frac{0}{\Delta t} = \mathbf{0}$$

The waveform for the average voltage across the coil is shown in Fig. 12.13. Note from the curve that

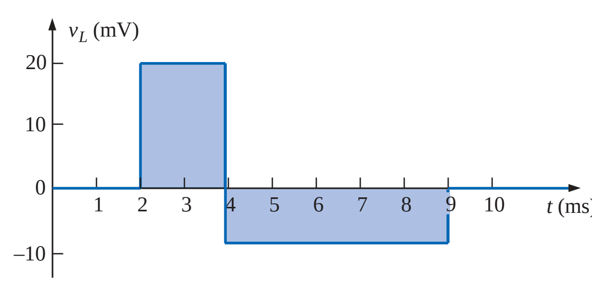

FIG. 12.13
Voltage across a 4-mH coil due to the current of Fig. 12.12.

the voltage across the coil is not determined solely by the magnitude of the change in current through the coil (Δi), but also by the rate of change of current through the coil ($\Delta i/\Delta t$).

A similar statement was made for the current of a capacitor due to a change in voltage across the capacitor.

A careful examination of Fig. 12.13 will also reveal that the area under the positive pulse from 2 ms to 4 ms equals the area under the negative pulse from 4 ms to 9 ms. In Section 12.13, we will find that the area under the curves represents the energy stored or released by the inductor. From 2 ms to 4 ms, the inductor is storing energy, whereas from 4 ms to 9 ms, the inductor is releasing the energy stored. For the full period zero to 10 ms, energy has simply been stored and released; there has been no dissipation as experienced for the resistive elements. Over a full cycle, both the ideal capacitor and inductor do not consume energy but simply store and release it in their respective forms.

12.7 *R-L* TRANSIENTS: STORAGE CYCLE

The changing voltages and current that result during the storing of energy in the form of a magnetic field by an inductor in a dc circuit can best be described using the circuit of Fig. 12.14. At the instant the switch is closed, the inductance of the coil will prevent an instantaneous change in current through the coil. The potential drop across the coil, v_L, will equal the impressed voltage E as determined by Kirchhoff's voltage law since $v_R = iR = (0)R = 0$ V. The current i_L will then build up from zero, establishing a voltage drop across the resistor and a corresponding drop in v_L. The current will continue to increase until the voltage across the inductor drops to zero volts and the full impressed voltage appears across the resistor. Initially, the current i_L increases quite rapidly, followed by a continually decreasing rate until it reaches its maximum value of E/R.

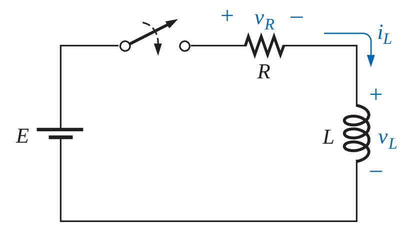

FIG. 12.14
Basic R-L transient network.

You will recall from the discussion of capacitors that a capacitor has a short-circuit equivalent when the switch is first closed and an open-circuit equivalent when steady-state conditions are established. The inductor assumes the opposite equivalents for each stage. The instant the switch of Fig. 12.14 is closed, the equivalent network will appear as shown in Fig. 12.15. Note the correspondence with the earlier comments regarding the levels of voltage and current. The inductor obviously meets all the requirements for an open-circuit equivalent: $v_L = E$ volts, and $i_L = 0$ A.

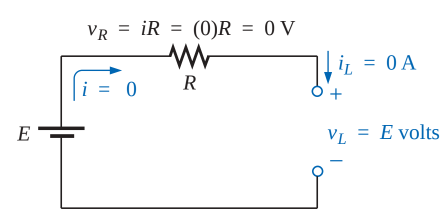

FIG. 12.15
Circuit of Fig. 12.14 the instant the switch is closed.

When steady-state conditions have been established and the storage phase is complete, the "equivalent" network will appear as shown in Fig. 12.16. The network clearly reveals the following:

An ideal inductor ($R_l = 0\ \Omega$) assumes a short-circuit equivalent in a dc network once steady-state conditions have been established.

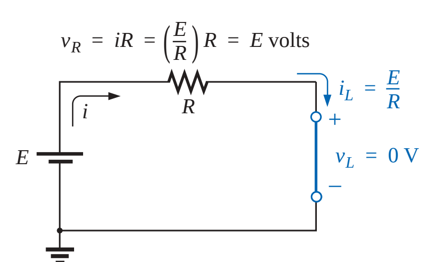

FIG. 12.16
Circuit of Fig. 12.14 under steady-state conditions.

Fortunately, the mathematical equations for the voltages and current for the storage phase are similar in many respects to those encountered for the *R-C* network. The experience gained with these equations in Chapter 10 will undoubtedly make the analysis of *R-L* networks somewhat easier to understand.

The equation for the current i_L during the storage phase is the following:

$$i_L = I_m(1 - e^{-t/\tau}) = \frac{E}{R}(1 - e^{-t/(L/R)}) \qquad \textbf{(12.8)}$$

Note the factor $(1 - e^{-t/\tau})$, which also appeared for the voltage v_C of a capacitor during the charging phase. A plot of the equation is given in Fig. 12.17, clearly indicating that the maximum steady-state value of i_L is E/R, and that the rate of change in current decreases as time passes. The abscissa is scaled in time constants, with τ for inductive circuits defined by the following:

$$\tau = \frac{L}{R} \qquad \text{(seconds, s)} \qquad \textbf{(12.9)}$$

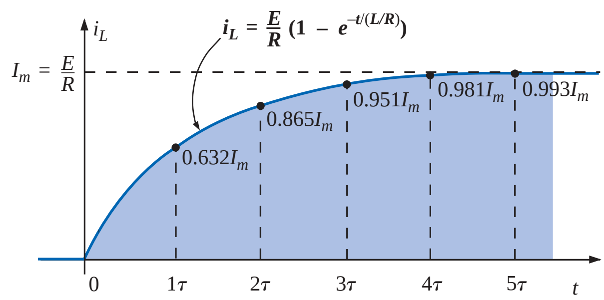

FIG. 12.17
Plotting the waveform for i_L during the storage cycle.

The fact that τ has the units of time can be verified by taking the equation for the induced voltage

$$v_L = L\frac{di}{dt}$$

and solving for L:

$$L = \frac{v_L}{di/dt}$$

which leads to the ratio

$$\tau = \frac{L}{R} = \frac{\dfrac{v_L}{di/dt}}{R} = \frac{v_L}{\dfrac{di}{dt}R} \longrightarrow \frac{V}{\dfrac{IR}{t}} = \frac{\cancel{V}}{\dfrac{\cancel{V}}{t}} = t \qquad \text{(s)}$$

Our experience with the factor $(1 - e^{-t/\tau})$ verifies the level of 63.2% after one time constant, 86.5% after two time constants, and so on. For convenience, Figure 10.29 is repeated as Fig. 12.18 to evaluate the functions $(1 - e^{-t/\tau})$ and $e^{-t/\tau}$ at various values of τ.

If we keep R constant and increase L, the ratio L/R increases and the rise time increases. The change in transient behavior for the current i_L is plotted in Fig. 12.19 for various values of L. Note again the duality between these curves and those obtained for the R-C network in Fig. 10.32.

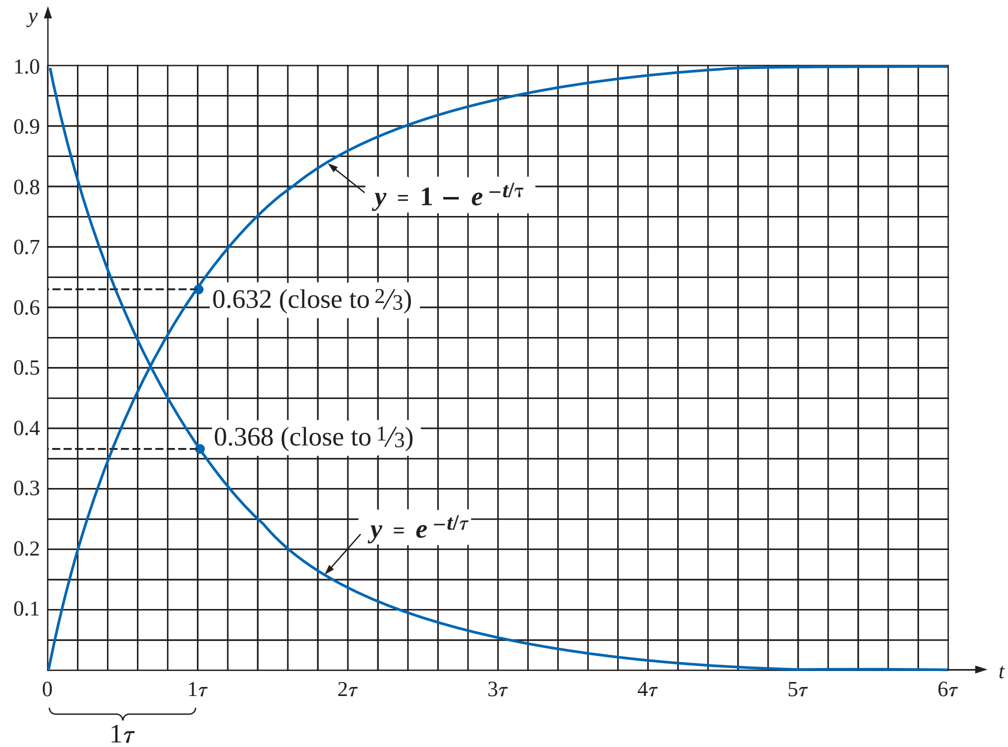

FIG. 12.18
Plotting the functions $y = 1 - e^{-t/\tau}$ and $y = e^{-t/\tau}$.

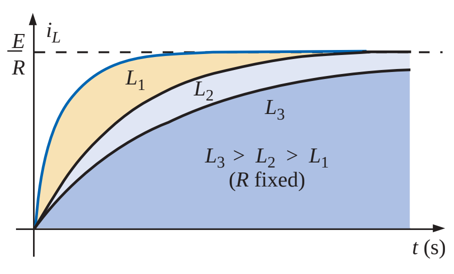

FIG. 12.19
Effect of L on the shape of the i_L storage waveform.

For most practical applications, we will assume that

the storage phase has passed and steady-state conditions have been established once a period of time equal to five time constants has occurred.

In addition, since L/R will always have some numerical value, even though it may be very small, the period 5τ will always be greater than zero, confirming the fact that

the current cannot change instantaneously in an inductive network.

In fact, the larger the inductance, the more the circuit will oppose a rapid buildup in current level.

Figures 12.15 and 12.16 clearly reveal that the voltage across the coil jumps to E volts when the switch is closed and decays to zero volts with time. The decay occurs in an exponential manner, and v_L during the storage phase can be described mathematically by the following equation:

$$v_L = Ee^{-t/\tau} \tag{12.10}$$

A plot of v_L appears in Fig. 12.20 with the time axis again divided into equal increments of τ. Obviously, the voltage v_L will decrease to zero volts at the same rate the current presses toward its maximum value.

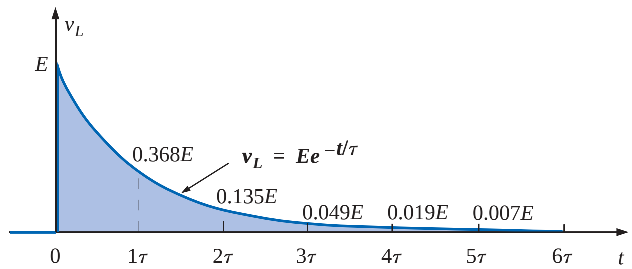

FIG. 12.20
Plotting the voltage v_R versus time for the network of Fig. 12.14.

In five time constants, $i_L = E/R$, $v_L = 0$ V, and the inductor can be replaced by its short-circuit equivalent.

Since $$v_R = i_R R = i_L R$$

then $$v_R = \left[\frac{E}{R}(1 - e^{-t/\tau})\right]R$$

and $$v_R = E(1 - e^{-t/\tau}) \tag{12.11}$$

and the curve for v_R will have the same shape as obtained for i_L.

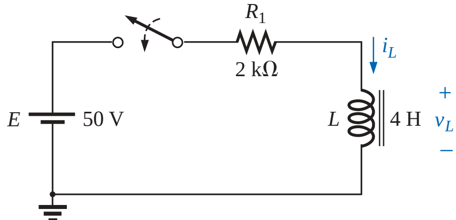

FIG. 12.21
Example 12.4.

EXAMPLE 12.4 Find the mathematical expressions for the transient behavior of i_L and v_L for the circuit of Fig. 12.21 after the closing of the switch. Sketch the resulting curves.

Solution:

$$\tau = \frac{L}{R_1} = \frac{4\text{ H}}{2\text{ k}\Omega} = 2\text{ ms}$$

By Eq. (12.8),

$$I_m = \frac{E}{R_1} = \frac{50}{2\text{ k}\Omega} = 25 \times 10^{-3}\text{ A} = 25\text{ mA}$$

and $$\boldsymbol{i_L = (25 \times 10^{-3})(1 - e^{-t/(2\times10^{-3})})}$$

By Eq. (12.10),

$$\boldsymbol{v_L = 50e^{-t/(2\times10^{-3})}}$$

Both waveforms appear in Fig. 12.22.

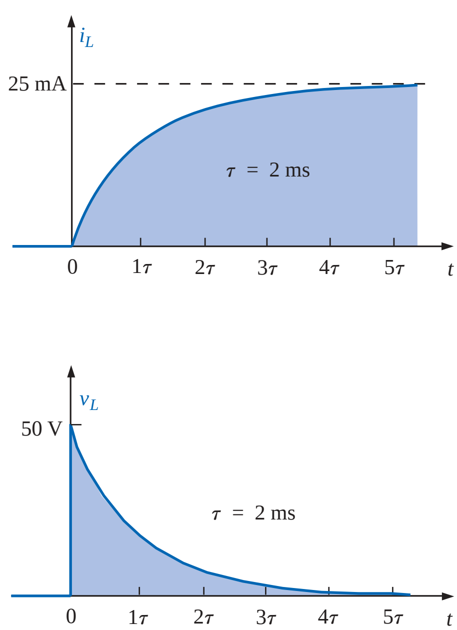

FIG. 12.22
i_L and v_L for the network of Fig. 12.21.

12.8 INITIAL VALUES

This section will parallel Section 10.9 (Initial Values—Capacitors) on the effect of *initial values* on the transient phase. Since the current through a coil cannot change instantaneously, the current through a coil will begin the *transient phase* at the *initial value* established by the network (note Fig. 12.23) before the switch was closed. It will then pass through the transient phase until it reaches the *steady-state* (or *final*) level after about five time constants. The steady-state level of the inductor current can be found by simply substituting its short-circuit equivalent (or R_l for the practical equivalent) and finding the resulting current through the element.

Using the transient equation developed in the previous section, an equation for the current i_L can be written for the entire time interval of Fig. 12.23; that is,

$$i_L = I_i + (I_f - I_i)(1 - e^{-t/\tau})$$

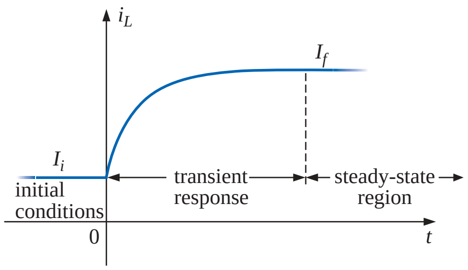

FIG. 12.23
Defining the three phases of a transient waveform.

with $(I_f - I_i)$ representing the total change during the transient phase. However, by multiplying through and rearranging terms:

$$i_L = I_i + I_f - I_f e^{-t/\tau} - I_i + I_i e^{-t/\tau}$$
$$= I_f - I_f e^{-t/\tau} + I_i e^{-t/\tau}$$

we find

$$i_L = I_f + (I_i - I_f)e^{-t/\tau} \quad \textbf{(12.12)}$$

If you are required to draw the waveform for the current i_L from initial value to final value, start by drawing a line at the initial value and steady-state levels, and then add the transient response (sensitive to the time constant) between the two levels. The following example will clarify the procedure.

