15

Series and Parallel ac Circuits

15.1 INTRODUCTION

In this chapter, phasor algebra will be used to develop a quick, direct method for solving both the series and the parallel ac circuits. The close relationship that exists between this method for solving for unknown quantities and the approach used for dc circuits will become apparent after a few simple examples are considered. Once this association is established, many of the rules (current divider rule, voltage divider rule, and so on) for dc circuits can be readily applied to ac circuits.

SERIES ac CIRCUITS

15.2 IMPEDANCE AND THE PHASOR DIAGRAM

Resistive Elements

In Chapter 14, we found, for the purely resistive circuit of Fig. 15.1, that v and i were in phase, and the magnitude

$$I_m = \frac{V_m}{R} \quad \text{or} \quad V_m = I_m R$$

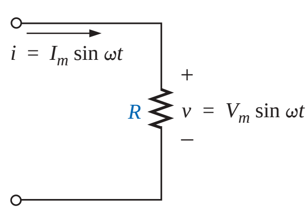


FIG. 15.1
Resistive ac circuit.

In phasor form,

$$v = V_m \sin \omega t \Rightarrow \mathbf{V} = V \angle 0°$$

where $V = 0.707V_m$.

Applying Ohm's law and using phasor algebra, we have

$$\mathbf{I} = \frac{V \angle 0°}{R \angle \theta_R} = \frac{V}{R} \underline{/0° - \theta_R}$$

Since i and v are in phase, the angle associated with i also must be 0°. To satisfy this condition, θ_R must equal 0°. Substituting $\theta_R = 0°$, we find

$$\mathbf{I} = \frac{V \angle 0°}{R \angle 0°} = \frac{V}{R} \underline{/0° - 0°} = \frac{V}{R} \angle 0°$$

so that in the time domain,

$$i = \sqrt{2}\left(\frac{V}{R}\right) \sin \omega t$$

The fact that $\theta_R = 0°$ will now be employed in the following polar format to ensure the proper phase relationship between the voltage and current of a resistor:

$$\boxed{\mathbf{Z}_R = R \angle 0°} \qquad \textbf{(15.1)}$$

The boldface roman quantity $\mathbf{Z}_R$, having both magnitude and an associated angle, is referred to as the *impedance* of a resistive element. It is measured in ohms and is a measure of how much the element will "impede" the flow of charge through the network. The above format will prove to be a useful "tool" when the networks become more complex and phase relationships become less obvious. It is important to realize, however, that $\mathbf{Z}_R$ is *not a phasor*, even though the format $R \angle 0°$ is very similar to the phasor notation for sinusoidal currents and voltages. The term *phasor* is reserved for quantities that vary with time, and R and its associated angle of 0° are fixed, nonvarying quantities.

EXAMPLE 15.1 Using complex algebra, find the current i for the circuit of Fig. 15.2. Sketch the waveforms of v and i.

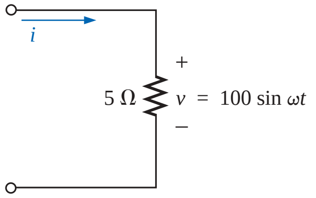


FIG. 15.2
Example 15.1.

Solution: Note Fig. 15.3:

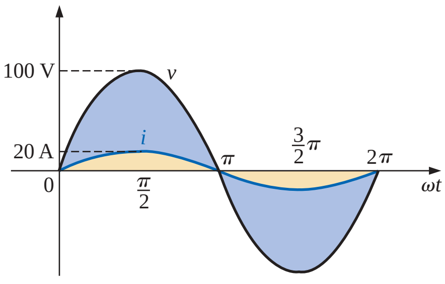


FIG. 15.3
Waveforms for Example 15.1.

$$v = 100 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 70.71 \text{ V} \angle 0°$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_R} = \frac{V \angle \theta}{R \angle 0°} = \frac{70.71 \text{ V} \angle 0°}{5\ \Omega \angle 0°} = 14.14 \text{ A} \angle 0°$$

and $$i = \sqrt{2}(14.14) \sin \omega t = \mathbf{20 \sin \omega t}$$

EXAMPLE 15.2 Using complex algebra, find the voltage v for the circuit of Fig. 15.4. Sketch the waveforms of v and i.

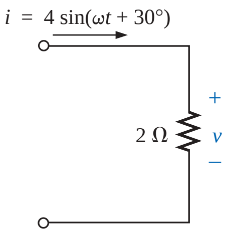


FIG. 15.4
Example 15.2.

Solution: Note Fig. 15.5:

$$i = 4 \sin(\omega t + 30°) \Rightarrow \text{phasor form } \mathbf{I} = 2.828 \text{ A} \angle 30°$$

$$\mathbf{V} = \mathbf{IZ}_R = (I \angle \theta)(R \angle 0°) = (2.828 \text{ A} \angle 30°)(2\ \Omega \angle 0°)$$
$$= 5.656 \text{ V} \angle 30°$$

and $$v = \sqrt{2}(5.656) \sin(\omega t + 30°) = \mathbf{8.0 \sin(\omega t + 30°)}$$

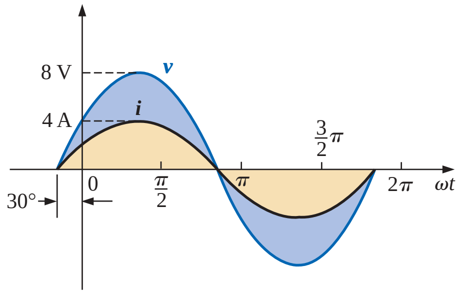


FIG. 15.5
Waveforms for Example 15.2.

It is often helpful in the analysis of networks to have a **phasor diagram**, which shows at a glance the *magnitudes* and *phase relations* among the various quantities within the network. For example, the phasor diagrams of the circuits considered in the two preceding examples would be as shown in Fig. 15.6. In both cases, it is immediately obvious that v and i are in phase since they both have the same phase angle.

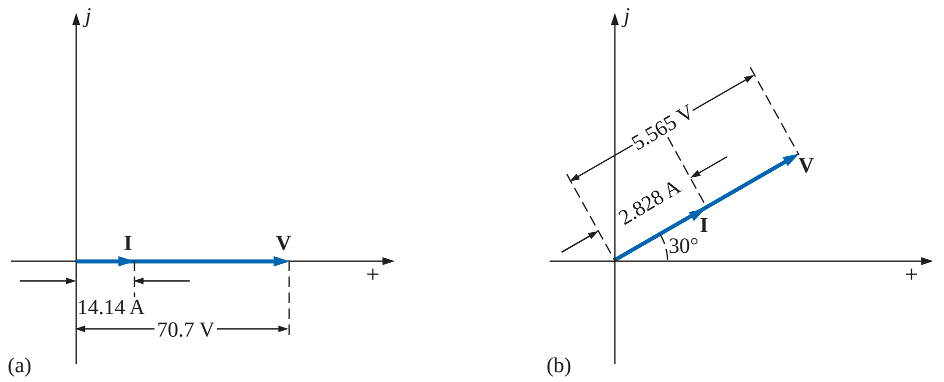


FIG. 15.6
Phasor diagrams for Examples 15.1 and 15.2.

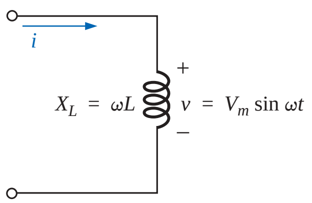


FIG. 15.7
Inductive ac circuit.

Inductive Reactance

It was learned in Chapter 13 that for the pure inductor of Fig. 15.7, the voltage leads the current by 90° and that the reactance of the coil X_L is determined by ωL.

$$v = V_m \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = V \angle 0°$$

By Ohm's law,

$$\mathbf{I} = \frac{V \angle 0°}{X_L \angle \theta_L} = \frac{V}{X_L} \angle 0° - \theta_L$$

Since v leads i by 90°, i must have an angle of −90° associated with it. To satisfy this condition, θ_L must equal +90°. Substituting $\theta_L = 90°$, we obtain

$$\mathbf{I} = \frac{V \angle 0°}{X_L \angle 90°} = \frac{V}{X_L} \angle 0° - 90° = \frac{V}{X_L} \angle -90°$$

so that in the time domain,

$$i = \sqrt{2}\left(\frac{V}{X_L}\right) \sin(\omega t - 90°)$$

The fact that $\theta_L = 90°$ will now be employed in the following polar format for inductive reactance to ensure the proper phase relationship between the voltage and current of an inductor.

$$\mathbf{Z}_L = X_L \angle 90° \qquad \textbf{(15.2)}$$

The boldface roman quantity $\mathbf{Z}_L$, having both magnitude and an associated angle, is referred to as the *impedance* of an inductive element. It is measured in ohms and is a measure of how much the inductive element will "control or impede" the level of current through the network (always keep in mind that inductive elements are storage devices and do not dissipate like resistors). The above format, like that defined for the resistive element, will prove to be a useful "tool" in the analysis of ac networks. Again, be aware that $\mathbf{Z}_L$ is not a phasor quantity, for the same reasons indicated for a resistive element.

EXAMPLE 15.3 Using complex algebra, find the current i for the circuit of Fig. 15.8. Sketch the v and i curves.

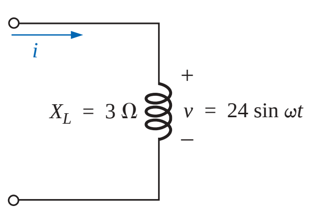


FIG. 15.8
Example 15.3.

Solution: Note Fig. 15.9:

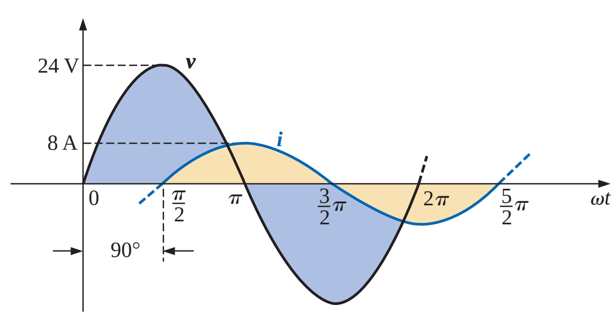


FIG. 15.9
Waveforms for Example 15.3.

$$v = 24 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 16.968 \text{ V} \angle 0°$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_L} = \frac{V \angle \theta}{X_L \angle 90°} = \frac{16.968 \text{ V} \angle 0°}{3 \ \Omega \angle 90°} = 5.656 \text{ A} \angle -90°$$

and $$i = \sqrt{2}(5.656) \sin(\omega t - 90°) = \mathbf{8.0 \sin(\omega t - 90°)}$$

EXAMPLE 15.4 Using complex algebra, find the voltage v for the circuit of Fig. 15.10. Sketch the v and i curves.

Solution: Note Fig. 15.11:

$$i = 5 \sin(\omega t + 30°) \Rightarrow \text{phasor form } \mathbf{I} = 3.535 \text{ A} \angle 30°$$

$$\mathbf{V} = \mathbf{IZ}_L = (I \angle \theta)(X_L \angle 90°) = (3.535 \text{ A} \angle 30°)(4 \ \Omega \angle +90°)$$
$$= 14.140 \text{ V} \angle 120°$$

and $$v = \sqrt{2}(14.140) \sin(\omega t + 120°) = \mathbf{20 \sin(\omega t + 120°)}$$

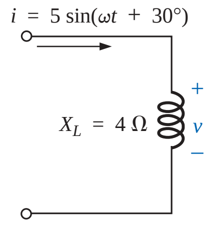


FIG. 15.10
Example 15.4.

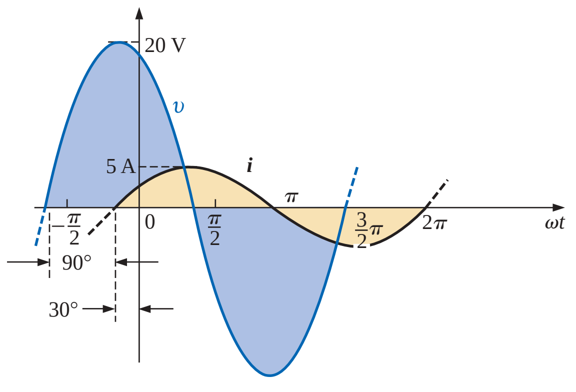


FIG. 15.11
Waveforms for Example 15.4.

The phasor diagrams for the two circuits of the two preceding examples are shown in Fig. 15.12. Both indicate quite clearly that the voltage leads the current by 90°.

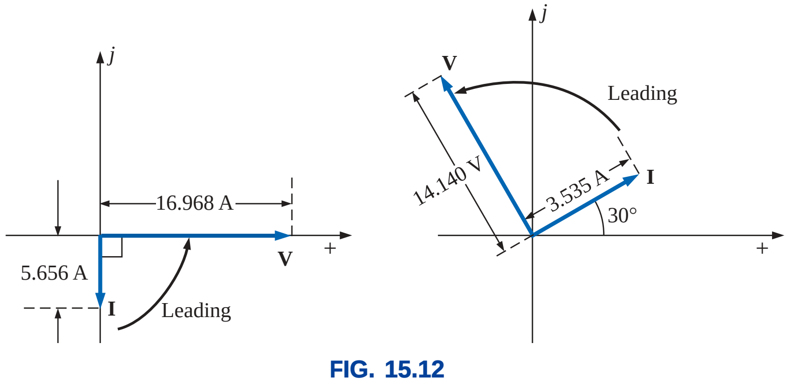


FIG. 15.12
Phasor diagrams for Examples 15.3 and 15.4.

Capacitive Reactance

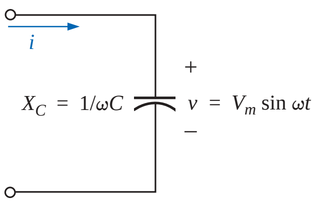


FIG. 15.13
Capacitive ac circuit.

It was learned in Chapter 13 that for the pure capacitor of Fig. 15.13, the current leads the voltage by 90° and that the reactance of the capacitor X_C is determined by $1/\omega C$.

$$v = V_m \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = V \angle 0°$$

Applying Ohm's law and using phasor algebra, we find

$$\mathbf{I} = \frac{V \angle 0°}{X_C \angle \theta_C} = \frac{V}{X_C} \underline{/0° - \theta_C}$$

Since i leads v by 90°, i must have an angle of +90° associated with it. To satisfy this condition, θ_C must equal −90°. Substituting $\theta_C = -90°$ yields

$$\mathbf{I} = \frac{V \angle 0°}{X_C \angle -90°} = \frac{V}{X_C} \underline{/0° - (-90°)} = \frac{V}{X_C} \angle 90°$$

so, in the time domain,

$$i = \sqrt{2}\left(\frac{V}{X_C}\right) \sin(\omega t + 90°)$$

The fact that $\theta_C = -90°$ will now be employed in the following polar format for capacitive reactance to ensure the proper phase relationship between the voltage and current of a capacitor.

$$\boxed{\mathbf{Z}_C = X_C \angle -90°} \tag{15.3}$$

The boldface roman quantity $\mathbf{Z}_C$, having both magnitude and an associated angle, is referred to as the *impedance* of a capacitive element. It is measured in ohms and is a measure of how much the capacitive element will "control or impede" the level of current through the network (always keep in mind that capacitive elements are storage devices and do not dissipate like resistors). The above format, like that defined for the resistive element, will prove a very useful "tool" in the analysis of ac networks. Again, be aware that $\mathbf{Z}_C$ is not a phasor quantity, for the same reasons indicated for a resistive element.

EXAMPLE 15.5 Using complex algebra, find the current i for the circuit of Fig. 15.14. Sketch the v and i curves.

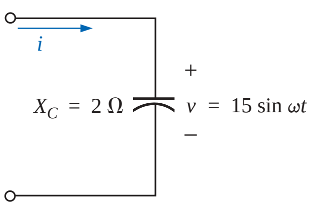


FIG. 15.14
Example 15.5.

Solution: Note Fig. 15.15:

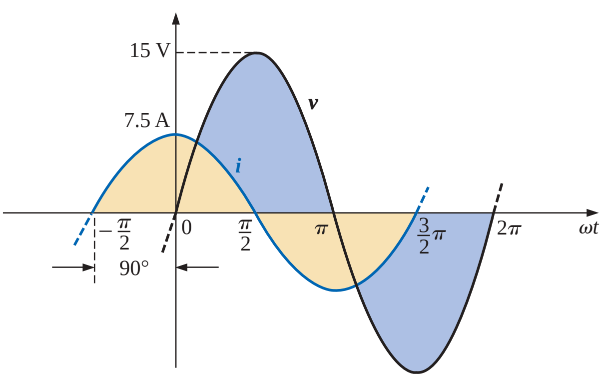


FIG. 15.15
Waveforms for Example 15.5.

$$v = 15 \sin \omega t \Rightarrow \text{phasor notation } \mathbf{V} = 10.605 \text{ V} \angle 0°$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_C} = \frac{V \angle \theta}{X_C \angle -90°} = \frac{10.605 \text{ V} \angle 0°}{2 \ \Omega \angle -90°} = 5.303 \text{ A} \angle 90°$$

and $$i = \sqrt{2}(5.303) \sin(\omega t + 90°) = \mathbf{7.5 \sin(\omega t + 90°)}$$

EXAMPLE 15.6 Using complex algebra, find the voltage v for the circuit of Fig. 15.16. Sketch the v and i curves.

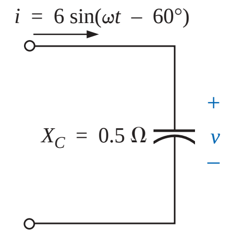


FIG. 15.16
Example 15.6.

Solution: Note Fig. 15.17:

$$i = 6 \sin(\omega t - 60°) \Rightarrow \text{phasor notation } \mathbf{I} = 4.242 \text{ A} \angle -60°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90°) = (4.242 \text{ A} \angle -60°)(0.5 \ \Omega \angle -90°)$$
$$= 2.121 \text{ V} \angle -150°$$

and $$v = \sqrt{2}(2.121) \sin(\omega t - 150°) = \mathbf{3.0 \sin(\omega t - 150°)}$$

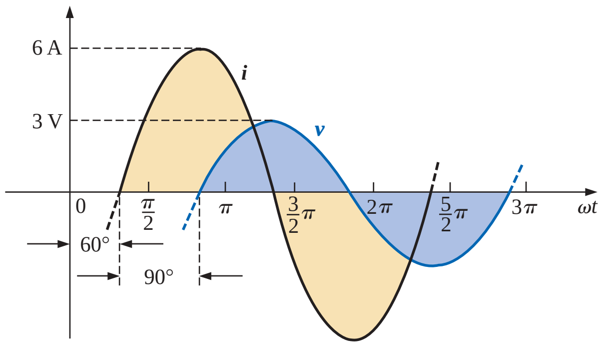


FIG. 15.17
Waveforms for Example 15.6.

The phasor diagrams for the two circuits of the two preceding examples are shown in Fig. 15.18. Both indicate quite clearly that the current i leads the voltage v by 90°.

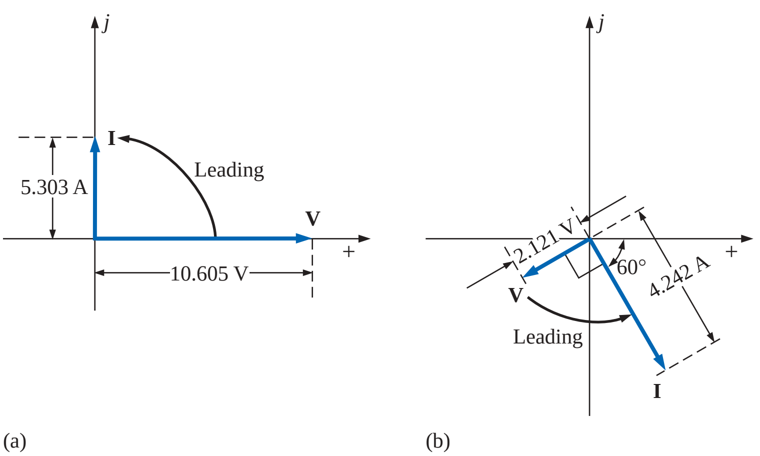


FIG. 15.18
Phasor diagrams for Examples 15.5 and 15.6.

Impedance Diagram

Now that an angle is associated with resistance, inductive reactance, and capacitive reactance, each can be placed on a complex plane dia-

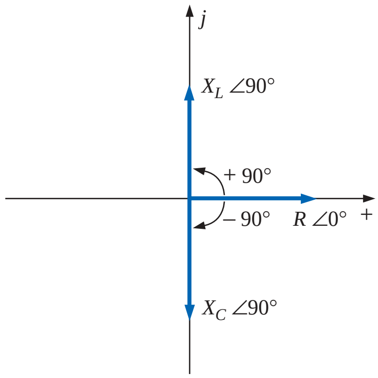


FIG. 15.19
Impedance diagram.

gram, as shown in Fig. 15.19. For any network, the resistance will *always* appear on the positive real axis, the inductive reactance on the positive imaginary axis, and the capacitive reactance on the negative imaginary axis. The result is an **impedance diagram** that can reflect the individual and total impedance levels of an ac network.

We will find in the sections and chapters to follow that networks combining different types of elements will have total impedances that extend from −90° to +90°. If the total impedance has an angle of 0°, it is said to be resistive in nature. If it is closer to 90°, it is inductive in nature; and if it is closer to −90°, it is capacitive in nature.

Of course, for single-element networks the angle associated with the impedance will be the same as that of the resistive or reactive element, as revealed by Eqs. (15.1) through (15.3). It is important to stay aware that impedance, like resistance or reactance, is not a phasor quantity representing a time-varying function with a particular phase shift. It is simply an operating "tool" that is extremely useful in determining the magnitude and angle of quantities in a sinusoidal ac network.

Once the total impedance of a network is determined, its magnitude will define the resulting current level (through Ohm's law), whereas its angle will reveal whether the network is primarily inductive or capacitive or simply resistive.

For any configuration (series, parallel, series-parallel, etc.), the angle associated with the total impedance is the angle by which the applied voltage leads the source current. For inductive networks, θ_T will be positive, whereas for capacitive networks, θ_T will be negative.

15.3 SERIES CONFIGURATION

The overall properties of series ac circuits (Fig. 15.20) are the same as those for dc circuits. For instance, the total impedance of a system is the sum of the individual impedances:

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N \quad \textbf{(15.4)}$$

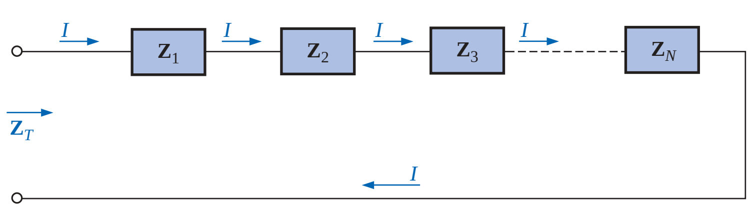


FIG. 15.20
Series impedances.

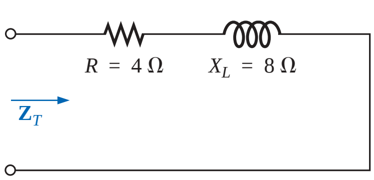


FIG. 15.21
Example 15.7.

EXAMPLE 15.7 Draw the impedance diagram for the circuit of Fig. 15.21, and find the total impedance.

Solution: As indicated by Fig. 15.22, the input impedance can be found graphically from the impedance diagram by properly scaling the real and imaginary axes and finding the length of the resultant vector Z_T and angle θ_T. Or, by using vector algebra, we obtain

$$\begin{aligned}\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \\ &= R\angle 0^\circ + X_L\angle 90^\circ \\ &= R + jX_L = 4\ \Omega + j8\ \Omega \\ \mathbf{Z}_T &= \mathbf{8.944\ \Omega\ \angle 63.43^\circ}\end{aligned}$$

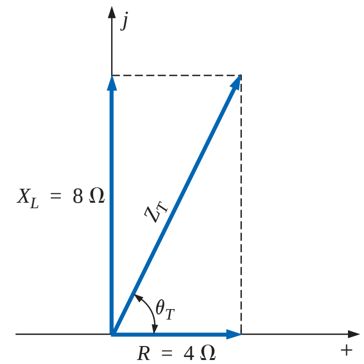


FIG. 15.22
Impedance diagram for Example 15.7.

EXAMPLE 15.8 Determine the input impedance to the series network of Fig. 15.23. Draw the impedance diagram.

Solution:

$$\begin{aligned}\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 \\ &= R\angle 0^\circ + X_L\angle 90^\circ + X_C\angle -90^\circ \\ &= R + jX_L - jX_C \\ &= R + j(X_L - X_C) = 6\ \Omega + j(10\ \Omega - 12\ \Omega) = 6\ \Omega - j2\ \Omega \\ \mathbf{Z}_T &= \mathbf{6.325\ \Omega\ \angle -18.43^\circ}\end{aligned}$$

The impedance diagram appears in Fig. 15.24. Note that in this example, series inductive and capacitive reactances are in direct opposition. For the circuit of Fig. 15.23, if the inductive reactance were equal to the capacitive reactance, the input impedance would be purely resistive. We will have more to say about this particular condition in a later chapter.

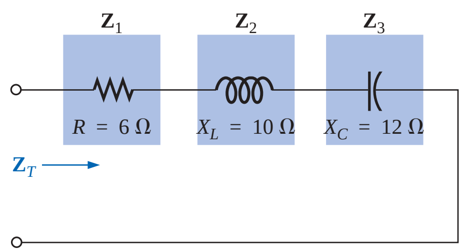


FIG. 15.23
Example 15.8

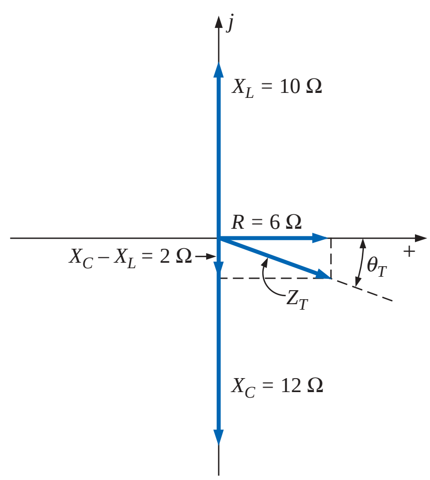


FIG. 15.24
Impedance diagram for Example 15.8.

For the representative **series ac configuration** of Fig. 15.25 having two impedances, *the current is the same through each element* (as it was for the series dc circuits) and is determined by Ohm's law:

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

and

$$\boxed{\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T}} \tag{15.5}$$

The voltage across each element can then be found by another application of Ohm's law:

$$\boxed{\mathbf{V}_1 = \mathbf{I}\mathbf{Z}_1} \tag{15.6a}$$

$$\boxed{\mathbf{V}_2 = \mathbf{I}\mathbf{Z}_2} \tag{15.6b}$$

Kirchhoff's voltage law can then be applied in the same manner as it is employed for dc circuits. However, keep in mind that we are now dealing with the algebraic manipulation of quantities that have both magnitude and direction.

$$\mathbf{E} - \mathbf{V}_1 - \mathbf{V}_2 = 0$$

or

$$\boxed{\mathbf{E} = \mathbf{V}_1 + \mathbf{V}_2} \tag{15.7}$$

The power to the circuit can be determined by

$$\boxed{P = EI\cos\theta_T} \tag{15.8}$$

where θ_T is the phase angle between **E** and **I**.

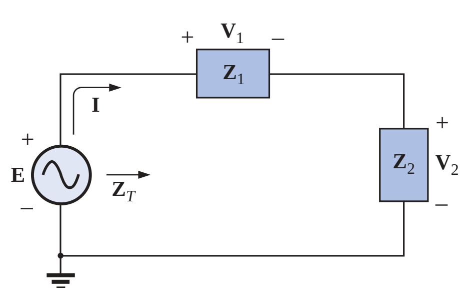


FIG. 15.25
Series ac circuit.

Now that a general approach has been introduced, the simplest of series configurations will be investigated in detail to further emphasize the similarities in the analysis of dc circuits. In many of the circuits to be considered, $3 + j4 = 5\ \angle 53.13°$ and $4 + j3 = 5\ \angle 36.87°$ will be used quite frequently to ensure that the approach is as clear as possible and not lost in mathematical complexity. Of course, the problems at the end of the chapter will provide plenty of experience with random values.

R-L

Refer to Fig. 15.26.

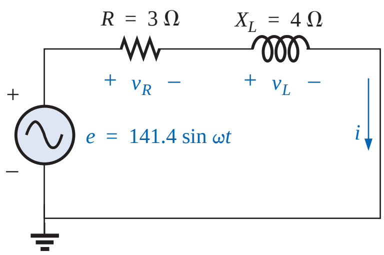


FIG. 15.26
Series R-L circuit.

Phasor Notation

$$e = 141.4 \sin \omega t \Rightarrow \mathbf{E} = 100 \text{ V} \angle 0°$$

Note Fig. 15.27.

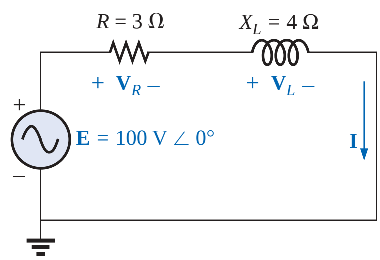


FIG. 15.27
Applying phasor notation to the network of Fig. 15.26.

$\mathbf{Z}_T$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 3\ \Omega \angle 0° + 4\ \Omega \angle 90° = 3\ \Omega + j4\ \Omega$$

and $$\mathbf{Z}_T = \mathbf{5\ \Omega \angle 53.13°}$$

Impedance diagram: See Fig. 15.28.

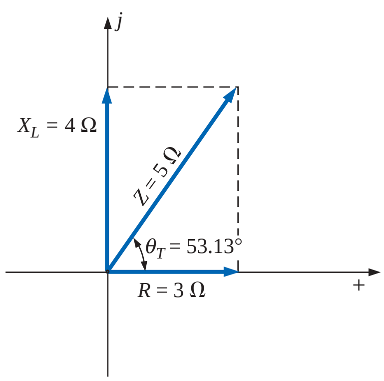


FIG. 15.28
Impedance diagram for the series R-L circuit of Fig. 15.26.

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100 \text{ V} \angle 0°}{5\ \Omega \angle 53.13°} = \mathbf{20 \text{ A} \angle -53.13°}$$

$\mathbf{V}_R$ and $\mathbf{V}_L$

Ohm's law:

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (20 \text{ A} \angle -53.13°)(3\ \Omega \angle 0°) = \mathbf{60 \text{ V} \angle -53.13°}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (20 \text{ A} \angle -53.13°)(4\ \Omega \angle 90°) = \mathbf{80 \text{ V} \angle 36.87°}$$

Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright} \mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L = 0$$

or $$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L$$

In rectangular form,

$$\mathbf{V}_R = 60\text{ V}\angle -53.13^\circ = 36\text{ V} - j\,48\text{ V}$$
$$\mathbf{V}_L = 80\text{ V}\angle +36.87^\circ = 64\text{ V} + j\,48\text{ V}$$

and

$$\begin{aligned}\mathbf{E} &= \mathbf{V}_R + \mathbf{V}_L = (36\text{ V} - j\,48\text{ V}) + (64\text{ V} + j\,48\text{ V}) = 100\text{ V} + j\,0\\ &= 100\text{ V}\angle 0^\circ\end{aligned}$$

as applied.

Phasor diagram: Note that for the phasor diagram of Fig. 15.29, **I** is in phase with the voltage across the resistor and lags the voltage across the inductor by 90°.

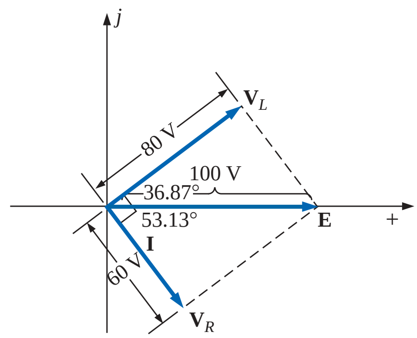


FIG. 15.29
Phasor diagram for the series R-L circuit of Fig. 15.26.

Power: The total power in watts delivered to the circuit is

$$\begin{aligned}P_T &= EI\cos\theta_T\\ &= (100\text{ V})(20\text{ A})\cos 53.13^\circ = (2000\text{ W})(0.6)\\ &= \mathbf{1200\ W}\end{aligned}$$

where E and I are effective values and θ_T is the phase angle between E and I, or

$$\begin{aligned}P_T &= I^2R\\ &= (20\text{ A})^2(3\ \Omega) = (400)(3)\\ &= \mathbf{1200\ W}\end{aligned}$$

where I is the effective value, or, finally,

$$\begin{aligned}P_T = P_R + P_L &= V_RI\cos\theta_R + V_LI\cos\theta_L\\ &= (60\text{ V})(20\text{ A})\cos 0^\circ + (80\text{ V})(20\text{ A})\cos 90^\circ\\ &= 1200\text{ W} + 0\\ &= \mathbf{1200\ W}\end{aligned}$$

where θ_R is the phase angle between $\mathbf{V}_R$ and **I**, and θ_L is the phase angle between $\mathbf{V}_L$ and **I**.

Power factor: The power factor F_p of the circuit is cos 53.13° = **0.6 lagging,** where 53.13° is the phase angle between **E** and **I.**

If we write the basic power equation $P = EI\cos\theta$ as follows:

$$\cos\theta = \frac{P}{EI}$$

where E and I are the input quantities and P is the power delivered to the network, and then perform the following substitutions from the basic series ac circuit:

$$\cos\theta = \frac{P}{EI} = \frac{I^2R}{EI} = \frac{IR}{E} = \frac{R}{E/I} = \frac{R}{Z_T}$$

we find

$$F_p = \cos\theta_T = \frac{R}{Z_T} \qquad \textbf{(15.9)}$$

Reference to Fig. 15.28 also indicates that θ is the impedance angle θ_T as written in Eq. (15.9), further supporting the fact that the impedance angle θ_T is also the phase angle between the input voltage and current for a series ac circuit. To determine the power factor, it is necessary only to form the ratio of the total resistance to the magnitude of the input impedance. For the case at hand,

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{3\ \Omega}{5\ \Omega} = \mathbf{0.6\ lagging}$$

as found above.

R-C

Refer to Fig. 15.30.

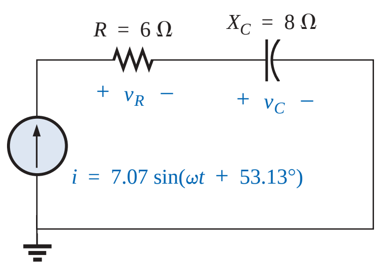


FIG. 15.30
Series R-C ac circuit.

Phasor Notation

$$i = 7.07 \sin(\omega t + 53.13°) \Rightarrow \mathbf{I} = 5\text{ A}\ \angle 53.13°$$

Note Fig. 15.31.

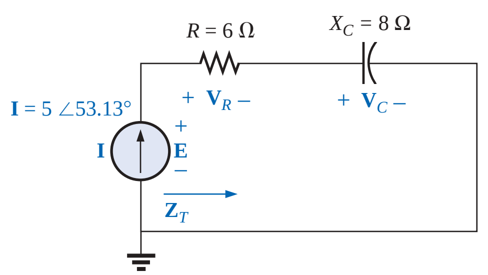


FIG. 15.31
Applying phasor notation to the circuit of Fig. 15.30.

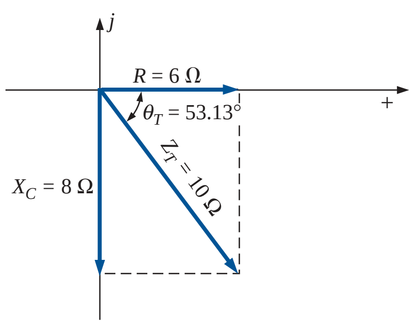


FIG. 15.32
Impedance diagram for the series R-C circuit of Fig. 15.30.

$\mathbf{Z}_T$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 6\ \Omega\ \angle 0° + 8\ \Omega\ \angle -90° = 6\ \Omega - j8\ \Omega$$

and $$\mathbf{Z}_T = \mathbf{10\ \Omega\ \angle -53.13°}$$

Impedance diagram: As shown in Fig. 15.32.

E

$$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = (5\text{ A}\ \angle 53.13°)(10\ \Omega\ \angle -53.13°) = \mathbf{50\ V\ \angle 0°}$$

$\mathbf{V}_R$ and $\mathbf{V}_C$

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I\ \angle\theta)(R\ \angle 0°) = (5\text{ A}\ \angle 53.13°)(6\ \Omega\ \angle 0°) = \mathbf{30\ V\ \angle 53.13°}$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I\ \angle\theta)(X_C\ \angle -90°) = (5\text{ A}\ \angle 53.13°)(8\ \Omega\ \angle -90°) = \mathbf{40\ V\ \angle -36.87°}$$

Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright} \mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_C = 0$$

or $$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_C$$

which can be verified by vector algebra as demonstrated for the *R-L* circuit.

Phasor diagram: Note on the phasor diagram of Fig. 15.33 that the current **I** is in phase with the voltage across the resistor and leads the voltage across the capacitor by 90°.

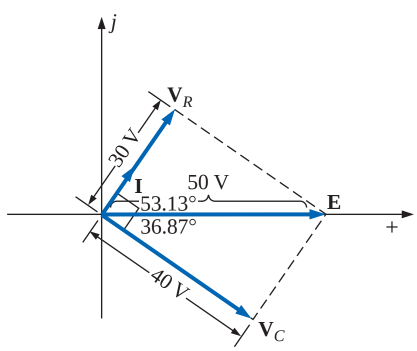


FIG. 15.33
Phasor diagram for the series R-C circuit of Fig. 15.30.

Time domain: In the time domain,

$$e = \sqrt{2}(50) \sin \omega t = \mathbf{70.70 \sin \omega t}$$
$$v_R = \sqrt{2}(30) \sin(\omega t + 53.13°) = \mathbf{42.42 \sin(\omega t + 53.13°)}$$
$$v_C = \sqrt{2}(40) \sin(\omega t - 36.87°) = \mathbf{56.56 \sin(\omega t - 36.87°)}$$

A plot of all of the voltages and the current of the circuit appears in Fig. 15.34. Note again that i and v_R are in phase and that v_C lags i by 90°.

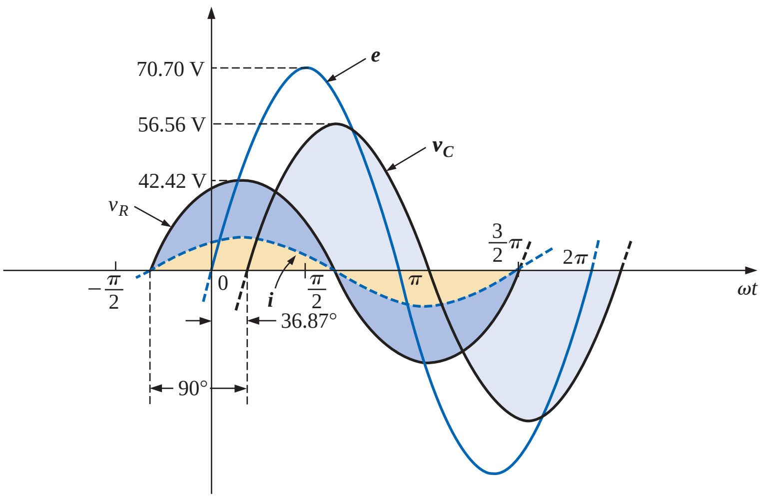


FIG. 15.34
Waveforms for the series R-C circuit of Fig. 15.30.

Power: The total power in watts delivered to the circuit is

$$P_T = EI \cos \theta_T = (50 \text{ V})(5 \text{ A}) \cos 53.13°$$
$$= (250)(0.6) = \mathbf{150 \text{ W}}$$

or

$$P_T = I^2R = (5 \text{ A})^2(6 \ \Omega) = (25)(6)$$
$$= \mathbf{150 \text{ W}}$$

or, finally,

$$P_T = P_R + P_C = V_R I \cos \theta_R + V_C I \cos \theta_C$$
$$= (30 \text{ V})(5 \text{ A}) \cos 0° + (40 \text{ V})(5 \text{ A}) \cos 90°$$
$$= 150 \text{ W} + 0$$
$$= \mathbf{150 \text{ W}}$$

Power factor: The power factor of the circuit is

$$F_p = \cos \theta = \cos 53.13° = \mathbf{0.6 \text{ leading}}$$

Using Eq. (15.9), we obtain

$$F_p = \cos \theta = \frac{R}{Z_T} = \frac{6 \ \Omega}{10 \ \Omega}$$
$$= \mathbf{0.6 \text{ leading}}$$

as determined above.

R-L-C

Refer to Fig. 15.35.

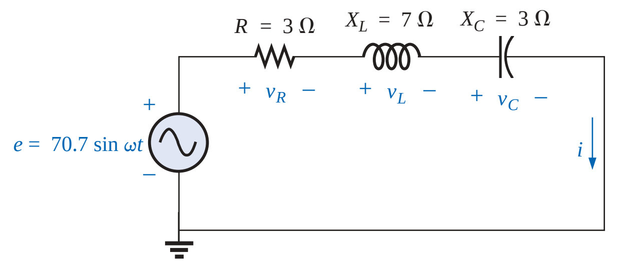


FIG. 15.35
Series R-L-C ac circuit.

Phasor Notation As shown in Fig. 15.36.

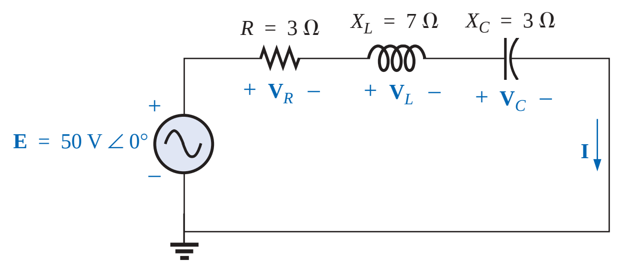


FIG. 15.36
Applying phasor notation to the circuit of Fig. 15.35.

$\mathbf{Z}_T$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R \angle 0° + X_L \angle 90° + X_C \angle -90°$$
$$= 3\ \Omega + j\,7\ \Omega - j\,3\ \Omega = 3\ \Omega + j\,4\ \Omega$$

and $$\mathbf{Z}_T = \mathbf{5\ \Omega \angle 53.13°}$$

Impedance diagram: As shown in Fig. 15.37.

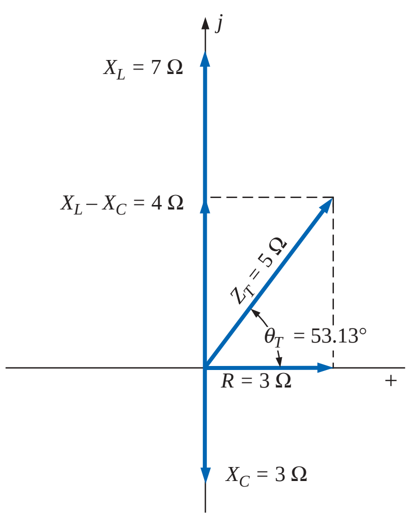


FIG. 15.37
Impedance diagram for the series R-L-C circuit of Fig. 15.35.

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\text{ V} \angle 0°}{5\ \Omega \angle 53.13°} = \mathbf{10\text{ A} \angle -53.13°}$$

$\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I \angle \theta)(R \angle 0°) = (10\text{ A} \angle -53.13°)(3\ \Omega \angle 0°)$$
$$= \mathbf{30\text{ V} \angle -53.13°}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I \angle \theta)(X_L \angle 90°) = (10\text{ A} \angle -53.13°)(7\ \Omega \angle 90°)$$
$$= \mathbf{70\text{ V} \angle 36.87°}$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90°) = (10\text{ A} \angle -53.13°)(3\ \Omega \angle -90°)$$
$$= \mathbf{30\text{ V} \angle -143.13°}$$

Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright} \mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L - \mathbf{V}_C = 0$$

or $$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C$$

which can also be verified through vector algebra.

Phasor diagram: The phasor diagram of Fig. 15.38 indicates that the current **I** is in phase with the voltage across the resistor, lags the voltage across the inductor by 90°, and leads the voltage across the capacitor by 90°.

Time domain:

$$i = \sqrt{2}(10)\sin(\omega t - 53.13°) = \mathbf{14.14\ sin(\boldsymbol{\omega} t - 53.13°)}$$
$$v_R = \sqrt{2}(30)\sin(\omega t - 53.13°) = \mathbf{42.42\ sin(\boldsymbol{\omega} t - 53.13°)}$$
$$v_L = \sqrt{2}(70)\sin(\omega t + 36.87°) = \mathbf{98.98\ sin(\boldsymbol{\omega} t + 36.87°)}$$
$$v_C = \sqrt{2}(30)\sin(\omega t - 143.13°) = \mathbf{42.42\ sin(\boldsymbol{\omega} t - 143.13°)}$$

A plot of all the voltages and the current of the circuit appears in Fig. 15.39.

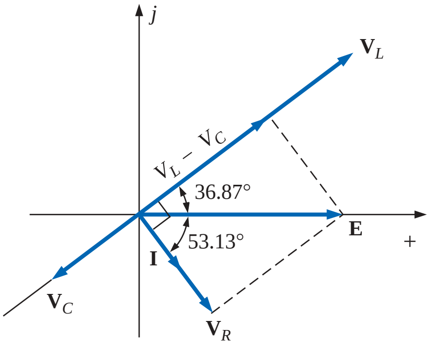


FIG. 15.38
Phasor diagram for the series R-L-C circuit of Fig. 15.35.

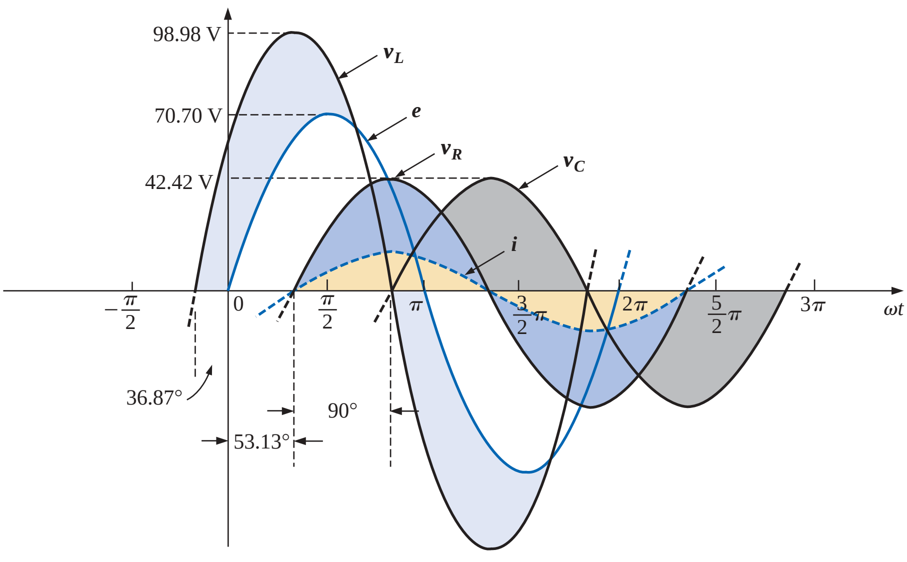


FIG. 15.39
Waveforms for the series R-L circuit of Fig. 15.35.

Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta_T = (50\text{ V})(10\text{ A})\cos 53.13° = (500)(0.6) = \mathbf{300\ W}$$

or $$P_T = I^2R = (10\text{ A})^2(3\ \Omega) = (100)(3) = \mathbf{300\ W}$$

or

$$\begin{aligned} P_T &= P_R + P_L + P_C \\ &= V_RI\cos\theta_R + V_LI\cos\theta_L + V_CI\cos\theta_C \\ &= (30\text{ V})(10\text{ A})\cos 0° + (70\text{ V})(10\text{ A})\cos 90° + (30\text{ V})(10\text{ A})\cos 90° \\ &= (30\text{ V})(10\text{ A}) + 0 + 0 = \mathbf{300\ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

Using Eq. (15.9), we obtain

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{3\ \Omega}{5\ \Omega} = \mathbf{0.6\ lagging}$$

15.4 VOLTAGE DIVIDER RULE

The basic format for the **voltage divider rule** in ac circuits is exactly the same as that for dc circuits:

$$\mathbf{V}_x = \frac{\mathbf{Z}_x\mathbf{E}}{\mathbf{Z}_T} \tag{15.10}$$

where $\mathbf{V}_x$ is the voltage across one or more elements in series that have total impedance $\mathbf{Z}_x$, $\mathbf{E}$ is the total voltage appearing across the series circuit, and $\mathbf{Z}_T$ is the total impedance of the series circuit.

EXAMPLE 15.9 Using the voltage divider rule, find the voltage across each element of the circuit of Fig. 15.40.

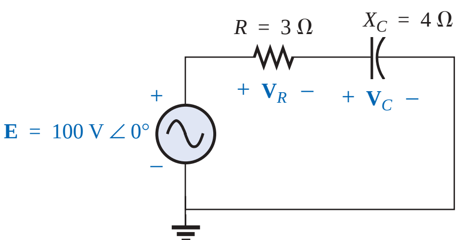


FIG. 15.40
Example 15.9.

Solution:

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(4\ \Omega \angle -90^\circ)(100\ \text{V} \angle 0^\circ)}{4\ \Omega \angle -90^\circ + 3\ \Omega \angle 0^\circ} = \frac{400 \angle -90^\circ}{3 - j\,4}$$

$$= \frac{400 \angle -90^\circ}{5 \angle -53.13^\circ} = \mathbf{80\ V \angle -36.87^\circ}$$

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(3\ \Omega \angle 0^\circ)(100\ \text{V} \angle 0^\circ)}{5\ \Omega \angle -53.13^\circ} = \frac{300 \angle 0^\circ}{5 \angle -53.13^\circ}$$

$$= \mathbf{60\ V \angle +53.13^\circ}$$

EXAMPLE 15.10 Using the voltage divider rule, find the unknown voltages $\mathbf{V}_R$, $\mathbf{V}_L$, $\mathbf{V}_C$, and $\mathbf{V}_1$ for the circuit of Fig. 15.41.

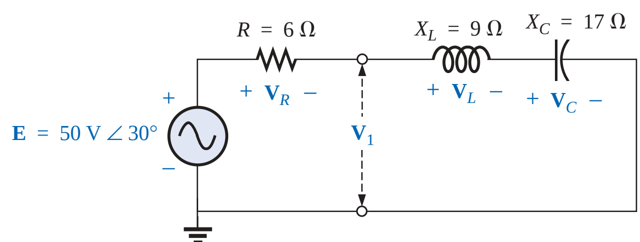


FIG. 15.41
Example 15.10.

Solution:

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\ \Omega \angle 0^\circ)(50\ \text{V} \angle 30^\circ)}{6\ \Omega \angle 0^\circ + 9\ \Omega \angle 90^\circ + 17\ \Omega \angle -90^\circ}$$

$$= \frac{300 \angle 30^\circ}{6 + j\,9 - j\,17} = \frac{300 \angle 30^\circ}{6 - j\,8}$$

$$= \frac{300 \angle 30^\circ}{10 \angle -53.13^\circ} = \mathbf{30\ V \angle 83.13^\circ}$$

Calculator The above calculation provides an excellent opportunity to demonstrate the power of today's calculators. Using the notation of the TI-86 calculator, the above calculation and the result are as follows:

```
(6∠0)*(50∠30)/((6∠0)+(9∠90)+(17∠−90))
        (3.588E0,29.785E0)
Ans ► Pol
        (30.000E0∠83.130E0)
```

CALC. 15.1

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ} = \frac{450\ \text{V}\angle 120^\circ}{10\ \angle -53.13^\circ}$$

$$= \mathbf{45\ V\ \angle 173.13^\circ}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ} = \frac{850\ \text{V}\angle -60^\circ}{10\ \angle -53^\circ}$$

$$= \mathbf{85\ V\ \angle -6.87^\circ}$$

$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90^\circ + 17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ}$$

$$= \frac{(8\ \angle -90^\circ)(50\ \angle 30^\circ)}{10\ \angle -53.13^\circ}$$

$$= \frac{400\ \angle -60^\circ}{10\ \angle -53.13^\circ} = \mathbf{40\ V\ \angle -6.87^\circ}$$

EXAMPLE 15.11 For the circuit of Fig. 15.42:

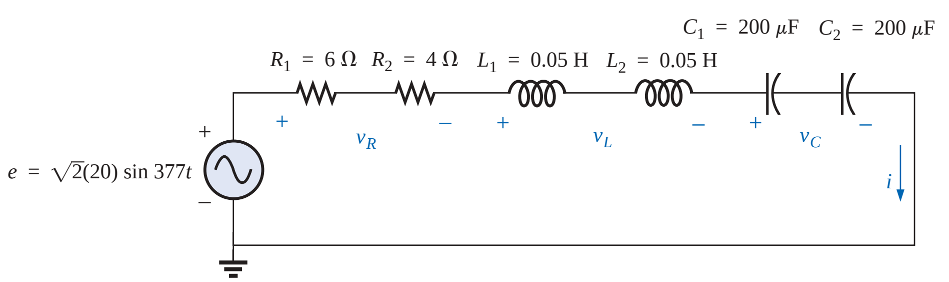


FIG. 15.42
Example 15.11.

a. Calculate **I**, $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ in phasor form.
b. Calculate the total power factor.
c. Calculate the average power delivered to the circuit.
d. Draw the phasor diagram.
e. Obtain the phasor sum of $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$, and show that it equals the input voltage **E**.
f. Find $\mathbf{V}_R$ and $\mathbf{V}_C$ using the voltage divider rule.

Solutions:

a. Combining common elements and finding the reactance of the inductor and capacitor, we obtain

$$R_T = 6\ \Omega + 4\ \Omega = 10\ \Omega$$

$$L_T = 0.05\ \text{H} + 0.05\ \text{H} = 0.1\ \text{H}$$

$$C_T = \frac{200\ \mu\text{F}}{2} = 100\ \mu\text{F}$$

$$X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.70\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(377 \text{ rad/s})(100 \times 10^{-6} \text{ F})} = \frac{10^6\ \Omega}{37{,}700} = 26.53\ \Omega$$

Redrawing the circuit using phasor notation results in Fig. 15.43.

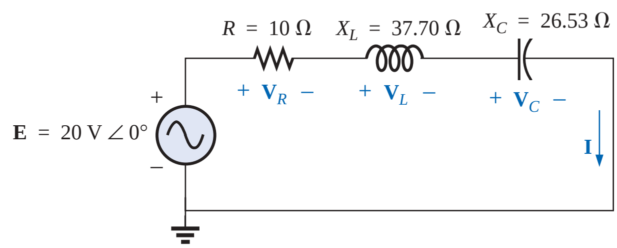


FIG. 15.43
Applying phasor notation to the circuit of Fig. 15.42.

For the circuit of Fig. 15.43,

$$\begin{aligned}\mathbf{Z}_T &= R \angle 0° + X_L \angle 90° + X_C \angle -90° \\ &= 10\ \Omega + j\,37.70\ \Omega - j\,26.53\ \Omega \\ &= 10\ \Omega + j\,11.17\ \Omega = \mathbf{15\ \Omega \angle 48.16°}\end{aligned}$$

The current $\mathbf{I}$ is

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{20 \text{ V} \angle 0°}{15\ \Omega \angle 48.16°} = \mathbf{1.33\ A \angle -48.16°}$$

The voltage across the resistor, inductor, and capacitor can be found using Ohm's law:

$$\begin{aligned}\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R &= (I \angle \theta)(R \angle 0°) = (1.33 \text{ A} \angle -48.16°)(10\ \Omega \angle 0°) \\ &= \mathbf{13.30\ V \angle -48.16°}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L &= (I \angle \theta)(X_L \angle 90°) = (1.33 \text{ A} \angle -48.16°)(37.70\ \Omega \angle 90°) \\ &= \mathbf{50.14\ V \angle 41.84°}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C &= (I \angle \theta)(X_C \angle -90°) = (1.33 \text{ A} \angle -48.16°)(26.53\ \Omega \angle -90°) \\ &= \mathbf{35.28\ V \angle -138.16°}\end{aligned}$$

b. The total power factor, determined by the angle between the applied voltage $\mathbf{E}$ and the resulting current $\mathbf{I}$, is 48.16°:

$$F_p = \cos \theta = \cos 48.16° = \mathbf{0.667\ lagging}$$

or
$$F_p = \cos \theta = \frac{R}{Z_T} = \frac{10\ \Omega}{15\ \Omega} = \mathbf{0.667\ lagging}$$

c. The total power in watts delivered to the circuit is

$$P_T = EI \cos \theta = (20 \text{ V})(1.33 \text{ A})(0.667) = \mathbf{17.74\ W}$$

d. The phasor diagram appears in Fig. 15.44.

e. The phasor sum of $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ is

$$\begin{aligned}\mathbf{E} &= \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C \\ &= 13.30 \text{ V} \angle -48.16° + 50.14 \text{ V} \angle 41.84° + 35.28 \text{ V} \angle -138.16° \\ \mathbf{E} &= 13.30 \text{ V} \angle -48.16° + 14.86 \text{ V} \angle 41.84°\end{aligned}$$

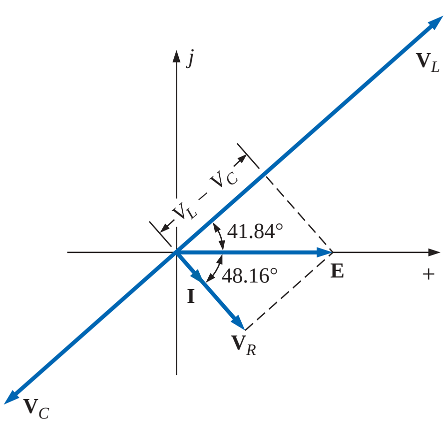


FIG. 15.44
Phasor diagram for the circuit of Fig. 15.42.

Therefore,

$$E = \sqrt{(13.30\text{ V})^2 + (14.86\text{ V})^2} = \mathbf{20\ V}$$

and $\theta_E = \mathbf{0°}$ (from phasor diagram)

and $\mathbf{E} = 20\ \angle 0°$

f. $$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_T} = \frac{(10\ \Omega\ \angle 0°)(20\text{ V}\ \angle 0°)}{15\ \Omega\ \angle 48.16°} = \frac{200\text{ V}\angle 0°}{15\ \angle 48.16°}$$
$$= \mathbf{13.3\ V\ \angle -48.16°}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(26.5\ \Omega\ \angle -90°)(20\text{ V}\ \angle 0°)}{15\ \Omega\ \angle 48.16°} = \frac{530.6\text{ V}\angle -90°}{15\ \angle 48.16°}$$
$$= \mathbf{35.37\ V\ \angle -138.16°}$$

15.5 FREQUENCY RESPONSE OF THE *R-C* CIRCUIT

Thus far, the analysis of series circuits has been limited to a particular frequency. We will now examine the effect of frequency on the response of an *R-C* series configuration such as that in Fig. 15.45. The magnitude of the source is fixed at 10 V, but the frequency range of analysis will extend from zero to 20 kHz.

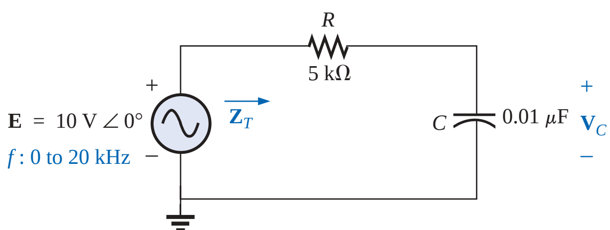


FIG. 15.45
Determining the frequency response of a series R-C circuit.

$\mathbf{Z}_T$ Let us first determine how the impedance of the circuit $\mathbf{Z}_T$ will vary with frequency for the specified frequency range of interest. Before getting into specifics, however, let us first develop a sense for what we should expect by noting the impedance-versus-frequency curve of each element, as drawn in Fig. 15.46.

At low frequencies the reactance of the capacitor will be quite high and considerably more than the level of the resistance R, suggesting that the total impedance will be primarily capacitive in nature. At high frequencies the reactance X_C will drop below the $R = 5\text{-k}\Omega$ level, and the network will start to shift toward one of a purely resistive nature (at 5 kΩ). The frequency at which $X_C = R$ can be determined in the following manner:

$$X_C = \frac{1}{2\pi f_1 C} = R$$

and
$$f_1 = \frac{1}{2\pi RC} \Big|_{X_C = R} \qquad \textbf{(15.11)}$$

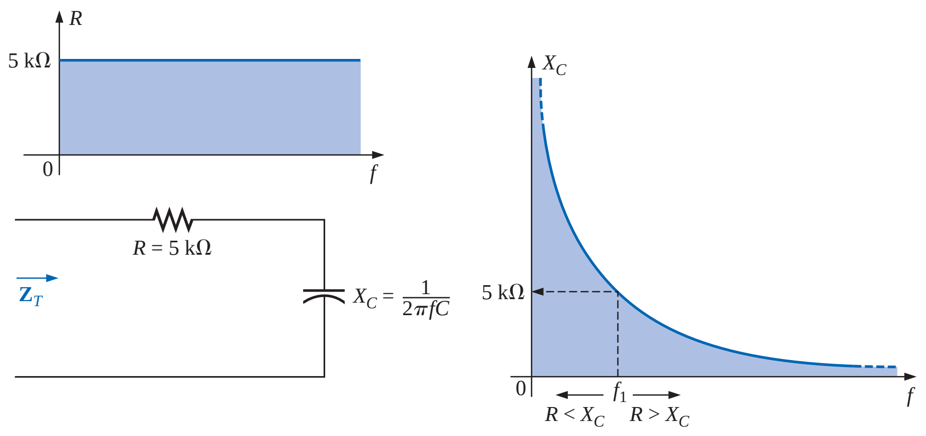


FIG. 15.46
The frequency response of the individual elements of a series R-C circuit.

which for the network of interest is

$$f_1 = \frac{1}{2\pi(5\ \text{k}\Omega)(0.01\ \mu\text{F})} \cong \mathbf{3183.1\ Hz}$$

For frequencies less than f_1, $X_C > R$, and for frequencies greater than f_1, $R > X_C$, as shown in Fig. 15.46.

Now for the details. The total impedance is determined by the following equation:

$$\mathbf{Z}_T = R - j X_C$$

and

$$\mathbf{Z}_T = Z_T \angle \theta_T = \sqrt{R^2 + X_C^2} \angle -\tan^{-1} \frac{X_C}{R} \tag{15.12}$$

The magnitude and angle of the total impedance can now be found at any frequency of interest by simply substituting into Eq. (15.12). The presence of the capacitor suggests that we start from a low frequency (100 Hz) and then open the spacing until we reach the upper limit of interest (20 kHz).

f = 100 Hz

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100\ \text{Hz})(0.01\ \mu\text{F})} = 159.16\ \text{k}\Omega$$

and $Z_T = \sqrt{R^2 + X_C^2} = \sqrt{(5\ \text{k}\Omega)^2 + (159.16\ \text{k}\Omega)^2} = 159.24\ \text{k}\Omega$

with $\theta_T = -\tan^{-1} \frac{X_C}{R} = -\tan^{-1} \frac{159.16\ \text{k}\Omega}{5\ \text{k}\Omega} = -\tan^{-1} 31.83$
$= -88.2°$

and

$$\mathbf{Z}_T = \mathbf{159.24\ k\Omega \angle -88.2°}$$

which compares very closely with $\mathbf{Z}_C = 159.16\ \text{k}\Omega \angle -90°$ if the circuit were purely capacitive ($R = 0\ \Omega$). Our assumption that the circuit is primarily capacitive at low frequencies is therefore confirmed.

***f* = 1 kHz**

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1\text{ kHz})(0.01\ \mu\text{F})} = 15.92\text{ k}\Omega$$

and $Z_T = \sqrt{R^2 + X_C^2} = \sqrt{(5\text{ k}\Omega)^2 + (15.92\text{ k}\Omega)^2} = 16.69\text{ k}\Omega$

with
$$\theta_T = -\tan^{-1}\frac{X_C}{R} = -\tan^{-1}\frac{15.92\text{ k}\Omega}{5\text{ k}\Omega}$$
$$= -\tan^{-1} 3.18 = -72.54°$$

and $\mathbf{Z}_T = \mathbf{16.69\ k\Omega \angle -72.54°}$

A noticeable drop in the magnitude has occurred, and the impedance angle has dropped almost 17° from the purely capacitive level.