EXAMPLE 12.5 The inductor of Fig. 12.24 has an initial current level of 4 mA in the direction shown. (Specific methods to establish the initial current will be presented in the sections and problems to follow.)

a. Find the mathematical expression for the current through the coil once the switch is closed.
b. Find the mathematical expression for the voltage across the coil during the same transient period.
c. Sketch the waveform for each from initial value to final value.

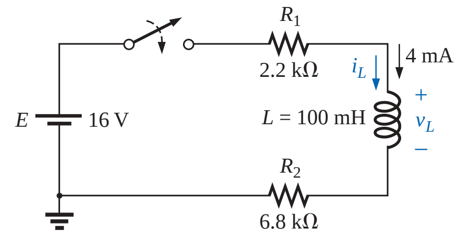

FIG. 12.24
Example 12.5.

Solutions:

a. Substituting the short-circuit equivalent for the inductor will result in a final or steady-state current determined by Ohm's law:

$$I_f = \frac{E}{R_1 + R_2} = \frac{16\text{ V}}{2.2\text{ k}\Omega + 6.8\text{ k}\Omega} = \frac{16\text{ V}}{9\text{ k}\Omega} = 1.78\text{ mA}$$

The time constant is determined by

$$\tau = \frac{L}{R_T} = \frac{100\text{ mH}}{2.2\text{ k}\Omega + 6.8\text{ k}\Omega} = \frac{100\text{ mH}}{9\text{ k}\Omega} = 11.11\ \mu\text{s}$$

Applying Eq. (12.12):

$$\begin{aligned} i_L &= I_f + (I_i - I_f)e^{-t/\tau} \\ &= 1.78\text{ mA} + (4\text{ mA} - 1.78\text{ mA})e^{-t/11.11\,\mu\text{s}} \\ &= \mathbf{1.78\text{ mA} + 2.22\text{ mA}e^{-t/11.11\,\mu s}} \end{aligned}$$

b. Since the current through the inductor is constant at 4 mA prior to the closing of the switch, the voltage (whose level is sensitive only to changes in current through the coil) must have an initial value of 0 V. At the instant the switch is closed, the current through the coil cannot change instantaneously, so the current through the resistive elements will be 4 mA. The resulting peak voltage at $t = 0$ s can then be found using Kirchhoff's voltage law as follows:

$$\begin{aligned} V_m &= E - V_{R_1} - V_{R_2} \\ &= 16\text{ V} - (4\text{ mA})(2.2\text{ k}\Omega) - (4\text{ mA})(6.8\text{ k}\Omega) \\ &= 16\text{ V} - 8.8\text{ V} - 27.2\text{ V} = 16\text{ V} - 36\text{ V} \\ &= -20\text{ V} \end{aligned}$$

Note the minus sign to indicate that the polarity of the voltage v_L is opposite to the defined polarity of Fig. 12.24.

The voltage will then decay (with the same time constant as the current i_L) to zero because the inductor is approaching its short-circuit equivalence.

The equation for v_L is therefore:

$$v_L = \mathbf{-20e^{-t/11.11\,\mu s}}$$

c. See Fig. 12.25. The initial and final values of the current were drawn first, and then the transient response was included between these levels. For the voltage, the waveform begins and ends at zero, with the peak value having a sign sensitive to the defined polarity of v_L in Fig. 12.24.

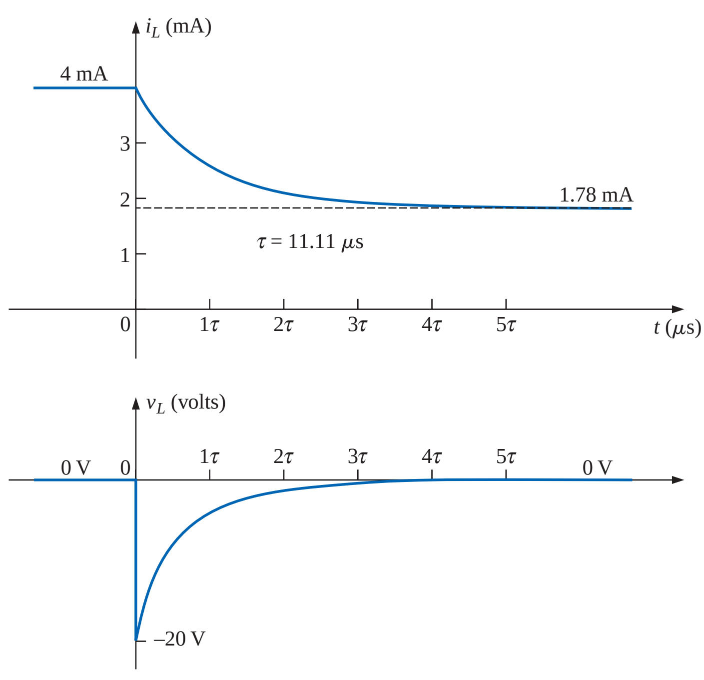

FIG. 12.25
i_L and v_L for the network of Fig. 12.24.

Let us now test the validity of the equation for i_L by substituting $t = 0$ s to reflect the instant the switch is closed.

$$e^{-t/\tau} = e^{-0} = 1$$

and $$i_L = 1.78 \text{ mA} + 2.22 \text{ mA}e^{-t/\tau} = 1.78 \text{ mA} + 2.22 \text{ mA} = 4 \text{ mA}$$

When $t > 5\tau$,

$$e^{-t/\tau} \cong 0$$

and $$i_L = 1.78 \text{ mA} + 2.22 \text{ mA}e^{-t/\tau} = 1.78 \text{ mA}$$

12.9 *R-L* TRANSIENTS: DECAY PHASE

In the analysis of *R-C* circuits, we found that the capacitor could hold its charge and store energy in the form of an electric field for a period of time determined by the leakage factors. In *R-L* circuits, the energy is stored in the form of a magnetic field established by the current through the coil. Unlike the capacitor, however, an isolated inductor cannot continue to store energy since the absence of a closed path would cause the current to drop to zero, releasing the energy stored in the form of a magnetic field. If the series *R-L* circuit of Fig. 12.26 had reached steady-state conditions and the switch were quickly opened, a spark would probably occur across the contacts due to the rapid change in current from a maximum of E/R to zero amperes. The change in current di/dt of the equation $v_L = L(di/dt)$ would establish a high voltage v_L across the coil that in conjunction with the applied voltage E appears across the points of the switch. This is the same mechanism as applied in the ignition system of a car to ignite the fuel in the cylinder. Some 25,000 V are generated by the rapid decrease in ignition coil current that occurs when the switch in the system is opened. (In older systems, the “points” in the distributor served as the switch.) This inductive reaction is significant when you consider that the only independent source in a car is a 12-V battery.

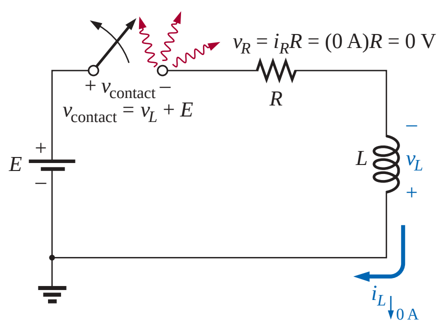

FIG. 12.26
Demonstrating the effect of opening a switch in series with an inductor with a steady-state current.

If opening the switch to move it to another position will cause such a rapid discharge in stored energy, how can the decay phase of an *R-L* circuit be analyzed in much the same manner as for the *R-C* circuit? The solution is to use a network such as that appearing in Fig. 12.27(a). When the switch is closed, the voltage across the resistor R_2 is E volts, and the *R-L* branch will respond in the same manner as described above, with the same waveforms and levels. A Thévenin network of E in parallel with R_2 would simply result in the source as

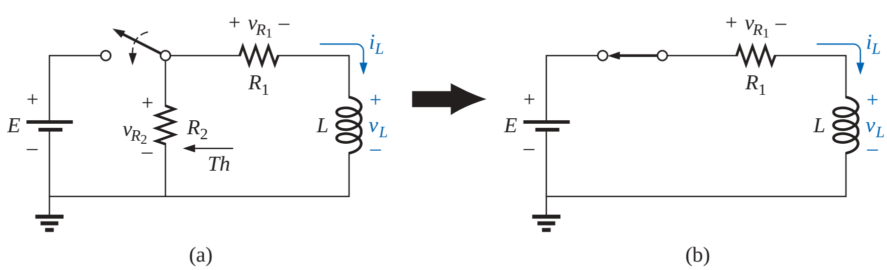

FIG. 12.27
Initiating the storage phase for the inductor L by closing the switch.

shown in Fig. 12.27(b) since R_2 would be shorted out by the short-circuit replacement of the voltage source E when the Thévenin resistance is determined.

After the storage phase has passed and steady-state conditions are established, the switch can be opened without the sparking effect or rapid discharge due to the resistor R_2, which provides a complete path for the current i_L. In fact, for clarity the discharge path is isolated in Fig. 12.28. The voltage v_L across the inductor will reverse polarity and have a magnitude determined by

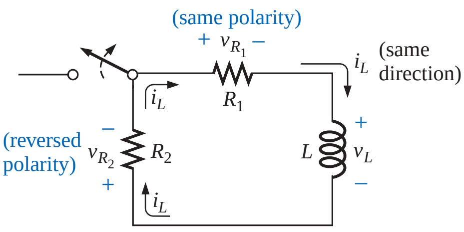

FIG. 12.28
Network of Fig. 12.27 the instant the switch is opened.

$$v_L = -(v_{R_1} + v_{R_2}) \qquad \textbf{(12.13)}$$

Recall that the voltage across an inductor can change instantaneously but the current cannot. The result is that the current i_L must maintain the same direction and magnitude as shown in Fig. 12.28. Therefore, the instant after the switch is opened, i_L is still $I_m = E/R_1$, and

$$\begin{aligned} v_L &= -(v_{R_1} + v_{R_2}) = -(i_1R_1 + i_2R_2) \\ &= -i_L(R_1 + R_2) = -\frac{E}{R_1}(R_1 + R_2) = -\left(\frac{R_1}{R_1} + \frac{R_2}{R_1}\right)E \end{aligned}$$

and

$$v_L = -\left(1 + \frac{R_2}{R_1}\right)E \qquad \textbf{(12.14)}$$

which is bigger than E volts by the ratio R_2/R_1. In other words, when the switch is opened, the voltage across the inductor will reverse polarity and drop instantaneously from E to $-[1 + (R_2/R_1)]E$ volts.

As an inductor releases its stored energy, the voltage across the coil will decay to zero in the following manner:

$$v_L = -V_i e^{-t/\tau'} \qquad \textbf{(12.15)}$$

with

$$V_i = \left(1 + \frac{R_2}{R_1}\right)E$$

and

$$\tau' = \frac{L}{R_T} = \frac{L}{R_1 + R_2}$$

The current will decay from a maximum of $I_m = E/R_1$ to zero. Using Eq. (12.20), $I_i = E/R_1$ and $I_f = 0$ A so that

$$\begin{aligned} i_L &= I_f + (I_i - I_f)e^{-t/\tau'} \\ &= 0\text{ A} + \left(\frac{E}{R_1} - 0\text{ A}\right)e^{-t/\tau'} \end{aligned}$$

and

$$i_L = \frac{E}{R_1}e^{-t/\tau'} \qquad \textbf{(12.16)}$$

with

$$\tau' = \frac{L}{R_1 + R_2}$$

The mathematical expression for the voltage across either resistor can then be determined using Ohm's law:

$$v_{R_1} = i_{R_1}R_1 = i_L R_1$$
$$= \frac{E}{R_1}R_1 e^{-t/\tau'}$$

and
$$v_{R_1} = Ee^{-t/\tau'} \qquad (12.17)$$

The voltage v_{R_1} has the same polarity as during the storage phase since the current i_L has the same direction. The voltage v_{R_2} is expressed as follows using the defined polarity of Fig. 12.27:

$$v_{R_2} = -i_{R_2}R_2 = -i_L R_2$$
$$= -\frac{E}{R_1}R_2 e^{-t/\tau'}$$

and
$$v_{R_2} = -\frac{R_2}{R_1}Ee^{-t/\tau'} \qquad (12.18)$$

EXAMPLE 12.6 The resistor R_2 was added to the network of Fig. 12.21, as shown in Fig. 12.29.

a. Find the mathematical expressions for i_L, v_L, v_{R_1}, and v_{R_2} for five time constants of the storage phase.
b. Find the mathematical expressions for i_L, v_L, v_{R_1}, and v_{R_2} if the switch is opened after five time constants of the storage phase.
c. Sketch the waveforms for each voltage and current for both phases covered by this example and Example 12.4 if five time constants pass between phases. Use the defined polarities of Fig. 12.27.

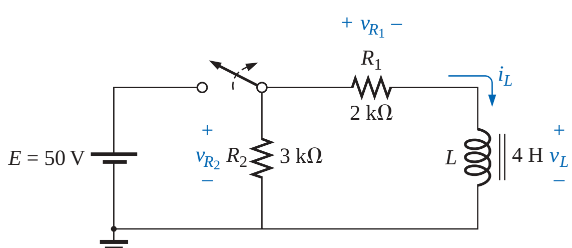

FIG. 12.29
Defined polarities for v_{R_1}, v_{R_2}, v_L, and current direction for i_L for Example 12.6.

Solutions:

a. $\tau = \dfrac{L}{R} = \dfrac{4\text{ H}}{2\text{ k}\Omega} = 2\text{ ms}$

Eq. (12.10): $v_L = Ee^{-t/\tau}$

$$v_L = \mathbf{50e^{-t/2\times 10^{-3}}}$$

Eq. (12.8): $i_L = I_m(1 - e^{-t/\tau})$

$$I_m = \frac{E}{R_1} = \frac{50\text{ V}}{2\text{ k}\Omega} = 25\text{ mA}$$

$$i_L = \mathbf{25 \times 10^{-3}(1 - e^{-t/2\times10^{-3}})}$$

Eq. (12.11): $v_{R_1} = E(1 - e^{-t/\tau})$

$$v_{R_1} = \mathbf{50(1 - e^{-t/2\times10^{-3}})}$$

$$v_{R_2} = \mathbf{50\text{ V}}$$

b. $\tau' = \dfrac{L}{R_1 + R_2} = \dfrac{4\text{ H}}{2\text{ k}\Omega + 3\text{ k}\Omega} = \dfrac{4\text{ H}}{5 \times 10^3\ \Omega} = 0.8 \times 10^{-3}\text{ s}$

$= 0.8\text{ ms}$

By Eq. (12.15),

$$V_i = \left(1 + \frac{R_2}{R_1}\right)E = \left(1 + \frac{3\text{ k}\Omega}{2\text{ k}\Omega}\right)(50\text{ V}) = 125\text{ V}$$

and $v_L = -V_i e^{-t/\tau'} = \mathbf{-125e^{-t/(0.8\times10^{-3})}}$

By Eq. (12.16),

$$I_i = I_m = \frac{E}{R_1} = \frac{50\text{ V}}{2\text{ k}\Omega} = 25\text{ mA}$$

and $i_L = \mathbf{(25 \times 10^{-3})e^{-t/(0.8\times10^{-3})}}$

By Eq. (12.17),

$$v_{R_1} = Ee^{-t/\tau'} = \mathbf{50e^{-t/(0.8\times10^{-3})}}$$

By Eq. (12.18),

$$v_{R_2} = -\frac{R_2}{R_1}Ee^{-t/\tau'} = -\frac{3\text{ k}\Omega}{2\text{ k}\Omega}(50\text{ V})e^{-t/\tau'} = \mathbf{-75e^{-t/(0.8\times10^{-3})}}$$

c. See Fig. 12.30 (opposite page).

In the preceding analysis, it was assumed that steady-state conditions were established during the charging phase and $I_m = E/R_1$, with $v_L = 0$ V. However, if the switch of Fig. 12.28 is opened before i_L reaches its maximum value, the equation for the decaying current of Fig. 12.28 must change to

$$i_L = I_i e^{-t/\tau'} \qquad \textbf{(12.19)}$$

where I_i is the starting or initial current. Equation (12.15) would be modified as follows:

$$v_L = -V_i e^{-t/\tau'} \qquad \textbf{(12.20)}$$

with $V_i = I_i(R_1 + R_2)$

12.10 INSTANTANEOUS VALUES

The development presented in Section 10.10 for capacitive networks can also be applied to *R-L* networks to determine instantaneous voltages, currents, and time. The instantaneous values of any voltage or current can be determined by simply inserting *t* into the equation and using

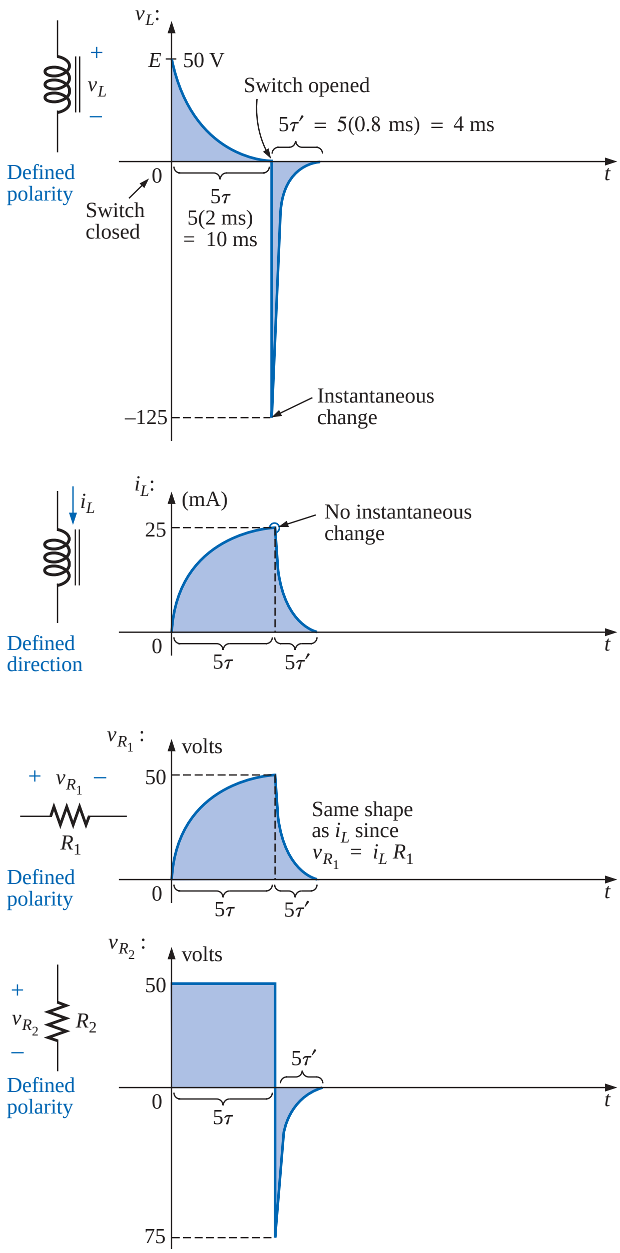

FIG. 12.30

The various voltages and the current for the network of Fig. 12.29.

a calculator or table to determine the magnitude of the exponential term.

The similarity between the equations $v_C = E(1 - e^{-t/\tau})$ and $i_L = I_m(1 - e^{-t/\tau})$ results in a derivation of the following for t that is identical to that used to obtain Eq. (10.24):

$$t = \tau \log_e\left(\frac{I_m}{I_m - i_L}\right) \qquad \textbf{(12.21)}$$

For the other form, the equation $v_C = Ee^{-t/\tau}$ is a close match with $v_L = Ee^{-t/\tau}$, permitting a derivation similar to that employed for Eq. (10.25):

$$t = \tau \log_e \frac{E}{v_L} \tag{12.22}$$

The similarities between the above and the equations in Chapter 10 should make the equation for t fairly easy to obtain.

12.11 THÉVENIN EQUIVALENT: $\tau = L/R_{Th}$

In Chapter 10 ("Capacitors"), we found that there are occasions when the circuit does not have the basic form of Fig. 12.14. The same is true for inductive networks. Again, it is necessary to find the Thévenin equivalent circuit before proceeding in the manner described in this chapter. Consider the following example.

EXAMPLE 12.7 For the network of Fig. 12.31:

a. Find the mathematical expression for the transient behavior of the current i_L and the voltage v_L after the closing of the switch (I_i = 0 mA).
b. Draw the resultant waveform for each.

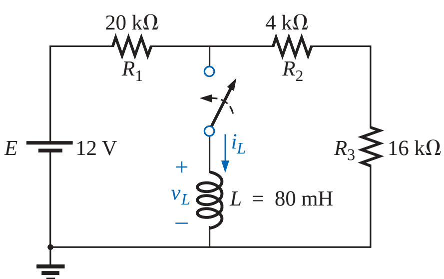

FIG. 12.31
Example 12.7.

Solutions:

a. Applying Thévenin's theorem to the 80-mH inductor (Fig. 12.32) yields

$$R_{Th} = \frac{R}{N} = \frac{20\text{ k}\Omega}{2} = 10\text{ k}\Omega$$

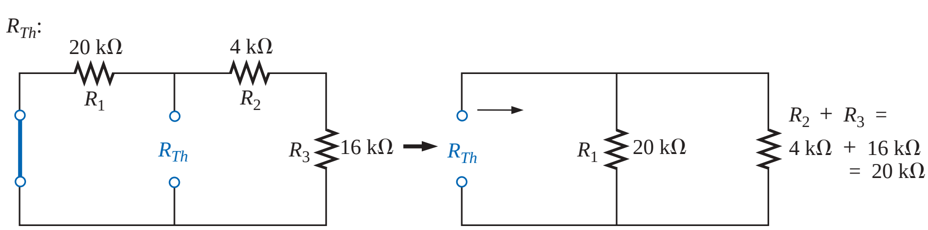

FIG. 12.32
Determining R_{Th} for the network of Fig. 12.31.

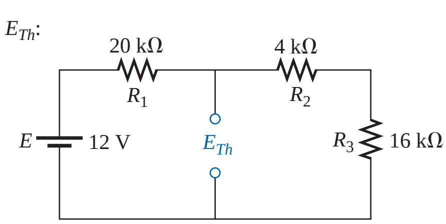

FIG. 12.33
Determining E_{Th} for the network of Fig. 12.31.

Applying the voltage divider rule (Fig. 12.33),

$$E_{Th} = \frac{(R_2 + R_3)E}{R_1 + R_2 + R_3}$$

$$= \frac{(4\text{ k}\Omega + 16\text{ k}\Omega)(12\text{ V})}{20\text{ k}\Omega + 4\text{ k}\Omega + 16\text{ k}\Omega} = \frac{(20\text{ k}\Omega)(12\text{ V})}{40\text{ k}\Omega} = 6\text{ V}$$

The Thévenin equivalent circuit is shown in Fig. 12.34. Using Eq. (12.8),

$$i_L = \frac{E_{Th}}{R}(1 - e^{-t/\tau})$$

$$\tau = \frac{L}{R_{Th}} = \frac{80 \times 10^{-3}\ \text{H}}{10 \times 10^{3}\ \Omega} = 8 \times 10^{-6}\ \text{s}$$

$$I_m = \frac{E_{Th}}{R_{Th}} = \frac{6\ \text{V}}{10 \times 10^{3}\ \Omega} = 0.6 \times 10^{-3}\ \text{A}$$

and $$\boldsymbol{i_L = (0.6 \times 10^{-3})(1 - e^{-t/(8\times10^{-6})})}$$

Using Eq. (12.10),

$$v_L = E_{Th}e^{-t/\tau}$$

so that $$\boldsymbol{v_L = 6e^{-t/(8\times10^{-6})}}$$

b. See Fig. 12.35.

Thévenin equivalent circuit:

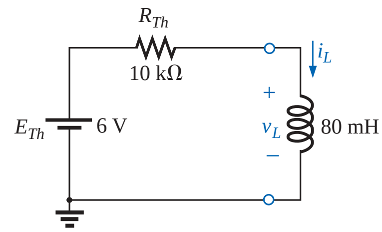

FIG. 12.34
The resulting Thévenin equivalent circuit for the network of Fig. 12.31.

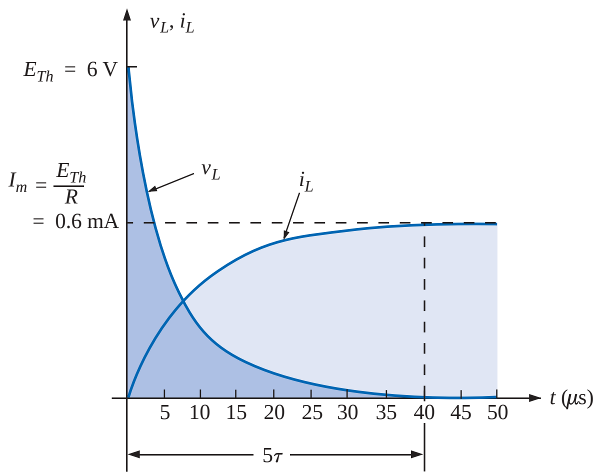

FIG. 12.35
The resulting waveforms for i_L and v_L for the network of Fig. 12.31.

EXAMPLE 12.8 The switch S_1 of Fig. 12.36 has been closed for a long time. At $t = 0$ s, S_1 is opened at the same instant S_2 is closed to avoid an interruption in current through the coil.

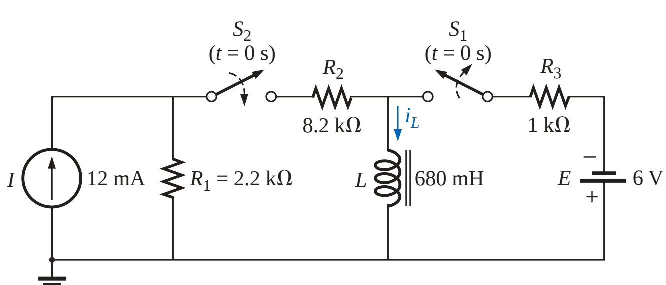

FIG. 12.36
Example 12.8.

a. Find the initial current through the coil. Pay particular attention to its direction.
b. Find the mathematical expression for the current i_L following the closing of the switch S_2.
c. Sketch the waveform for i_L.

Solutions:

a. Using Ohm's law, the initial current through the coil is determined by

$$I_i = -\frac{E}{R_3} = -\frac{6\text{ V}}{1\text{ k}\Omega} = -6\text{ mA}$$

b. Applying Thévenin's theorem:

$$R_{Th} = R_1 + R_2 = 2.2\text{ k}\Omega + 8.2\text{ k}\Omega = 10.4\text{ k}\Omega$$
$$E_{Th} = IR_1 = (12\text{ mA})(2.2\text{ k}\Omega) = 26.4\text{ V}$$

The Thévenin equivalent network appears in Fig. 12.37.

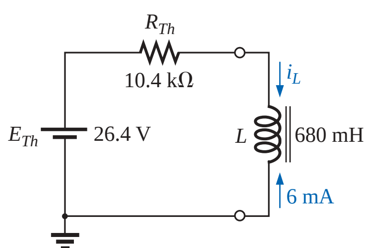

FIG. 12.37
Thévenin equivalent circuit for the network of Fig. 12.36 for $t \geq 0$ s.

The steady-state current can then be determined by substituting the short-circuit equivalent for the inductor:

$$I_f = \frac{E}{R_{Th}} = \frac{26.4\text{ V}}{10.4\text{ k}\Omega} = 2.54\text{ mA}$$

The time constant:

$$\tau = \frac{L}{R_{Th}} = \frac{680\text{ mH}}{10.4\text{ k}\Omega} = 65.39\ \mu\text{s}$$

Applying Eq. (12.12):

$$\begin{aligned} i_L &= I_f + (I_i - I_f)e^{-t/\tau} \\ &= 2.54\text{ mA} + (-6\text{ mA} - 2.54\text{ mA})e^{-t/65.39\,\mu\text{s}} \\ &= \mathbf{2.54\ mA - 8.54\ mA}\boldsymbol{e^{-t/(65.39\,\mu s)}} \end{aligned}$$

c. Note Fig. 12.38.

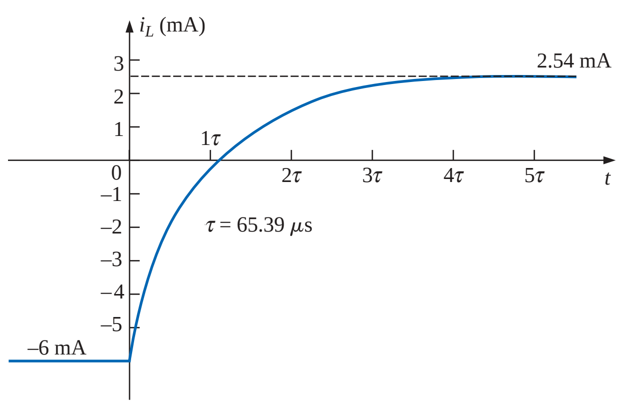

FIG. 12.38
The current i_L for the network of Fig. 12.37.

12.12 INDUCTORS IN SERIES AND PARALLEL

Inductors, like resistors and capacitors, can be placed in series or parallel. Increasing levels of inductance can be obtained by placing inductors in series, while decreasing levels can be obtained by placing inductors in parallel.

For inductors in series, the total inductance is found in the same manner as the total resistance of resistors in series (Fig. 12.39):

$$L_T = L_1 + L_2 + L_3 + \cdots + L_N \tag{12.23}$$

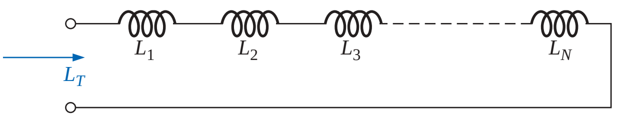

FIG. 12.39
Inductors in series.

For inductors in parallel, the total inductance is found in the same manner as the total resistance of resistors in parallel (Fig. 12.40):

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N} \tag{12.24}$$

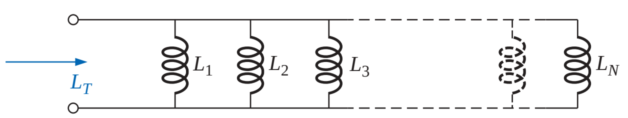

FIG. 12.40
Inductors in parallel.

For two inductors in parallel,

$$L_T = \frac{L_1 L_2}{L_1 + L_2} \tag{12.25}$$

EXAMPLE 12.9 Reduce the network of Fig. 12.41 to its simplest form.

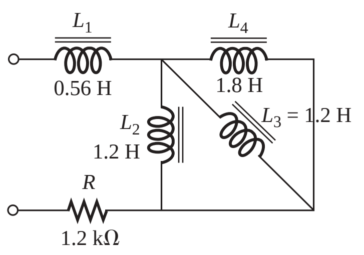

FIG. 12.41
Example 12.9.

Solution: The inductors L_2 and L_3 are equal in value and they are in parallel, resulting in an equivalent parallel value of

$$L'_T = \frac{L}{N} = \frac{1.2\text{ H}}{2} = 0.6\text{ H}$$

The resulting 0.6 H is then in parallel with the 1.8-H inductor, and

$$L''_T = \frac{(L'_T)(L_4)}{L'_T + L_4} = \frac{(0.6\text{ H})(1.8\text{ H})}{0.6\text{ H} + 1.8\text{ H}}$$
$$= 0.45\text{ H}$$

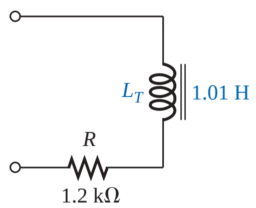

FIG. 12.42
Terminal equivalent of the network of Fig. 12.41.

The inductor L_1 is then in series with the equivalent parallel value, and

$$L_T = L_1 + L''_T = 0.56\ \text{H} + 0.45\ \text{H}$$
$$= 1.01\ \text{H}$$

The reduced equivalent network appears in Fig. 12.42.