Continuing:

$f = 5$ kHz: $\mathbf{Z}_T = \mathbf{5.93\ k\Omega \angle -32.48°}$
$f = 10$ kHz: $\mathbf{Z}_T = \mathbf{5.25\ k\Omega \angle -17.66°}$
$f = 15$ kHz: $\mathbf{Z}_T = \mathbf{5.11\ k\Omega \angle -11.98°}$
$f = 20$ kHz: $\mathbf{Z}_T = \mathbf{5.06\ k\Omega \angle -9.04°}$

Note how close the magnitude of Z_T at $f = 20$ kHz is to the resistance level of 5 kΩ. In addition, note how the phase angle is approaching that associated with a pure resistive network (0°).

A plot of Z_T versus frequency in Fig. 15.47 completely supports our assumption based on the curves of Fig. 15.46. The plot of θ_T versus frequency in Fig. 15.48 further suggests the fact that the total impedance made a transition from one of a capacitive nature ($\theta_T = -90°$) to one with resistive characteristics ($\theta_T = 0°$).

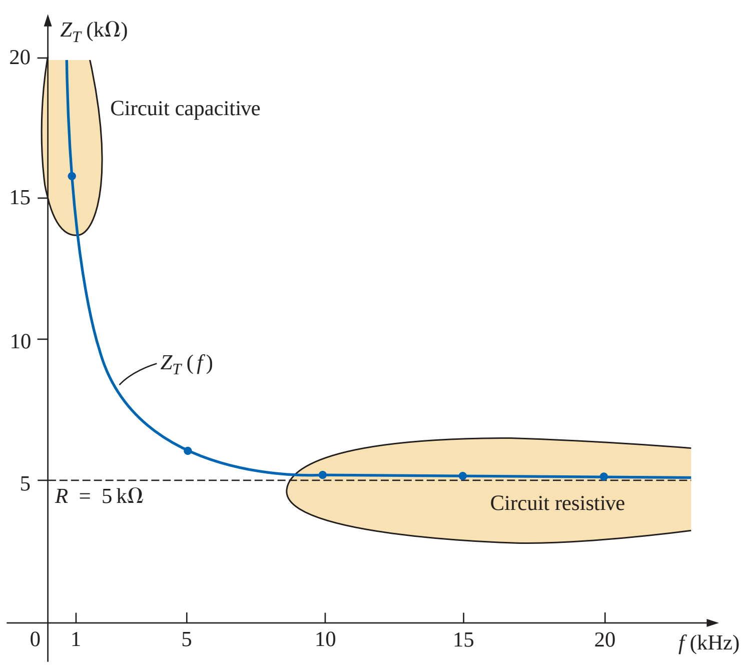


FIG. 15.47
The magnitude of the input impedance versus frequency for the circuit of Fig. 15.45.

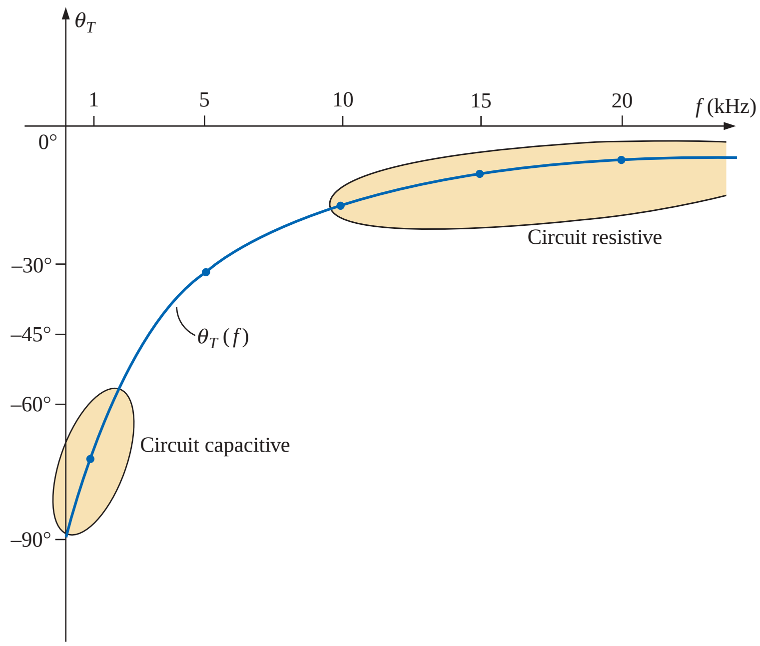


FIG. 15.48
The phase angle of the input impedance versus frequency for the circuit of Fig. 15.45.

Applying the voltage divider rule to determine the voltage across the capacitor in phasor form yields

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_C}$$
$$= \frac{(X_C \angle -90°)(E \angle 0°)}{R - j\,X_C} = \frac{X_C E \angle -90°}{R - j\,X_C}$$
$$= \frac{X_C E \angle -90°}{\sqrt{R^2 + X_C^2}\ \angle -\tan^{-1} X_C/R}$$

or
$$\mathbf{V}_C = V_C \angle \theta_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} \angle -90° + \tan^{-1}(X_C/R)$$

The magnitude of $\mathbf{V}_C$ is therefore determined by

$$V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} \tag{15.13}$$

and the phase angle θ_C by which $\mathbf{V}_C$ leads $\mathbf{E}$ is given by

$$\theta_C = -90° + \tan^{-1}\frac{X_C}{R} = -\tan^{-1}\frac{R}{X_C} \tag{15.14}$$

To determine the frequency response, X_C must be calculated for each frequency of interest and inserted into Eqs. (15.13) and (15.14).

To begin our analysis, it makes good sense to consider the case of $f = 0$ Hz (dc conditions).

***f* = 0 Hz**

$$X_C = \frac{1}{2\pi(0)C} = \frac{1}{0} \Rightarrow \text{very large value}$$

Applying the open-circuit equivalent for the capacitor based on the above calculation will result in the following:

$$\mathbf{V}_C = \mathbf{E} = 10\text{ V}\angle 0°$$

If we apply Eq. (15.13), we find

$$X_C^2 >> R^2$$

and
$$\sqrt{R^2 + X_C^2} \cong \sqrt{X_C^2} = X_C$$

and
$$V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} = \frac{X_C E}{X_C} = E$$

with
$$\theta_C = -\tan^{-1}\frac{R}{X_C} = -\tan^{-1} 0 = 0°$$

verifying the above conclusions.

***f* = 1 kHz** Applying Eq. (15.13):

$$X_C = \frac{1}{2\pi fC} = \frac{1}{(2\pi)(1 \times 10^3\text{ Hz})(0.01 \times 10^{-6}\text{ F})} \cong \mathbf{15.92\ k\Omega}$$

$$\sqrt{R^2 + X_C^2} = \sqrt{(5\text{ k}\Omega)^2 + (15.92\text{ k}\Omega)^2} \cong 16.69\text{ k}\Omega$$

and
$$V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} = \frac{(15.92\text{ k}\Omega)(10)}{16.69\text{ k}\Omega} = \mathbf{9.54\ V}$$

Applying Eq. (15.14):

$$\theta_C = -\tan^{-1}\frac{R}{X_C} = -\tan^{-1}\frac{5\text{ k}\Omega}{15.9\text{ k}\Omega}$$

$$= -\tan^{-1} 0.314 = \mathbf{-17.46°}$$

and
$$\mathbf{V}_C = \mathbf{9.53\ V}\angle\mathbf{-17.46°}$$

As expected, the high reactance of the capacitor at low frequencies has resulted in the major part of the applied voltage appearing across the capacitor.

If we plot the phasor diagrams for $f = 0$ Hz and $f = 1$ kHz, as shown in Fig. 15.49, we find that $\mathbf{V}_C$ is beginning a clockwise rotation with an increase in frequency that will increase the angle θ_C and decrease the phase angle between **I** and **E.** Recall that for a purely capacitive net-

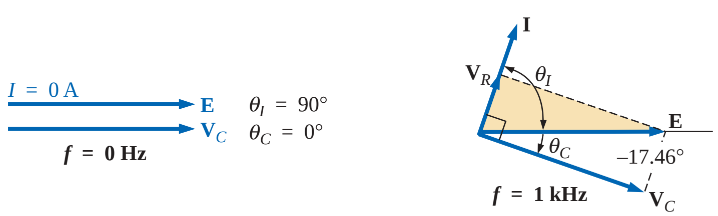


FIG. 15.49

The phasor diagram for the circuit of Fig. 15.45 for f = 0 Hz and 1 kHz.

work, **I** leads **E** by 90°. As the frequency increases, therefore, the capacitive reactance is decreasing, and eventually $R >> X_C$ with $\theta_C = -90°$, and the angle between **I** and **E** will approach 0°. Keep in mind as we proceed through the other frequencies that θ_C is the phase angle between $\mathbf{V}_C$ and **E** and that the magnitude of the angle by which **I** leads **E** is determined by

$$|\theta_I| = 90° - |\theta_C| \tag{15.15}$$

***f* = 5 kHz** Applying Eq. (15.13):

$$X_C = \frac{1}{2\pi fC} = \frac{1}{(2\pi)(5 \times 10^3\ \text{Hz})(0.01 \times 10^{-6}\ \text{F})} \cong \mathbf{3.18\ k\Omega}$$

Note the dramatic drop in X_C from 1 kHz to 5 kHz. In fact, X_C is now less than the resistance R of the network, and the phase angle determined by $\tan^{-1}(X_C/R)$ must be less than 45°. Here,

$$V_C = \frac{X_C E}{\sqrt{R^2 + X_C^2}} = \frac{(3.18\ \text{k}\Omega)(10\ \text{V})}{\sqrt{(5\ \text{k}\Omega)^2 + (3.18\ \text{k}\Omega)^2}} = \mathbf{5.37\ V}$$

with $$\theta_C = -\tan^{-1}\frac{R}{X_C} = -\tan^{-1}\frac{5\ \text{k}\Omega}{3.2\ \text{k}\Omega}$$

$$= -\tan^{-1} 1.56 = \mathbf{-57.38°}$$

***f* = 10 kHz**

$$X_C \cong \mathbf{1.59\ k\Omega} \qquad V_C = \mathbf{3.03\ V} \qquad \theta_C = \mathbf{-72.34°}$$

***f* = 15 kHz**

$$X_C \cong \mathbf{1.06\ k\Omega} \qquad V_C = \mathbf{2.07\ V} \qquad \theta_C = \mathbf{-78.02°}$$

***f* = 20 kHz**

$$X_C \cong \mathbf{795.78\ \Omega} \qquad V_C = \mathbf{1.57\ V} \qquad \theta_C = \mathbf{-80.96°}$$

The phasor diagrams for $f = 5$ kHz and $f = 20$ kHz appear in Fig. 15.50 to show the continuing rotation of the $\mathbf{V}_C$ vector.

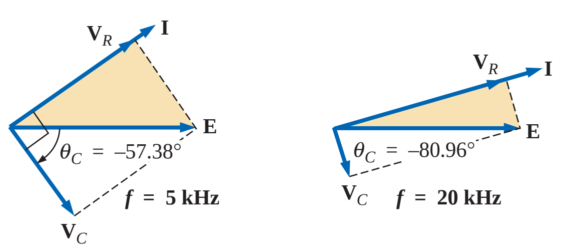


FIG. 15.50
The phasor diagram for the circuit of Fig. 15.45 for f = 5 kHz and 20 kHz.

Note also from Figs. 15.49 and 15.50 that the vector $\mathbf{V}_R$ and the current **I** have grown in magnitude with the reduction in the capacitive reactance. Eventually, at very high frequencies X_C will approach zero ohms and the short-circuit equivalent can be applied, resulting in $V_C \cong 0$ V and $\theta_C \cong -90°$, and producing the phasor diagram of Fig. 15.51. The network is then resistive, the phase angle between **I** and **E** is essentially zero degrees, and V_R and I are their maximum values.

A plot of V_C versus frequency appears in Fig. 15.52. At low frequencies $X_C >> R$, and V_C is very close to E in magnitude. As the applied frequency increases, X_C decreases in magnitude along with V_C as V_R captures more of the applied voltage. A plot of θ_C versus frequency is provided in Fig. 15.53. At low frequencies the phase angle

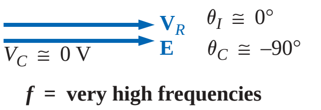


FIG. 15.51
The phasor diagram for the circuit of Fig. 15.45 at very high frequencies.

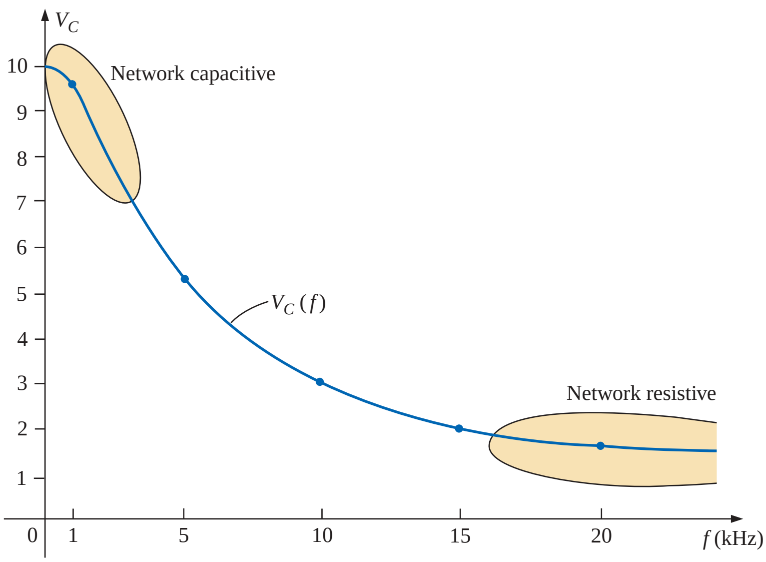


FIG. 15.52
The magnitude of the voltage V_C versus frequency for the circuit of Fig. 15.45.

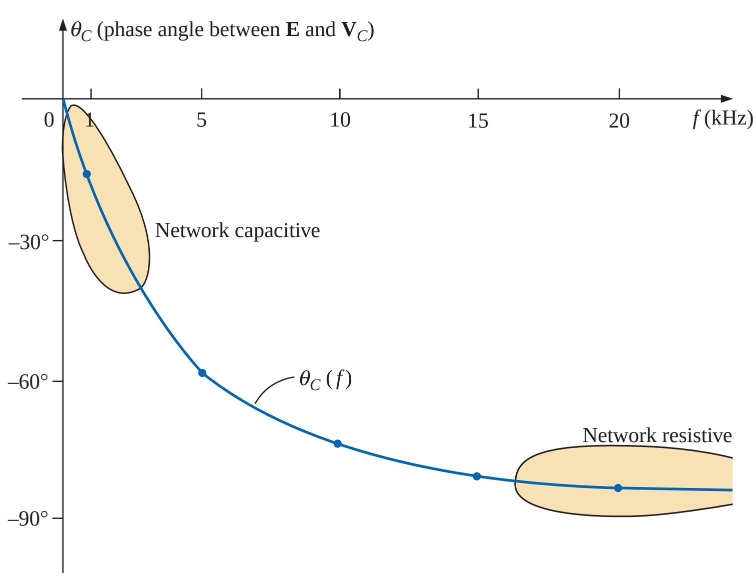


FIG. 15.53
*The phase angle between **E** and $\mathbf{V}_C$ versus frequency for the circuit of Fig. 15.45.*

between $\mathbf{V}_C$ and $\mathbf{E}$ is very small since $\mathbf{V}_C \cong \mathbf{E}$. Recall that if two phasors are equal, they must have the same angle. As the applied frequency increases, the network becomes more resistive and the phase angle between $\mathbf{V}_C$ and $\mathbf{E}$ approaches 90°. Keep in mind that, at high frequencies, $\mathbf{I}$ and $\mathbf{E}$ are approaching an in-phase situation and the angle between $\mathbf{V}_C$ and $\mathbf{E}$ will approach that between $\mathbf{V}_C$ and $\mathbf{I}$, which we know must be 90° ($\mathbf{I}_C$ leading $\mathbf{V}_C$).

A plot of V_R versus frequency would approach E volts from zero volts with an increase in frequency, but remember $V_R \neq E - V_C$ due to the vector relationship. The phase angle between **I** and **E** could be plotted directly from Fig. 15.53 using Eq. (15.15).

In Chapter 23, the analysis of this section will be extended to a much wider frequency range using a log axis for frequency. It will be demonstrated that an *R-C* circuit such as that in Fig. 15.45 can be used as a filter to determine which frequencies will have the greatest impact on the stage to follow. From our current analysis, it is obvious that any network connected across the capacitor will receive the greatest potential level at low frequencies and be effectively "shorted out" at very high frequencies.

The analysis of a series *R-L* circuit would proceed in much the same manner, except that X_L and V_L would increase with frequency and the angle between **I** and **E** would approach 90° (voltage leading the current) rather than 0°. If $\mathbf{V}_L$ were plotted versus frequency, $\mathbf{V}_L$ would approach **E**, and X_L would eventually attain a level at which the open-circuit equivalent would be appropriate.

15.6 SUMMARY: SERIES ac CIRCUITS

The following is a review of important conclusions that can be derived from the discussion and examples of the previous sections. The list is not all-inclusive, but it does emphasize some of the conclusions that should be carried forward in the future analysis of ac systems.

For series ac circuits with reactive elements:

1. ***The total impedance will be frequency dependent.***
2. ***The impedance of any one element can be greater than the total impedance of the network.***
3. ***The inductive and capacitive reactances are always in direct opposition on an impedance diagram.***
4. ***Depending on the frequency applied, the same circuit can be either predominantly inductive or predominantly capacitive.***
5. ***At lower frequencies the capacitive elements will usually have the most impact on the total impedance, while at high frequencies the inductive elements will usually have the most impact.***
6. ***The magnitude of the voltage across any one element can be greater than the applied voltage.***
7. ***The magnitude of the voltage across an element compared to the other elements of the circuit is directly related to the magnitude of its impedance; that is, the larger the impedance of an element, the larger the magnitude of the voltage across the element.***
8. ***The voltages across a coil or capacitor are always in direct opposition on a phasor diagram.***
9. ***The current is always in phase with the voltage across the resistive elements, lags the voltage across all the inductive***

elements by 90°, and leads the voltage across all the capacitive elements by 90°.

10. The larger the resistive element of a circuit compared to the net reactive impedance, the closer the power factor is to unity.

PARALLEL ac CIRCUITS

15.7 ADMITTANCE AND SUSCEPTANCE

The discussion for **parallel ac circuits** will be very similar to that for dc circuits. In dc circuits, *conductance* (G) was defined as being equal to $1/R$. The total conductance of a parallel circuit was then found by adding the conductance of each branch. The total resistance R_T is simply $1/G_T$.

In ac circuits, we define **admittance** ($\mathbf{Y}$) as being equal to $1/\mathbf{Z}$. The unit of measure for admittance as defined by the SI system is *siemens*, which has the symbol S. Admittance is a measure of how well an ac circuit will *admit*, or allow, current to flow in the circuit. The larger its value, therefore, the heavier the current flow for the same applied potential. The total admittance of a circuit can also be found by finding the sum of the parallel admittances. The total impedance $\mathbf{Z}_T$ of the circuit is then $1/\mathbf{Y}_T$; that is, for the network of Fig. 15.54:

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N \tag{15.16}$$

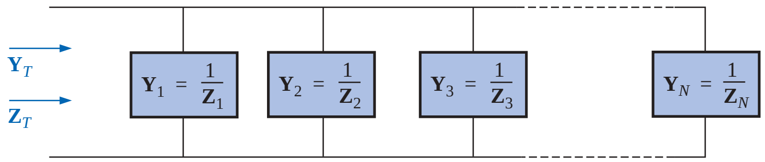


FIG. 15.54
Parallel ac network.

or, since $\mathbf{Z} = 1/\mathbf{Y}$,

$$\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N} \tag{15.17}$$

For two impedances in parallel,

$$\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}$$

If the manipulations used in Chapter 6 to find the total resistance of two parallel resistors are now applied, the following similar equation will result:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \tag{15.18}$$

For three parallel impedances,

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3} \qquad \textbf{(15.19)}$$

As pointed out in the introduction to this section, conductance is the reciprocal of resistance, and

$$\mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ \qquad \textbf{(15.20)}$$

The reciprocal of reactance ($1/X$) is called **susceptance** and is a measure of how *susceptible* an element is to the passage of current through it. Susceptance is also measured in *siemens* and is represented by the capital letter B.

For the inductor,

$$\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = \frac{1}{X_L\angle 90^\circ} = \frac{1}{X_L}\angle -90^\circ \qquad \textbf{(15.21)}$$

Defining $$B_L = \frac{1}{X_L} \quad \text{(siemens, S)} \qquad \textbf{(15.22)}$$

we have $$\mathbf{Y}_L = B_L\angle -90^\circ \qquad \textbf{(15.23)}$$

Note that for inductance, an increase in frequency or inductance will result in a decrease in susceptance or, correspondingly, in admittance.

For the capacitor,

$$\mathbf{Y}_C = \frac{1}{\mathbf{Z}_C} = \frac{1}{X_C\angle -90^\circ} = \frac{1}{X_C}\angle 90^\circ \qquad \textbf{(15.24)}$$

Defining $$B_C = \frac{1}{X_C} \quad \text{(siemens, S)} \qquad \textbf{(15.25)}$$

we have $$\mathbf{Y}_C = B_C\angle 90^\circ \qquad \textbf{(15.26)}$$

For the capacitor, therefore, an increase in frequency or capacitance will result in an increase in its susceptibility.

For parallel ac circuits, the **admittance diagram** is used with the three admittances, represented as shown in Fig. 15.55.

Note in Fig. 15.55 that the conductance (like resistance) is on the positive real axis, whereas inductive and capacitive susceptances are in direct opposition on the imaginary axis.

For any configuration (series, parallel, series-parallel, etc.), the angle associated with the total admittance is the angle by which the source current leads the applied voltage. For inductive networks, θ_T is negative, whereas for capacitive networks, θ_T is positive.

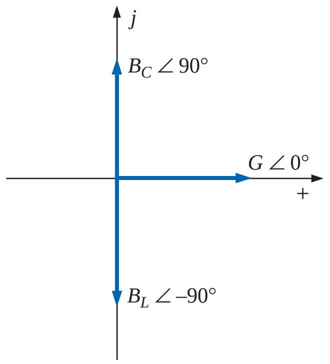


FIG. 15.55
Admittance diagram.

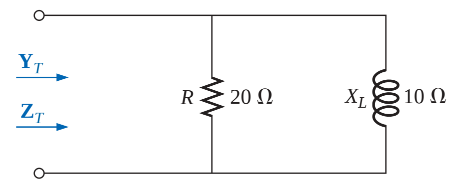


FIG. 15.56
Example 15.12.

EXAMPLE 15.12 For the network of Fig. 15.56:

a. Find the admittance of each parallel branch.
b. Determine the input admittance.
c. Calculate the input impedance.
d. Draw the admittance diagram.

Solutions:

a. $\mathbf{Y}_R = G\,\angle 0° = \dfrac{1}{R}\,\angle 0° = \dfrac{1}{20\,\Omega}\,\angle 0°$

$= \mathbf{0.05\ S\ \angle 0° = 0.05\ S} + \boldsymbol{j}\ \mathbf{0}$

$\mathbf{Y}_L = B_L\,\angle -90° = \dfrac{1}{X_L}\,\angle -90° = \dfrac{1}{10\,\Omega}\,\angle -90°$

$= \mathbf{0.1\ S\ \angle -90° = 0} - \boldsymbol{j}\ \mathbf{0.1\ S}$

b. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L = (0.05\text{ S} + j\,0) + (0 - j\,0.1\text{ S})$

$= \mathbf{0.05\ S} - \boldsymbol{j}\ \mathbf{0.1\ S} = G - jB_L$

c. $\mathbf{Z}_T = \dfrac{1}{\mathbf{Y}_T} = \dfrac{1}{0.05\text{ S} - j\,0.1\text{ S}} = \dfrac{1}{0.112\text{ S}\,\angle -63.43°}$

$= \mathbf{8.93\ \Omega\ \angle 63.43°}$

or Eq. (15.17):

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(20\,\Omega\,\angle 0°)(10\,\Omega\,\angle 90°)}{20\,\Omega + j\,10\,\Omega}$$

$$= \frac{200\,\Omega\,\angle 90°}{22.36\,\angle 26.57°} = \mathbf{8.93\ \Omega\ \angle 63.43°}$$

d. The admittance diagram appears in Fig. 15.57.

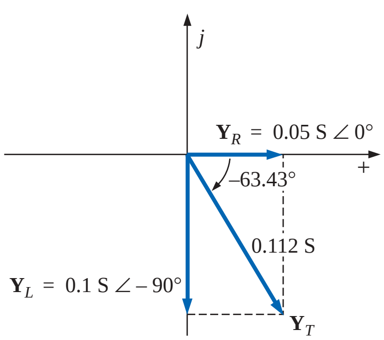


FIG. 15.57
Admittance diagram for the network of Fig. 15.56.

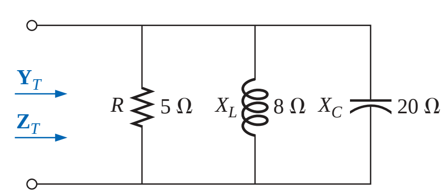


FIG. 15.58
Example 15.13.

EXAMPLE 15.13 Repeat Example 15.12 for the parallel network of Fig. 15.58.

Solutions:

a. $\mathbf{Y}_R = G\,\angle 0° = \dfrac{1}{R}\,\angle 0° = \dfrac{1}{5\,\Omega}\,\angle 0°$

$= \mathbf{0.2\ S\ \angle 0° = 0.2\ S} + \boldsymbol{j}\ \mathbf{0}$

$$\mathbf{Y}_L = B_L \angle -90^\circ = \frac{1}{X_L} \angle -90^\circ = \frac{1}{8\ \Omega} \angle -90^\circ$$
$$= \mathbf{0.125\ S \angle -90^\circ} = \mathbf{0 - j\, 0.125\ S}$$
$$\mathbf{Y}_C = B_C \angle 90^\circ = \frac{1}{X_C} \angle 90^\circ = \frac{1}{20\ \Omega} \angle 90^\circ$$
$$= \mathbf{0.050\ S \angle +90^\circ} = \mathbf{0 + j\, 0.050\ S}$$

b. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C$
$$= (0.2\ \text{S} + j\,0) + (0 - j\,0.125\ \text{S}) + (0 + j\,0.050\ \text{S})$$
$$= 0.2\ \text{S} - j\,0.075\ \text{S} = \mathbf{0.2136\ S \angle -20.56^\circ}$$

c. $\mathbf{Z}_T = \dfrac{1}{0.2136\ \text{S} \angle -20.56^\circ} = \mathbf{4.68\ \Omega \angle 20.56^\circ}$

or

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L\mathbf{Z}_C}{\mathbf{Z}_R\mathbf{Z}_L + \mathbf{Z}_L\mathbf{Z}_C + \mathbf{Z}_R\mathbf{Z}_C}$$
$$= \frac{(5\ \Omega \angle 0^\circ)(8\ \Omega \angle 90^\circ)(20\ \Omega \angle -90^\circ)}{(5\ \Omega \angle 0^\circ)(8\ \Omega \angle 90^\circ) + (8\ \Omega \angle 90^\circ)(20\ \Omega \angle -90^\circ) + (5\ \Omega \angle 0^\circ)(20\ \Omega \angle -90^\circ)}$$
$$= \frac{800\ \Omega \angle 0^\circ}{40 \angle 90^\circ + 160 \angle 0^\circ + 100 \angle -90^\circ}$$
$$= \frac{800\ \Omega}{160 + j\,40 - j\,100} = \frac{800\ \Omega}{160 - j\,60}$$
$$= \frac{800\ \Omega}{170.88 \angle -20.56^\circ}$$
$$= \mathbf{4.68\ \Omega \angle 20.56^\circ}$$

d. The admittance diagram appears in Fig. 15.59.

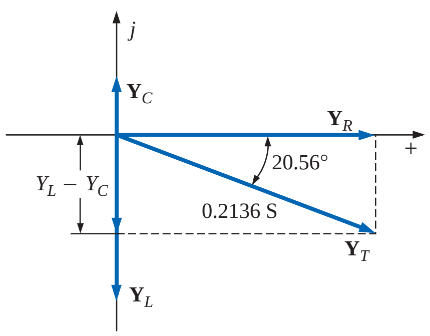


FIG. 15.59
Admittance diagram for the network of Fig. 15.58.

On many occasions, the inverse relationship $\mathbf{Y}_T = 1/\mathbf{Z}_T$ or $\mathbf{Z}_T = 1/\mathbf{Y}_T$ will require that we divide the number 1 by a complex number having a real and an imaginary part. This division, if not performed in the polar form, requires that we multiply the numerator and denominator by the conjugate of the denominator, as follows:

$$\mathbf{Y}_T = \frac{1}{\mathbf{Z}_T} = \frac{1}{4\ \Omega + j\,6\ \Omega} = \left(\frac{1}{4\ \Omega + j\,6\ \Omega}\right)\left(\frac{(4\ \Omega - j\,6\ \Omega)}{(4\ \Omega - j\,6\ \Omega)}\right) = \frac{4 - j\,6}{4^2 + 6^2}$$

and
$$\mathbf{Y}_T = \frac{4}{52}\ \text{S} - j\,\frac{6}{52}\ \text{S}$$

To avoid this laborious task each time we want to find the reciprocal of a complex number in rectangular form, a format can be developed using the following complex number, which is symbolic of any impedance or admittance in the first or fourth quadrant:

$$\frac{1}{a_1 \pm j\,b_1} = \left(\frac{1}{a_1 \pm j\,b_1}\right)\left(\frac{a_1 \mp j\,b_1}{a_1 \mp j\,b_1}\right) = \frac{a_1 \mp j\,b_1}{a_1^2 + b_1^2}$$

or
$$\frac{1}{a_1 \pm j\,b_1} = \frac{a_1}{a_1^2 + b_1^2} \mp j\,\frac{b_1}{a_1^2 + b_1^2} \qquad \textbf{(15.27)}$$

Note that the denominator is simply the sum of the squares of each term. The sign is inverted between the real and imaginary parts. A few examples will develop some familiarity with the use of this equation.

EXAMPLE 15.14 Find the admittance of each set of series elements in Fig. 15.60.

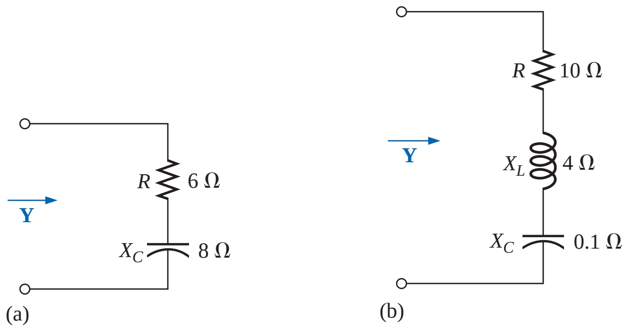


FIG. 15.60
Example 15.14.

Solutions:

a. $\mathbf{Z} = R - j\,X_C = 6\ \Omega - j\,8\ \Omega$

Eq. (15.27):

$$\mathbf{Y} = \frac{1}{6\ \Omega - j\,8\ \Omega} = \frac{6}{(6)^2 + (8)^2} + j\frac{8}{(6)^2 + (8)^2}$$

$$= \mathbf{\frac{6}{100}\ S + j\frac{8}{100}\ S}$$

b. $\mathbf{Z} = 10\ \Omega + j\,4\ \Omega + (-j\,0.1\ \Omega) = 10\ \Omega + j\,3.9\ \Omega$

Eq. (15.27):

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{10\ \Omega + j\,3.9\ \Omega} = \frac{10}{(10)^2 + (3.9)^2} - j\frac{3.9}{(10)^2 + (3.9)^2}$$

$$= \frac{10}{115.21} - j\frac{3.9}{115.21} = \mathbf{0.087\ S - j\,0.034\ S}$$

15.8 PARALLEL ac NETWORKS

For the representative parallel ac network of Fig. 15.61, the total impedance or admittance is determined as described in the previous section, and the source current is determined by Ohm's law as follows:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T \tag{15.28}$$

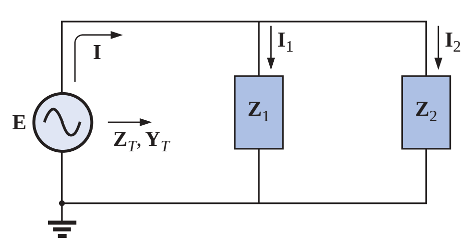


FIG. 15.61
Parallel ac network.