12.13 *R-L* AND *R-L-C* CIRCUITS WITH dc INPUTS

We found in Section 12.7 that, for all practical purposes, an inductor can be replaced by a short circuit in a dc circuit after a period of time greater than five time constants has passed. If in the following circuits we assume that all of the currents and voltages have reached their final values, the current through each inductor can be found by replacing each inductor with a short circuit. For the circuit of Fig. 12.43, for example,

$$I_1 = \frac{E}{R_1} = \frac{10\ \text{V}}{2\ \Omega} = \mathbf{5\ A}$$

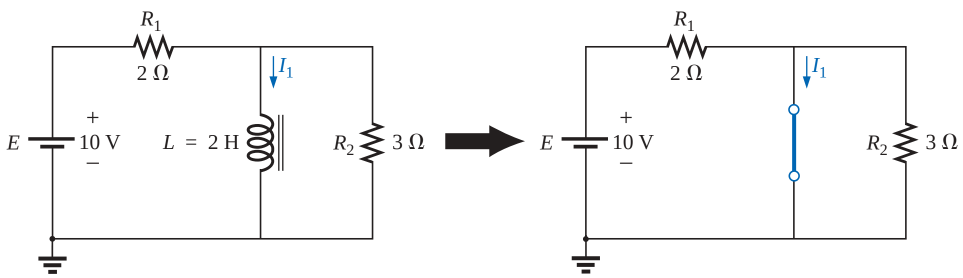

FIG. 12.43
Substituting the short-circuit equivalent for the inductor for t > 5τ.

For the circuit of Fig. 12.44,

$$I = \frac{E}{R_2 \parallel R_3} = \frac{21\ \text{V}}{2\ \Omega} = \mathbf{10.5\ A}$$

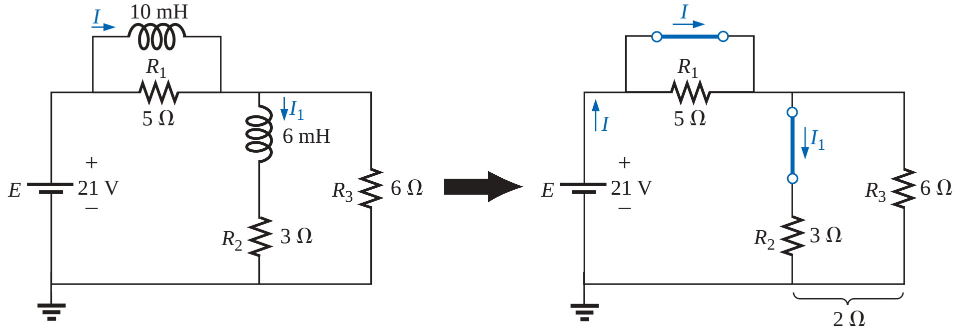

FIG. 12.44
Establishing the equivalent network for t > 5τ.

Applying the current divider rule,

$$I_1 = \frac{R_3 I}{R_3 + R_2} = \frac{(6\ \Omega)(10.5\ \text{A})}{6\ \Omega + 3\ \Omega} = \frac{63\ \text{A}}{9} = \mathbf{7\ A}$$

In the following examples we will assume that the voltage across the capacitors and the current through the inductors have reached their final values. Under these conditions, the inductors can be replaced with short circuits, and the capacitors can be replaced with open circuits.

EXAMPLE 12.10 Find the current I_L and the voltage V_C for the network of Fig. 12.45.

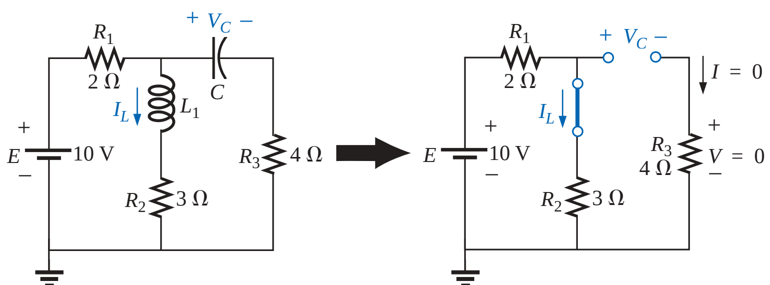

FIG. 12.45
Example 12.10.

Solution:

$$I_L = \frac{E}{R_1 + R_2} = \frac{10\ \text{V}}{5\ \Omega} = \mathbf{2\ A}$$

$$V_C = \frac{R_2 E}{R_2 + R_1} = \frac{(3\ \Omega)(10\ \text{V})}{3\ \Omega + 2\ \Omega} = \mathbf{6\ V}$$

EXAMPLE 12.11 Find the currents I_1 and I_2 and the voltages V_1 and V_2 for the network of Fig. 12.46.

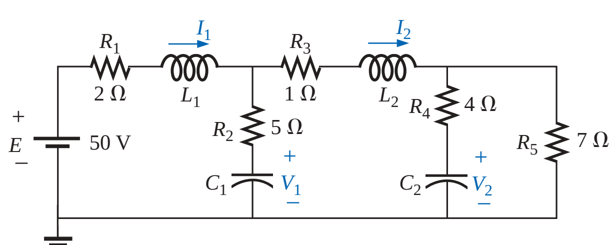

FIG. 12.46
Example 12.11.

Solution: Note Fig. 12.47:

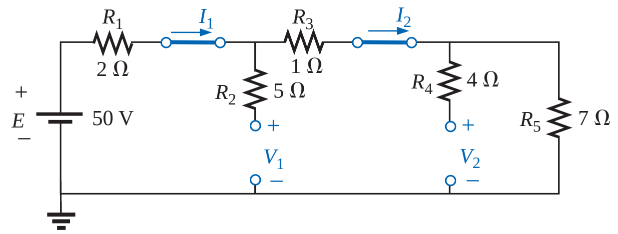

FIG. 12.47
Substituting the short-circuit equivalents for the inductors and the open-circuit equivalents for the capacitor for $t > 5\tau$.

$$I_1 = I_2$$

$$I_1 = \frac{E}{R_1 + R_3 + R_5} = \frac{50\text{ V}}{2\ \Omega + 1\ \Omega + 7\ \Omega} = \frac{50\text{ V}}{10\ \Omega} = \mathbf{5\ A}$$

$$V_2 = I_2 R_5 = (5\text{ A})(7\ \Omega) = \mathbf{35\ V}$$

Applying the voltage divider rule,

$$V_1 = \frac{(R_3 + R_5)E}{R_1 + R_3 + R_5} = \frac{(1\ \Omega + 7\ \Omega)(50\text{ V})}{2\ \Omega + 1\ \Omega + 7\ \Omega} = \frac{(8\ \Omega)(50\text{ V})}{10\ \Omega} = \mathbf{40\ V}$$

12.14 ENERGY STORED BY AN INDUCTOR

The ideal inductor, like the ideal capacitor, does not dissipate the electrical energy supplied to it. It stores the energy in the form of a magnetic field. A plot of the voltage, current, and power to an inductor is shown in Fig. 12.48 during the buildup of the magnetic field surrounding the inductor. The energy stored is represented by the shaded area under the power curve. Using calculus, we can show that the evaluation of the area under the curve yields

$$W_{\text{stored}} = \frac{1}{2}LI_m^2 \qquad \text{(joules, J)} \qquad \textbf{(12.26)}$$

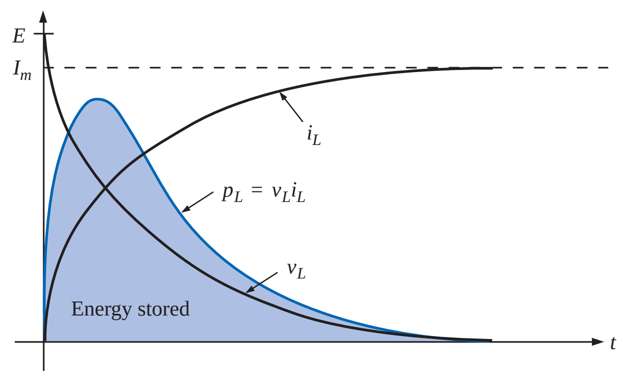

FIG. 12.48
The power curve for an inductive element under transient conditions.

EXAMPLE 12.12 Find the energy stored by the inductor in the circuit of Fig. 12.49 when the current through it has reached its final value.

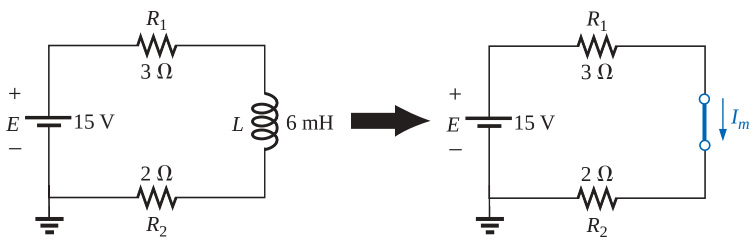

FIG. 12.49
Example 12.12.

Solution:

$$I_m = \frac{E}{R_1 + R_2} = \frac{15\text{ V}}{3\ \Omega + 2\ \Omega} = \frac{15\text{ V}}{5\ \Omega} = 3\text{ A}$$

$$W_{\text{stored}} = \frac{1}{2}LI_m^2 = \frac{1}{2}(6 \times 10^{-3}\text{ H})(3\text{ A})^2 = \frac{54}{2} \times 10^{-3}\text{ J}$$

$$= \mathbf{27\ mJ}$$

12.15 APPLICATIONS

Camera Flash Lamp and Line Conditioner

The inductor (or coil as some prefer to call it) played important roles in both the camera flash lamp circuitry and the line conditioner (surge protector) described in the Applications section of Chapter 10 on capacitors. For the camera it was the important component that resulted in the high spike voltage across the trigger coil which was then magnified by the autotransformer action of the secondary to generate the 4000 V necessary to ignite the flash lamp. Recall that the capacitor in parallel with the trigger coil charged up to 300 V using the low-resistance path provided by the SCR. However, once the capacitor was fully charged, the short-circuit path to ground provided by the SCR was removed, and the capacitor immediately started to discharge through the trigger coil. Since the only resistance in the time constant for the inductive network is the relatively low resistance of the coil itself, the current through the coil grew at a very rapid rate. A significant voltage was then developed across the coil as defined by Eq. (12.6): $v_L = L(di/dt)$. This voltage was in turn increased by transformer action to the secondary coil of the autotransformer, and the flash lamp was ignited. That high voltage generated across the trigger coil will also appear directly across the capacitor of the trigger network. The result is that it will begin to charge up again until the generated voltage across the coil drops to zero volts. However, when it does drop, the capacitor will again discharge through the coil, establish another charging current through the coil, and again develop a voltage across the coil. The high-frequency exchange of energy between the coil and capacitor is called *flyback* because of the "flying back" of energy from one storage element to the other. It will begin to decay with time because of the resistive elements in the loop.

The more resistance, the more quickly it will die out. If the capacitor-inductor pairing were isolated and "tickled" along the way with the application of a dc voltage, the high frequency-generated voltage across the coil could be maintained and put to good use. In fact, it is this flyback effect that is used to generate a steady dc voltage (using rectification to convert the oscillating waveform to one of a steady dc nature) that is commonly used in TVs.

In the line conditioner, the primary purpose of the inductors is to "choke" out spikes of current that may come down the line using the effect described under the discussion of Lenz's law in this chapter. Inductors are such that a rapidly changing current through a coil will result in the development of a current in the coil that will oppose the current that established the induced effect in the first place. This effect is so strong that it can squelch current spikes of a significant number of amperes in the line. An undesirable result in line conditioners, however, is the voltage across the coil that will develop when it "chokes" this rapidly changing current through the coil. However, as mentioned in Chapter 10, there are two coils in the system that will generate opposing emf's so that the net voltage to ground is zero. This is fairly clear when you carefully examine the two coils on the ferromagnetic core and note that they are wound in a way to develop opposing fields. The reaction of the coils in the line conditioner to different frequencies and their ability to help out with the blocking of EMI and RFI disturbances will have to wait until we discuss the effect of frequency on the reaction of an inductor in a later chapter.

Household Dimmer Switch

Inductors can be found in a wide variety of common electronic circuits in the home. The typical household dimmer uses an inductor to protect the other components and the applied load from "rush" currents—currents that increase at very high rates and often to excessively high levels. This feature is particularly important for dimmers since they are most commonly used to control the light intensity of an incandescent lamp. At "turn on," the resistance of incandescent lamps is typically very low, and relatively high currents may flow for short periods of time until the filament of the bulb heats up. The inductor is also effective in blocking high-frequency noise (RFI) generated by the switching action of the triac in the dimmer. A capacitor is also normally included from line to neutral to prevent any voltage spikes from affecting the operation of the dimmer and the applied load (lamp, etc.) and to assist with the suppression of RFI disturbances.

A photograph of one of the most common dimmers is provided in Fig. 12.50(a), with an internal view shown in Fig. 12.50(b). The basic components of most commercially available dimmers appear in the schematic of Fig. 12.50(c). In this design, a 14.5-μH inductor is used in the "choking" capacity described above, with a 0.068-μF capacitor for the "bypass" operation. Note the size of the inductor with its heavy wire and large ferromagnetic core and the relatively large size of the two 0.068-μF capacitors. Both suggest that they are designed to absorb high-energy disturbances.

The general operation of the dimmer is shown in Fig. 12.51. The controlling network is in series with the lamp and will essentially act as an impedance that can vary between very low and very high levels: very low impedance levels resembling a short circuit so that the majority of

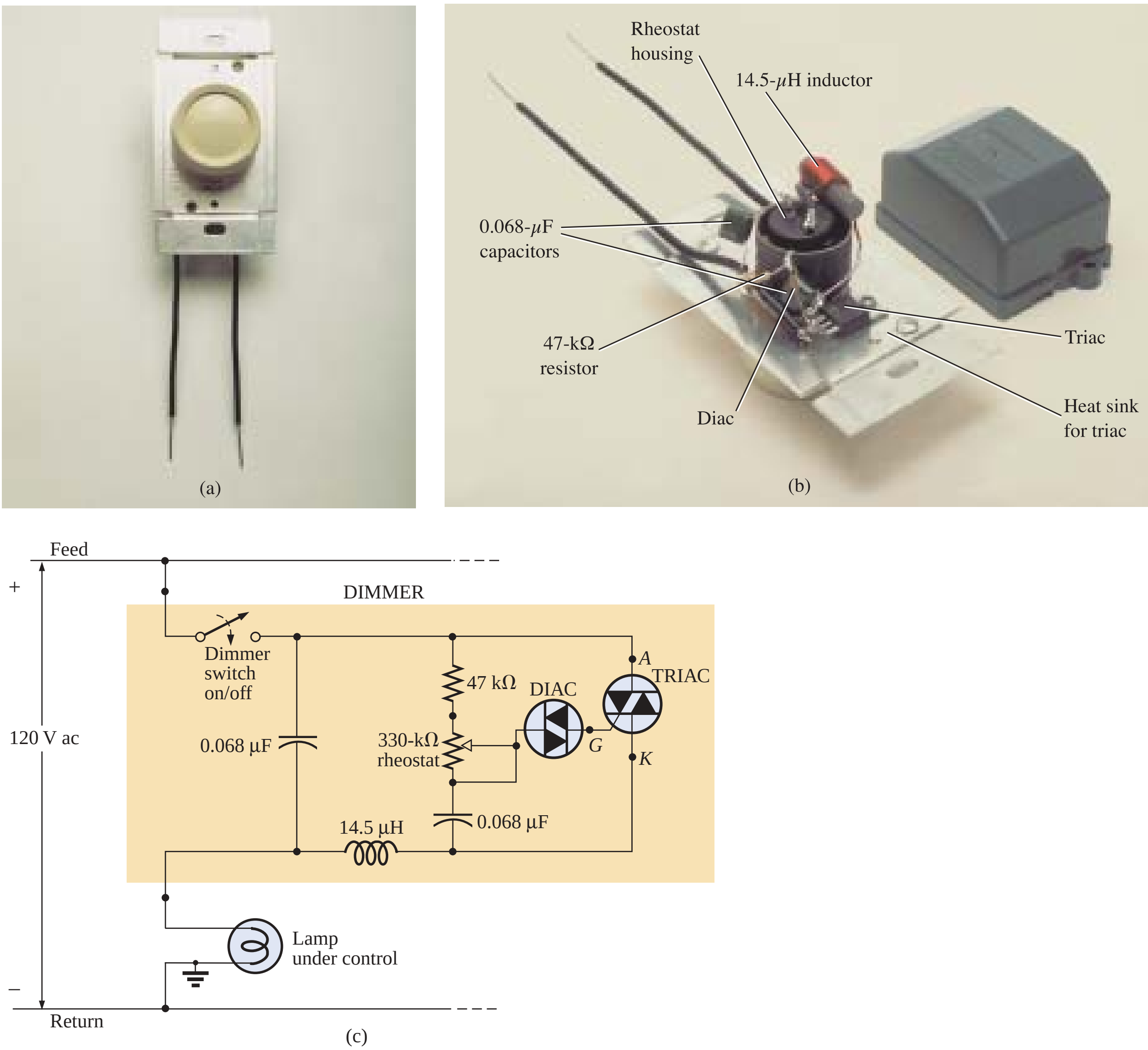

FIG. 12.50

Dimmer control: (a) external appearance; (b) internal construction; (c) schematic.

the applied voltage appears across the lamp [Fig. 12.51(a)] and very high impedances approaching open circuit where very little voltage appears across the lamp [Fig. 12.51(b)]. Intermediate levels of impedance will control the terminal voltage of the bulb accordingly. For instance, if the controlling network has a very high impedance (open-circuit equivalent) through half the cycle as shown in Fig. 12.51(c), the brightness of the bulb will be less than full voltage but not 50% due to the nonlinear relationship between the brightness of a bulb and the applied voltage. There is also a lagging effect present in the actual operation of the dimmer, but this subject will have to wait until leading and lagging networks are examined in the ac chapters.