Since the voltage is the same across parallel elements, the current through each branch can then be found through another application of Ohm's law:

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \mathbf{E}\mathbf{Y}_1 \tag{15.29a}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \mathbf{E}\mathbf{Y}_2 \tag{15.29b}$$

Kirchhoff's current law can then be applied in the same manner as employed for dc networks. However, keep in mind that we are now dealing with the algebraic manipulation of quantities that have both magnitude and direction.

$$\mathbf{I} - \mathbf{I}_1 - \mathbf{I}_2 = 0$$

or

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 \quad \textbf{(15.30)}$$

The power to the network can be determined by

$$P = EI \cos \theta_T \quad \textbf{(15.31)}$$

where θ_T is the phase angle between **E** and **I**.

Let us now look at a few examples carried out in great detail for the first exposure.

R-L

Refer to Fig. 15.62.

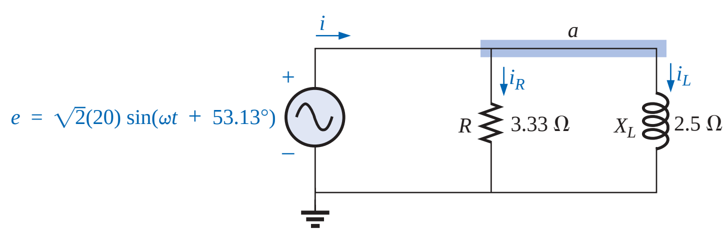


FIG. 15.62
Parallel R-L network.

Phasor Notation As shown in Fig. 15.63.

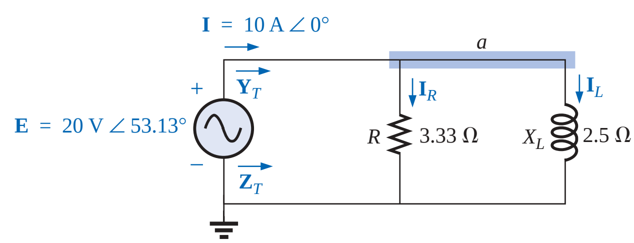


FIG. 15.63
Applying phasor notation to the network of Fig. 15.62.

$\mathbf{Y}_T$ and $\mathbf{Z}_T$

$$\begin{aligned}
\mathbf{Y}_T &= \mathbf{Y}_R + \mathbf{Y}_L \\
&= G \angle 0° + B_L \angle -90° = \frac{1}{3.33\ \Omega} \angle 0° + \frac{1}{2.5\ \Omega} \angle -90° \\
&= 0.3\ \text{S} \angle 0° + 0.4\ \text{S} \angle -90° = 0.3\ \text{S} - j\,0.4\ \text{S} \\
&= \mathbf{0.5\ S \angle -53.13°}
\end{aligned}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\ \text{S} \angle -53.13°} = \mathbf{2\ \Omega \angle 53.13°}$$

Admittance diagram: As shown in Fig. 15.64.

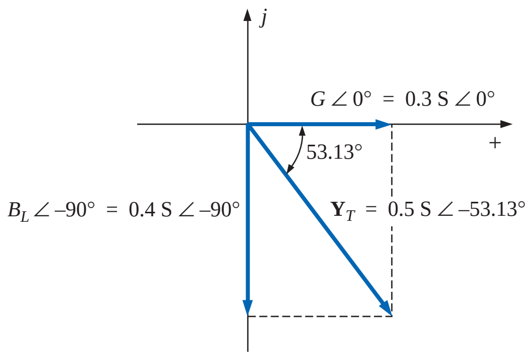


FIG. 15.64
Admittance diagram for the parallel R-L network of Fig. 15.62.

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (20 \text{ V} \angle 53.13°)(0.5 \text{ S} \angle -53.13°) = \mathbf{10 \text{ A} \angle 0°}$$

I_R and I_L

$$\mathbf{I}_R = \frac{E \angle \theta}{R \angle 0°} = (E \angle \theta)(G \angle 0°)$$
$$= (20 \text{ V} \angle 53.13°)(0.3 \text{ S} \angle 0°) = \mathbf{6 \text{ A} \angle 53.13°}$$
$$\mathbf{I}_L = \frac{E \angle \theta}{X_L \angle 90°} = (E \angle \theta)(B_L \angle -90°)$$
$$= (20 \text{ V} \angle 53.13°)(0.4 \text{ S} \angle -90°)$$
$$= \mathbf{8 \text{ A} \angle -36.87°}$$

Kirchhoff's current law: At node *a*,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L = 0$$

or

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L$$
$$10 \text{ A} \angle 0° = 6 \text{ A} \angle 53.13° + 8 \text{ A} \angle -36.87°$$
$$10 \text{ A} \angle 0° = (3.60 \text{ A} + j\,4.80 \text{ A}) + (6.40 \text{ A} - j\,4.80 \text{ A}) = 10 \text{ A} + j\,0$$

and $\mathbf{10 \text{ A} \angle 0° = 10 \text{ A} \angle 0°}$ (checks)

Phasor diagram: The phasor diagram of Fig. 15.65 indicates that the applied voltage $\mathbf{E}$ is in phase with the current $\mathbf{I}_R$ and leads the current $\mathbf{I}_L$ by 90°.

Power: The total power in watts delivered to the circuit is

$$P_T = EI \cos \theta_T$$
$$= (20 \text{ V})(10 \text{ A}) \cos 53.13° = (200 \text{ W})(0.6)$$
$$= \mathbf{120 \text{ W}}$$

or $$P_T = I^2R = \frac{V_R^2}{R} = V_R^2 G = (20 \text{ V})^2(0.3 \text{ S}) = \mathbf{120 \text{ W}}$$

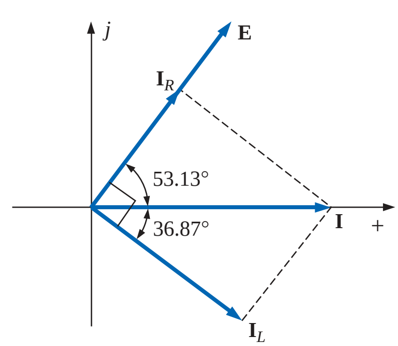


FIG. 15.65
Phasor diagram for the parallel R-L network of Fig. 15.62.

or, finally,

$$\begin{aligned}P_T &= P_R + P_L = EI_R \cos\theta_R + EI_L \cos\theta_L \\ &= (20\text{ V})(6\text{ A})\cos 0° + (20\text{ V})(8\text{ A})\cos 90° = 120\text{ W} + 0 \\ &= \mathbf{120\ W}\end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

or, through an analysis similar to that employed for a series ac circuit,

$$\cos\theta_T = \frac{P}{EI} = \frac{E^2/R}{EI} = \frac{EG}{I} = \frac{G}{I/V} = \frac{G}{Y_T}$$

and

$$F_p = \cos\theta_T = \frac{G}{Y_T} \qquad \textbf{(15.32)}$$

where G and Y_T are the magnitudes of the total conductance and admittance of the parallel network. For this case,

$$F_p = \cos\theta_T = \frac{0.3\text{ S}}{0.5\text{ S}} = \mathbf{0.6\ lagging}$$

Impedance approach: The current **I** can also be found by first finding the total impedance of the network:

$$\begin{aligned}\mathbf{Z}_T &= \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(3.33\ \Omega\ \angle 0°)(2.5\ \Omega\ \angle 90°)}{3.33\ \Omega\ \angle 0° + 2.5\ \Omega\ \angle 90°} \\ &= \frac{8.325\ \angle 90°}{4.164\ \angle 36.87°} = \mathbf{2\ \Omega\ \angle 53.13°}\end{aligned}$$

And then, using Ohm's law, we obtain

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{20\text{ V}\ \angle 53.13°}{2\ \Omega\ \angle 53.13°} = \mathbf{10\ A\ \angle 0°}$$

R-C

Refer to Fig. 15.66.

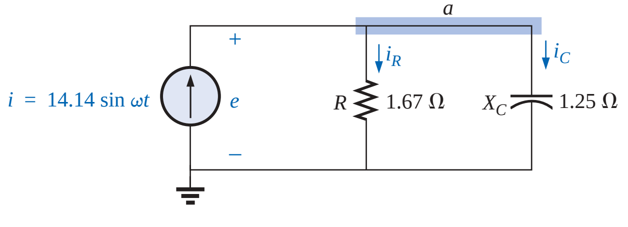


FIG. 15.66
Parallel R-C network.

Phasor Notation As shown in Fig. 15.67.

$\mathbf{Y}_T$ and $\mathbf{Z}_T$

$$\begin{aligned}\mathbf{Y}_T &= \mathbf{Y}_R + \mathbf{Y}_C = G\ \angle 0° + B_C\ \angle 90° = \frac{1}{1.67\ \Omega}\ \angle 0° + \frac{1}{1.25\ \Omega}\ \angle 90° \\ &= 0.6\text{ S}\ \angle 0° + 0.8\text{ S}\ \angle 90° = 0.6\text{ S} + j\,0.8\text{ S} = \mathbf{1.0\ S\ \angle 53.13°}\end{aligned}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{1.0\text{ S}\ \angle 53.13°} = \mathbf{1\ \Omega\ \angle{-53.13°}}$$

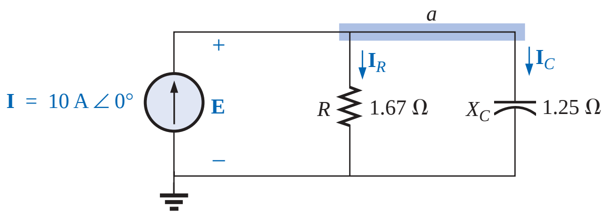


FIG. 15.67
Applying phasor notation to the network of Fig. 15.66.

Admittance diagram: As shown in Fig. 15.68.

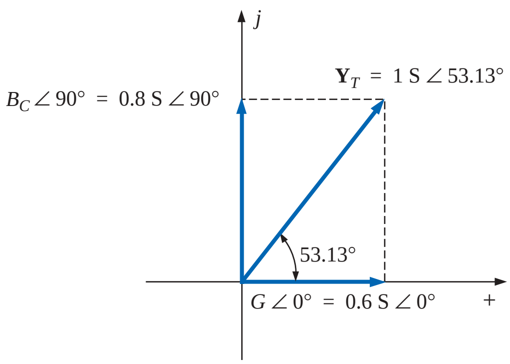


FIG. 15.68
Admittance diagram for the parallel R-C network of Fig. 15.66.

E

$$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = \frac{\mathbf{I}}{\mathbf{Y}_T} = \frac{10 \text{ A} \angle 0°}{1 \text{ S} \angle 53.13°} = \mathbf{10 \text{ V} \angle -53.13°}$$

$\mathbf{I}_R$ and $\mathbf{I}_C$

$$\mathbf{I}_R = (E \angle \theta)(G \angle 0°)$$
$$= (10 \text{ V} \angle -53.13°)(0.6 \text{ S} \angle 0°) = \mathbf{6 \text{ A} \angle -53.13°}$$
$$\mathbf{I}_C = (E \angle \theta)(B_C \angle 90°)$$
$$= (10 \text{ V} \angle -53.13°)(0.8 \text{ S} \angle 90°) = \mathbf{8 \text{ A} \angle 36.87°}$$

Kirchhoff's current law: At node *a*,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_C = 0$$

or

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_C$$

which can also be verified (as for the *R-L* network) through vector algebra.

Phasor diagram: The phasor diagram of Fig. 15.69 indicates that **E** is in phase with the current through the resistor $\mathbf{I}_R$ and lags the capacitive current $\mathbf{I}_C$ by 90°.

Time domain:

$$e = \sqrt{2}(10) \sin(\omega t - 53.13°) = \mathbf{14.14 \sin(\omega t - 53.13°)}$$
$$i_R = \sqrt{2}(6) \sin(\omega t - 53.13°) = \mathbf{8.48 \sin(\omega t - 53.13°)}$$
$$i_C = \sqrt{2}(8) \sin(\omega t + 36.87°) = \mathbf{11.31 \sin(\omega t + 36.87°)}$$

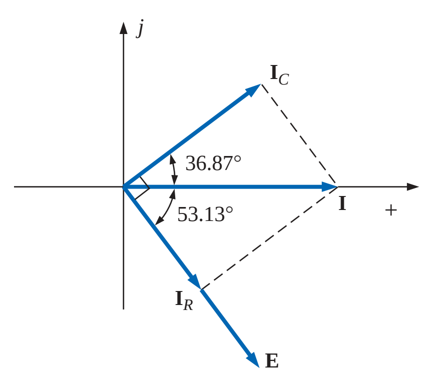


FIG. 15.69
Phasor diagram for the parallel R-C network of Fig. 15.66.

A plot of all of the currents and the voltage appears in Fig. 15.70. Note that e and i_R are in phase and e lags i_C by 90°.

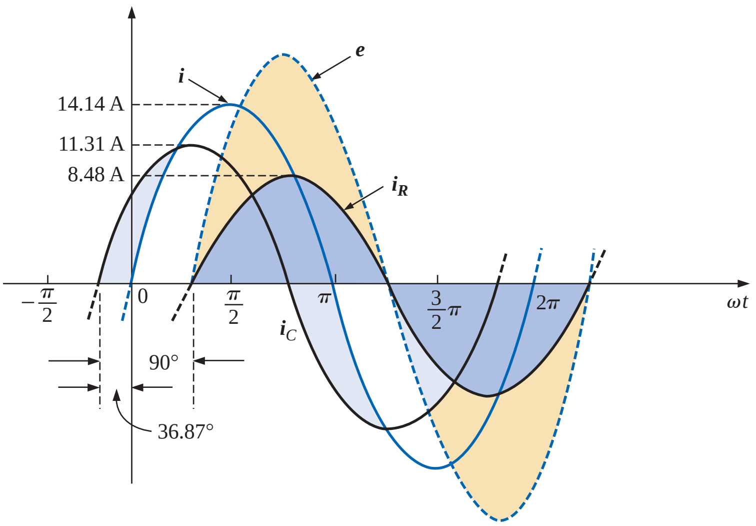


FIG. 15.70
Waveforms for the parallel R-C network of Fig. 15.66.

Power:

$$P_T = EI\cos\theta = (10\text{ V})(10\text{ A})\cos 53.13° = (10)^2(0.6)$$
$$= \mathbf{60\ W}$$

or
$$P_T = E^2G = (10\text{ V})^2(0.6\text{ S}) = \mathbf{60\ W}$$

or, finally,

$$P_T = P_R + P_C = EI_R\cos\theta_R + EI_C\cos\theta_C$$
$$= (10\text{ V})(6\text{ A})\cos 0° + (10\text{ V})(8\text{ A})\cos 90°$$
$$= \mathbf{60\ W}$$

Power factor: The power factor of the circuit is

$$F_p = \cos 53.13° = \mathbf{0.6\ leading}$$

Using Eq. (15.32), we have

$$F_p = \cos\theta_T = \frac{G}{Y_T} = \frac{0.6\text{ S}}{1.0\text{ S}} = \mathbf{0.6\ leading}$$

Impedance approach: The voltage **E** can also be found by first finding the total impedance of the circuit:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_C}{\mathbf{Z}_R + \mathbf{Z}_C} = \frac{(1.67\ \Omega\ \angle 0°)(1.25\ \Omega\ \angle -90°)}{1.67\ \Omega\ \angle 0° + 1.25\ \Omega\ \angle -90°}$$
$$= \frac{2.09\ \angle -90°}{2.09\ \angle -36.81°} = \mathbf{1\ \Omega\ \angle -53.19°}$$

and then, using Ohm's law, we find

$$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = (10\text{ A}\ \angle 0°)(1\ \Omega\ \angle -53.19°) = \mathbf{10\ V\ \angle -53.19°}$$

R-L-C

Refer to Fig. 15.71.

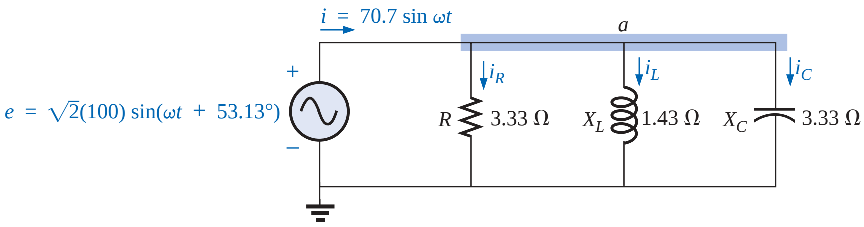


FIG. 15.71
Parallel R-L-C ac network.

Phasor notation: As shown in Fig. 15.72.

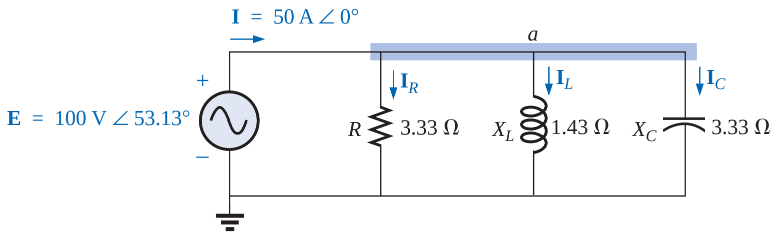


FIG. 15.72
Applying phasor notation to the network of Fig. 15.71.

$\mathbf{Y}_T$ and $\mathbf{Z}_T$

$$\begin{aligned}\mathbf{Y}_T &= \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C = G\angle 0^\circ + B_L\angle -90^\circ + B_C\angle 90^\circ\\ &= \frac{1}{3.33\ \Omega}\angle 0^\circ + \frac{1}{1.43\ \Omega}\angle -90^\circ + \frac{1}{3.33\ \Omega}\angle 90^\circ\\ &= 0.3\text{ S}\angle 0^\circ + 0.7\text{ S}\angle -90^\circ + 0.3\text{ S}\angle 90^\circ\\ &= 0.3\text{ S} - j\,0.7\text{ S} + j\,0.3\text{ S}\\ &= 0.3\text{ S} - j\,0.4\text{ S} = \mathbf{0.5\ S\angle -53.13^\circ}\end{aligned}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\text{ S}\angle -53.13^\circ} = \mathbf{2\ \Omega\angle 53.13^\circ}$$

Admittance diagram: As shown in Fig. 15.73.

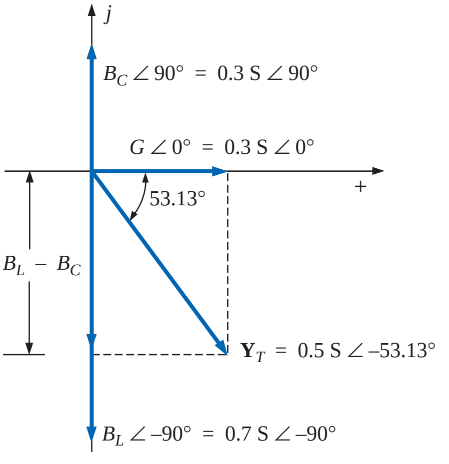


FIG. 15.73
Admittance diagram for the parallel R-L-C network of Fig. 15.71.

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (100\text{ V}\angle 53.13^\circ)(0.5\text{ S}\angle -53.13^\circ) = \mathbf{50\ A\angle 0^\circ}$$

$\mathbf{I}_R$, $\mathbf{I}_L$, and $\mathbf{I}_C$

$$\begin{aligned}\mathbf{I}_R &= (E\angle\theta)(G\angle 0^\circ)\\ &= (100\text{ V}\angle 53.13^\circ)(0.3\text{ S}\angle 0^\circ) = \mathbf{30\ A\angle 53.13^\circ}\end{aligned}$$

$$\begin{aligned}\mathbf{I}_L &= (E\angle\theta)(B_L\angle -90^\circ)\\ &= (100\text{ V}\angle 53.13^\circ)(0.7\text{ S}\angle -90^\circ) = \mathbf{70\ A\angle -36.87^\circ}\end{aligned}$$

$$\begin{aligned}\mathbf{I}_C &= (E\angle\theta)(B_C\angle 90^\circ)\\ &= (100\text{ V}\angle 53.13^\circ)(0.3\text{ S}\angle +90^\circ) = \mathbf{30\ A\angle 143.13^\circ}\end{aligned}$$

Kirchhoff's current law: At node *a*,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L - \mathbf{I}_C = 0$$

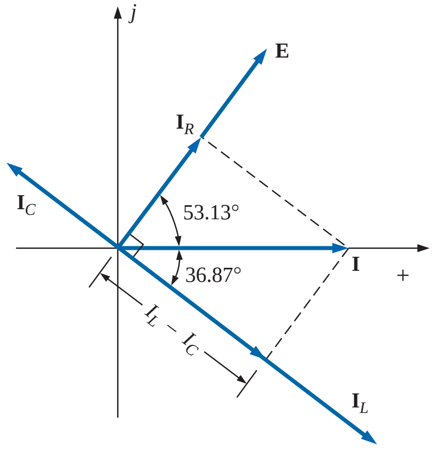


FIG. 15.74
Phasor diagram for the parallel R-L-C network of Fig. 15.71.

or
$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C$$

Phasor diagram: The phasor diagram of Fig. 15.74 indicates that the impressed voltage **E** is in phase with the current $\mathbf{I}_R$ through the resistor, leads the current $\mathbf{I}_L$ through the inductor by 90°, and lags the current $\mathbf{I}_C$ of the capacitor by 90°.

Time domain:

$$i = \sqrt{2}(50)\sin\omega t = \mathbf{70.70\ sin\ \omega t}$$
$$i_R = \sqrt{2}(30)\sin(\omega t + 53.13°) = \mathbf{42.42\ sin(\omega t + 53.13°)}$$
$$i_L = \sqrt{2}(70)\sin(\omega t - 36.87°) = \mathbf{98.98\ sin(\omega t - 36.87°)}$$
$$i_C = \sqrt{2}(30)\sin(\omega t + 143.13°) = \mathbf{42.42\ sin(\omega t + 143.13°)}$$

A plot of all of the currents and the impressed voltage appears in Fig. 15.75.

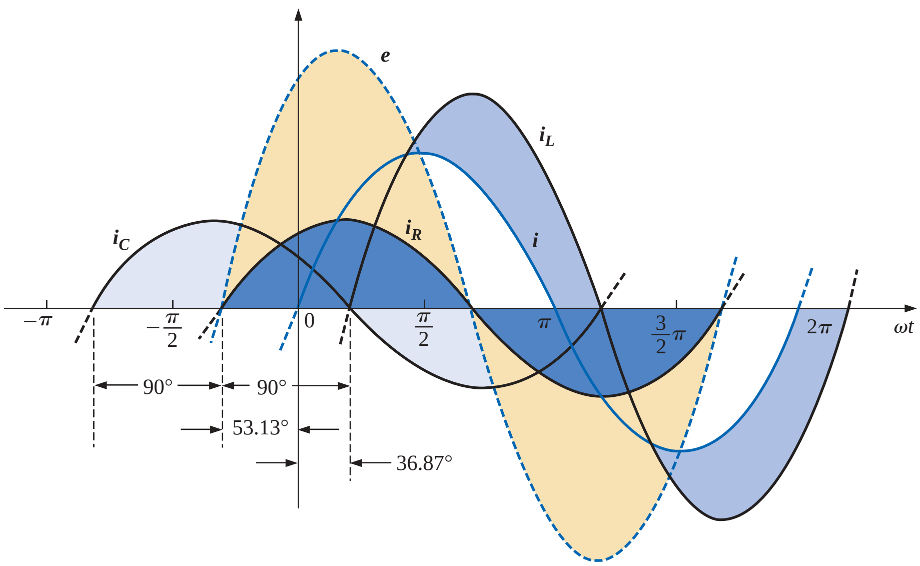


FIG. 15.75
Waveforms for the parallel R-L-C network of Fig. 15.71.

Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta = (100\text{ V})(50\text{ A})\cos 53.13° = (5000)(0.6)$$
$$= \mathbf{3000\ W}$$

or
$$P_T = E^2G = (100\text{ V})^2(0.3\text{ S}) = \mathbf{3000\ W}$$

or, finally,

$$P_T = P_R + P_L + P_C$$
$$= EI_R\cos\theta_R + EI_L\cos\theta_L + EI_C\cos\theta_C$$
$$= (100\text{ V})(30\text{ A})\cos 0° + (100\text{ V})(70\text{ A})\cos 90° + (100\text{ V})(30\text{ A})\cos 90°$$
$$= 3000\text{ W} + 0 + 0$$
$$= \mathbf{3000\ W}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

Using Eq. (15.32), we obtain

$$F_p = \cos\theta_T = \frac{G}{Y_T} = \frac{0.3\ \text{S}}{0.5\ \text{S}} = \mathbf{0.6\ lagging}$$

Impedance approach: The input current **I** can also be determined by first finding the total impedance in the following manner:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L\mathbf{Z}_C}{\mathbf{Z}_R\mathbf{Z}_L + \mathbf{Z}_L\mathbf{Z}_C + \mathbf{Z}_R\mathbf{Z}_C} = \mathbf{2\ \Omega\ \angle 53.13^\circ}$$

and, applying Ohm's law, we obtain

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\ \text{V}\ \angle 53.13^\circ}{2\ \Omega\ \angle 53.13^\circ} = \mathbf{50\ A\ \angle 0^\circ}$$

15.9 CURRENT DIVIDER RULE

The basic format for the **current divider rule** in ac circuits is exactly the same as that for dc circuits; that is, for two parallel branches with impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ as shown in Fig. 15.76,

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \textbf{(15.33)}$$

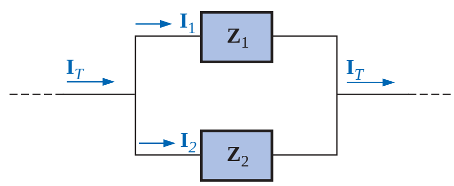


FIG. 15.76
Applying the current divider rule.

EXAMPLE 15.15 Using the current divider rule, find the current through each impedance of Fig. 15.77.

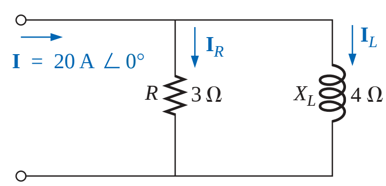


FIG. 15.77
Example 15.15.

Solution:

$$\mathbf{I}_R = \frac{\mathbf{Z}_L\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(4\ \Omega\ \angle 90^\circ)(20\ \text{A}\ \angle 0^\circ)}{3\ \Omega\ \angle 0^\circ + 4\ \Omega\ \angle 90^\circ} = \frac{80\ \text{A}\ \angle 90^\circ}{5\ \angle 53.13^\circ}$$
$$= \mathbf{16\ A\ \angle 36.87^\circ}$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_R\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(3\ \Omega\ \angle 0^\circ)(20\ \text{A}\ \angle 0^\circ)}{5\ \Omega\ \angle 53.13^\circ} = \frac{60\ \text{A}\ \angle 0^\circ}{5\ \angle 53.13^\circ}$$
$$= \mathbf{12\ A\ \angle -53.13^\circ}$$

EXAMPLE 15.16 Using the current divider rule, find the current through each parallel branch of Fig. 15.78.

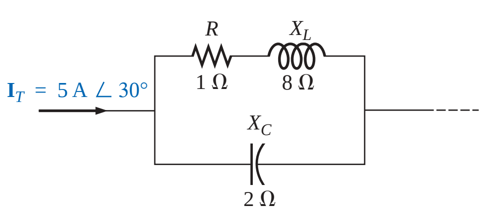


FIG. 15.78
Example 15.16.

Solution:

$$\mathbf{I}_{R\text{-}L} = \frac{\mathbf{Z}_C\mathbf{I}_T}{\mathbf{Z}_C + \mathbf{Z}_{R\text{-}L}} = \frac{(2\ \Omega\ \angle -90^\circ)(5\ \text{A}\ \angle 30^\circ)}{-j\,2\ \Omega + 1\ \Omega + j\,8\ \Omega} = \frac{10\ \text{A}\ \angle -60^\circ}{1 + j\,6}$$
$$= \frac{10\ \text{A}\ \angle -60^\circ}{6.083\ \angle 80.54^\circ} \cong \mathbf{1.644\ A\ \angle -140.54^\circ}$$

$$\mathbf{I}_C = \frac{\mathbf{Z}_{R\text{-}L}\mathbf{I}_T}{\mathbf{Z}_{R\text{-}L} + \mathbf{Z}_C} = \frac{(1\ \Omega + j\ 8\ \Omega)(5\ \text{A} \angle 30^\circ)}{6.08\ \Omega \angle 80.54^\circ}$$

$$= \frac{(8.06 \angle 82.87^\circ)(5\ \text{A} \angle 30^\circ)}{6.08 \angle 80.54^\circ} = \frac{40.30\ \text{A} \angle 112.87^\circ}{6.083 \angle 80.54^\circ}$$

$$= \mathbf{6.625\ A \angle 32.33^\circ}$$

15.10 FREQUENCY RESPONSE OF THE PARALLEL *R-L* NETWORK

In Section 15.5 the frequency response of a series *R-C* circuit was analyzed. Let us now note the impact of frequency on the total impedance and inductive current for the parallel *R-L* network of Fig. 15.79 for a frequency range of zero through 40 kHz.

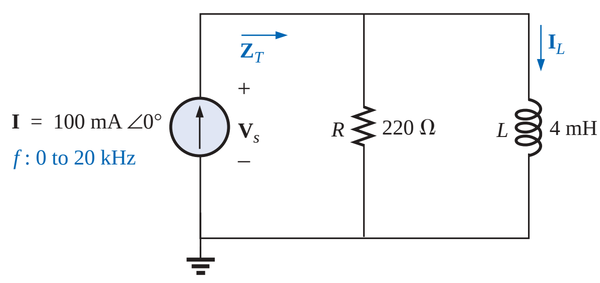


FIG. 15.79
Determining the frequency response of a parallel R-L network.

$\mathbf{Z}_T$ Before getting into specifics, let us first develop a "sense" for the impact of frequency on the network of Fig. 15.79 by noting the impedance-versus-frequency curves of the individual elements, as shown in Fig. 15.80. The fact that the elements are now in parallel requires that we consider their characteristics in a different manner than occurred for the series *R-C* circuit of Section 15.5. Recall that for parallel elements, the element with the smallest impedance will have the greatest impact

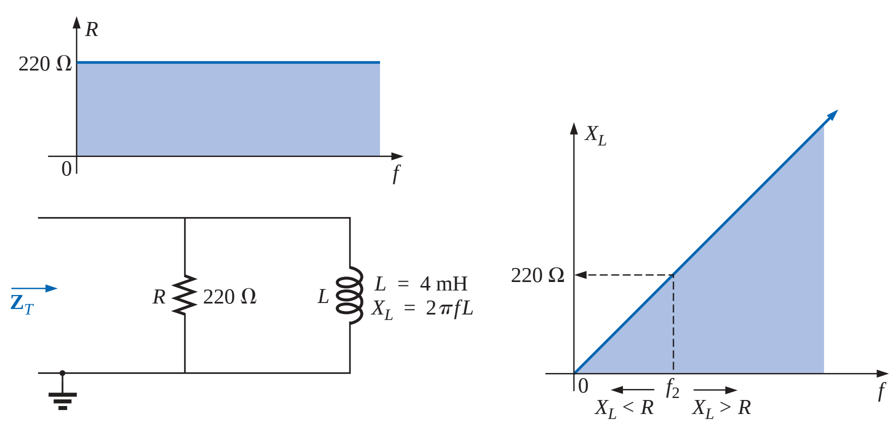


FIG. 15.80
The frequency response of the individual elements of a parallel R-L network.

on the total impedance at that frequency. In Fig. 15.80, for example, X_L is very small at low frequencies compared to R, establishing X_L as the predominant factor in this frequency range. In other words, at low frequencies the network will be primarily inductive, and the angle associated with the total impedance will be close to 90°, as with a pure inductor. As the frequency increases, X_L will increase until it equals the impedance of the resistor (220 Ω). The frequency at which this situation occurs can be determined in the following manner:

$$X_L = 2\pi f_2 L = R$$

and

$$f_2 = \frac{R}{2\pi L} \tag{15.34}$$

which for the network of Fig. 15.79 is

$$f_2 = \frac{R}{2\pi L} = \frac{220\ \Omega}{2\pi(4 \times 10^{-3}\ \text{H})}$$
$$\cong \mathbf{8.75\ kHz}$$

which falls within the frequency range of interest.

For frequencies less than f_2, $X_L < R$, and for frequencies greater than f_2, $X_L > R$, as shown in Fig. 15.80. A general equation for the total impedance in vector form can be developed in the following manner:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R \mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L}$$
$$= \frac{(R \angle 0°)(X_L \angle 90°)}{R + j X_L} = \frac{R X_L \angle 90°}{\sqrt{R^2 + X_L^2} \angle \tan^{-1} X_L/R}$$

and

$$\mathbf{Z}_T = \frac{R X_L}{\sqrt{R^2 + X_L^2}} \angle 90° - \tan^{-1} X_L/R$$

so that

$$\mathbf{Z}_T = \frac{R X_L}{\sqrt{R^2 + X_L^2}} \tag{15.35}$$

and

$$\theta_T = 90° - \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{R}{X_L} \tag{15.36}$$

The magnitude and angle of the total impedance can now be found at any frequency of interest simply by substituting Eqs. (15.35) and (15.36).

f = 1 kHz

$$X_L = 2\pi f L = 2\pi(1\ \text{kHz})(4 \times 10^{-3}\ \text{H}) = 25.12\ \Omega$$

and

$$Z_T = \frac{R X_L}{\sqrt{R^2 + X_L^2}} = \frac{(220\ \Omega)(25.12\ \Omega)}{\sqrt{(220\ \Omega)^2 + (25.12\ \Omega)^2}} = \mathbf{24.96\ \Omega}$$

with

$$\theta_T = \tan^{-1}\frac{R}{X_L} = \tan^{-1}\frac{220\ \Omega}{25.12\ \Omega}$$
$$= \tan^{-1} 8.76 = 83.49°$$

and $\mathbf{Z}_T = \mathbf{24.96\ \Omega\ \angle 83.49°}$

This value compares very closely with $X_L = 25.12\ \Omega\ \angle 90°$, which it would be if the network were purely inductive ($R = \infty\ \Omega$). Our assumption that the network is primarily inductive at low frequencies is therefore confirmed.

Continuing:

$$f = 5\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{109.1\ \Omega\ \angle 60.23°}$$
$$f = 10\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{165.5\ \Omega\ \angle 41.21°}$$
$$f = 15\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{189.99\ \Omega\ \angle 30.28°}$$
$$f = 20\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{201.53\ \Omega\ \angle 23.65°}$$
$$f = 30\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{211.19\ \Omega\ \angle 16.27°}$$
$$f = 40\text{ kHz:}\quad \mathbf{Z}_T = \mathbf{214.91\ \Omega\ \angle 12.35°}$$

At $f = 40$ kHz, note how closely the magnitude of Z_T has approached the resistance level of 220 Ω and how the associated angle with the total impedance is approaching zero degrees. The result is a network with terminal characteristics that are becoming more and more resistive as the frequency increases, which further confirms the earlier conclusions developed by the curves of Fig. 15.80.

Plots of Z_T versus frequency in Fig. 15.81 and θ_T in Fig. 15.82 clearly reveal the transition from an inductive network to one that has resistive characteristics. Note that the transition frequency of 8.75 kHz occurs right in the middle of the knee of the curves for both Z_T and θ_T.

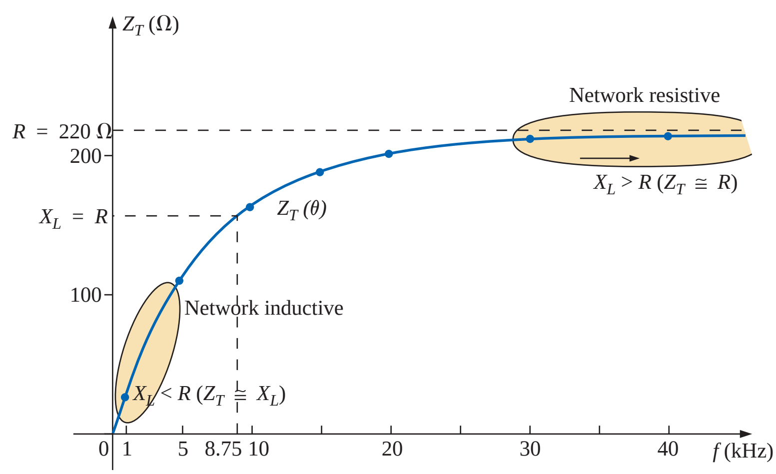


FIG. 15.81

The magnitude of the input impedance versus frequency for the network of Fig. 15.79.

A review of Figs. 15.47 and 15.81 will reveal that a series *R-C* and a parallel *R-L* network will have an impedance level that approaches the resistance of the network at high frequencies. The capacitive circuit approaches the level from above, whereas the inductive network does the same from below. For the series *R-L* circuit and the parallel *R-C* network, the total impedance will begin at the resistance level and then display the characteristics of the reactive elements at high frequencies.

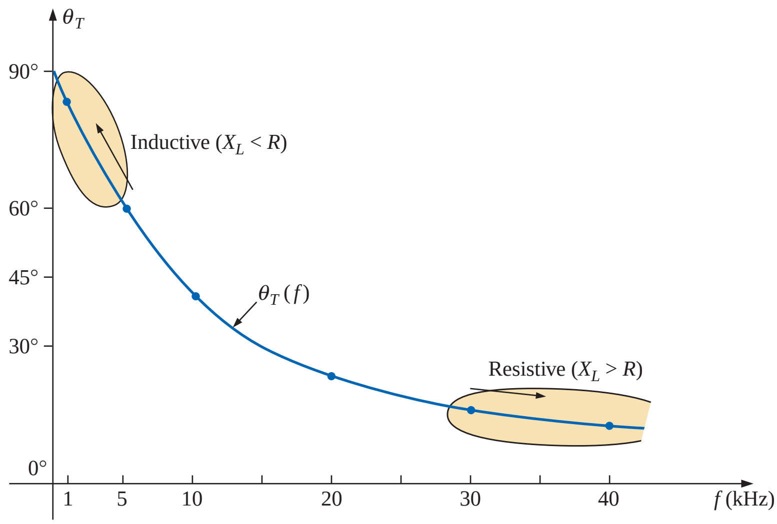


FIG. 15.82
The phase angle of the input impedance versus frequency for the network of Fig. 15.79.

I_L Applying the current divider rule to the network of Fig. 15.79 will result in the following:

$$\mathbf{I}_L = \frac{\mathbf{Z}_R \mathbf{I}}{\mathbf{Z}_R + \mathbf{Z}_L}$$
$$= \frac{(R \angle 0°)(I \angle 0°)}{R + j X_L} = \frac{RI \angle 0°}{\sqrt{R^2 + X_L^2} \;\angle \tan^{-1} X_L/R}$$

and
$$\mathbf{I}_L = I_L \angle \theta_L = \frac{RI}{\sqrt{R^2 + X_L^2}} \angle -\tan^{-1} X_L/R$$

The magnitude of I_L is therefore determined by

$$I_L = \frac{RI}{\sqrt{R^2 + X_L^2}} \tag{15.37}$$

and the phase angle θ_L, by which $\mathbf{I}_L$ leads $\mathbf{I}$, is given by

$$\theta_L = -\tan^{-1} \frac{X_L}{R} \tag{15.38}$$

Because θ_L is always negative, the magnitude of θ_L is, in actuality, the angle by which $\mathbf{I}_L$ lags $\mathbf{I}$.

To begin our analysis, let us first consider the case of $f = 0$ Hz (dc conditions).

f = 0 Hz

$$X_L = 2\pi f L = 2\pi(0 \text{ Hz})L = 0\ \Omega$$

Applying the short-circuit equivalent for the inductor in Fig. 15.79 would result in

$$\mathbf{I}_L = \mathbf{I} = 100 \text{ mA} \angle 0°$$

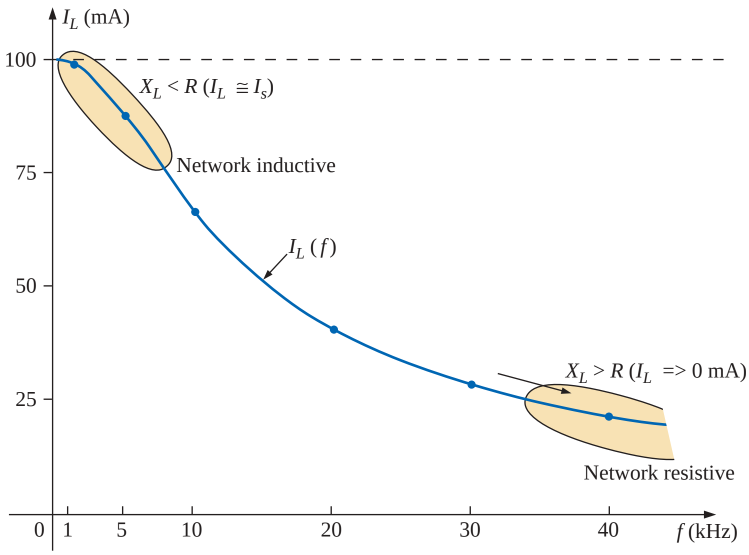


FIG. 15.83

The magnitude of the current $\mathbf{I}_L$ *versus frequency for the parallel R-L network of Fig. 15.79.*

as appearing in Figs. 15.83 and 15.84.

f = 1 kHz Applying Eq. (15.37):

$$X_L = 2\pi fL = 2\pi(1\text{ kHz})(4\text{ mH}) = 25.12\ \Omega$$

and
$$\sqrt{R^2 + X_L^2} = \sqrt{(220\ \Omega)^2 + (25.12\ \Omega)^2} = 221.43\ \Omega$$

and
$$I_L = \frac{RI}{\sqrt{R^2 + X_L^2}} = \frac{(220\ \Omega)(100\text{ mA})}{221.43\ \Omega} = \mathbf{99.35\ mA}$$

with

$$\theta_L = \tan^{-1}\frac{X_L}{R} = -\tan^{-1}\frac{25.12\ \Omega}{220\ \Omega} = -\tan^{-1} 0.114 = \mathbf{-6.51°}$$

and
$$\mathbf{I}_L = \mathbf{99.35\ mA\ \angle -6.51°}$$

The result is a current $\mathbf{I}_L$ that is still very close to the source current $\mathbf{I}$ in both magnitude and phase.

Continuing:

$$\begin{aligned}
f &= 5\text{ kHz:} & \mathbf{I}_L &= \mathbf{86.84\ mA\ \angle -29.72°}\\
f &= 10\text{ kHz:} & \mathbf{I}_L &= \mathbf{65.88\ mA\ \angle -48.79°}\\
f &= 15\text{ kHz:} & \mathbf{I}_L &= \mathbf{50.43\ mA\ \angle -59.72°}\\
f &= 20\text{ kHz:} & \mathbf{I}_L &= \mathbf{40.11\ mA\ \angle -66.35°}\\
f &= 30\text{ kHz:} & \mathbf{I}_L &= \mathbf{28.02\ mA\ \angle -73.73°}\\
f &= 40\text{ kHz:} & \mathbf{I}_L &= \mathbf{21.38\ mA\ \angle -77.65°}
\end{aligned}$$

The plot of the magnitude of I_L versus frequency is provided in Fig. 15.83 and reveals that the current through the coil dropped from its maximum of 100 mA to almost 20 mA at 40 kHz. As the reactance of the coil increased with frequency, more of the source current chose the

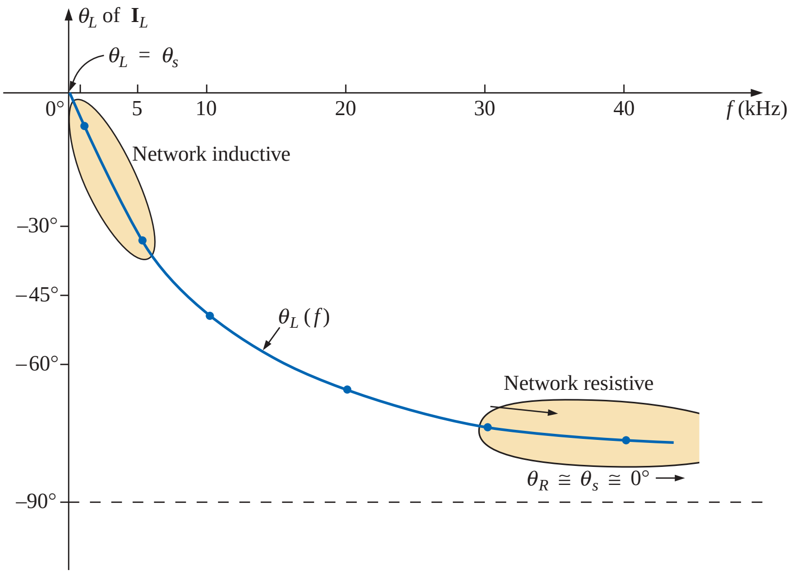


FIG. 15.84
The phase angle of the current $\mathbf{I}_L$ versus frequency for the parallel R-L network of Fig. 15.79.

lower-resistance path of the resistor. The magnitude of the phase angle between $\mathbf{I}_L$ and $\mathbf{I}$ is approaching 90° with an increase in frequency, as shown in Fig. 15.84, leaving its initial value of zero degrees at $f = 0$ Hz far behind.

At $f = 1$ kHz, the phasor diagram of the network appears as shown in Fig. 15.85. First note that the magnitude and the phase angle of $\mathbf{I}_L$ are very close to those of $\mathbf{I}$. Since the voltage across a coil must lead the current through a coil by 90°, the voltage $\mathbf{V}_s$ appears as shown. The voltage across a resistor is in phase with the current through the resistor, resulting in the direction of $\mathbf{I}_R$ shown in Fig. 15.85. Of course, at this frequency $R > X_L$, and the current I_R is relatively small in magnitude.

At $f = 40$ kHz, the phasor diagram changes to that appearing in Fig. 15.86. Note that now $\mathbf{I}_R$ and $\mathbf{I}$ are close in magnitude and phase because $X_L > R$. The magnitude of $\mathbf{I}_L$ has dropped to very low levels, and the phase angle associated with $\mathbf{I}_L$ is approaching $-90°$. The network is now more "resistive" compared to its "inductive" characteristics at low frequencies.

The analysis of a parallel *R-C* or *R-L-C* network would proceed in much the same manner, with the inductive impedance predominating at low frequencies and the capacitive reactance predominating at high frequencies.

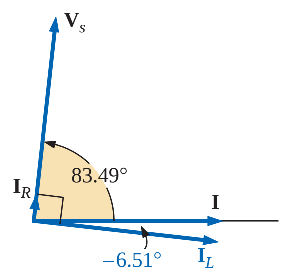


FIG. 15.85
The phasor diagram for the parallel R-L network of Fig. 15.79 at $f = 1$ kHz.

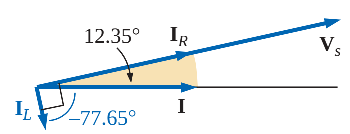


FIG. 15.86
The phasor diagram for the parallel R-L network of Fig. 15.79 at $f = 40$ kHz.

15.11 SUMMARY: PARALLEL ac NETWORKS

The following is a review of important conclusions that can be derived from the discussion and examples of the previous sections. The list is not all-inclusive, but it does emphasize some of the conclusions that should be carried forward in the future analysis of ac systems.

For parallel ac networks with reactive elements:

1. ***The total admittance (impedance) will be frequency dependent.***
2. ***The impedance of any one element can be less than the total impedance (recall that for dc circuits the total resistance must always be less than the smallest parallel resistor).***
3. ***The inductive and capacitive susceptances are in direct opposition on an admittance diagram.***
4. ***Depending on the frequency applied, the same network can be either predominantly inductive or predominantly capacitive.***
5. ***At lower frequencies the inductive elements will usually have the most impact on the total impedance, while at high frequencies the capacitive elements will usually have the most impact.***
6. ***The magnitude of the current through any one branch can be greater than the source current.***
7. ***The magnitude of the current through an element, compared to the other elements of the network, is directly related to the magnitude of its impedance; that is, the smaller the impedance of an element, the larger the magnitude of the current through the element.***
8. ***The current through a coil is always in direct opposition with the current through a capacitor on a phasor diagram.***
9. ***The applied voltage is always in phase with the current through the resistive elements, leads the voltage across all the inductive elements by 90°, and lags the current through all capacitive elements by 90°.***
10. ***The smaller the resistive element of a network compared to the net reactive susceptance, the closer the power factor is to unity.***

15.12 EQUIVALENT CIRCUITS

In a series ac circuit, the total impedance of two or more elements in series is often equivalent to an impedance that can be achieved with fewer elements of different values, the elements and their values being determined by the frequency applied. This is also true for parallel circuits. For the circuit of Fig. 15.87(a),

$$\mathbf{Z}_T = \frac{\mathbf{Z}_C\mathbf{Z}_L}{\mathbf{Z}_C + \mathbf{Z}_L} = \frac{(5\ \Omega \angle -90°)(10\ \Omega \angle 90°)}{5\ \Omega \angle -90° + 10\ \Omega \angle 90°} = \frac{50 \angle 0°}{5 \angle 90°}$$
$$= 10\ \Omega \angle -90°$$

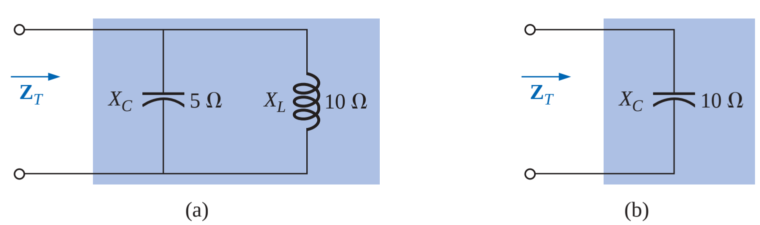


FIG. 15.87
Defining the equivalence between two networks at a specific frequency.

The total impedance at the frequency applied is equivalent to a capacitor with a reactance of 10 Ω, as shown in Fig. 15.87(b). Always keep in mind that this equivalence is true only at the applied frequency. If the frequency changes, the reactance of each element changes, and the equivalent circuit will change—perhaps from capacitive to inductive in the above example.

Another interesting development appears if the impedance of a parallel circuit, such as the one of Fig. 15.88(a), is found in rectangular form. In this case,

$$\mathbf{Z}_T = \frac{\mathbf{Z}_L\mathbf{Z}_R}{\mathbf{Z}_L + \mathbf{Z}_R} = \frac{(4\ \Omega\ \angle 90°)(3\ \Omega\ \angle 0°)}{4\ \Omega\ \angle 90° + 3\ \Omega\ \angle 0°}$$

$$= \frac{12\ \angle 90°}{5\ \angle 53.13°} = 2.40\ \Omega\ \angle 36.87°$$

$$= 1.920\ \Omega + j\,1.440\ \Omega$$

which is the impedance of a series circuit with a resistor of 1.92 Ω and an inductive reactance of 1.44 Ω, as shown in Fig. 15.88(b).

The current **I** will be the same in each circuit of Fig. 15.87 or Fig. 15.88 if the same input voltage **E** is applied. For a parallel circuit of one resistive element and one reactive element, the series circuit with the same input impedance will always be composed of one resistive and one reactive element. The impedance of each element of the series circuit will be different from that of the parallel circuit, but the reactive elements will always be of the same type; that is, an *R-L* circuit and an *R-C* parallel circuit will have an equivalent *R-L* and *R-C* series circuit, respectively. The same is true when converting from a series to a parallel circuit. In the discussion to follow, keep in mind that

the term* equivalent *refers only to the fact that for the same applied potential, the same impedance and input current will result.

To formulate the equivalence between the series and parallel circuits, the equivalent series circuit for a resistor and reactance in parallel can be found by determining the total impedance of the circuit in rectangular form; that is, for the circuit of Fig. 15.89(a),

$$\mathbf{Y}_p = \frac{1}{R_p} + \frac{1}{\pm j\,X_p} = \frac{1}{R_p} \mp j\frac{1}{X_p}$$

and

$$\mathbf{Z}_p = \frac{1}{\mathbf{Y}_p} = \frac{1}{(1/R_p) \mp j\,(1/X_p)}$$

$$= \frac{1/R_p}{(1/R_p)^2 + (1/X_p)^2} \pm j\frac{1/X_p}{(1/R_p)^2 + (1/X_p)^2}$$

Multiplying the numerator and denominator of each term by $R_p^2X_p^2$ results in

$$\mathbf{Z}_p = \frac{R_pX_p^2}{X_p^2 + R_p^2} \pm j\frac{R_p^2X_p}{X_p^2 + R_p^2}$$

$$= R_s \pm j\,X_s \qquad \text{[Fig. 15.89(b)]}$$

and

$$R_s = \frac{R_pX_p^2}{X_p^2 + R_p^2} \qquad \textbf{(15.39)}$$

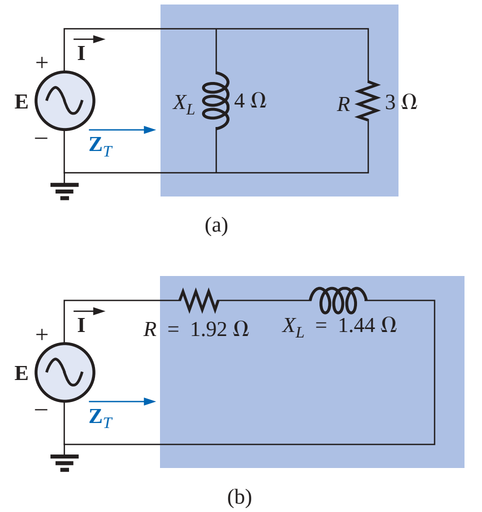


FIG. 15.88
Finding the series equivalent circuit for a parallel R-L network.

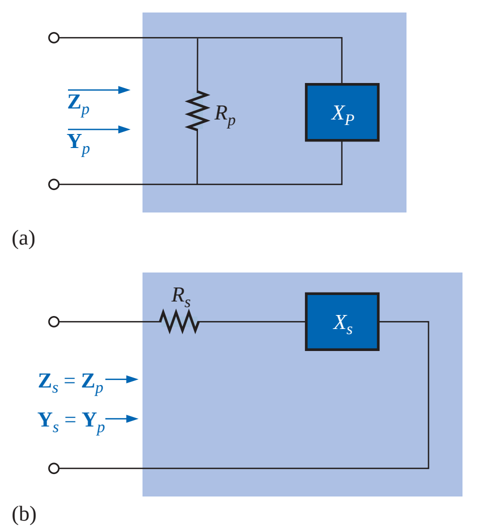


FIG. 15.89
Defining the parameters of equivalent series and parallel networks.

with

$$X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2} \tag{15.40}$$

For the network of Fig. 15.88,

$$R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2} = \frac{(3\ \Omega)(4\ \Omega)^2}{(4\ \Omega)^2 + (3\ \Omega)^2} = \frac{48\ \Omega}{25} = \mathbf{1.920\ \Omega}$$

and

$$X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2} = \frac{(3\ \Omega)^2(4\ \Omega)}{(4\ \Omega)^2 + (3\ \Omega)^2} = \frac{36\ \Omega}{25} = \mathbf{1.440\ \Omega}$$

which agrees with the previous result.

The equivalent parallel circuit for a circuit with a resistor and reactance in series can be found by simply finding the total admittance of the system in rectangular form; that is, for the circuit of Fig. 15.89(b),

$$\mathbf{Z}_s = R_s \pm j X_s$$

$$\mathbf{Y}_s = \frac{1}{\mathbf{Z}_s} = \frac{1}{R_s \pm j X_s} = \frac{R_s}{R_s^2 + X_s^2} \mp j \frac{X_s}{R_s^2 + X_s^2}$$

$$= G_p \mp j B_p = \frac{1}{R_p} \mp j \frac{1}{X_p} \qquad \text{[Fig. 15.89(a)]}$$

or

$$R_p = \frac{R_s^2 + X_s^2}{R_s} \tag{15.41}$$

with

$$X_p = \frac{R_s^2 + X_s^2}{X_s} \tag{15.42}$$

For the above example,

$$R_p = \frac{R_s^2 + X_s^2}{R_s} = \frac{(1.92\ \Omega)^2 + (1.44\ \Omega)^2}{1.92\ \Omega} = \frac{5.76\ \Omega}{1.92} = \mathbf{3.0\ \Omega}$$

and

$$X_p = \frac{R_s^2 + X_s^2}{X_s} = \frac{5.76\ \Omega}{1.44} = \mathbf{4.0\ \Omega}$$

as shown in Fig. 15.88(a).

EXAMPLE 15.17 Determine the series equivalent circuit for the network of Fig. 15.90.

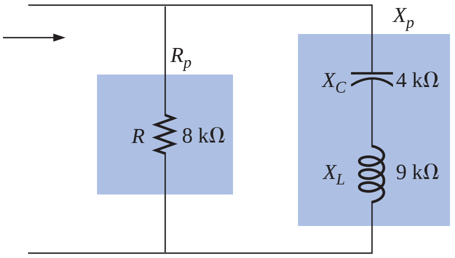


FIG. 15.90
Example 15.17.

Solution:

$$R_p = 8\text{ k}\Omega$$
$$X_p\text{ (resultant)} = |X_L - X_C| = |9\text{ k}\Omega - 4\text{ k}\Omega| = 5\text{ k}\Omega$$

and

$$R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2} = \frac{(8\text{ k}\Omega)(5\text{ k}\Omega)^2}{(5\text{ k}\Omega)^2 + (8\text{ k}\Omega)^2} = \frac{200\text{ k}\Omega}{89} = \mathbf{2.247\text{ k}\Omega}$$

with

$$X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2} = \frac{(8\text{ k}\Omega)^2(5\text{ k}\Omega)}{(5\text{ k}\Omega)^2 + (8\text{ k}\Omega)^2} = \frac{320\text{ k}\Omega}{89} = \mathbf{3.596\text{ k}\Omega} \quad \textbf{(inductive)}$$

The equivalent series circuit appears in Fig. 15.91.

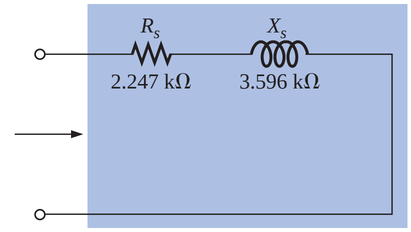


FIG. 15.91
The equivalent series circuit for the parallel network of Fig. 15.90.

EXAMPLE 15.18 For the network of Fig. 15.92:

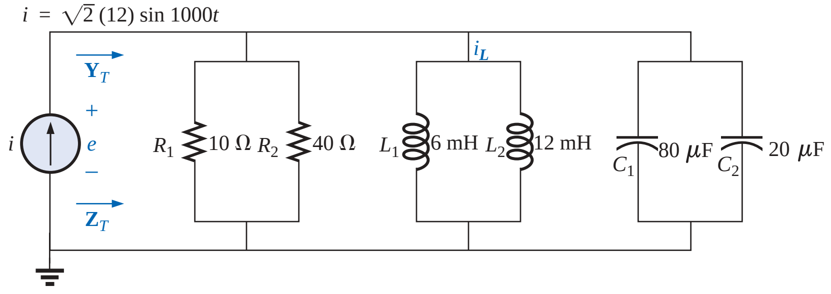


FIG. 15.92
Example 15.18.

a. Determine $\mathbf{Y}_T$.
b. Sketch the admittance diagram.
c. Find $\mathbf{E}$ and $\mathbf{I}_L$.
d. Compute the power factor of the network and the power delivered to the network.
e. Determine the equivalent series circuit as far as the terminal characteristics of the network are concerned.
f. Using the equivalent circuit developed in part (e), calculate $\mathbf{E}$, and compare it with the result of part (c).
g. Determine the power delivered to the network, and compare it with the solution of part (d).
h. Determine the equivalent parallel network from the equivalent series circuit, and calculate the total admittance $\mathbf{Y}_T$. Compare the result with the solution of part (a).

Solutions:

a. Combining common elements and finding the reactance of the inductor and capacitor, we obtain

$$R_T = 10\ \Omega \parallel 40\ \Omega = 8\ \Omega$$
$$L_T = 6\text{ mH} \parallel 12\text{ mH} = 4\text{ mH}$$
$$C_T = 80\ \mu\text{F} + 20\ \mu\text{F} = 100\ \mu\text{F}$$

$$X_L = \omega L = (1000\text{ rad/s})(4\text{ mH}) = 4\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(1000\text{ rad/s})(100\ \mu\text{F})} = 10\ \Omega$$

The network is redrawn in Fig. 15.93 with phasor notation. The total admittance is

$$\begin{aligned}
\mathbf{Y}_T &= \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C \\
&= G\angle 0^\circ + B_L\angle -90^\circ + B_C\angle +90^\circ \\
&= \frac{1}{8\ \Omega}\angle 0^\circ + \frac{1}{4\ \Omega}\angle -90^\circ + \frac{1}{10\ \Omega}\angle +90^\circ \\
&= 0.125\text{ S}\angle 0^\circ + 0.25\text{ S}\angle -90^\circ + 0.1\text{ S}\angle +90^\circ \\
&= 0.125\text{ S} - j\,0.25\text{ S} + j\,0.1\text{ S} \\
&= 0.125\text{ S} - j\,0.15\text{ S} = \mathbf{0.195\ S\angle -50.194^\circ}
\end{aligned}$$

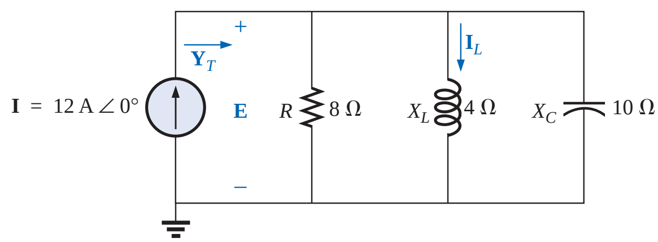


FIG. 15.93
Applying phasor notation to the network of Fig. 15.92.