The controlling knob, slide, or whatever other method is used on the face of the switch to control the light intensity is connected directly to the rheostat in the branch parallel to the triac. Its setting will determine

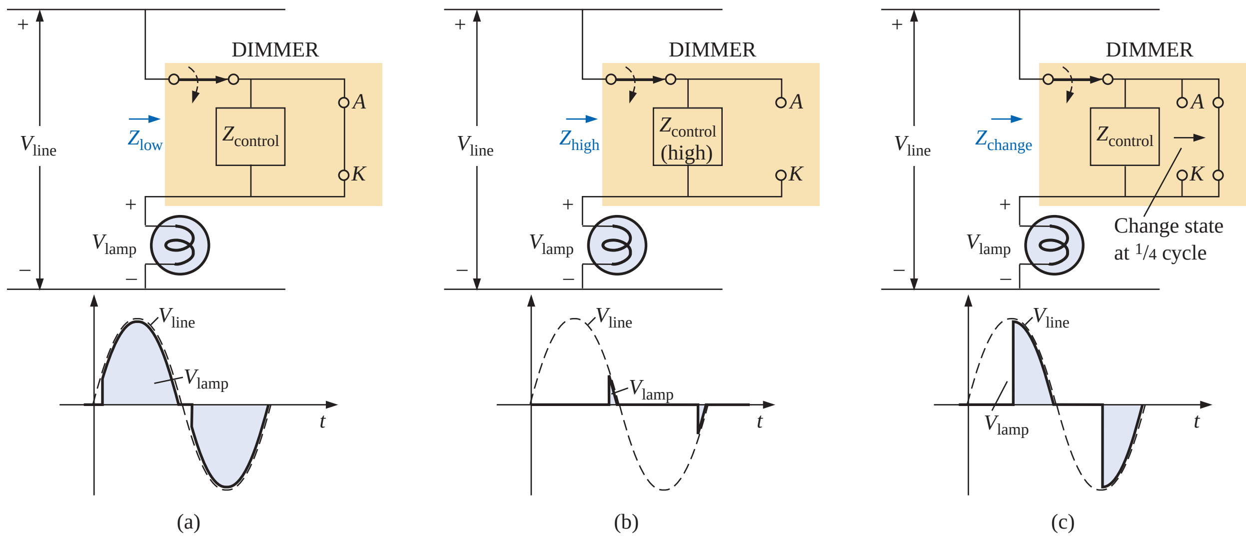

FIG. 12.51
Basic operation of the dimmer of Fig. 12.50: (a) full voltage to the lamp; (b) approaching the cutoff point for the bulb; (c) reduced illumination of the lamp.

when the voltage across the capacitor reaches a sufficiently high level to turn on the diac (a bidirectional diode) and establish a voltage at the gate (G) of the triac to turn it on. When it does, it will establish a very low resistance path from the anode (A) to the cathode (K), and the applied voltage will appear directly across the lamp. A more detailed explanation of this operation will appear in a later chapter following the examination of some important concepts for ac networks. During the period the SCR is off, its terminal resistance between anode and cathode will be very high and can be approximated by an open circuit. During this period the applied voltage will not reach the load (lamp). During such intervals the impedance of the parallel branch containing the rheostat, fixed resistor, and capacitor is sufficiently high compared to the load that it can also be ignored, completing the open-circuit equivalent in series with the load. Note the placement of the elements in the photograph of Fig. 12.50 and the fact that the metal plate to which the triac is connected is actually a heat sink for the device. The on/off switch is in the same housing as the rheostat. The total design is certainly well planned to maintain a relatively small size for the dimmer.

Since the effort here is simply to control the amount of power getting to the load, the question is often asked, Why don't we simply use a rheostat in series with the lamp? The question is best answered by examining Fig. 12.52, which shows a rather simple network with a rheostat in series with the lamp. At full wattage, a 60-W bulb on a 120-V line theoretically has an internal resistance of $R = V^2/P$ (from the equation $P = V^2/R$) $= (120\text{ V})^2/60\text{ W} = 240\ \Omega$. Although the resistance is sensitive to the applied voltage, we will assume this level for the following calculations. If we consider the case where the rheostat is set for the same level as the bulb, as shown in Fig. 12.52, there will be 60 V across the rheostat and the bulb. The power to each element will then be $P = V^2/R = (60\text{ V})^2/240\ \Omega = 15$ W. The bulb is certainly quite dim, but the rheostat inside the dimmer switch would be dissipating 15 W of power on a continuous basis. When you consider the size of a 2-W potentiometer in your laboratory, you can imagine the size rheostat you

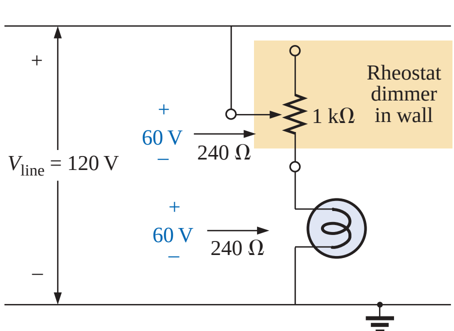

FIG. 12.52
Direct rheostat control of the brightness of a 60-W bulb.

would need for 15 W, not to mention the purchase cost, although the biggest concern would probably be all the heat developed in the walls of the house. You would certainly be paying for electric power that would not be performing a useful function. Also, if you had four dimmers set at the same level, you would actually be wasting sufficient power to fully light another 60-W bulb.

On occasion, especially when the lights are set very low by the dimmer, a faint “singing” can sometimes be heard from the light bulb. This effect will sometimes occur when the conduction period of the dimmer is very small. The short, repetitive voltage pulse applied to the bulb will set the bulb into a condition that could be likened to a resonance (Chapter 20) state. The short pulses are just enough to heat up the filament and its supporting structures, and then the pulses are removed to allow the filament to cool down again for a longer period of time. This repetitive heating and cooling cycle can set the filament in motion, and the “singing” can be heard in a quiet environment. Incidentally, the longer the filament, the louder the “singing.” A further condition for this effect is that the filament be in the shape of a coil and not a straight wire so that the “slinky” effect can develop.

TV or PC Monitor Yolk

Inductors and capacitors play a multitude of roles in the operation of a TV or PC monitor. However, the most obvious use of the coil is in the yolk assembly wrapped around the neck of the tube as shown in Fig. 12.53. As shown in the figure, the tube itself, in addition to providing the screen for viewing, is actually a large capacitor which plays an integral part in establishing the high dc voltage for the proper operation of the monitor.

A photograph of the yolk assembly of a black-and-white TV tube is provided in Fig. 12.54(a). It is constructed of four 28-mH coils with two sets of two coils connected at one point [Fig. 12.54(b)] so that they will share the same current and will establish the same magnetic field. The purpose of the yolk assembly is to control the direction of the electron beam from the cathode to the screen of the tube. When the cathode

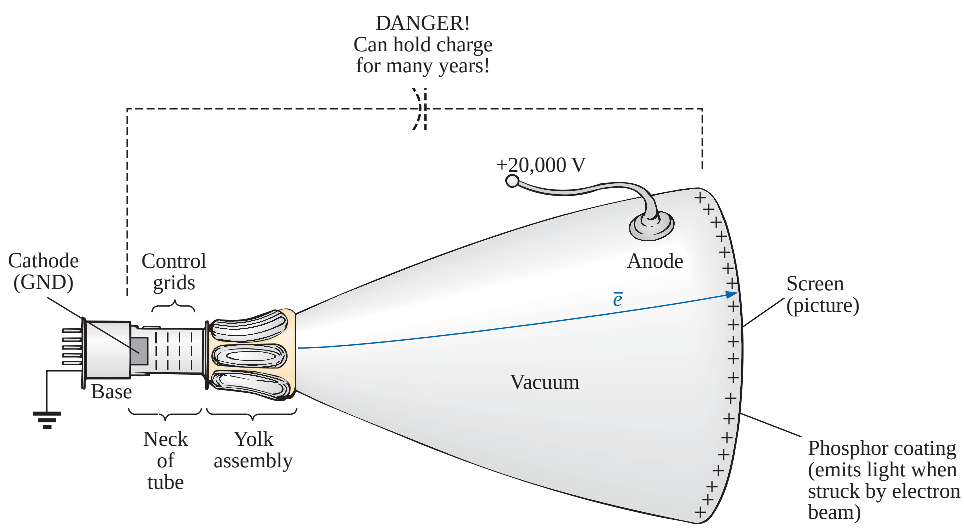

FIG. 12.53
Yolk assembly for a TV or PC tube.

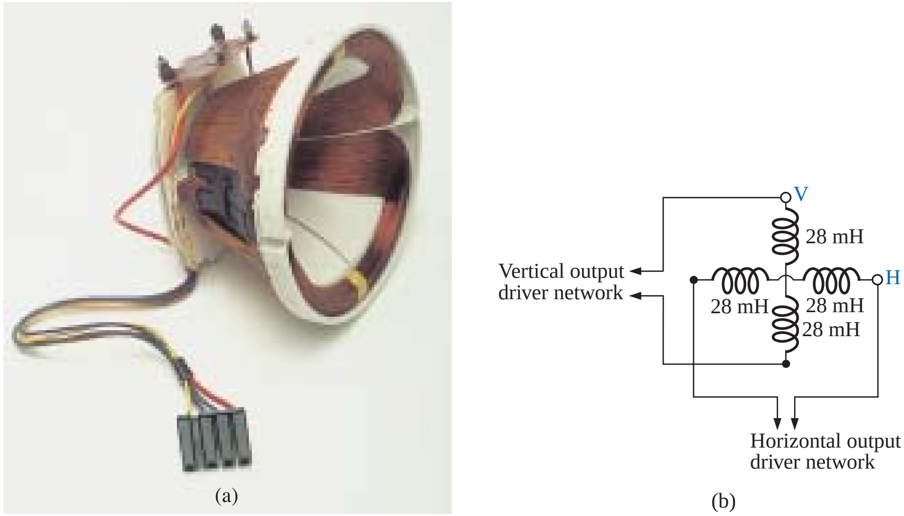

FIG. 12.54
(a) Black-and-white TV yolk assembly; (b) schematic representation.

is heated to a very high temperature by a filament internal to the structure, electrons are emitted into the surrounding media. The placement of a very high positive potential (10 kV to 25 kV dc) to the conductive coating on the face of the tube will attract the emitted electrons at a very high speed and therefore at a high level of kinetic energy. When the electrons hit the phosphorescent coating (usually white, green, or amber) on the screen, light will be emitted which can be seen by someone facing the monitor. The beam characteristics (such as intensity, focus, and shape) are controlled by a series of grids placed relatively close to the cathode in the neck of the tube. The grid is such that the negatively charged electrons can easily pass through, but the number and speed with which they pass can be controlled by a negative potential applied to the grid. The grids cannot have a positive potential because the negatively charged electrons would be attracted to the grid structure and would eventually disintegrate from the high rate of conduction. Negative potentials on the grids control the flow of electrons by repulsion and by masking the attraction for the large positive potential applied to the face of the tube.

Once the beam has been established with the desired intensity and shape, it must be directed to a particular location on the screen using the yolk assembly. For vertical control, the two coils on the side establish a magnetic flux pattern as shown in Fig. 12.55(a). The resulting direction of the magnetic field is from left to right as shown in Figs. 12.55(a) and 12.55(b). Using the right hand, with the index finger pointing in the direction of the magnetic field and the middle finger (at right angles to the index finger) in the direction of electron flow, will result in the thumb (also at right angles to the index finger) pointing in the direction of the force on the electron beam. The result is a bending of the beam as shown in Fig. 12.53. The stronger the magnetic field of the coils as determined by the current through the coils, the greater the deflection of the beam.

Before continuing, it is important to realize that when the electron beam hits the phosphorescent screen as shown in Fig. 12.56, it is moving with sufficient velocity to cause a secondary emission of X rays that

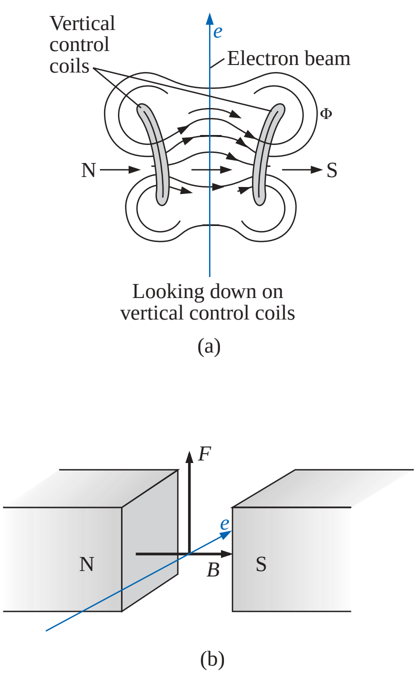

FIG. 12.55
Deflection coils: (a) vertical control; (b) right-hand-rule (RHR) for electron flow.

will scatter to all sides of the monitor. Even though the X rays will die off exponentially with distance from the source, there is some concern about safety, and all modern-day monitors have shields all around the outside surface of the tube as shown in Fig. 12.56. It is therefore interesting that it is not the direct viewing that is of some concern but rather viewing by individuals to the side, above, or below the screen. Monitors are currently limited to 25 kV at the anode because the application of voltages in excess of 25 kV can result in a direct emission of X rays. Internally, all monitors currently have a safety shutoff to ensure that this level is never attained in the operating system.

Time and space do not permit a detailed discussion of the full operation of a monitor, but there are some facts about its operation that reveal the sophistication of the design. When an image is generated on a screen, it is done one *pixel* at a time along one horizontal line at a time. A pixel is one point on the screen. Pixels are black (no signal) or white (with signal) for black-and-white (monochromic) TVs or black and white or some color for color TVs. For EGA monitors the resolution is 640 pixels wide and 35 pixels high, whereas VGA monitors are also 640 pixels wide but 480 pixels high. Obviously the more pixels in the same area, the sharper the image. A typical scan rate is 31.5 kHz which means that 31,500 lines can be drawn in 1 s, or one line of 640 pixels can be drawn in about 31.7 μs.

Patterns on the screen are developed by the sequence of lines appearing in Fig. 12.57. Starting at the top left, the image moves across the screen down to the next line until it ends at the bottom right of the screen, at which point there is a rapid retrace (invisible) back to the starting point. Typical scanning rates (full image generated) extend from 60 frames per second to 80 frames per second. The slower the rate, the higher the possibility of flickering in the images. At 60 frames per second, one entire frame is generated every 1/60 = 16.67 ms = 0.017 s.

Color monitors are particularly interesting because all colors on the screen are generated by the colors red, blue, and green. The reason is that the human eye is responding to the wavelengths and energy levels of the various colors. The absence of any color is black, and the result of full energy to each of the three colors is white. The color yellow is a combination of red and green with no blue, and pink is primarily red energy with smaller amounts of blue and green. An in-depth description of this "additive" type of color generation must be left as an exercise for the reader.

The fact that three colors define the resulting color requires that there be three cathodes in a color monitor to generate three electron beams. However, the three beams must sweep the screen in the same relative positions. Each pixel is now made up of three color dots in the same relative position for each pixel, as shown in Fig. 12.58. Each dot has a phosphorescent material that will generate the desired color when hit with an electron beam. For situations where the desired color has no green, the electron beam associated with the color green will be turned off. In fact, between each pixel, each beam is shut down to provide definition between the color pixels. The dots within the pixel are so close that the human eye cannot pick up the individual colors but simply the color that would result from the "additive" process.

During the entire "on" time of a monitor, a full 10 kV to 25 kV are applied to the conductor on the screen to attract electrons. Over time there will naturally be an accumulation of negative charge on the screen which will remain after the power is turned off—a typical capacitive storage charge. For a brief period of time, it will sit with 25 kV across

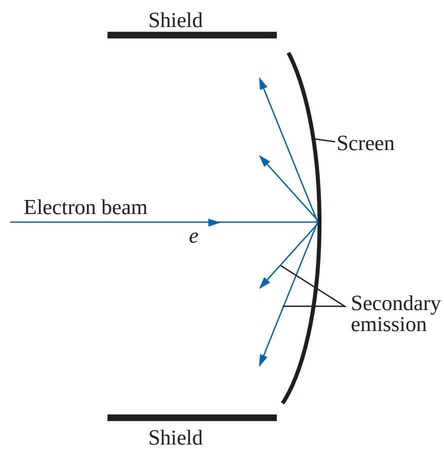

FIG. 12.56
Secondary emission from and protective measures for a TV or PC monitor.

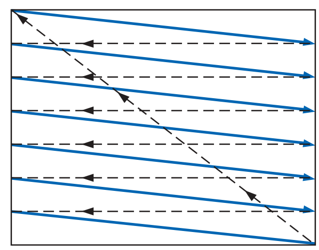

FIG. 12.57
Pattern generation.

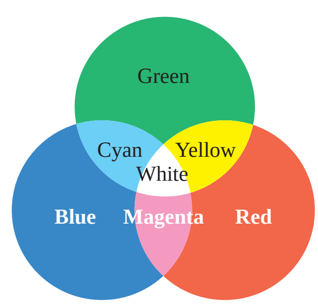

FIG. 12.58
Color television pixels.

the plates which will drop as the "capacitor" begins to discharge. However, the lack of a low-resistance path will often result in a storage of the charge for a fairly long period of time. This stored charge and the associated voltage across the plates are sufficiently high to cause severe damage. It is therefore paramount that TVs and monitors be repaired or investigated only by someone who is well versed in how to discharge the tube. One commonly applied procedure is to attach a long lead from the metal shaft of a flat-edge screwdriver to a good ground connection. Then leave the anode connection to the tube in place, and simply insert the screwdriver under the cap until it touches the metal clip of the cap. You will probably hear a loud snap when discharge occurs. Because of the enormous amount of residual charge, it is recommended that the above procedure be repeated two or three times. Even then, treat the tube with a great deal of respect. In short, until you become familiar with the discharge procedure, leave the investigation of TVs and monitors to someone with the necessary experience. A further concern is the very high pulse voltages generated in an operating system. Be aware that they are of a magnitude that could destroy standard test equipment.

The capacitive effect of the tube is an integral part of developing the high dc anode potential. Its filtering action smooths out the repetitive, high-voltage pulses generated by the flyback action of the TV. Otherwise, the screen would simply be a flickering pattern as the anode potential switched on and off with the pulsating signal.

12.16 COMPUTER ANALYSIS

PSpice

Transient *RL* Response The computer analysis will begin with a transient analysis of the network of parallel inductive elements in Fig. 12.59. The inductors are picked up from the **ANALOG** library in the **Place Part** dialog box. As noted in Fig. 12.59, the inductor is displayed with its terminal identification which is helpful for identifying nodes when calling for specific output plots and values. In general, when an element is first placed on a schematic, the number 1 is assigned to the left end on a horizontal display and to the top on a vertical display. Similarly, the number 2 is assigned to the right end of an element in the horizontal position and to the bottom in the vertical position. Be aware, however, that the option **Rotate** rotates the element in the CCW direction, so taking a horizontal resistor to the vertical position requires three rotations to get the number 1 to the top again. In previous chapters you may have noted that a number of the outputs were taken off terminal 2 because a single rotation placed this terminal at the top of the vertical display. Also note in Fig. 12.59 the need for a series resistor R_l within the parallel loop of inductors. In PSpice, inductors must have a series resistor to reflect real-world conditions. The chosen value of 1 mΩ is so small, however, that it will not affect the response of the system. For **VPulse,** the rise time was selected as 0.01 ms, and the pulse width was chosen as 10 ms because the time constant of the network is $\tau = L_T/R = (4\text{ H} \parallel 12\text{ H})/2\text{ k}\Omega = 1.5\text{ ms}$ and $5\tau = 7.5\text{ ms}$.

The simulation is the same as applied when obtaining the transient response of capacitive networks. In condensed form, the sequence to obtain a plot of the voltage across the coils versus time is as follows: **New SimulationProfile**key-**TransientRL-Create-TimeDomain(Transient)-Run to time:**10ms-**Start saving data after:**0s and **Maximum step size:**5μs-**OK-Run PSpice** key-**Add Trace** key-**V1(L2)-OK.** The result-

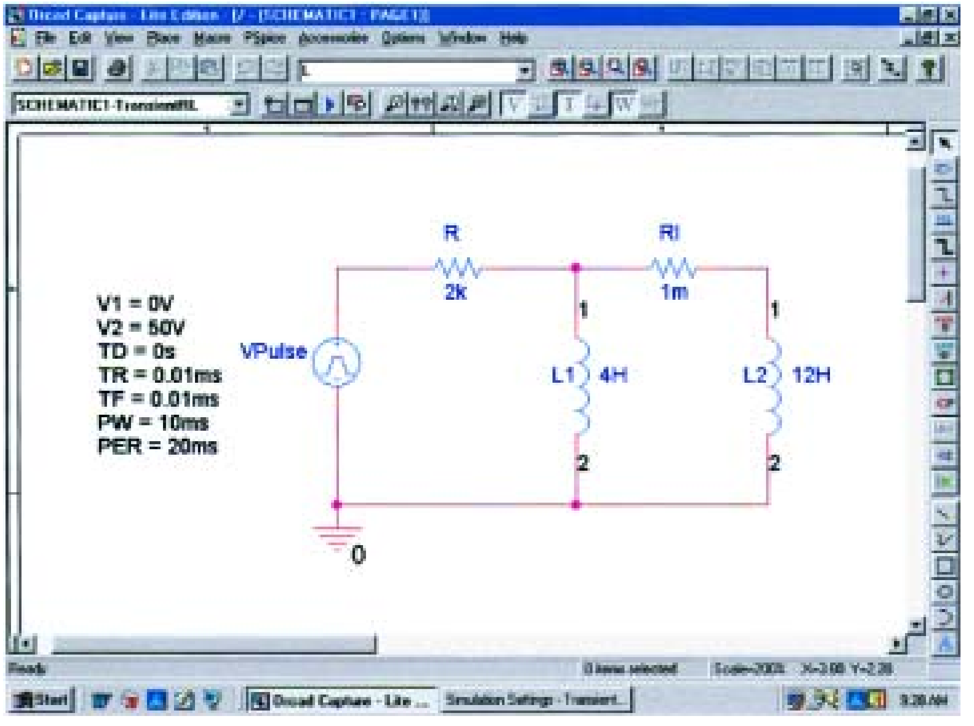

FIG. 12.59

Using PSpice to obtain the transient response of a parallel inductive network due to an applied pulse of 50 V.

ing trace appears in the bottom of Fig. 12.60. A maximum step size of 5 μs was chosen to ensure that it was less than the rise or fall times of 10 μs. Note that the voltage across the coil jumps to the 50-V level almost immediately; then it decays to 0 V in about 8 ms. A plot of the total current through the parallel coils can be obtained through **Plot-Plot to Window-Add Trace** key-**I(R)-OK,** resulting in the trace appearing at the top of Fig. 12.60. When the trace first appeared, the vertical scale extended from 0 A to 40 mA even though the maximum value of i_R was 25 mA. To bring the maximum value to the top of the graph, **Plot** was selected followed by **Axis Settings-Y Axis-User Defined-0A to 25mA-OK.**

For values, the voltage plot was selected, **SEL>>,** followed by the **Toggle cursor** key and a click on the screen to establish the crosshairs. The left-click cursor was set on one time constant to reveal a value of 18.461 V for **A1** (about 36.8% of the maximum as defined by the exponential waveform). The right-click cursor was set at 7.5 ms or five time constants, resulting in a relatively low 0.338 V for **A2.**

Transient Response with Initial Conditions The next application will verify the results of Example 12.5 which has an initial condition associated with the inductive element. **VPULSE** is again employed with the parameters appearing in Fig. 12.61. Since $\tau = L/R = 100$ mH/(2.2 kΩ + 6.8 kΩ) = 100 mH/9 kΩ = 11.11 μs and $5\tau = 55.55$ μs, the pulse width (**PW**) was set to 100 μs. The rise and fall times were set at 100 μs/1000 = 0.1 μs. Note again that the labels 1 and 2 appear with the inductive element.

Setting the initial conditions for the inductor requires a procedure that has not been described as yet. First double-click on the inductor symbol to obtain the **Property Editor** dialog box. Then select **Parts** at

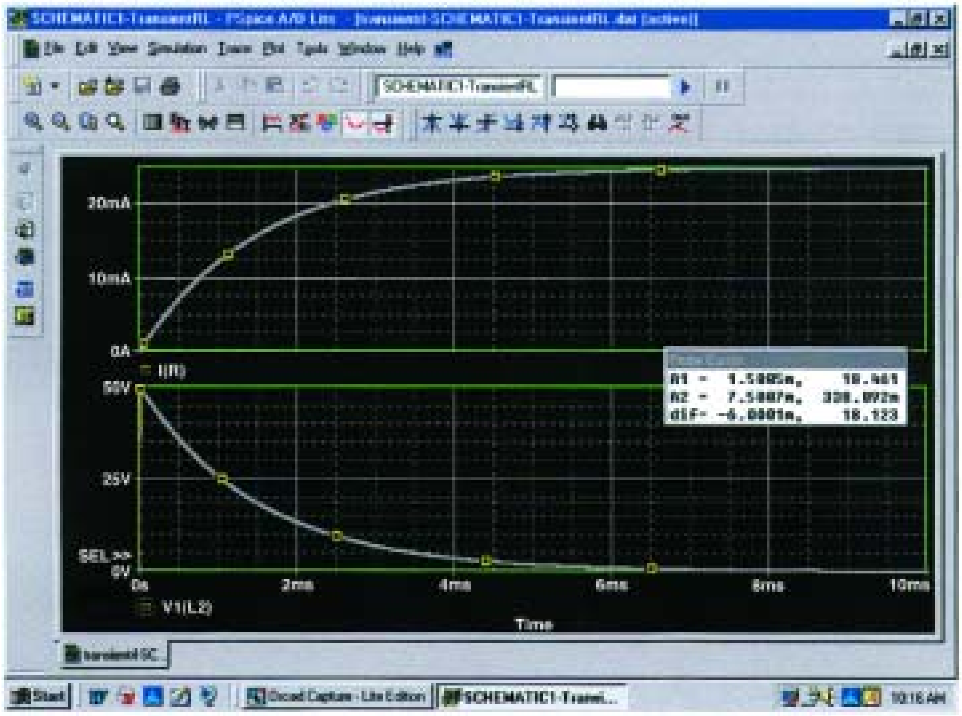

FIG. 12.60
The transient response of v_L and i_R for the network of Fig. 12.59.

the bottom of the dialog box, and select **New Column** to obtain the **Add New Column** dialog box. Under **Name,** enter **IC** (an abbreviation for "initial condition"—not "capacitive current") followed by the initial condition of 4 mA under **Value;** then click **OK.** The **Property Editor** dialog box will appear again, but now the initial condition appears as a **New Column** in the horizontal listing dedicated to the inductive element. Now select **Display** to obtain the **Display Properties** dialog box, and under **Display Format** choose **Name and Value** so that both **IC** and **4mA** will appear. Click **OK,** and we return to the **Property Editor** dialog box. Finally, click on **Apply** and exit the dialog box (**X**). The result is the display of Fig. 12.61 for the inductive element.

Now for the simulation. First select the **New Simulation Profile** key, insert the name **InitialCond(L),** and follow up with **Create.** Then in the **Simulation Settings** dialog box, select **Time Domain(Transient)** for the **Analysis type** and **General Settings** for the **Options.** The **Run to time** should be 200 μs so that we can see the full effect of the pulse source on the transient response. The **Start saving data after** should remain at 0 s, and the **Maximum step size** should be 200 μs/1000 = 200 ns. Click **OK** and then select the **Run PSpice** key. The result will be a screen with an *x*-axis extending from 0 to 200 μs. Selecting **Trace** to get to the **Add Traces** dialog box and then selecting **I(L)** followed by **OK** will result in the display of Fig. 12.62. The plot for **I(L)** clearly starts at the initial value of 4 mA and then decays to 1.78 mA as defined by the left-click cursor. The right-click cursor reveals that the current has dropped to 0.222 μA (essentially 0 A) after the pulse source has dropped to 0 V for 100 μs. The **VPulse** source was placed in the same figure through **Plot-Add Plot to Window-Trace-Add Trace-V(VPulse:+)-OK** to permit a comparison between the applied voltage and the resulting inductor current.

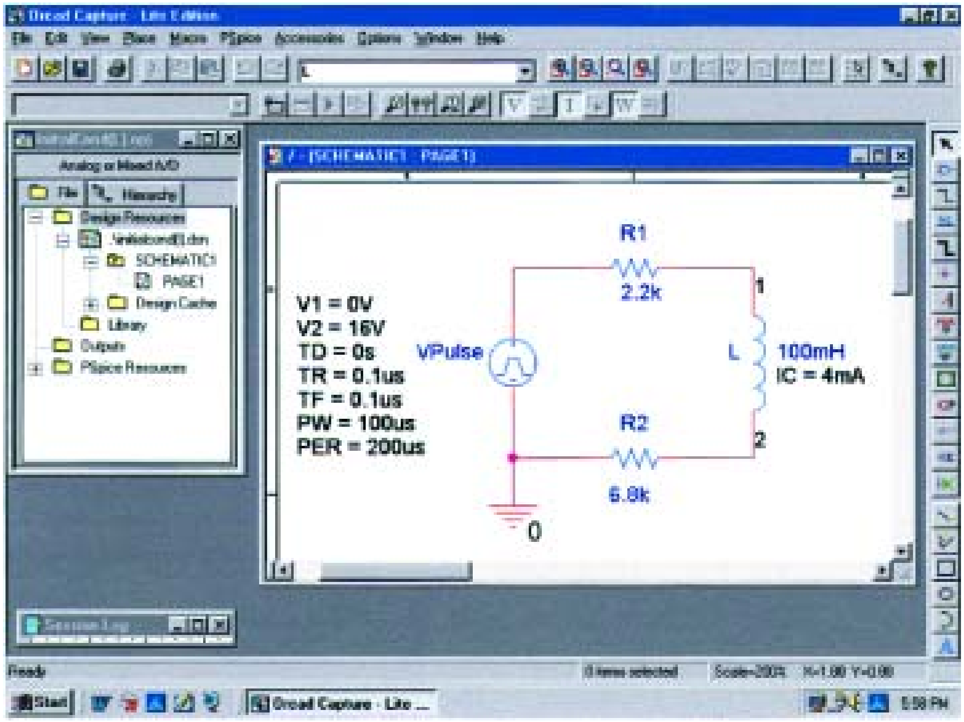

FIG. 12.61
Using PSpice to determine the transient response for a circuit in which the inductive element has an initial condition.