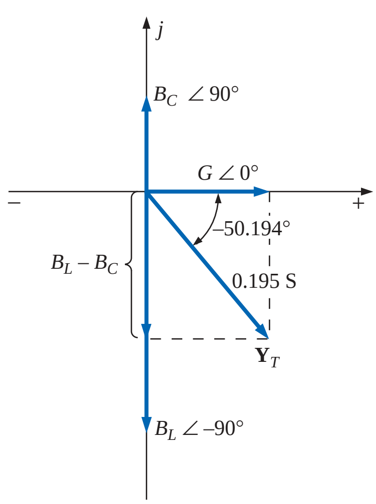


FIG. 15.94
Admittance diagram for the parallel R-L-C network of Fig. 15.92.

b. See Fig. 15.94.

c. $\mathbf{E} = \mathbf{I}\mathbf{Z}_T = \dfrac{\mathbf{I}}{\mathbf{Y}_T} = \dfrac{12\text{ A}\angle 0^\circ}{0.195\text{ S}\angle -50.194^\circ} = \mathbf{61.538\ V\angle 50.194^\circ}$

$\mathbf{I}_L = \dfrac{\mathbf{V}_L}{\mathbf{Z}_L} = \dfrac{\mathbf{E}}{\mathbf{Z}_L} = \dfrac{61.538\text{ V}\angle 50.194^\circ}{4\ \Omega\angle 90^\circ} = \mathbf{15.385\ A\angle -39.81^\circ}$

d. $F_p = \cos\theta = \dfrac{G}{Y_T} = \dfrac{0.125\text{ S}}{0.195\text{ S}} = \mathbf{0.641}$ **lagging** (**E** leads **I**)

$P = EI\cos\theta = (61.538\text{ V})(12\text{ A})\cos 50.194^\circ$
$= \mathbf{472.75\ W}$

e. $\mathbf{Z}_T = \dfrac{1}{\mathbf{Y}_T} = \dfrac{1}{0.195\text{ S}\angle -50.194^\circ} = 5.128\ \Omega\angle +50.194^\circ$
$= 3.283\ \Omega + j3.939\ \Omega$
$= R + j\,X_L$

$X_L = 3.939\ \Omega = \omega L$

$L = \dfrac{3.939\ \Omega}{\omega} = \dfrac{3.939\ \Omega}{1000\text{ rad/s}} = \mathbf{3.939\ mH}$

The series equivalent circuit appears in Fig. 15.95.

f. $\mathbf{E} = \mathbf{I}\mathbf{Z}_T = (12\text{ A}\angle 0^\circ)(5.128\ \Omega\angle 50.194^\circ)$
$= \mathbf{61.536\ V\angle 50.194^\circ}$ (as above)

g. $P = I^2R = (12\text{ A})^2(3.283\ \Omega) = \mathbf{472.75\ W}$ (as above)

h. $R_p = \dfrac{R_s^2 + X_s^2}{R_s} = \dfrac{(3.283\ \Omega)^2 + (3.939\ \Omega)^2}{3.283\ \Omega} = \mathbf{8\ \Omega}$

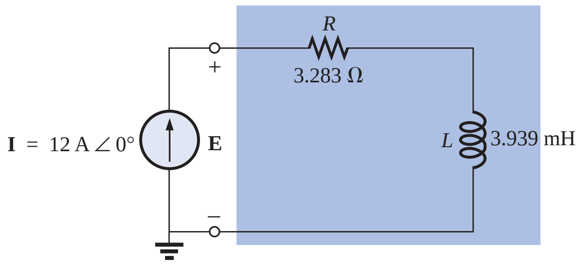


FIG. 15.95
Series equivalent circuit for the parallel R-L-C network of Fig. 15.92 with ω = 1000 rad/s.

$$X_p = \frac{R_s^2 + X_s^2}{X_s} = \frac{(3.283\ \Omega)^2 + (3.939\ \Omega)^2}{3.939\ \Omega} = \mathbf{6.675\ \Omega}$$

The parallel equivalent circuit appears in Fig. 15.96.

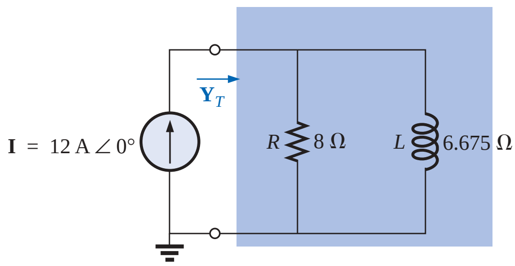


FIG. 15.96
Parallel equivalent of the circuit of Fig. 15.95.

$$\mathbf{Y}_T = G\ \angle 0° + B_L\ \angle -90° = \frac{1}{8\ \Omega}\ \angle 0° + \frac{1}{6.675\ \Omega}\ \angle -90°$$
$$= 0.125\text{ S}\ \angle 0° + 0.15\text{ S}\ \angle -90°$$
$$= 0.125\text{ S} - j\,0.15\text{ S} = \mathbf{0.195\ S\ \angle -50.194°} \qquad \text{(as above)}$$

15.13 PHASE MEASUREMENTS (DUAL-TRACE OSCILLOSCOPE)

The phase shift between the voltages of a network or between the voltages and currents of a network can be found using a dual-trace (two signals displayed at the same time) oscilloscope. Phase-shift measurements can also be performed using a single-trace oscilloscope by properly interpreting the resulting Lissajous patterns obtained on the screen. This latter approach, however, will be left for the laboratory experience.

In Fig. 15.97, channel 1 of the dual-trace oscilloscope is hooked up to display the applied voltage *e*. Channel 2 is connected to display the voltage across the inductor v_L. Of particular importance is the fact that the ground of the scope is connected to the ground of the oscilloscope for both channels. In other words, there is only one common ground for the circuit and oscilloscope. The resulting waveforms may appear as shown in Fig. 15.98.

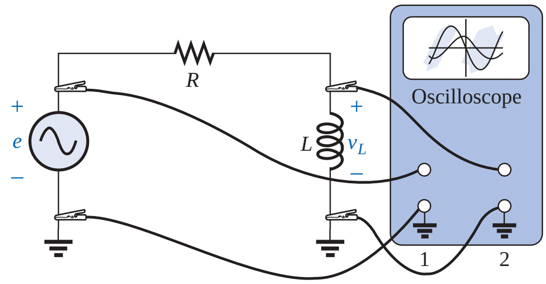


FIG. 15.97
Determining the phase relationship between e and v_L.

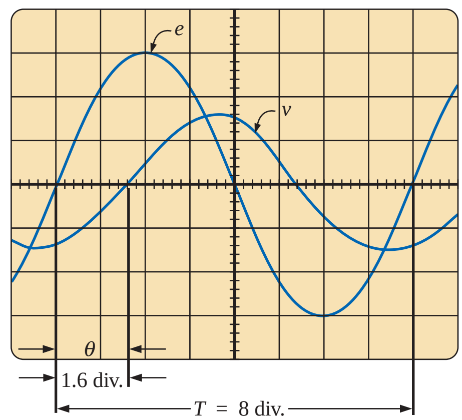


FIG. 15.98
Determining the phase angle between e and v_L.

For the chosen horizontal sensitivity, each waveform of Fig. 15.98 has a period T defined by eight horizontal divisions, and the phase angle between the two waveforms is defined by $1\frac{1}{2}$ divisions. Using the fact that each period of a sinusoidal waveform encompasses 360°, the following ratios can be set up to determine the phase angle θ:

$$\frac{8\text{ div.}}{360^\circ} = \frac{1.6\text{ div.}}{\theta}$$

and
$$\theta = \left(\frac{1.6}{8}\right)360^\circ = \mathbf{72^\circ}$$

In general,

$$\boxed{\theta = \frac{(\text{div. for } \theta)}{(\text{div. for } T)} \times 360^\circ} \qquad \textbf{(15.43)}$$

If the phase relationship between e and v_R is required, the oscilloscope *must not* be hooked up as shown in Fig. 15.99. Points a and b have a common ground that will establish a zero-volt drop between the two points; this drop will have the same effect as a short-circuit connection between a and b. The resulting short circuit will "short out" the inductive element, and the current will increase due to the drop in impedance for the circuit. A dangerous situation can arise if the inductive element has a high impedance and the resistor has a relatively low

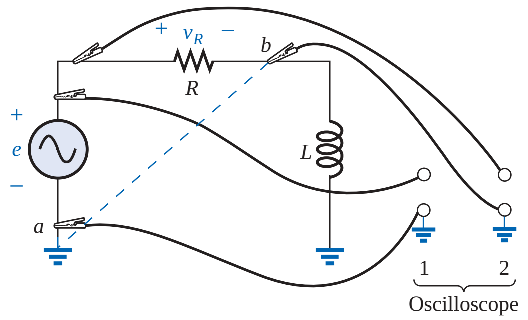


FIG. 15.99
An improper phase-measurement connection.

impedance. The current, controlled solely by the resistance R, could jump to dangerous levels and damage the equipment.

The phase relationship between e and v_R can be determined by simply interchanging the positions of the coil and resistor or by introducing a sensing resistor, as shown in Fig. 15.100. A sensing resistor is exactly that: introduced to "sense" a quantity without adversely affecting the behavior of the network. In other words, the sensing resistor must be small enough compared to the other impedances of the network not to cause a significant change in the voltage and current levels or phase relationships. Note that the sensing resistor is introduced in a way that will result in one end being connected to the common ground of the network. In Fig. 15.100, channel 2 will display the voltage v_{R_s}, which is in phase with the current i. However, the current i is also in phase with the voltage v_R across the resistor R. The net result is that the voltages v_{R_s} and v_R are in phase and the phase relationship between e and v_R can be determined from the waveforms e and v_{R_s}. Since v_{R_s} and i are in phase, the above procedure will also determine the phase angle between the applied voltage e and the source current i. If the magnitude of R_s is sufficiently small compared to R or X_L, the phase measurements of Fig. 15.97 can be performed with R_s in place. That is, channel 2 can be connected to the top of the inductor and to ground, and the effect of R_s can be ignored. In the above application, the sensing resistor will not reveal the magnitude of the voltage v_R but simply the phase relationship between e and v_R.

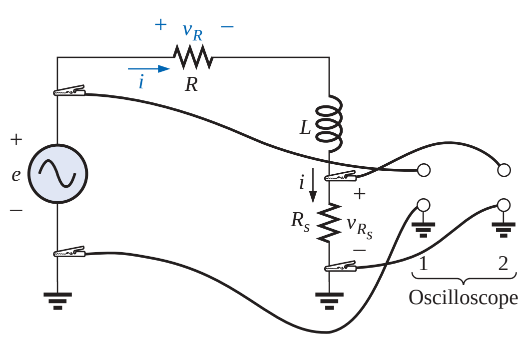


FIG. 15.100
Determining the phase relationship between e and v_R or e and i using a sensing resistor.

For the parallel network of Fig. 15.101, the phase relationship between two of the branch currents, i_R and i_L, can be determined using a sensing resistor, as shown in the figure. Channel 1 will display the voltage v_R, and channel 2 will display the voltage v_{R_s}. Since v_R is in phase with i_R, and v_{R_s} is in phase with the current i_L, the phase relationship between v_R and v_{R_s} will be the same as that between i_R and i_L. In this case, the magnitudes of the current levels can be determined using Ohm's law and the resistance levels R and R_s, respectively.

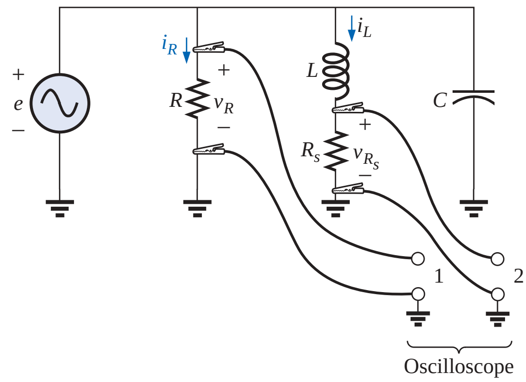


FIG. 15.101
Determining the phase relationship between i_R and i_L.

If the phase relationship between e and i_s of Fig. 15.101 is required, a sensing resistor can be employed, as shown in Fig. 15.102.

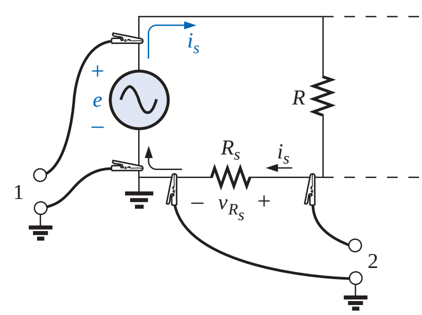


FIG. 15.102
Determining the phase relationship between e and i_s.

In general, therefore, for dual-trace measurements of phase relationships, be particularly careful of the grounding arrangement, and fully utilize the in-phase relationship between the voltage and current of a resistor.

15.14 APPLICATIONS

Home Wiring

An expanded view of house wiring is provided in Fig. 15.103 to permit a discussion of the entire system. The house panel has been included with the "feed" and the important grounding mechanism. In addition, a

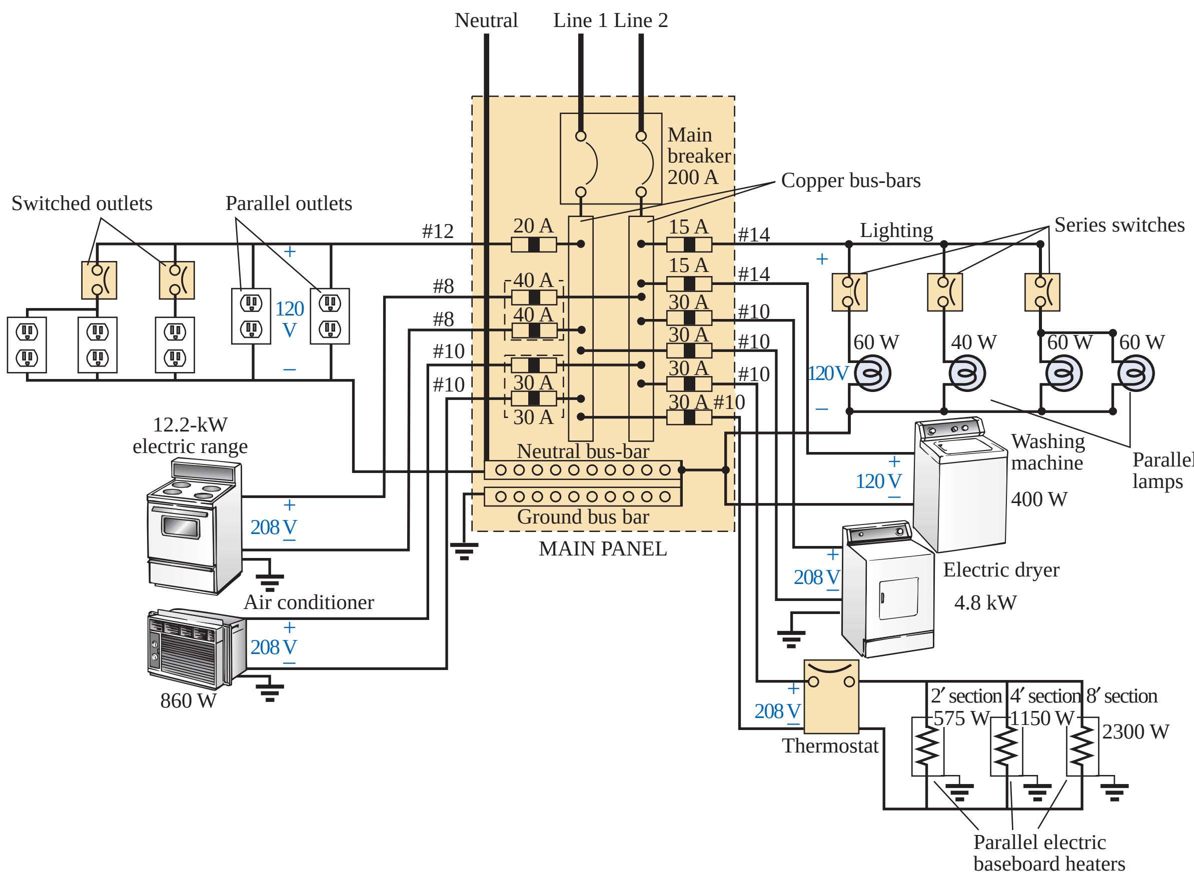


FIG. 15.103
Home wiring diagram.

number of typical circuits found in the home have been included to provide a sense for the manner in which the total power is distributed.

First note how the copper bars in the panel are laid out to provide both 120 V and 208 V. Between any one bar and ground is the single-phase 120-V supply. However, the bars have been arranged so that 208 V can be obtained between two vertical adjacent bars using a double-gang circuit breaker. When time permits, examine your own panel (but do not remove the cover), and note the dual circuit breaker arrangement for the 208-V supply.

For appliances such as fixtures and heaters that have a metal casing, the ground wire is connected to the metal casing to provide a direct path to ground path for a “shorting” or errant current as described in Section 7.7. For outlets and such that do not have a conductive casing, the ground lead is connected to a point on the outlet that distributes to all important points of the outlet.

Note the series arrangement between the thermostat and the heater but the parallel arrangement between heaters on the same circuit. In addition, note the series connection of switches to lights in the upper-right corner but the parallel connection of lights and outlets. Due to high current demand the air conditioner, heaters, and electric stove have 30-A breakers. Keep in mind that the total current does not equal the product of the two (or 60 A) since each breaker is in a line and the same current will flow through each breaker.

In general, you now have a surface understanding of the general wiring in your home. You may not be a qualified, licensed electrician, but at least you should now be able to converse with some intelligence about the system.

Speaker Systems

The best reproduction of sound is obtained using a different speaker for the low-, mid-, and high-frequency regions. Although the typical audio range for the human ear is from about 100 Hz to 20 kHz, speakers are available from 20 Hz to 40 kHz. For the low-frequency range usually extending from about 20 Hz to 300 Hz, a speaker referred to as a *woofer* is used. Of the three speakers, it is normally the largest. The mid-range speaker is typically smaller in size and covers the range from about 100 Hz to 5 kHz. The *tweeter,* as it is normally called, is usually the smallest of the three speakers and typically covers the range from about 2 kHz to 25 kHz. There is an overlap of frequencies to ensure that frequencies aren't lost in those regions where the response of one drops off and the other takes over. A great deal more about the range of each speaker and their dB response (a term you may have heard when discussing speaker response) will be covered in detail in Chapter 23.

One popular method for hooking up the three speakers is the *crossover* configuration of Fig. 15.104. Note that it is nothing more than a parallel network with a speaker in each branch and full applied voltage across each branch. The added elements (inductors and capacitors) were carefully chosen to set the range of response for each speaker. Note that each speaker is labeled with an impedance level and associated frequency. This type of information is typical when purchasing a quality speaker. It immediately identifies the type of speaker and reveals at which frequency it will have its maximum response. A detailed analysis of the same network will be included in Section 23.15. For now, however, it should prove interesting to determine the total impedance of each branch at specific frequencies to see if indeed the response of one will far outweigh the response of the other two. Since an amplifier with an output impedance of 8 Ω is to be employed, maximum

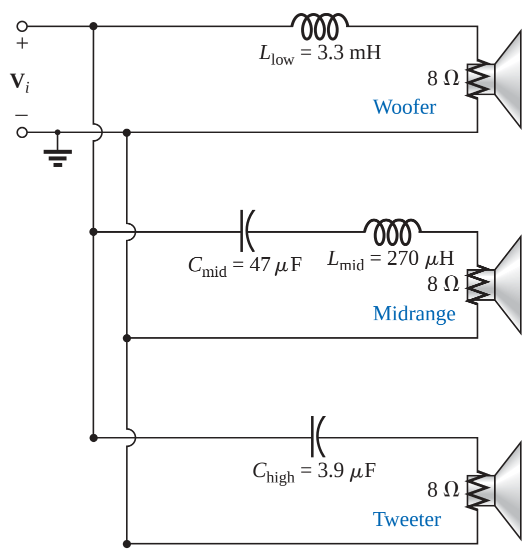


FIG. 15.104
Crossover speaker system.

transfer of power (see Section 18.5 for ac networks) to the speaker will result when the impedance of the branch is equal to or very close to 8 Ω.

Let us begin by examining the response of the frequencies to be carried primarily by the mid-range speaker since it represents the greatest portion of the human hearing range. Since the mid-range speaker branch is rated at 8 Ω at 1.4 kHz, let us test the effect of applying 1.4 kHz to all branches of the crossover network.

For the mid-range speaker:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.4\text{ kHz})(47\ \mu\text{F})} = 2.42\ \Omega$$

$$X_L = 2\pi fL = 2\pi(1.4\text{ kHz})(270\ \mu\text{H}) = 2.78\ \Omega$$

$$R = 8\ \Omega$$

and
$$\begin{aligned}\mathbf{Z}_{\text{mid-range}} &= R + j(X_L - X_C) = 8\ \Omega + j(2.78\ \Omega - 2.42\ \Omega)\\ &= 8\ \Omega + j0.36\ \Omega\\ &= 8.008\ \Omega\ \angle -2.58^\circ \cong 8\ \Omega\ \angle 0^\circ = R\end{aligned}$$

In Fig. 15.105(a), the amplifier with the output impedance of 8 Ω has been applied across the mid-range speaker at a frequency of 1.4 kHz. Since the total reactance offered by the two series reactive elements is so small compared to the 8-Ω resistance of the speaker, we can essentially replace the series combination of the coil and capacitor by a short circuit of 0 Ω. We are then left with a situation where the load impedance is an exact match with the output impedance of the amplifier, and maximum power will be delivered to the speaker. Because of the equal series impedances, each will capture half the applied voltage or 6 V. The power to the speaker is then $V^2/R = (6\text{ V})^2/8\ \Omega = 4.5$ W.

At a frequency of 1.4 kHz we would expect the woofer and tweeter to have minimum impact on the generated sound. We will now test the validity of this statement by determining the impedance of each branch at 1.4 kHz.

For the woofer:

$$X_L = 2\pi fL = 2\pi(1.4\text{ kHz})(3.3\text{ mH}) = 29.03\ \Omega$$

and
$$\begin{aligned}\mathbf{Z}_{\text{woofer}} &= R + jX_L = 8\ \Omega + j29.03\ \Omega\\ &= 30.11\ \Omega\ \angle 74.59^\circ\end{aligned}$$

which is a poor match with the output impedance of the amplifier. The resulting network is shown in Fig. 15.105(b).

The total load on the source of 12 V is

$$\begin{aligned}\mathbf{Z}_T &= 8\ \Omega + 8\ \Omega + j29.03\ \Omega = 16\ \Omega + j29.03\ \Omega\\ &= 33.15\ \Omega\ \angle 61.14^\circ\end{aligned}$$

and the current is

$$\begin{aligned}\mathbf{I} &= \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{12\text{ V}\ \angle 0^\circ}{33.15\ \Omega\ \angle 61.14^\circ}\\ &= 362\text{ mA}\ \angle -61.14^\circ\end{aligned}$$

The power to the 8-Ω speaker is then

$$P_{\text{woofer}} = I^2R = (362\text{ mA})^2 8\ \Omega = \mathbf{1.048\ W}$$

or about 1 W.

Consequently, the sound generated by the mid-range speaker will far outweigh the response of the woofer (as it should).

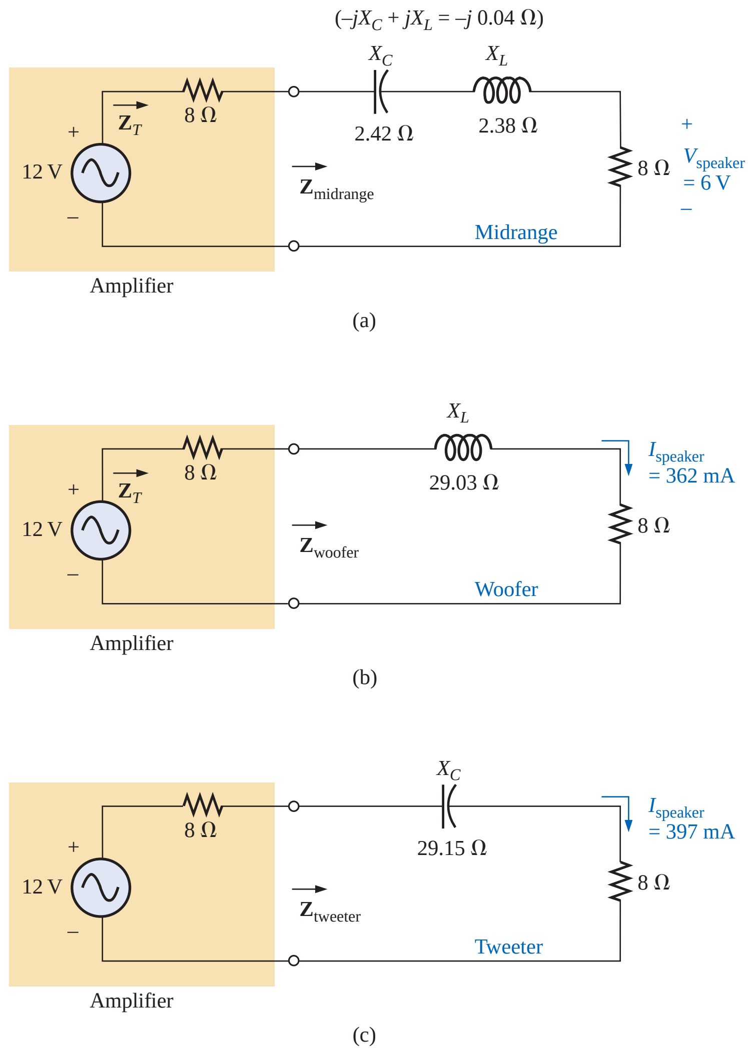


FIG. 15.105

Crossover network: (a) mid-range speaker at 1.4 kHz; (b) woofer at 1.4 kHz; (c) tweeter.

For the tweeter:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.4\text{ kHz})(3.9\ \mu\text{F})} = 29.15\ \Omega$$

and

$$\mathbf{Z}_{\text{tweeter}} = R - jX_C = 8\ \Omega - j\,29.15\ \Omega$$
$$= 30.23\ \Omega \angle -74.65^\circ$$

which, as for the woofer, is a poor match with the output impedance of the amplifier. The current

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{12\text{ V}\angle 0^\circ}{30.23\ \Omega \angle -74.65^\circ}$$
$$= 397\text{ mA}\angle 74.65^\circ$$

The power to the 8-Ω speaker is then

$$P_{\text{tweeter}} = I^2R = (397\text{ mA})^2(8\ \Omega) = \mathbf{1.261\ W}$$

or about 1.3 W.

Consequently, the sound generated by the mid-range speaker will far outweigh the response of the tweeter also.

All in all, the mid-range speaker predominates at a frequency of 1.4 kHz for the crossover network of Fig. 15.104.

Just for interest sake, let us now determine the impedance of the tweeter at 20 kHz and the impact of the woofer at this frequency.

For the tweeter:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(20\text{ kHz})(3.9\ \mu\text{F})} = 2.04\ \Omega$$

with $$\mathbf{Z}_{\text{tweeter}} = 8\ \Omega - j\,2.04\ \Omega = 8.26\ \Omega\ \angle -14.31°$$

Even though the magnitude of the impedance of the branch is not exactly 8 Ω, it is very close, and the speaker will receive a high level of power (actually 4.43 W).

For the woofer:

$$X_L = 2\pi fL = 2\pi(20\text{ kHz})(3.3\text{ mH}) = 414.69\ \Omega$$

with $$\mathbf{Z}_{\text{woofer}} = 8\ \Omega - j\,414.69\ \Omega = 414.77\ \Omega\ \angle 88.9°$$

which is a terrible match with the output impedance of the amplifier. Therefore, the speaker will receive a very low level of power (6.69 mW ≅ 0.007 W).

For all the calculations, note that the capacitive elements predominate at low frequencies, and the inductive elements at high frequencies. For the low frequencies, the reactance of the coil will be quite small, permitting a full transfer of power to the speaker. For the high-frequency tweeter, the reactance of the capacitor is quite small, providing a direct path for power flow to the speaker.

Phase-Shift Power Control

In Chapter 12 the internal structure of a light dimmer was examined and its basic operation described. We can now turn our attention to how the power flow to the bulb is controlled.

If the dimmer were composed of simply resistive elements, all the voltages of the network would be in phase as shown in Fig. 15.106(a). If we assume that 20 V are required to turn on the triac of Fig. 12.49, then the power will be distributed to the bulb for the period highlighted by the blue area of Fig. 15.106(a). For this situation, the bulb is close to full brightness since the applied voltage is available to the bulb for almost the entire cycle. To reduce the power to the bulb (and therefore reduce its brightness), the controlling voltage would have to have a lower peak voltage as shown in Fig. 15.106(b). In fact, the waveform of Fig. 15.106(b) is such that the turn-on voltage is not reached until the peak value occurs. In this case power is delivered to the bulb for only half the cycle, and the brightness of the bulb will be reduced. The problem with using only resistive elements in a dimmer now becomes apparent: The bulb can be made no dimmer than the situation depicted by Fig. 15.106(b). Any further reduction in the controlling voltage would reduce its peak value below the trigger level, and the bulb would never turn on.

This dilemma can be resolved by using a series combination of elements such as shown in Fig. 15.107(a) from the dimmer of Fig. 12.49. Note that the controlling voltage is the voltage across the capacitor,

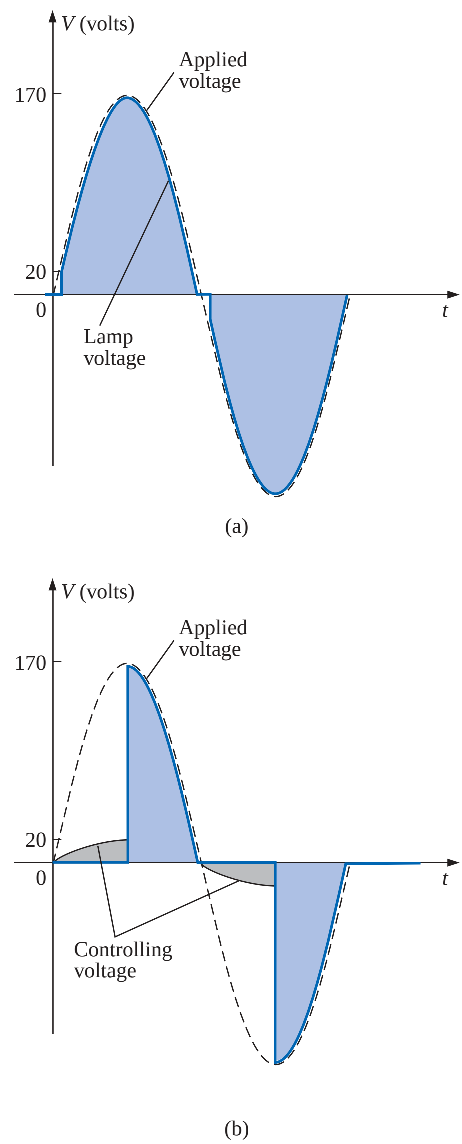


FIG. 15.106
Light dimmer: (a) with purely resistive elements; (b) half-cycle power distribution.

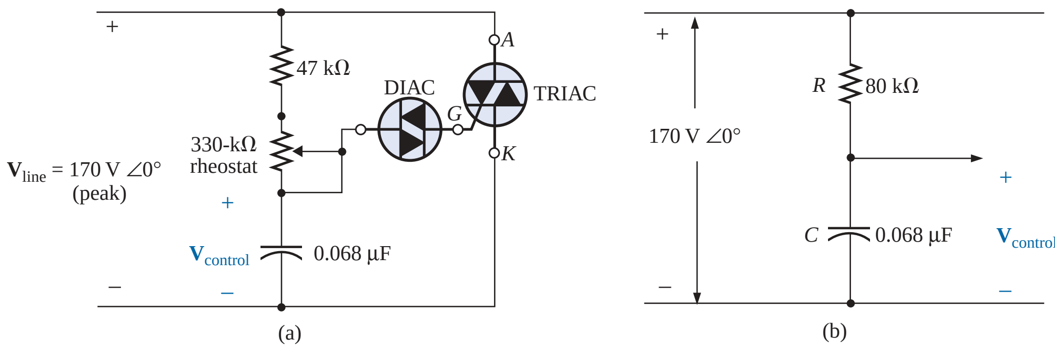


FIG. 15.107
Light dimmer: (a) from Fig. 12.49; (b) with rheostat set at 33 kΩ.

while the full line voltage of 120 V rms, 170 V peak, is across the entire branch. To describe the behavior of the network, let us examine the case defined by setting the potentiometer (used as a rheostat) to 1/10 its maximum value, or 33 kΩ. Combining the 33 kΩ with the fixed resistance of 47 kΩ will result in a total resistance of 80 kΩ and the equivalent network of Fig. 15.107(b).

At 60 Hz, the reactance of the capacitor is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60\text{ Hz})(62\ \mu\text{F})} = 42.78\text{ k}\Omega$$

Applying the voltage divider rule:

$$\begin{aligned}\mathbf{V}_{\text{control}} &= \frac{\mathbf{Z}_C\mathbf{V}_s}{\mathbf{Z}_R + \mathbf{Z}_C} \\ &= \frac{(42.78\text{ k}\Omega \angle -90°)(V_s \angle 0°)}{80\text{ k}\Omega - j\,42.78\text{ k}\Omega} = \frac{42.78\text{ k}\Omega\ V_s \angle -90°}{90.72\text{ k}\Omega \angle -28.14°} \\ &= 0.472V_s \angle -61.86°\end{aligned}$$

Using a peak value of 170 V:

$$\begin{aligned}\mathbf{V}_{\text{control}} &= 0.472(170\text{ V}) \angle -61.86° \\ &= 80.24\text{ V} \angle -61.86°\end{aligned}$$

producing the waveform of Fig. 15.108(a). The result is a waveform with a phase shift of 61.86° (lagging the applied line voltage) and a relatively high peak value. The high peak value will result in a quick transition to the 20-V turn-on level, and power will be distributed to the bulb for the major portion of the applied signal. Recall from the discussion of Chapter 12 that the response in the negative region is a replica of that achieved in the positive region. If we reduced the potentiometer resistance further, the phase angle would be reduced, and the bulb would burn brighter. The situation is now very similar to that described for the response of Fig. 15.106(a). In other words, nothing has been gained thus far by using the capacitive element in the control network. However, let us now increase the potentiometer resistance to 200 kΩ and note the effect on the controlling voltage.