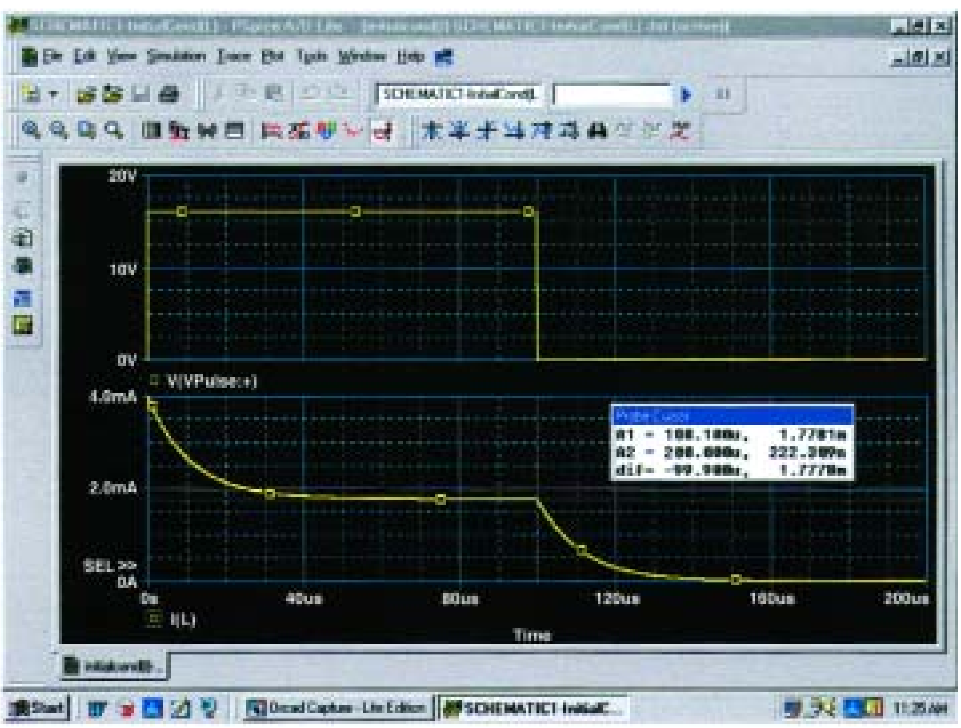

FIG. 12.62
A plot of the applied pulse and resulting current for the circuit of Fig. 12.61.

Electronics Workbench

The transient response of an *R-L* network can also be obtained using Electronics Workbench. The circuit to be examined appears in Fig. 12.63 with a pulse voltage source to simulate the closing of a switch at $t = 0$ s. The source, referred to as **PULSE_VOLTAGE_SOURCE** in the **Source** listing, is the near the bottom left of the **Sources** parts bin. When selected, it will appear with a label, an initial voltage, a step voltage, and a frequency. All can be changed by simply double-clicking on the source symbol to obtain the **Pulse Voltage** dialog box. As shown in Fig. 12.63, the **Pulsed Value** will be set at 20 V, and the **Delay Time** to 0 s. The **Rise Time** and **Fall Time** will both remain at the default levels of 1 ns. For our analysis we want a **Pulse Width** that is at least twice the 5τ transient period of the circuit. For the chosen values of R and L, $\tau = L/R = 10\text{ mH}/100\ \Omega = 0.1\text{ ms} = 100\ \mu\text{s}$. The transient period of 5τ is therefore 500 μs or 0.5 ms. Thus, a **Pulse Width** of 1 ms would seem appropriate with a **Period** of 2 ms. The result is a frequency of $f = 1/T = 1/2\text{ ms} = 500$ Hz. When all have been set and selected, the parameters of the pulse source will appear as shown in Fig. 12.63. Next the resistor, inductor, and ground are placed on the screen to complete the circuit.

This time we will want to see the node names so that we can call for them when we set up the simulation process. This is accomplished through **Options-Preferences-Show node names.** In this case we have two—one at the positive terminal of the supply **(1)** and the other at the top end of the inductor **(2)** representing the voltage across the inductor.

The simulation process is initiated by the following sequence: **Simulate-Analyses-Transient Analysis.** The result is the **Transient Analysis** dialog box in which **Analysis Parameters** is chosen first. Under

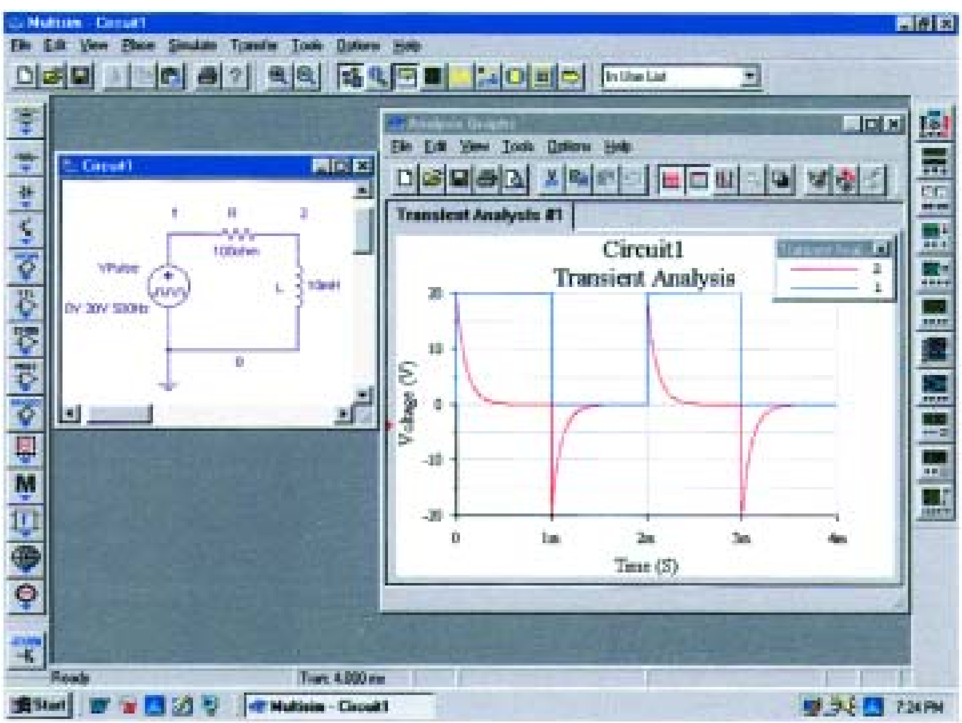

FIG. 12.63
Using Electronics Workbench to obtain the transient response for an inductive circuit.

Parameters, use 0 s as the **Start time** and 4 ms as the **End time** so that we get two full cycles of the applied voltage. After enabling the **Maximum time step settings(TMAX),** we set the **Minimum number of time points** at 1000 to get a reasonably good plot during the rapidly changing transient period. Next, the **Output variables** section must be selected and the program told which voltage and current levels we are interested in. On the left side of the dialog box is a list of **Variables** that have been defined for the circuit. On the right is a list of **Selected variables for analysis.** In between you see a **Plot during simulation** or **Remove.** To move a variable from the left to the right column, simply select it in the left column and choose **Plot during simulation.** It will then appear in the right column. For our purposes it seems appropriate that we plot both the applied voltage and the voltage across the coil, so **1** and **2** were moved to the right column. Then **Simulate** is selected, and a window titled **Analysis Graphs** will appear with the selected plots as shown in Fig. 12.63. Click on the **Show/Hide Grid** key (a red grid on a black axis), and the grid lines will appear. Then selecting the **Show/Hide Legend** key on the immediate right will result in the small **Transient Anal** dialog box that will identify the color that goes with each nodal voltage. In our case, blue is the color of the applied voltage, and red is the color of the voltage across the coil.

The source voltage appears as expected with its transition to 20 V, 50% duty cycle, and the period of 2 ms. The voltage across the coil jumped immediately to the 20-V level and then began its decay to 0 V in about 0.5 ms as predicted. When the source voltage dropped to zero, the voltage across the coil reversed polarity to maintain the same direction of current in the inductive circuit. Remember that for a coil, the voltage can change instantaneously, but the inductor will "choke" any instantaneous change in current. By reversing its polarity, the voltage across the coil ensures the same polarity of voltage across the resistor and therefore the same direction of current through the coil and circuit.

PROBLEMS

SECTION 12.2 Faraday's Law of Electromagnetic Induction

1. If the flux linking a coil of 50 turns changes at a rate of 0.085 Wb/s, what is the induced voltage across the coil?
2. Determine the rate of change of flux linking a coil if 20 V are induced across a coil of 40 turns.
3. How many turns does a coil have if 42 mV are induced across the coil by a change of flux of 0.003 Wb/s?

SECTION 12.4 Self-Inductance

4. Find the inductance L in henries of the inductor of Fig. 12.64.
5. Repeat Problem 4 with $l = 4$ in. and $d = 0.25$ in.

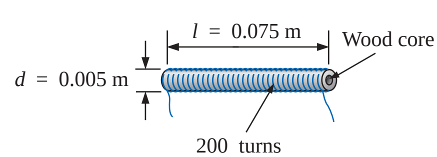

FIG. 12.64
Problems 4 and 5.

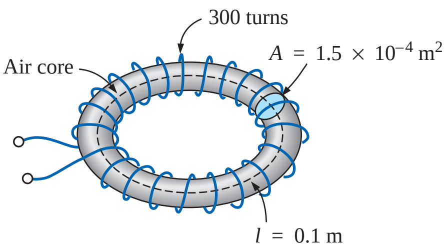

FIG. 12.65
Problem 6.

6. **a.** Find the inductance L in henries of the inductor of Fig. 12.65.
   b. Repeat part (a) if a ferromagnetic core is added having a μ_r of 2000.

SECTION 12.6 Induced Voltage

7. Find the voltage induced across a coil of 5 H if the rate of change of current through the coil is
   a. 0.5 A/s
   b. 60 mA/s
   c. 0.04 A/ms
8. Find the induced voltage across a 50-mH inductor if the current through the coil changes at a rate of 0.1 mA/μs.
9. Find the waveform for the voltage induced across a 200-mH coil if the current through the coil is as shown in Fig. 12.66.

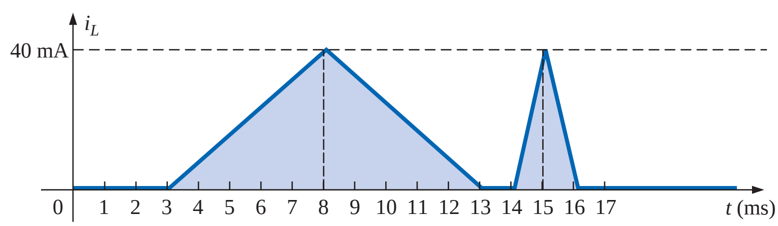

FIG. 12.66
Problem 9.

10. Sketch the waveform for the voltage induced across a 0.2-H coil if the current through the coil is as shown in Fig. 12.67.

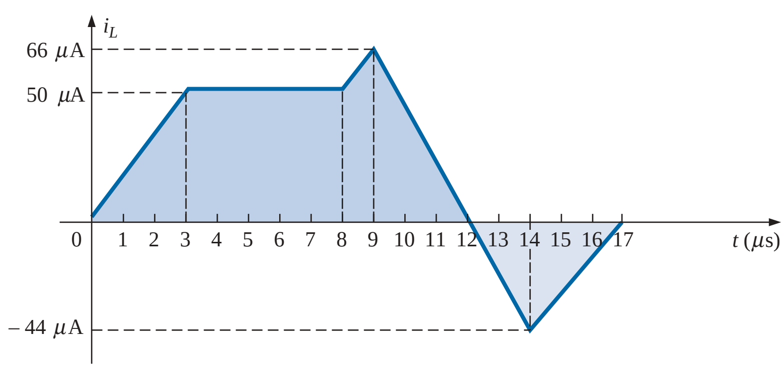

FIG. 12.67
Problem 10.

***11.** Find the waveform for the current of a 10-mH coil if the voltage across the coil follows the pattern of Fig. 12.68. The current i_L is 4 mA at $t = 0$ s.

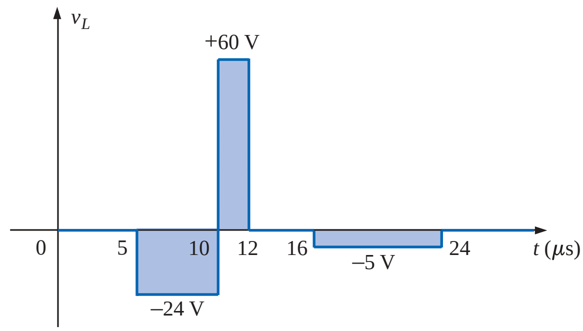

FIG. 12.68
Problem 11.

SECTION 12.7 *R-L* Transients: Storage Cycle

12. For the circuit of Fig. 12.69:
- **a.** Determine the time constant.
- **b.** Write the mathematical expression for the current i_L after the switch is closed.
- **c.** Repeat part (b) for v_L and v_R.
- **d.** Determine i_L and v_L at one, three, and five time constants.
- **e.** Sketch the waveforms of i_L, v_L, and v_R.

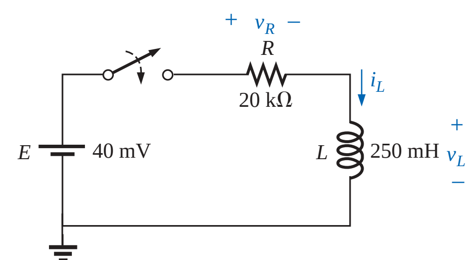

FIG. 12.69
Problem 12.

13. For the circuit of Fig. 12.70:
- **a.** Determine τ.
- **b.** Write the mathematical expression for the current i_L after the switch is closed at $t = 0$ s.
- **c.** Write the mathematical expressions for v_L and v_R after the switch is closed at $t = 0$ s.
- **d.** Determine i_L and v_L at $t = 1\tau$, 3τ, and 5τ.
- **e.** Sketch the waveforms of i_L, v_L, and v_R for the storage phase.

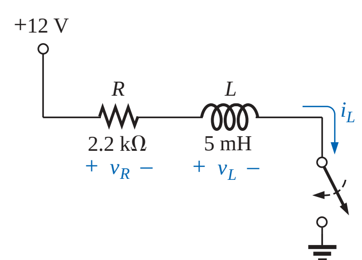

FIG. 12.70
Problem 13.

SECTION 12.8 Initial Values

14. For the network of Fig. 12.71:
- **a.** Write the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch. Note the magnitude and direction of the initial current.
- **b.** Sketch the waveform of i_L and v_L for the entire period from initial value to steady-state level.

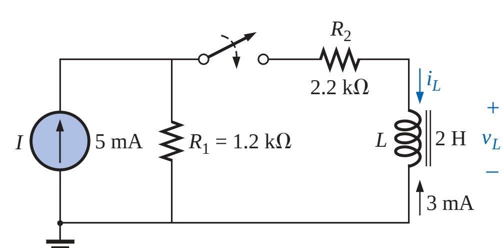

FIG. 12.71
Problem 14.

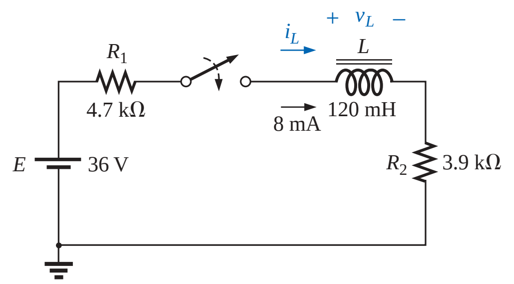

FIG. 12.72
Problem 15.

15. For the network of Fig. 12.72:
- **a.** Write the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch. Note the magnitude and direction of the initial current.
- **b.** Sketch the waveform of i_L and v_L for the entire period from initial value to steady-state level.

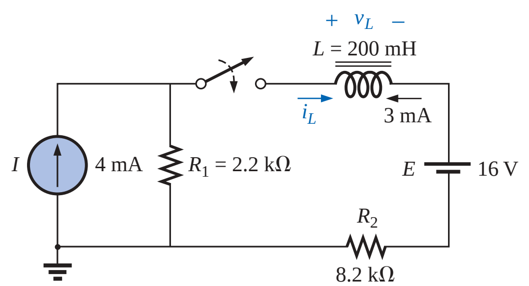

FIG. 12.73
Problem 16.

***16.** For the network of Fig. 12.73:
- **a.** Write the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch. Note the magnitude and direction of the initial current.
- **b.** Sketch the waveform of i_L and v_L for the entire period from initial value to steady-state level.

SECTION 12.9 *R-L* Transients: Decay Phase

17. For the network of Fig. 12.74:
- **a.** Determine the mathematical expressions for the current i_L and the voltage v_L when the switch is closed.
- **b.** Repeat part (a) if the switch is opened after a period of five time constants has passed.
- **c.** Sketch the waveforms of parts (a) and (b) on the same set of axes.

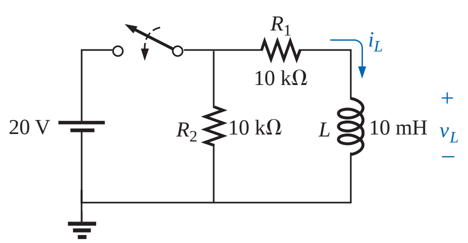

FIG. 12.74
Problems 17, 45, and 46.