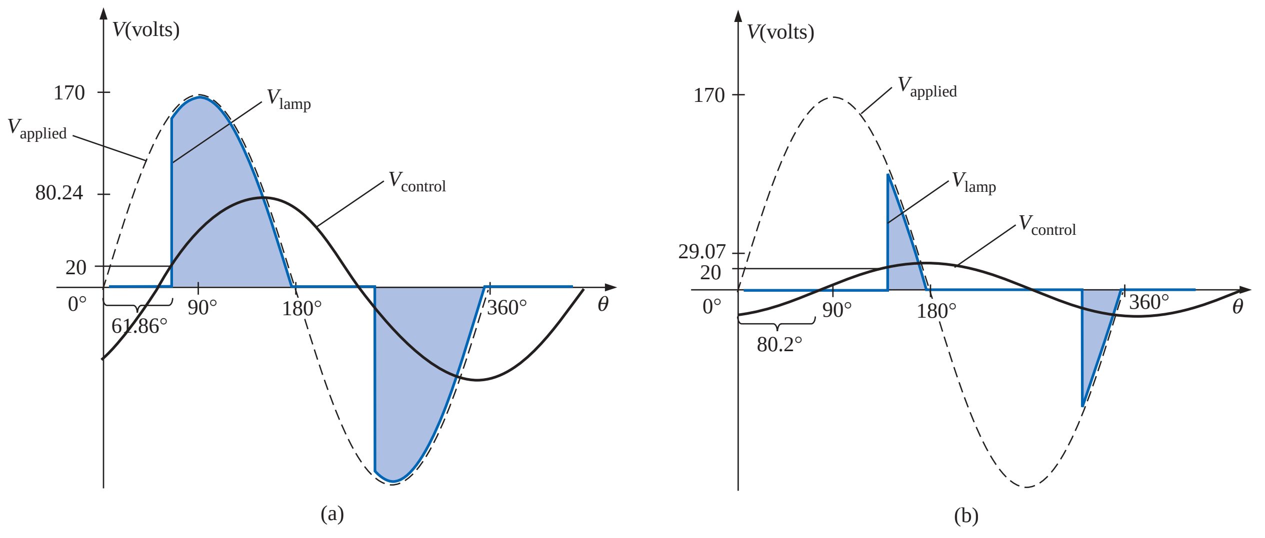


FIG. 15.108
Light dimmer of Fig. 12.49: (a) rheostat set at 33 kΩ; (b) rheostat set at 200 kΩ.

That is,

$$R_T = 200\text{ k}\Omega + 47\text{ k}\Omega = 247\text{ k}\Omega$$

$$\mathbf{V}_{\text{control}} = \frac{\mathbf{Z}_C\mathbf{V}_s}{\mathbf{Z}_R + \mathbf{Z}_C}$$

$$= \frac{(42.78\text{ k}\Omega \angle -90°)(V_s \angle 0°)}{247\text{ k}\Omega - j42.78\text{ k}\Omega} = \frac{42.78\text{ k}\Omega\ V_s \angle -90°}{250.78\text{ k}\Omega \angle -9.8°}$$

$$= 0.171V_s \angle -80.2°$$

and using a peak value of 170 V, we have

$$\mathbf{V}_{\text{control}} = 0.171(170\text{ V}) \angle -80.2°$$
$$= 29.07\text{ V} \angle -80.2°$$

The peak value has been substantially reduced to only 29.07 V, and the phase-shift angle has increased to 80.2°. The result, as depicted by Fig. 15.108(b), is that the firing potential of 20 V is not reached until near the end of the positive region of the applied voltage. Power is delivered to the bulb for only a very short period of time, causing the bulb to be quite dim, significantly dimmer than obtained from the response of Fig. 15.106(b).

A conduction angle less than 90° is therefore possible due only to the phase shift introduced by the series *R-C* combination. Thus, it is possible to construct a network of some significance with a rather simple pair of elements.

15.15 COMPUTER ANALYSIS

PSpice

Series *R-L-C* Circuit The *R-L-C* network of Fig. 15.35 will now be analyzed using OrCAD Capture. Since the inductive and capacitive reactances cannot be entered onto the screen, the associated inductive and capacitive levels were first determined as follows:

$$X_L = 2\pi f L \Rightarrow L = \frac{X_L}{2\pi f} = \frac{7\ \Omega}{2\pi(1\ \text{kHz})} = 1.114\ \text{mH}$$

$$X_C = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(1\ \text{kHz})3\ \Omega} = 53.05\ \mu\text{F}$$

The values were then entered into the schematic as shown in Fig. 15.109. For the ac source, the sequence is **Place part** icon-**SOURCE-VSIN-OK** with **VOFF** set at 0 V, **VAMPL** set at 70.7 V (the peak value of the applied sinusoidal source in Fig. 15.35), and **FREQ** = 1 kHz. If we double-click on the source symbol, the **Property Editor** will appear, confirming the above choices and showing that **DF** = 0 s, **PHASE** = 0°, and **TD** = 0 s as set by the default levels. We are now ready to do an analysis of the circuit for the fixed frequency of 1 kHz.

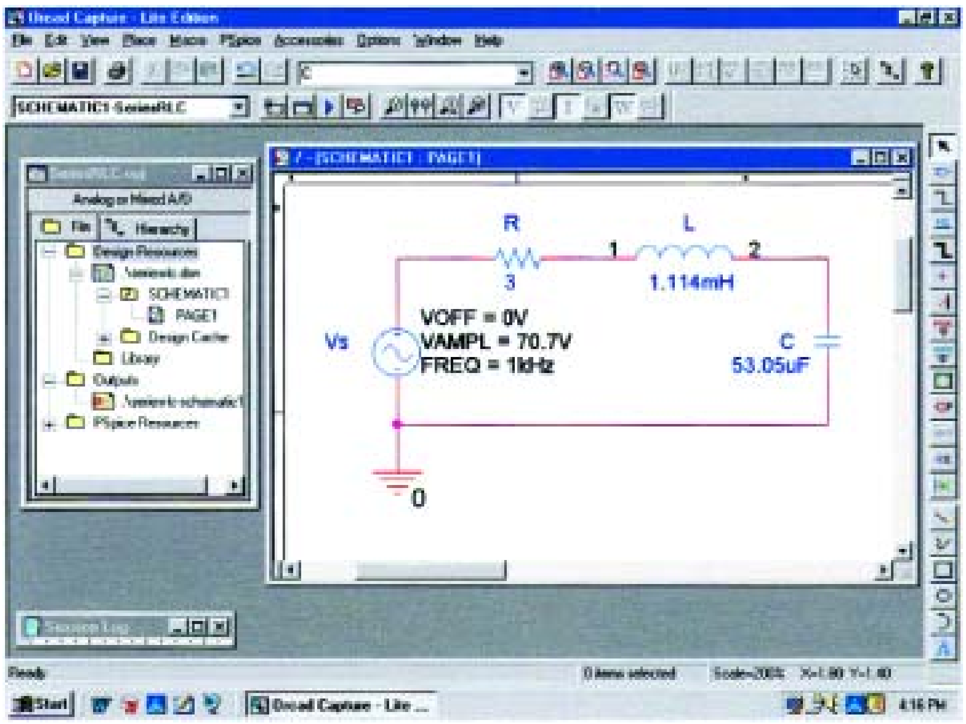


FIG. 15.109
Using PSpice to analyze a series R-L-C ac circuit.

The simulation process is initiated by first selecting the **New Simulation Profile** icon and inserting **SeriesRLC** as the **Name** followed by **Create.** The **Simulation Settings** dialog will now appear, and since we are continuing to plot the results against time, the **Time Domain(Transient)** option is selected under **Analysis type.** Since the period of each **cycle** of the applied source is 1 ms, the **Run to time** will be set at 5 ms so that five cycles will appear. The **Start saving data after** will be left at 0 s even though there will be an oscillatory period for the reactive elements before the circuit settles down. The **Maximum step size** will be set at 5 ms/1000 = 5 μs. Finally **OK** is selected followed by the

Run PSpice key. The result will be a blank screen with an *x*-axis extending from 0 s to 5 ms.

The first quantity of interest is the current through the circuit, so **Trace-Add-Trace** is selected followed by **I(R)** and **OK.** The resulting plot of Fig. 15.110 clearly shows that there is a period of storing and discharging of the reactive elements before a steady-state level is established. It would appear that after 3 ms, steady-state conditions have been essentially established. Select the **Toggle cursor** key, and left-click the mouse; a cursor will result that can be moved along the axis near the maximum value around 1.4 ms. In fact, the cursor reveals a maximum value of 16.4 A which exceeds the steady-state solution by over 2 A. A right click of the mouse will establish a second cursor on the screen that can be placed near the steady-state peak around 4.4 ms. The resulting peak value is about 14.15 A which is a match with the longhand solution for Fig. 15.35. We will therefore assume that steady-state conditions have been established for the circuit after 4 ms.

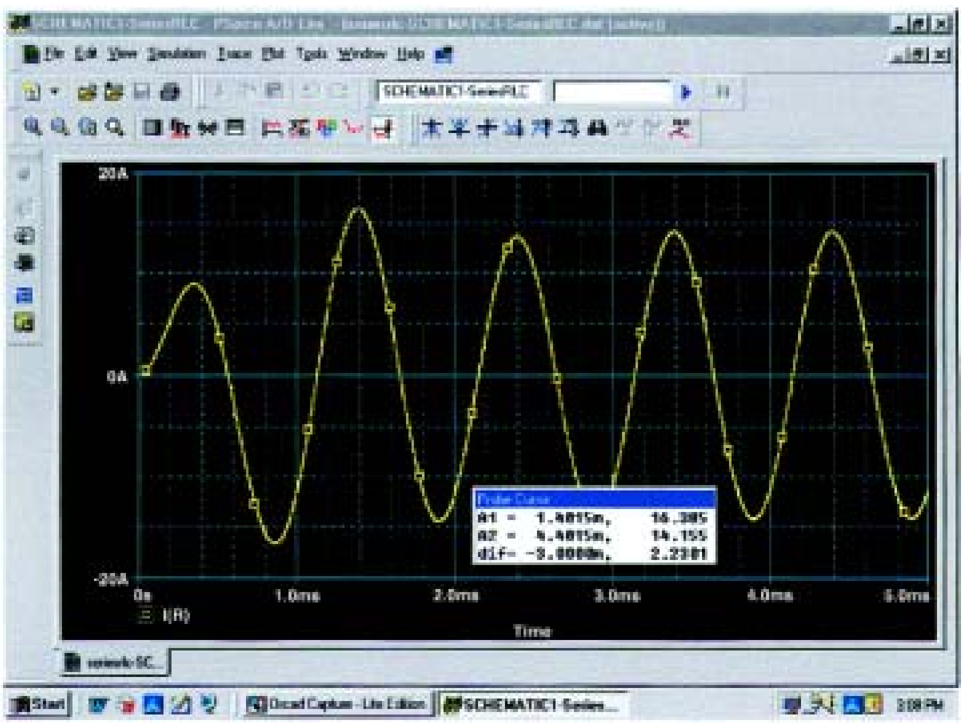


FIG. 15.110

A plot of the current for the circuit of Fig. 15.109 showing the transition from the transient state to the steady-state response.

Let us now add the source voltage through **Trace-Add Trace-V(Vs:+)-OK** to obtain the multiple plot at the bottom of Fig. 15.111. For the voltage across the coil, the sequence **Plot-Add Plot to Window-Trace-Add Trace-V(L:1)-V(L:2)** will result in the plot appearing at the top of Fig. 15.111. Take special note of the fact that the **Trace Expression** is **V(L:1)−V(L:2)** rather than just **V(L:1)** because **V(L:1)** would be the voltage from that point to ground which would include the voltage across the capacitor. In addition, the **−** sign between the two comes from the **Functions or Macros** list at right of the **Add Traces**

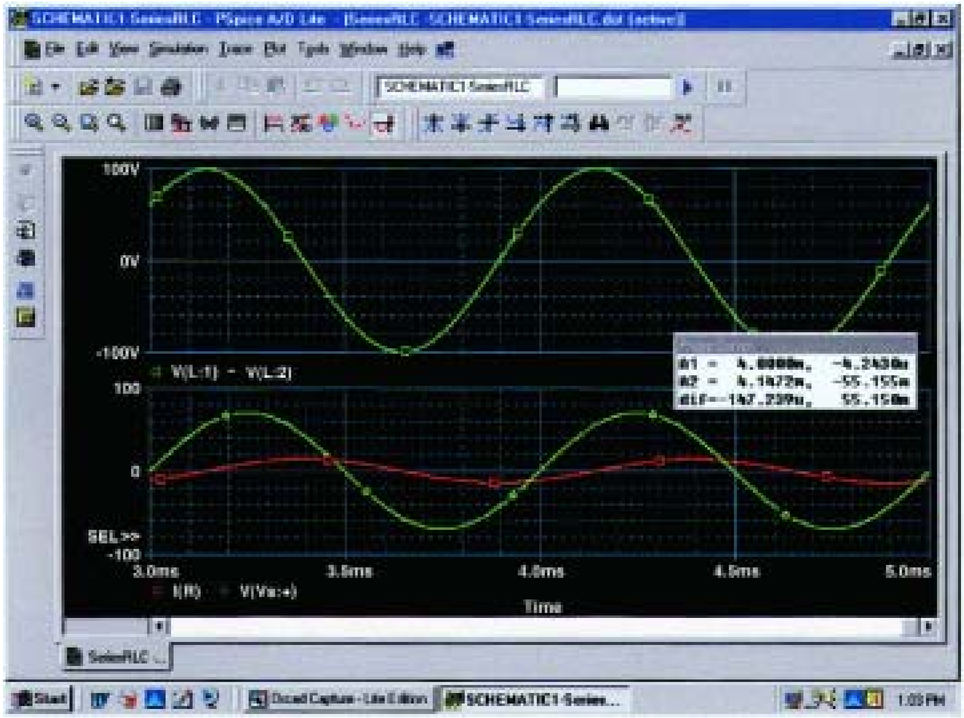


FIG. 15.111

A plot of the steady-state response ($t > 3$ ms) for v_L, v_s, and i for the circuit of Fig. 15.109.

dialog box. Finally, since we know that the waveforms are fairly steady after 3 ms, let us cut away the waveforms before 3 ms with **Plot-Axis Settings-X axis-User Defined-3ms to 5ms-OK** to obtain the two cycles of Fig. 15.111. Now you can clearly see that the peak value of the voltage across the coil is 100 V to match the analysis of Fig. 15.35. It is also clear that the applied voltage leads the input current by an angle that can be determined using the cursors. First activate the cursor option by selecting the cursor key (a red plot through the origin) in the second toolbar down from the menu bar. Then select **V(Vs:+)** at the bottom left of the screen with a left click of the mouse, and set it at that point where the applied voltage passes through the horizontal axis with a positive slope. The result is **A1** = 4 ms at $-4.243\ \mu$V $\cong$ 0 V. Then select **I(R)** at the bottom left of the screen with a right click of the mouse, and place it at the point where the current waveform passes through the horizontal axis with a positive slope. The result is **A2** = 4.15 ms at -55.15 mA = 0.55 A $\cong$ 0 A (compared to a peak value of 14.14 A). At the bottom of the **Probe Cursor** dialog box, the time difference is 147.24 μs.

Now set up the ratio

$$\frac{147.24\ \mu\text{s}}{1000\ \mu\text{s}} = \frac{\theta}{360^\circ}$$

$$\theta = 52.99^\circ$$

The phase angle by which the applied voltage leads the source is 52.99° which is very close to the theoretical solution of 53.13° obtained in Fig. 15.39. Increasing the number of data points for the plot would have increased the accuracy level and brought the results closer to 53.13°.

Electronics Workbench

We will now examine the response of a network versus frequency rather than time using the network of Fig. 15.79 which now appears on the schematic of Fig. 15.112. The ac current source appears as **AC_CURRENT_SOURCE** in the **Sources** tool bin next to the ac voltage source. Note that the current source was given an amplitude of 1 A to establish a magnitude match between the response of the voltage across the network and the impedance of the network. That is,

$$|Z_T| = \left|\frac{V_s}{I_s}\right| = \left|\frac{V_s}{1\text{ A}}\right| = |V_s|$$

Before applying computer methods, we should develop a rough idea of what to expect so that we have something to which to compare the computer solution. At very high frequencies such as 1 MHz, the impedance of the inductive element will be about 25 kΩ which when placed in parallel with the 220 Ω will look like an open circuit. The result is that as the frequency gets very high, we should expect the impedance of the network to approach the 220-Ω level of the resistor. In addition, since the network will take on resistive characteristics at very high frequencies, the angle associated with the input impedance should also approach 0 Ω. At very low frequencies the reactance of the inductive element will be much less than the 220 Ω of the resistor, and the network will take on inductive characteristics. In fact, at, say, 10 Hz, the reactance of the inductor is only about 0.25 Ω which is very close to a short-circuit equivalent compared to the parallel 220-Ω resistor. The result is that the impedance of the network is very close to 0 Ω at very low frequencies. Again, since the inductive effects are so strong at low

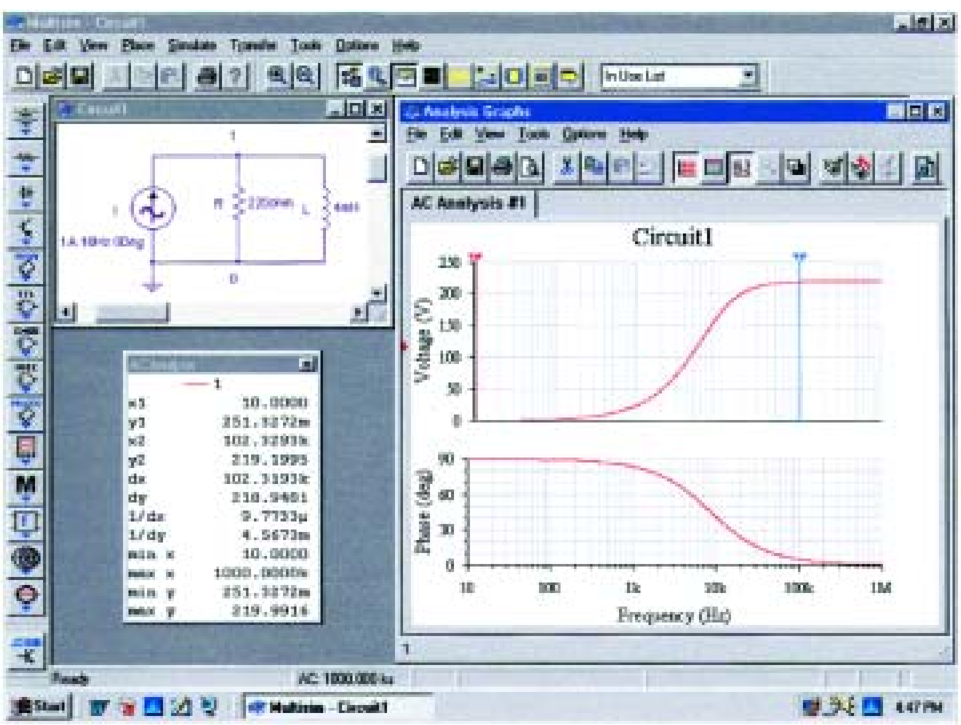


FIG. 15.112

Obtaining an impedance plot for a parallel R-L network using Electronics Workbench.

frequencies, the phase angle associated with the input impedance should be very close to 90°.

Now for the computer analysis. The current source, the resistor element, and the inductor are all placed and connected using procedures described in detail in earlier chapters. However, there is one big difference this time that the user must be aware of: Since the output will be plotted versus frequency, the **Analysis Setup** heading must be selected in the **AC Current** dialog box for the current source. When selected, the **AC Magnitude** must be set to the value of the ac source. In this case, the default level of **1A** matches that of the applied source, so we were set even if we failed to check the setting. In the future, however, a voltage or current source may be used that does not have an amplitude of 1, and proper entries must be made to this listing.

For the simulation the sequence **Simulate-Analyses-AC Analysis** is first applied to obtain the **AC Analysis** dialog box. The **Start frequency** will be set at **10 Hz** so that we have entries at very low frequencies, and the **Stop frequency** will be set at **1MHz** so that we have data points at the other end of the spectrum. The **Sweep type** can remain **Decade,** but the number of points per decade will be 1000 so that we obtain a detailed plot. The **Vertical scale** will be set on **Linear.** Within **Output variables** we find that only one node, **1,** is defined. Shifting it over to the **Selected variables for analysis** column using the **Plot during simulation** key pad and then hitting the **Simulate** key will result in the two plots of Fig. 15.112. The **Show/Hide Grid** key was selected to place the grid on the graph, and the **Show/Hide Cursors** key was selected to place the **AC Analysis** dialog box appearing in Fig. 15.112. Since two graphs are present, we must define the one we are working on by clicking on the **Voltage** or **Phase** heading on the left side of each plot. A small red arrow will appear when selected to keep us aware of the active plot. When setting up the cursors, be sure that you have activated the correct plot. When the red cursor is moved to 10 Hz **(x1),** we find that the voltage across the network is only 0.251 V **(y1),** resulting in an input impedance of only 0.25 Ω—quite small and matching our theoretical prediction. In addition, note that the phase angle is essentially at 90° in the other plot, confirming our other assumption above—a totally inductive network. If we set the blue cursor near 100 kHz (**x2** = 102.3 kHz), we find that the impedance at 219.2 Ω **(y2)** is closing in on the resistance of the parallel resistor of 220 Ω, again confirming the preliminary analysis above. As noted in the bottom of the **AC Analysis** box, the maximum value of the voltage is 219.99 Ω or essentially 220 Ω at 1 MHz. Before leaving the plot, note the advantages of using a log axis when you want a response over a wide frequency range.

PROBLEMS

SECTION 15.2 Impedance and the Phasor Diagram

1. Express the impedances of Fig. 15.113 in both polar and rectangular forms.

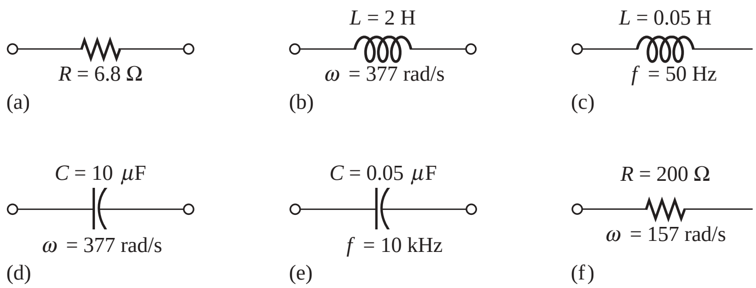


FIG. 15.113
Problem 1.

2. Find the current i for the elements of Fig. 15.114 using complex algebra. Sketch the waveforms for v and i on the same set of axes.

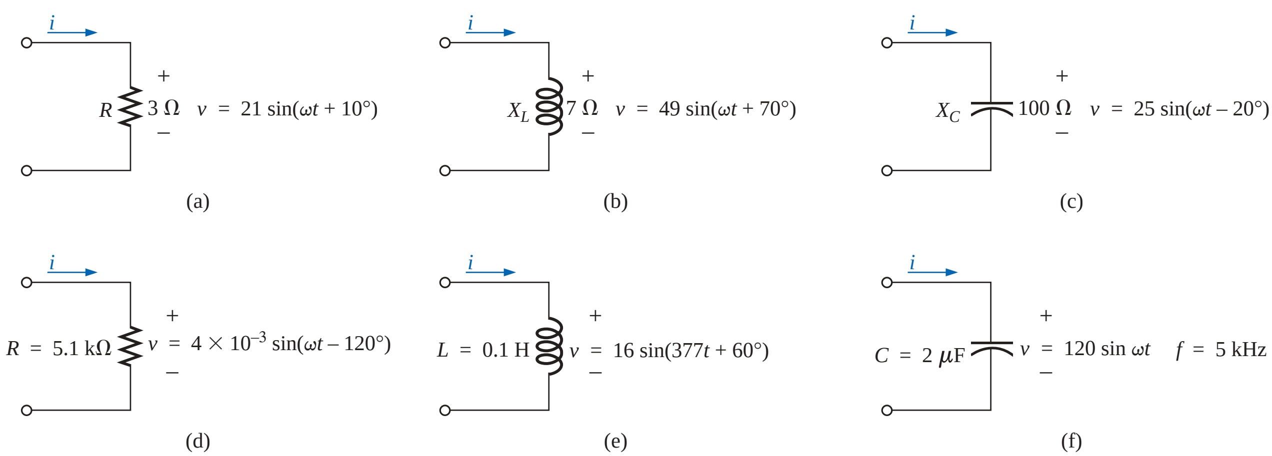


FIG. 15.114
Problem 2.

3. Find the voltage v for the elements of Fig. 15.115 using complex algebra. Sketch the waveforms of v and i on the same set of axes.

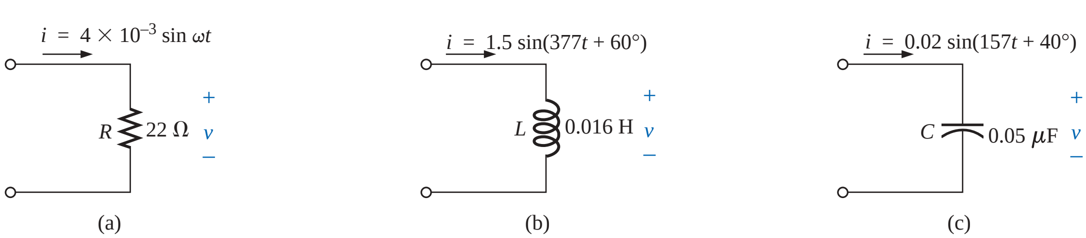


FIG. 15.115
Problem 3.

SECTION 15.3 Series Configuration

4. Calculate the total impedance of the circuits of Fig. 15.116. Express your answer in rectangular and polar forms, and draw the impedance diagram.

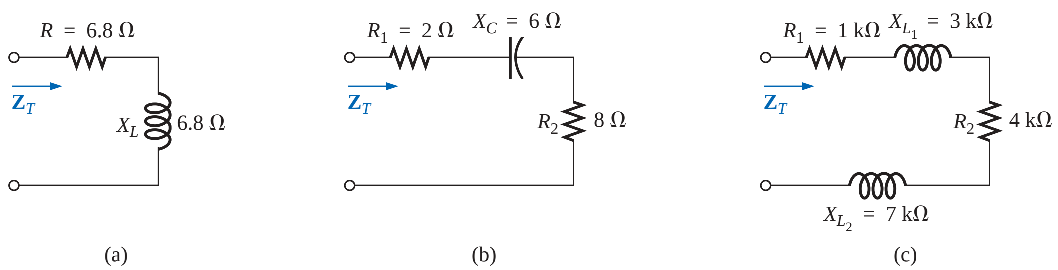


FIG. 15.116
Problem 4.

5. Calculate the total impedance of the circuits of Fig. 15.117. Express your answer in rectangular and polar forms, and draw the impedance diagram.

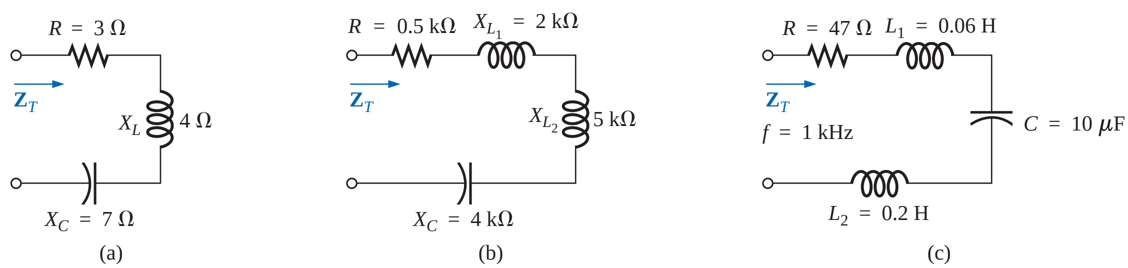


FIG. 15.117
Problem 5.

6. Find the type and impedance in ohms of the series circuit elements that must be in the closed container of Fig. 15.118 for the indicated voltages and currents to exist at the input terminals. (Find the simplest series circuit that will satisfy the indicated conditions.)

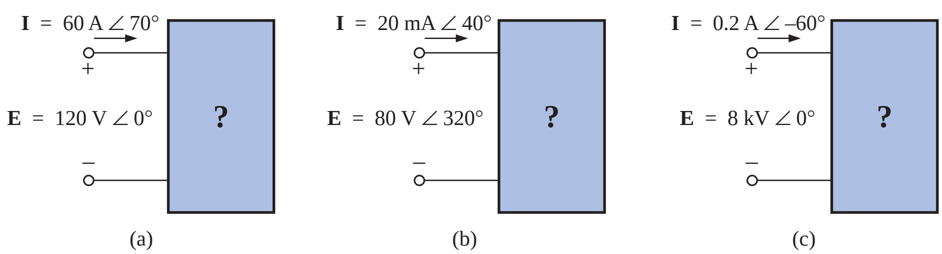


FIG. 15.118
Problems 6 and 26.

7. For the circuit of Fig. 15.119:
   a. Find the total impedance $\mathbf{Z}_T$ in polar form.
   b. Draw the impedance diagram.
   c. Find the current $\mathbf{I}$ and the voltages $\mathbf{V}_R$ and $\mathbf{V}_L$ in phasor form.
   d. Draw the phasor diagram of the voltages $\mathbf{E}$, $\mathbf{V}_R$, and $\mathbf{V}_L$, and the current $\mathbf{I}$.
   e. Verify Kirchhoff's voltage law around the closed loop.
   f. Find the average power delivered to the circuit.
   g. Find the power factor of the circuit, and indicate whether it is leading or lagging.
   h. Find the sinusoidal expressions for the voltages and current if the frequency is 60 Hz.
   i. Plot the waveforms for the voltages and current on the same set of axes.

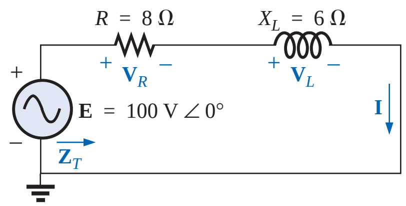


FIG. 15.119
Problems 7 and 47.

8. Repeat Problem 7 for the circuit of Fig. 15.120, replacing $\mathbf{V}_L$ with $\mathbf{V}_C$ in parts (c) and (d).

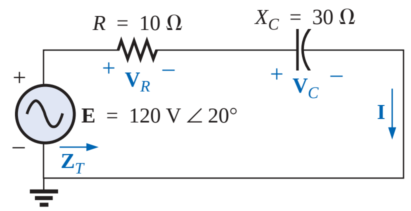


FIG. 15.120
Problem 8.

9. Given the network of Fig. 15.121:
   a. Determine $\mathbf{Z}_T$.
   b. Find $\mathbf{I}$.
   c. Calculate $\mathbf{V}_R$ and $\mathbf{V}_L$.
   d. Find P and F_p.

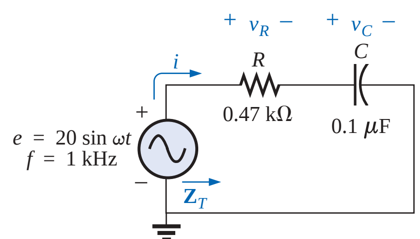


FIG. 15.121
Problems 9 and 49.

10. For the circuit of Fig. 15.122:
    a. Find the total impedance $\mathbf{Z}_T$ in polar form.
    b. Draw the impedance diagram.
    c. Find the value of C in microfarads and L in henries.
    d. Find the current $\mathbf{I}$ and the voltages $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ in phasor form.
    e. Draw the phasor diagram of the voltages $\mathbf{E}$, $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$, and the current $\mathbf{I}$.
    f. Verify Kirchhoff's voltage law around the closed loop.
    g. Find the average power delivered to the circuit.
    h. Find the power factor of the circuit, and indicate whether it is leading or lagging.
    i. Find the sinusoidal expressions for the voltages and current.
    j. Plot the waveforms for the voltages and current on the same set of axes.