***18.** For the network of Fig. 12.75:
- **a.** Write the mathematical expression for the current i_L and the voltage v_L following the closing of the switch.
- **b.** Determine the mathematical expressions for i_L and v_L if the switch is opened after a period of five time constants has passed.
- **c.** Sketch the waveforms of i_L and v_L for the time periods defined by parts (a) and (b).
- **d.** Sketch the waveform for the voltage across R_2 for the same period of time encompassed by i_L and v_L. Take careful note of the defined polarities and directions of Fig. 12.75.

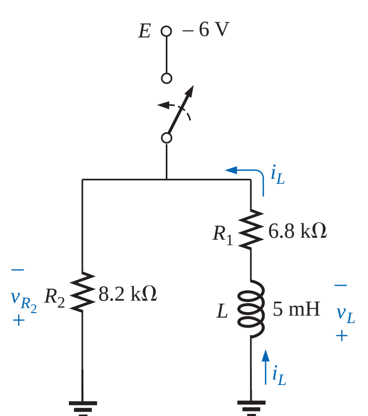

FIG. 12.75
Problem 18.

***19.** For the network of Fig. 12.76:

a. Determine the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch.

b. Repeat part (a) if the switch is opened at $t = 1\ \mu\text{s}$.

c. Sketch the waveforms of parts (a) and (b) on the same set of axes.

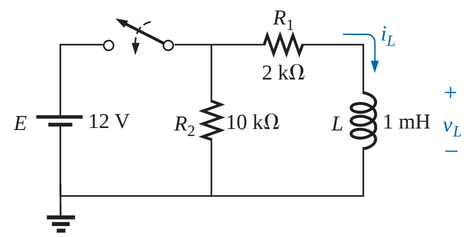

FIG. 12.76
Problem 19.

SECTION 12.10 Instantaneous Values

20. Referring to the solution to Example 12.4, determine the time when the current i_L reaches a level of 10 mA. Then determine the time when the voltage drops to a level of 10 V.

21. Referring to the solution to Example 12.5, determine the time when the current i_L drops to 2 mA.

SECTION 12.11 Thévenin Equivalent: $\tau = L/R_{Th}$

22. a. Determine the mathematical expressions for i_L and v_L following the closing of the switch in Fig. 12.77.

b. Determine i_L and v_L at $t = 100$ ns.

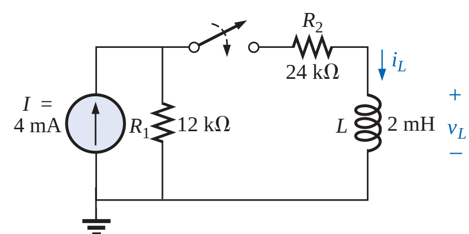

FIG. 12.77
Problem 22.

***23. a.** Determine the mathematical expressions for i_L and v_L following the closing of the switch in Fig. 12.78.

b. Calculate i_L and v_L at $t = 10\ \mu\text{s}$.

c. Write the mathematical expressions for the current i_L and the voltage v_L if the switch is opened at $t = 10\ \mu\text{s}$.

d. Sketch the waveforms of i_L and v_L for parts (a) and (c).

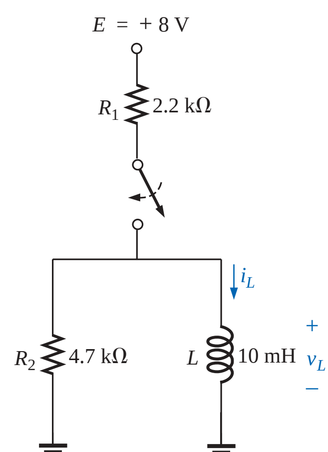

FIG. 12.78
Problem 23.

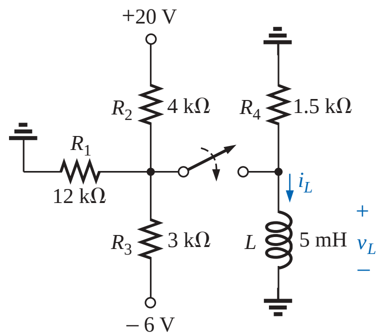

FIG. 12.79
Problem 24.

***24. a.** Determine the mathematical expressions for i_L and v_L following the closing of the switch in Fig. 12.79.

b. Determine i_L and v_L after two time constants of the storage phase.

c. Write the mathematical expressions for the current i_L and the voltage v_L if the switch is opened at the instant defined by part (b).

d. Sketch the waveforms of i_L and v_L for parts (a) and (c).

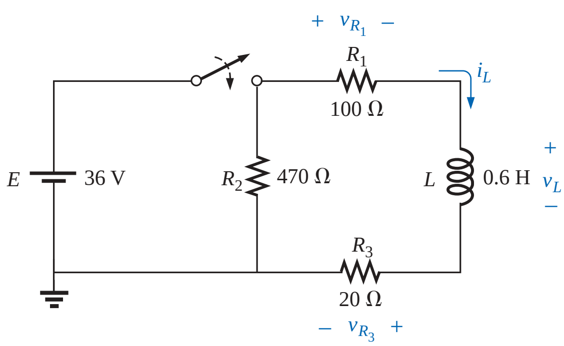

FIG. 12.80
Problem 25.

***25.** For the network of Fig. 12.80, the switch is closed at $t = 0$ s.

a. Determine v_L at $t = 25$ ms.

b. Find v_L at $t = 1$ ms.

c. Calculate v_{R_1} at $t = 1\tau$.

d. Find the time required for the current i_L to reach 100 mA.

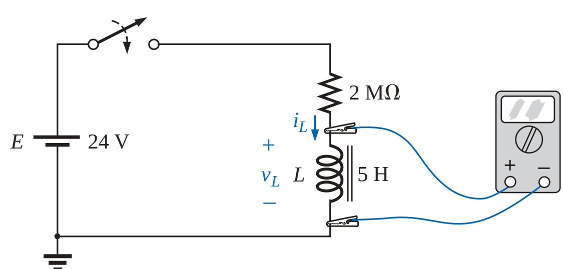

FIG. 12.81
Problems 26 and 27.

***26.** The switch for the network of Fig. 12.81 has been closed for about 1 h. It is then opened at the time defined as $t =$ 0 s.

a. Determine the time required for the current i_R to drop to 1 mA.

b. Find the voltage v_L at $t = 1$ ms.

c. Calculate v_{R_3} at $t = 5\tau$.

27. The network of Fig. 12.81 employs a DMM with an internal resistance of 10 MΩ in the voltmeter mode. The switch is closed at $t = 0$ s.

a. Find the voltage across the coil the instant after the switch is closed.

b. What is the final value of the current i_L?

c. How much time must pass before i_L reaches 10 μA?

d. What is the voltmeter reading at $t = 12$ μs?

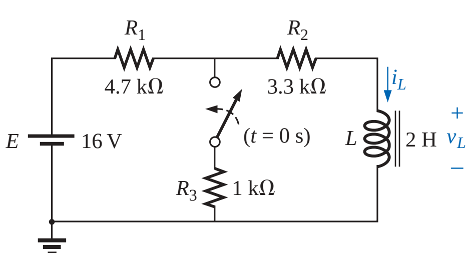

FIG. 12.82
Problem 28.

***28.** The switch in Fig. 12.82 has been open for a long time. It is then closed at $t = 0$ s.

a. Write the mathematical expression for the current i_L and the voltage v_L after the switch is closed.

b. Sketch the waveform of i_L and v_L from the initial value to the steady-state level.

***29.** The switch of Fig. 12.83 has been closed for a long time. It is then opened at $t = 0$ s.

a. Write the mathematical expression for the current i_L and the voltage v_L after the switch is opened.

b. Sketch the waveform of i_L and v_L from initial value to the steady-state level.

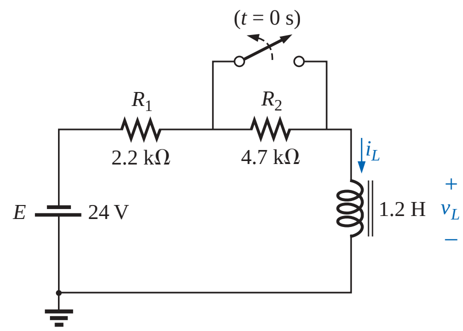

FIG. 12.83
Problem 29.

***30.** The switch of Fig. 12.84 has been open for a long time. It is then closed at $t = 0$ s.

a. Write the mathematical expression for the current i_L and the voltage v_L after the switch is closed.

b. Sketch the waveform of i_L and v_L from initial value to the steady-state level.

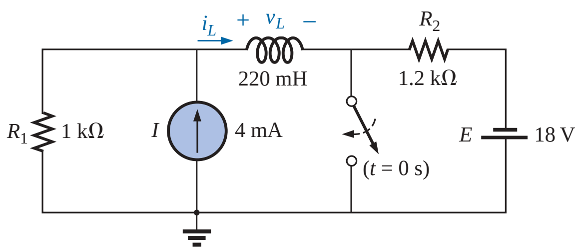

FIG. 12.84
Problems 30 and 43.

SECTION 12.12 Inductors in Series and Parallel

31. Find the total inductance of the circuits of Fig. 12.85.

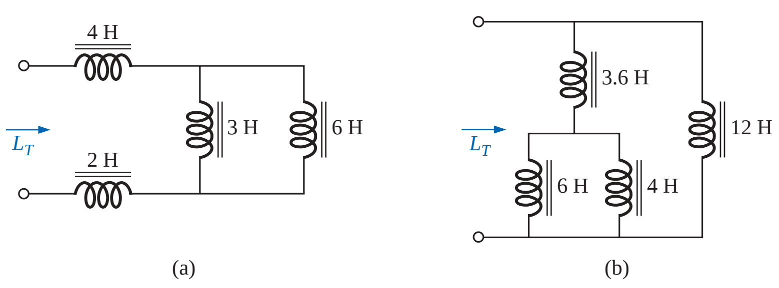

FIG. 12.85
Problem 31.

32. Reduce the networks of Fig. 12.86 to the fewest elements.

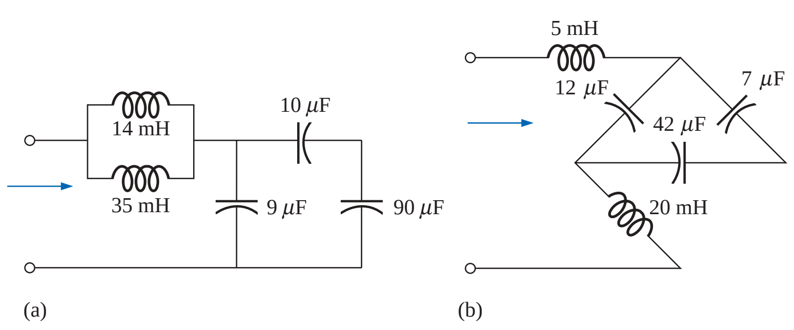

FIG. 12.86
Problem 32.

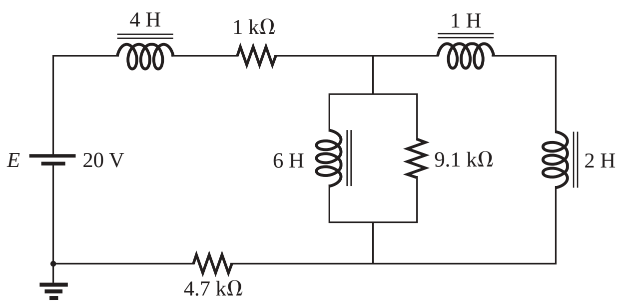

FIG. 12.87
Problem 33.

33. Reduce the network of Fig. 12.87 to the fewest number of components.

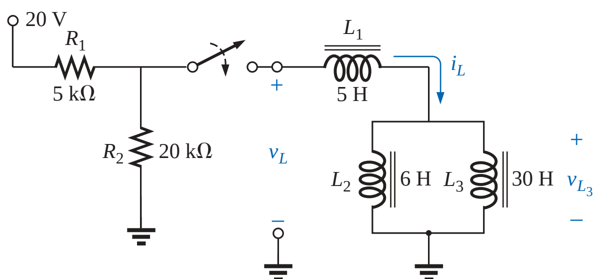

FIG. 12.88
Problem 34.

***34.** For the network of Fig. 12.88:

a. Find the mathematical expressions for the voltage v_L and the current i_L following the closing of the switch.

b. Sketch the waveforms of v_L and i_L obtained in part (a).

c. Determine the mathematical expression for the voltage v_{L_3} following the closing of the switch, and sketch the waveform.

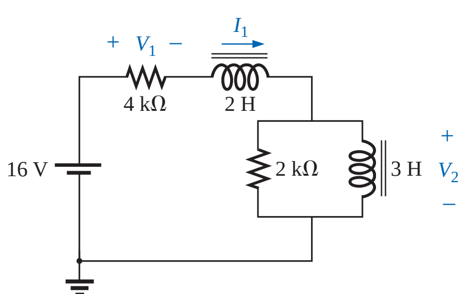

FIG. 12.89
Problems 35 and 38.

SECTION 12.13 *R-L* and *R-L-C* Circuits with dc Inputs

For Problems 35 through 37, assume that the voltage across each capacitor and the current through each inductor have reached their final values.

35. Find the voltages V_1 and V_2 and the current I_1 for the circuit of Fig. 12.89.

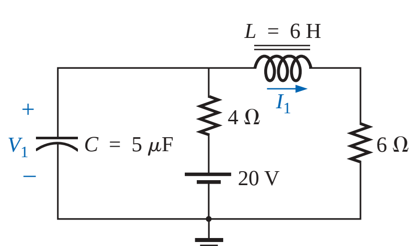

FIG. 12.90
Problems 36 and 39.

36. Find the current I_1 and the voltage V_1 for the circuit of Fig. 12.90.

37. Find the voltage V_1 and the current through each inductor in the circuit of Fig. 12.91.

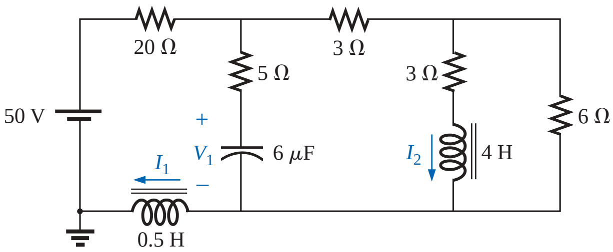

FIG. 12.91
Problems 37 and 40.

SECTION 12.14 Energy Stored by an Inductor

38. Find the energy stored in each inductor of Problem 35.

39. Find the energy stored in the capacitor and inductor of Problem 36.

40. Find the energy stored in each inductor of Problem 37.

SECTION 12.16 Computer Analysis

PSpice or Electronics Workbench

***41.** Verify the results of Example 12.6 using the VPULSE function and a pulse width (*PW*) equal to five time constants of the charging network.

***42.** Verify the results of Example 12.3 using the VPULSE function and a *PW* equal to 1 ns.

***43.** Verify the results of Problem 30 using the VPULSE function and the appropriate initial current.

Programming Language (C++, QBASIC, Pascal, etc.)

44. Write a program to provide a general solution for the circuit of Fig. 12.14; that is, given the network parameters, generate the equations for i_L, v_L, and v_R.

45. Write a program that will provide a general solution for the storage and decay phase of the network of Fig. 12.74; that is, given the network values, generate the equations for i_L and v_L for each phase. In this case, assume that the storage phase has passed through five time constants before the decay phase begins.

46. Repeat Problem 45, but assume that the storage phase was not completed, requiring that the instantaneous values of i_L and v_L be determined when the switch is opened.

GLOSSARY

Choke A term often applied to an inductor, due to the ability of an inductor to resist a change in current through it.

Faraday's law A law relating the voltage induced across a coil to the number of turns in the coil and the rate at which the flux linking the coil is changing.

Inductor A fundamental element of electrical systems constructed of numerous turns of wire around a ferromagnetic core or an air core.

Lenz's law A law stating that an induced effect is always such as to oppose the cause that produced it.

Self-inductance (*L*) A measure of the ability of a coil to oppose any change in current through the coil and to store energy in the form of a magnetic field in the region surrounding the coil.