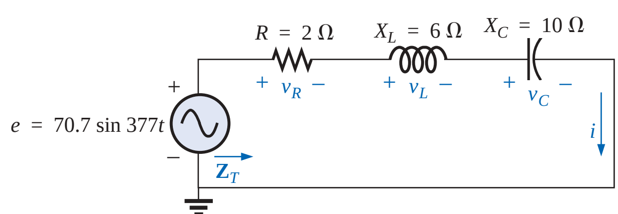


FIG. 15.122
Problem 10.

11. Repeat Problem 10 for the circuit of Fig. 15.123.

12. Using the oscilloscope reading of Fig. 15.124, determine the resistance R.

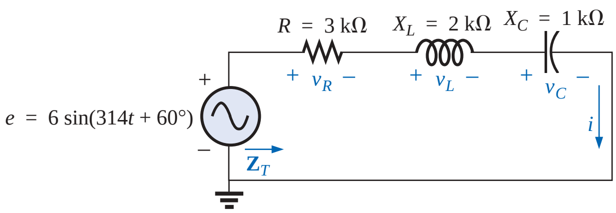


FIG. 15.123
Problem 11.

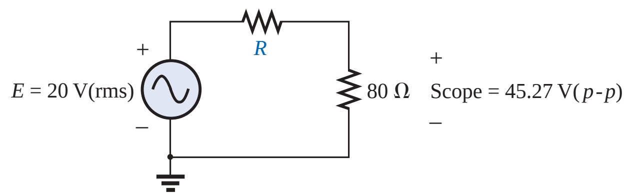


FIG. 15.124
Problem 12.

***13.** Using the DMM current reading and the oscilloscope measurement of Fig. 15.125:
a. Determine the inductance L.
b. Find the resistance R.

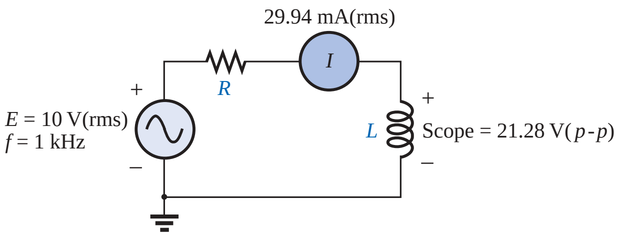


FIG. 15.125
Problem 13.

***14.** Using the oscilloscope reading of Fig. 15.126, determine the capacitance C.

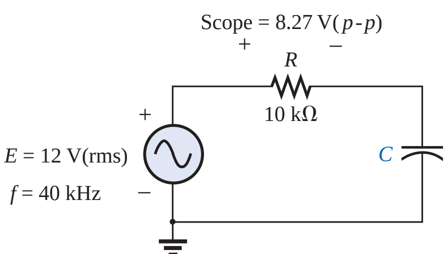


FIG. 15.126
Problem 14.

SECTION 15.4 Voltage Divider Rule

15. Calculate the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ for the circuit of Fig. 15.127 in phasor form using the voltage divider rule.

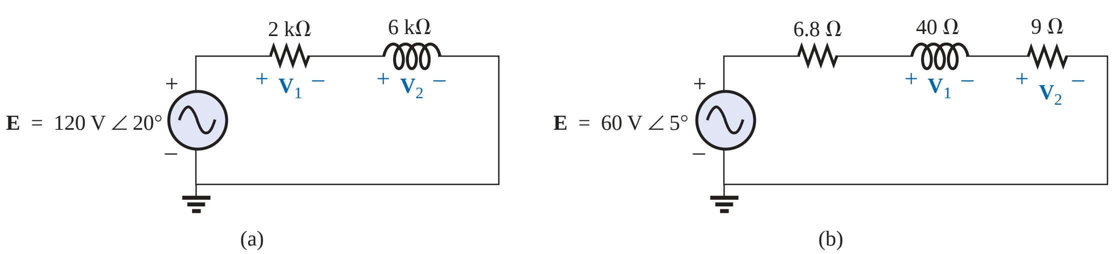


FIG. 15.127
Problem 15.

16. Calculate the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ for the circuit of Fig. 15.128 in phasor form using the voltage divider rule.

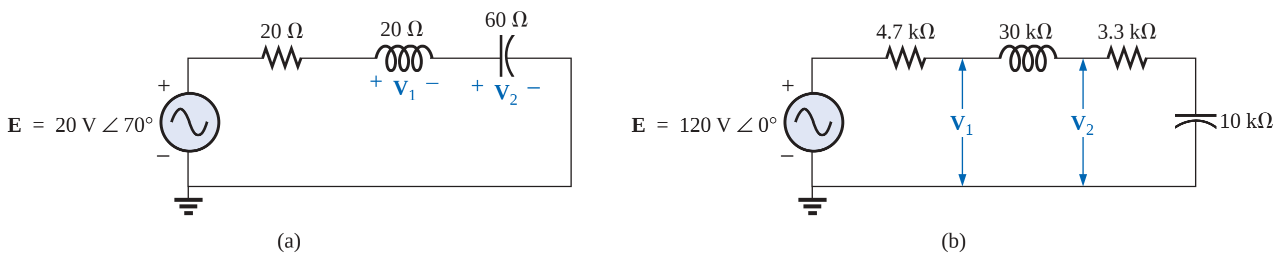


FIG. 15.128
Problem 16.

***17.** For the circuit of Fig. 15.129:

a. Determine **I**, $\mathbf{V}_R$, and $\mathbf{V}_C$ in phasor form.
b. Calculate the total power factor, and indicate whether it is leading or lagging.
c. Calculate the average power delivered to the circuit.
d. Draw the impedance diagram.
e. Draw the phasor diagram of the voltages **E**, $\mathbf{V}_R$, and $\mathbf{V}_C$, and the current **I**.
f. Find the voltages $\mathbf{V}_R$ and $\mathbf{V}_C$ using the voltage divider rule, and compare them with the results of part (a) above.
g. Draw the equivalent series circuit of the above as far as the total impedance and the current i are concerned.

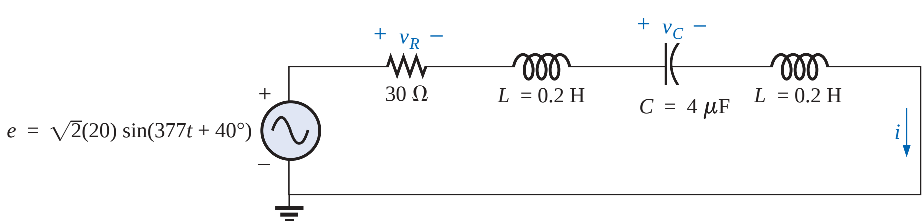


FIG. 15.129
Problems 17, 18, and 50.

***18.** Repeat Problem 17 if the capacitance is changed to 1000 μF.

19. An electrical load has a power factor of 0.8 lagging. It dissipates 8 kW at a voltage of 200 V. Calculate the impedance of this load in rectangular coordinates.

***20.** Find the series element or elements that must be in the enclosed container of Fig. 15.130 to satisfy the following conditions:

a. Average power to circuit = 300 W.
b. Circuit has a lagging power factor.

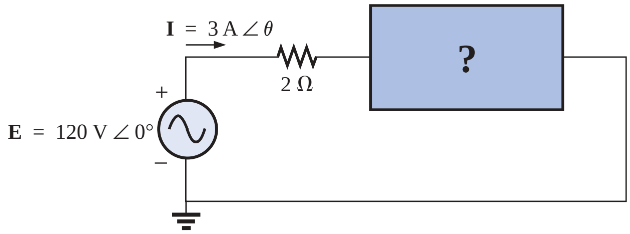


FIG. 15.130
Problem 20.

SECTION 15.5 Frequency Response of the *R-C* Circuit

***21.** For the circuit of Fig. 15.131:

a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 20 kHz.
b. Plot V_L versus frequency for the frequency range of part (a).
c. Plot θ_L versus frequency for the frequency range of part (a).
d. Plot V_R versus frequency for the frequency range of part (a).

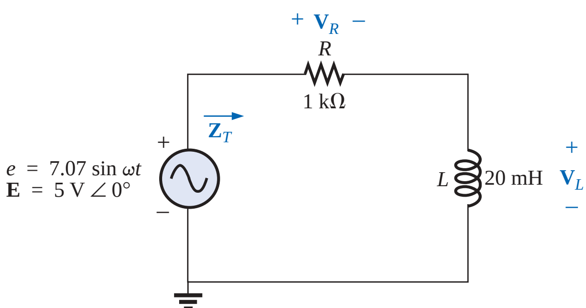


FIG. 15.131
Problem 21.

***22.** For the circuit of Fig. 15.132:

a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 10 kHz.
b. Plot V_C versus frequency for the frequency range of part (a).
c. Plot θ_C versus frequency for the frequency range of part (a).
d. Plot V_R versus frequency for the frequency range of part (a).

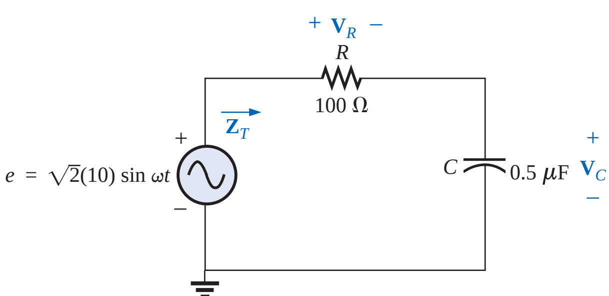


FIG. 15.132
Problem 22.

***23.** For the series *R-L-C* circuit of Fig. 15.133:

a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 20 kHz in increments of 1 kHz.
b. Plot V_C (magnitude only) versus frequency for the same frequency range of part (a).
c. Plot *I* (magnitude only) versus frequency for the same frequency range of part (a).

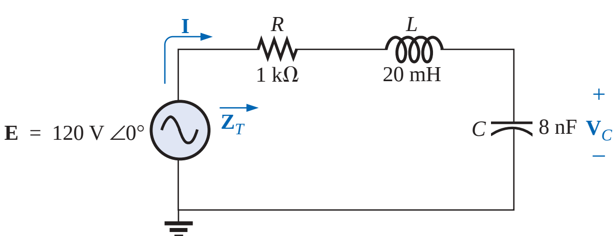


FIG. 15.133
Problem 23.

SECTION 15.7 Admittance and Susceptance

24. Find the total admittance and impedance of the circuits of Fig. 15.134. Identify the values of conductance and susceptance, and draw the admittance diagram.

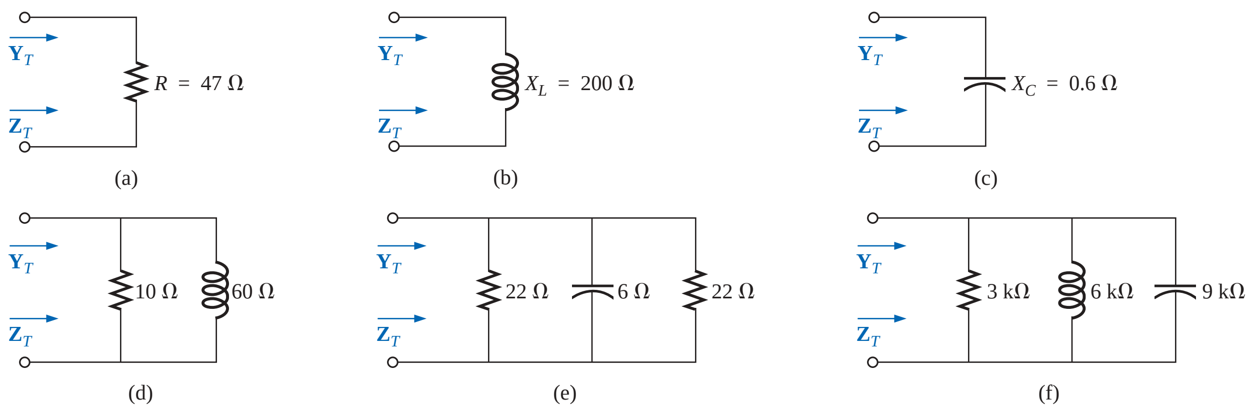


FIG. 15.134
Problem 24.

25. Find the total admittance and impedance of the circuits of Fig. 15.135. Identify the values of conductance and susceptance, and draw the admittance diagram.

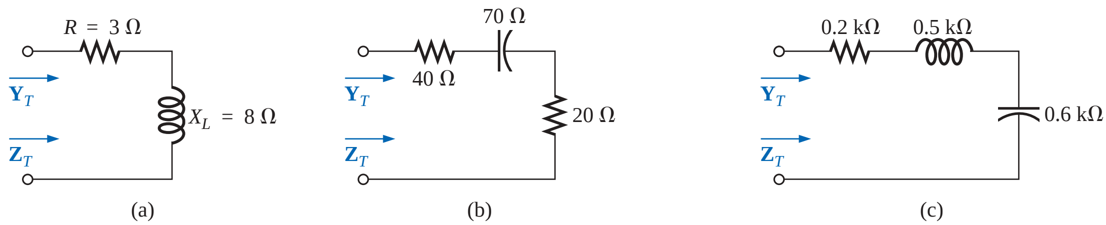


FIG. 15.135
Problem 25.

26. Repeat Problem 6 for the parallel circuit elements that must be in the closed container for the same voltage and current to exist at the input terminals. (Find the simplest parallel circuit that will satisfy the conditions indicated.)

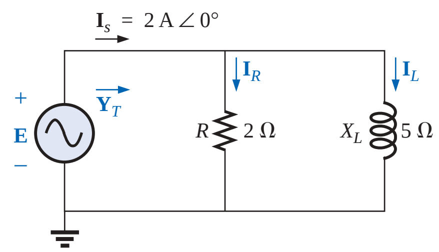


FIG. 15.136
Problem 27.

SECTION 15.8 Parallel ac Networks

27. For the circuit of Fig. 15.136:
- **a.** Find the total admittance $\mathbf{Y}_T$ in polar form.
- **b.** Draw the admittance diagram.
- **c.** Find the voltage **E** and the currents $\mathbf{I}_R$ and $\mathbf{I}_L$ in phasor form.
- **d.** Draw the phasor diagram of the currents $\mathbf{I}_s$, $\mathbf{I}_R$, and $\mathbf{I}_L$, and the voltage **E**.
- **e.** Verify Kirchhoff's current law at one node.
- **f.** Find the average power delivered to the circuit.
- **g.** Find the power factor of the circuit, and indicate whether it is leading or lagging.
- **h.** Find the sinusoidal expressions for the currents and voltage if the frequency is 60 Hz.
- **i.** Plot the waveforms for the currents and voltage on the same set of axes.

28. Repeat Problem 27 for the circuit of Fig. 15.137, replacing $\mathbf{I}_L$ with $\mathbf{I}_C$ in parts (c) and (d).

29. Repeat Problem 27 for the circuit of Fig. 15.138, replacing **E** with $\mathbf{I}_s$ in part (c).

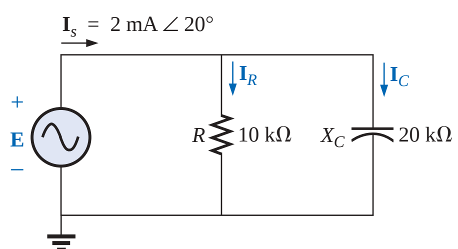


FIG. 15.137
Problem 28.

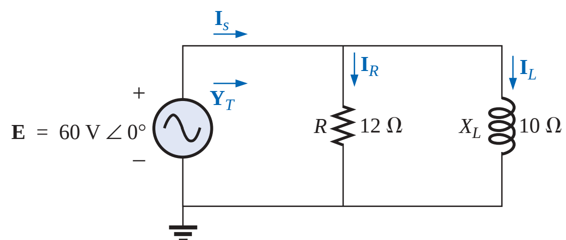


FIG. 15.138
Problems 29 and 48.

30. For the circuit of Fig. 15.139:
- **a.** Find the total admittance $\mathbf{Y}_T$ in polar form.
- **b.** Draw the admittance diagram.
- **c.** Find the value of C in microfarads and L in henries.
- **d.** Find the voltage **E** and currents $\mathbf{I}_R$, $\mathbf{I}_L$, and $\mathbf{I}_C$ in phasor form.
- **e.** Draw the phasor diagram of the currents $\mathbf{I}_s$, $\mathbf{I}_R$, $\mathbf{I}_L$, and $\mathbf{I}_C$, and the voltage **E.**
- **f.** Verify Kirchhoff's current law at one node.
- **g.** Find the average power delivered to the circuit.
- **h.** Find the power factor of the circuit, and indicate whether it is leading or lagging.
- **i.** Find the sinusoidal expressions for the currents and voltage.
- **j.** Plot the waveforms for the currents and voltage on the same set of axes.

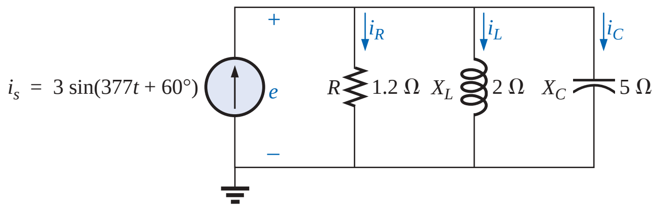


FIG. 15.139
Problem 30.

31. Repeat Problem 30 for the circuit of Fig. 15.140.

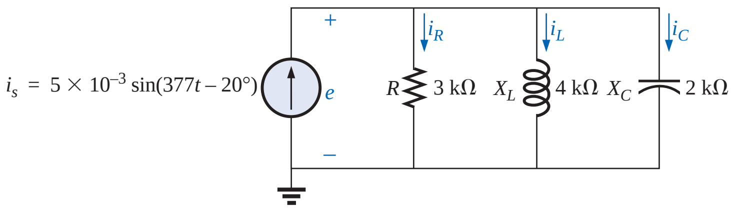


FIG. 15.140
Problem 31.

32. Repeat Problem 30 for the circuit of Fig. 15.141, replacing e with i_s in part (d).

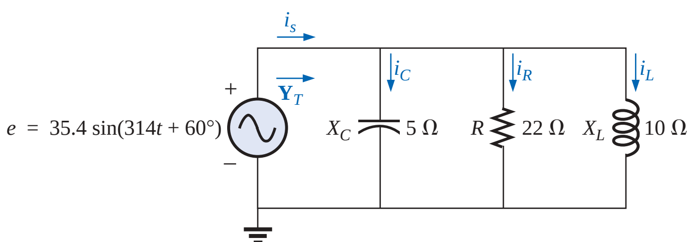


FIG. 15.141
Problem 32.

SECTION 15.9 Current Divider Rule

33. Calculate the currents $\mathbf{I}_1$ and $\mathbf{I}_2$ of Fig. 15.142 in phasor form using the current divider rule.

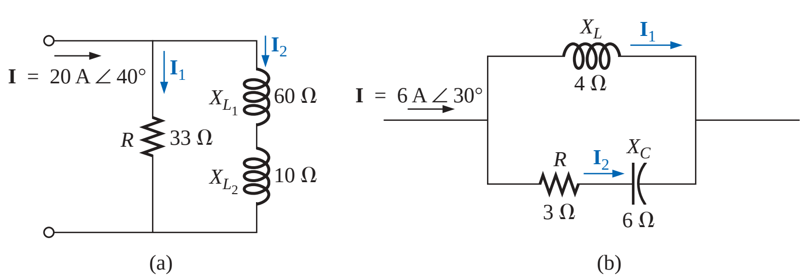


FIG. 15.142
Problem 33.

SECTION 15.10 Frequency Response of the Parallel *R-L* Network

***34.** For the parallel *R-C* network of Fig. 15.143:

a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 20 kHz.

b. Plot V_C versus frequency for the frequency range of part (a).

c. Plot I_R versus frequency for the frequency range of part (a).

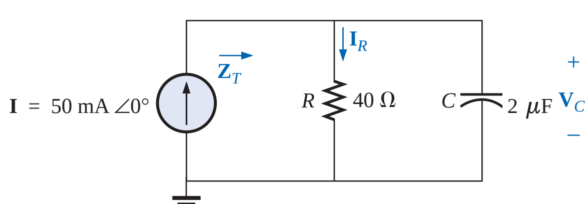


FIG. 15.143
Problems 34 and 36.

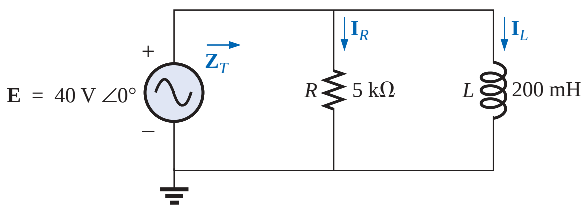


FIG. 15.144
Problems 35 and 37.

***35.** For the parallel *R-L* network of Fig. 15.144:

a. Plot Z_T and θ_T versus frequency for a frequency range of zero to 10 kHz.

b. Plot I_L versus frequency for the frequency range of part (a).

c. Plot I_R versus frequency for the frequency range of part (a).

36. Plot Y_T and θ_T (of $\mathbf{Y}_T = Y_T \angle \theta_T$) for a frequency range of zero to 20 kHz for the network of Fig. 15.143.

37. Plot Y_T and θ_T (of $\mathbf{Y}_T = Y_T \angle \theta_T$) for a frequency range of zero to 10 kHz for the network of Fig. 15.144.

38. For the parallel *R-L-C* network of Fig. 15.145:

a. Plot Y_T and θ_T (of $\mathbf{Y}_T = Y_T \angle \theta_T$) for a frequency range of zero to 20 kHz.

b. Repeat part (a) for Z_T and θ_T (of $\mathbf{Z}_T = Z_T \angle \theta_T$).

c. Plot V_C versus frequency for the frequency range of part (a).

d. Plot I_L versus frequency for the frequency range of part (a).

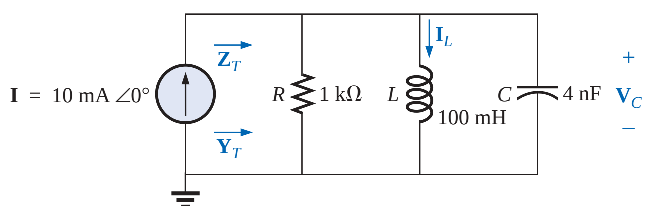


FIG. 15.145
Problem 38.

SECTION 15.12 Equivalent Circuits

39. For the series circuits of Fig. 15.146, find a parallel circuit that will have the same total impedance ($\mathbf{Z}_T$).

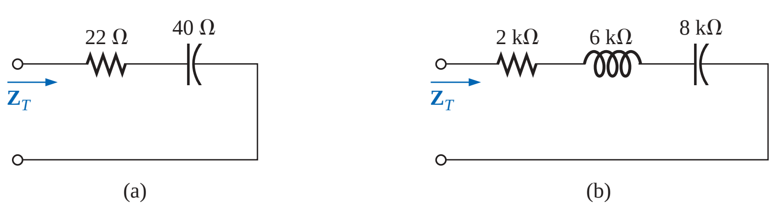


FIG. 15.146
Problem 39.

40. For the parallel circuits of Fig. 15.147, find a series circuit that will have the same total impedance.

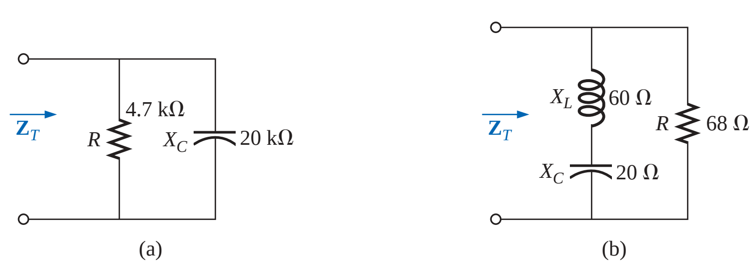


FIG. 15.147
Problem 40.

41. For the network of Fig. 15.148:

a. Calculate $\mathbf{E}$, $\mathbf{I}_R$, and $\mathbf{I}_L$ in phasor form.

b. Calculate the total power factor, and indicate whether it is leading or lagging.

c. Calculate the average power delivered to the circuit.

d. Draw the admittance diagram.

e. Draw the phasor diagram of the currents $\mathbf{I}_s$, $\mathbf{I}_R$, and $\mathbf{I}_L$, and the voltage $\mathbf{E}$.

f. Find the current $\mathbf{I}_C$ for each capacitor using only Kirchhoff's current law.

g. Find the series circuit of one resistive and reactive element that will have the same impedance as the original circuit.

***42.** Repeat Problem 41 if the inductance is changed to 1 H.

43. Find the element or elements that must be in the closed container of Fig. 15.149 to satisfy the following conditions. (Find the simplest parallel circuit that will satisfy the indicated conditions.)

a. Average power to the circuit = 3000 W.

b. Circuit has a lagging power factor.

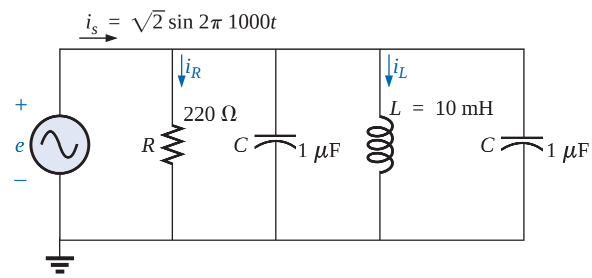


FIG. 15.148
Problems 41 and 42.

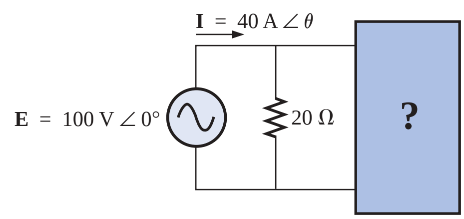


FIG. 15.149
Problem 43.

SECTION 15.13 Phase Measurements (Dual-Trace Oscilloscope)

44. For the circuit of Fig. 15.150, determine the phase relationship between the following using a dual-trace oscilloscope. The circuit can be reconstructed differently for each part, but do not use sensing resistors. Show all connections on a redrawn diagram.

a. e and v_C

b. e and i_s

c. e and v_L

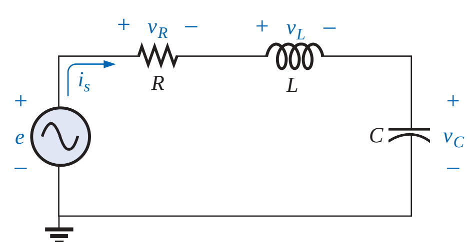


FIG. 15.150
Problem 44.

45. For the network of Fig. 15.151, determine the phase relationship between the following using a dual-trace oscilloscope. The network must remain as constructed in Fig. 15.151, but sensing resistors can be introduced. Show all connections on a redrawn diagram.

a. e and v_{R_2}

b. e and i_s

c. i_L and i_C

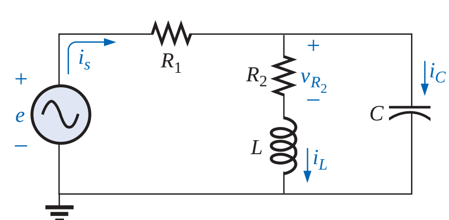
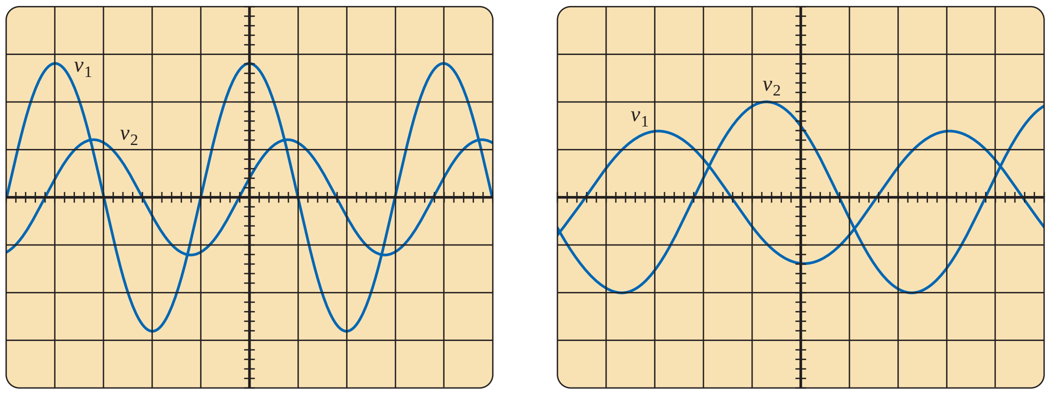


FIG. 15.151
Problem 45.

46. For the oscilloscope traces of Fig. 15.152:
  a. Determine the phase relationship between the waveforms, and indicate which one leads or lags.
  b. Determine the peak-to-peak and rms values of each waveform.
  c. Find the frequency of each waveform.



Vertical sensitivity = 0.5 V/div.
Horizontal sensitivity = 0.2 ms/div.
(I)

Vertical sensitivity = 2 V/div.
Horizontal sensitivity = 10 μs/div.
(II)

FIG. 15.152
Problem 46.

SECTION 15.15 Computer Analysis

PSpice or Electronics Workbench

47. For the network of Fig. 15.119 (use $f = 1$ kHz):
  a. Determine the rms values of the voltages $\mathbf{V}_R$ and $\mathbf{V}_L$ and the current $\mathbf{I}$.
  b. Plot v_R, v_L, and i versus time on separate plots.
  c. Place e, v_R, v_L, and i on the same plot, and label accordingly.

48. For the network of Fig. 15.138:
  a. Determine the rms values of the currents $\mathbf{I}_s$, $\mathbf{I}_R$, and $\mathbf{I}_L$.
  b. Plot i_s, i_R, and i_L versus time on separate plots.
  c. Place e, i_s, i_R, and i_L on the same plot, and label accordingly.

49. For the network of Fig. 15.121:
  a. Plot the impedance of the network versus frequency from 0 to 10 kHz.
  b. Plot the current i versus frequency for the frequency range zero to 10 kHz.

*50. For the network of Fig. 15.129:
  a. Find the rms values of the voltages v_R and v_C at a frequency of 1 kHz.
  b. Plot v_C versus frequency for the frequency range zero to 10 kHz.
  c. Plot the phase angle between e and i for the frequency range zero to 10 kHz.

Programming Language (C++, QBASIC, Pascal, etc.)

51. Write a program to generate the sinusoidal expression for the current of a resistor, inductor, or capacitor given the value of R, L, or C and the applied voltage in sinusoidal form.

52. Given the impedance of each element in rectangular form, write a program to determine the total impedance in rectangular form of any number of series elements.

53. Given two phasors in polar form in the first quadrant, write a program to generate the sum of the two phasors in polar form.

GLOSSARY

Admittance A measure of how easily a network will "admit" the passage of current through that system. It is measured in siemens, abbreviated S, and is represented by the capital letter *Y*.

Admittance diagram A vector display that clearly depicts the magnitude of the admittance of the conductance, capacitive susceptance, and inductive susceptance, and the magnitude and angle of the total admittance of the system.

Current divider rule A method by which the current through either of two parallel branches can be determined in an ac network without first finding the voltage across the parallel branches.

Equivalent circuits For every series ac network there is a parallel ac network (and vice versa) that will be "equivalent" in the sense that the input current and impedance are the same.

Impedance diagram A vector display that clearly depicts the magnitude of the impedance of the resistive, reactive, and capacitive components of a network, and the magnitude and angle of the total impedance of the system.

Parallel ac circuits A connection of elements in an ac network in which all the elements have two points in common. The voltage is the same across each element.

Phasor diagram A vector display that provides at a glance the magnitude and phase relationships among the various voltages and currents of a network.

Series ac configuration A connection of elements in an ac network in which no two impedances have more than one terminal in common and the current is the same through each element.

Susceptance A measure of how "susceptible" an element is to the passage of current through it. It is measured in siemens, abbreviated S, and is represented by the capital letter *B*.

Voltage divider rule A method through which the voltage across one element of a series of elements in an ac network can be determined without first having to find the current through the elements.